Electron beam excitation of regular oscillations in an inhomogeneous plasma waveguide

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The stimulated and self-consistent problems of interaction between a monoenergetic electron beam and plasma waveguide with non-uniform density are considered. The dependence of the gain of a slow charge-density wave on the magnitude and sign of the density gradient, and the efficiency of transformation of the wave into a quasi-transverse wave in the plasma-beam waveguide, are determined analytically and numerically.

In devising methods of controlling beam instabilities[1,2] it is necessary to bring about controlled variation of the plasma density to ensure amplification or attenuation of these instabilities. This is the reason for the recently begun theoretical[3-6] and experimental[6-9] investigations of the dependence of beam-plasma interaction characteristics on the degree of inhomogeneity of the plasma.

Physically, the possibility of controlling beam instabilities with plasma density gradients is based on the dependence of the dispersion properties of the medium and the energy losses in the elementary radiation act on the plasma density[1] and gradient[10]. A consequence of this dependence is the change in the collective characteristics of the interaction between the beam and an inhomogeneous plasma, i.e., in the increments (gains). For example, in the case of irregular oscillations described by the quasilinear theory[3] plasma inhomogeneity causes a decrease in the increments and leads to production of field-accelerated particles at positive values of the density gradient.

In many applications it is of interest to investigate the conditions for efficient excitation or cutoff of regular oscillations in an inhomogeneous plasma. This paper is devoted to these questions.

We consider a uniform-density monoenergetic particle beam of radius a traveling parallel to the axis of a cylindrical waveguide having conducting walls of the same radius and filled with an inhomogeneous plasma. The external magnetic field is parallel to the waveguide axis and the plasma density gradient; its intensity is rather high: $\omega_H^2 \gg \omega_p^2 \gg \omega_b^2$, where ω_H, ω_p and ω_b are the gyrofrequency, the plasma Langmuir frequency, and the beam frequency, respectively.

The complete system of equations for this problem in the hydrodynamic approximation is

$$\begin{gathered} m_0 dv_p / dt = -eE_\parallel - m_0 \nu v_p, \quad m_\parallel dv_b / dt = -eE_\parallel, \\ m_\parallel = m_0\gamma^3, \quad \gamma = (1-\beta^2)^{-1/2}, \quad \beta = V_0/c, \\ \frac{\partial n_b}{\partial t} + \frac{\partial}{\partial z}(n_b v_b) = 0, \quad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \\ \operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi e}{c}(n_p v_p + I_b), \\ n_b = n_0 + \tilde{N}, \quad v_b = V_0 + \tilde{V}, \end{gathered} \tag{1}$$

where m_0 is the electron mass, ν is the collision frequency, and $v_{p,b}$ and $n_{p,b}$ are the respective velocities and densities of the beam particles and the plasma particles.

We seek a solution of system (1) by separating the variables. The field E_z, for example, is in the form $\mathscr{E}_\parallel = \mathscr{E}_\parallel(z)J_0(\lambda_n r/a)\exp(-i\omega t)$, where λ_n are the roots of the Bessel function ($J_0(\lambda_n) = 0$), and an unknown function $\mathscr{E}_\parallel(z)$ describes the dependence of the field $E_\parallel$ on the longitudinal coordinate z. By substituting the fields thus chosen in the initial equations (1) we reduce the problem of finding the field amplitudes, velocities and densities to that of solving a system of first-order total differential equations with variable coefficients.

$$\frac{d\mathscr{H}_\perp}{dz} = k_0\mathscr{E}_\perp, \quad \frac{d\mathscr{E}_\perp}{dz} + k_\perp\mathscr{E}_\parallel = ik_0\mathscr{H}_\perp, \tag{2a}$$

$$\begin{gathered} k_\perp\mathscr{H}_\perp = -ik_0\varepsilon(z)\mathscr{E}_\parallel - 4\pi c^{-1}e\tilde{I}_b, \quad \tilde{I}_b = n_0\tilde{V} + V_0\tilde{N}, \\ \left(-i\omega + V_0\frac{d}{dz}\right)\tilde{V} = -\frac{e}{m}\mathscr{E}_\parallel, \quad \varepsilon(z) = 1 - \frac{\omega_p^2(z)}{\omega(\omega+i\nu)}, \\ \left(-i\omega + V_0\frac{d}{dz}\right)\tilde{N} + n_0\frac{d}{dz}\tilde{V} = 0, \quad k_\perp = \frac{\lambda_n}{a}, \quad k_0 = \frac{\omega}{c}, \end{gathered} \tag{2b}$$

where V_0 and n_0 are the equilibrium values of the beam velocity and density.

Equations (2) have four linearly independent solutions corresponding to guided waves in the plasma-beam system. These waves are in general are neither purely longitudinal nor purely transverse.[1)] However, when $n_0 \ll n_p$,[2)] which is the most important case in practice, two of these waves differ from the transverse waves in the anisotropic plasma waveguide without the beam by corrections on the order of n_0/n_p. These solutions will henceforth be called quasitransverse waves, while the two remaining solutions (which vanish in the limit as $n_0 \to 0$) will be referred to as quasilongitudinal waves or beam-charge-density waves. The effect of inhomogeneity on the dispersion and excitation of quasitransverse waves and on the amplification and transformation of quasi-longitudinal waves is studied below.

We consider first the dependence of the natural-frequency spectrum of the plasma resonator on the degree of plasma inhomogeneity. In the simplest case of a constant plasma density gradient ($n_p(z) = \bar{n}(1 + z/L)$) the solution of system (2) at $n_0 = 0$ is (see[11], Sec. 27):

$$\mathscr{E}_\parallel(z) = [\varepsilon(z)]^{-1/2}\left\{G_1H_1^{(1)}\left[\frac{2k_\perp}{|\varepsilon'|}\sqrt{-\varepsilon(z)}\right] + G_2H_1^{(2)}\left[\frac{2k_\perp}{|\varepsilon'|}\sqrt{-\varepsilon(z)}\right]\right\},$$
$$\varepsilon' \equiv \frac{d}{dz}\varepsilon(z), \quad \operatorname{Re}\varepsilon < 0, \quad k_0^2|\varepsilon| \ll k_\perp^2. \tag{3}$$

Calculating $\mathscr{E}_\perp(z)$ with the aid of (3) and (2), we obtain the equation for the spectrum from the conditions $\mathscr{E}_\perp(z = \pm l) = 0$ at the resonator end faces:

$$\begin{gathered} H_0^{(1)}(k_-)H_0^{(2)}(k_+) - H_0^{(1)}(k_+)H_0^{(2)}(k_-) = 0, \\ k_\pm \equiv 2k_\perp[-\varepsilon(\pm l)]^{1/2}/|\varepsilon'|. \end{gathered} \tag{4}$$

In the presence of weak inhomogeneity, ($\nu/\omega \ll ma/\lambda_n l \ll L/l$), explicit expressions for the natural frequencies

ω_{mn} of the resonator under consideration can be found from (4) by iteration:

$$\omega_{mn}^2=\left(\frac{\pi ma}{2\lambda_n l}\right)^2\omega_p^2(0)\left[1+\left(\varepsilon'\frac{\pi ma}{2\sqrt{2}\lambda_n}\right)^2\right]-i\nu\omega_{mn}. \qquad (5)$$

Here m and n are the indices of the longitudinal and transverse wave numbers, respectively.

Thus, the plasma inhomogeneity increases the natural frequencies of the plasma resonator, while the corresponding frequency shift is, as expected, independent of the sign of the density gradient. Relation (5) is valid on the condition that the point of plasma resonance at which the plasma frequency $\omega_p(z)$ is equal to the working frequency ω does not lie within the resonator. In the contrary case, a damping term appears in the right-hand side of (5) and is due to absorption of the quasitransverse waves ([11], Secs. 20 and 27).

We consider now the problem of such a resonator excited by a given modulated current of the external beam $I_b(z, t) \equiv I_0 \exp[ik_M z - i\omega_M t]$, $\omega_M \equiv k_M V_0$. The current forced solution of system (2) is then of the form

$$D\equiv\varepsilon\mathscr{E}_{\parallel}=\frac{\pi i I_0 k_M^2 D_1(z)}{\omega_M\sin(k_- - k_+)}\int\limits_{-l}^{+l}D_2(z')\exp(ik_M z')\,dz', \qquad (6)$$

where

$$D_s(z)\equiv(z-z_0)^{1/2}H_1^{(s)}\left[2k_\perp\left(\frac{z_0-z}{\varepsilon'}\right)^{1/2}\right],\quad s=1,2,$$

and z_0 is a complex number determined by solving the equation $\epsilon(z_0) = 0$.

The last expression permits us to find the dependence of the pattern of the beam-excited field on the degree of plasma inhomogeneity and the beam modulation frequency ω_M:

$$\mathscr{E}_{\parallel}(\omega_M,\Delta)=\frac{\pi i}{2}(-1)^m I_0\left(\frac{z_*-z_0}{|\varepsilon'|}\right)^{1/4}\cdot$$
$$\times\frac{k_M^2\exp[i(k_M\Delta-\zeta^2\Delta^2)]}{\omega_M k_\perp^{3/2}\sin(k_- - k_+)}2\Phi\left[\frac{k_\perp l}{2(z_*-z_0)^{3/4}|\varepsilon'|^{1/4}}\right], \qquad (7)$$

where

$$\Delta\equiv z-z_*,\quad \zeta^2\equiv\frac{k_\perp|\varepsilon(z_*)|^{1/2}}{4|\varepsilon'|(z_*-z_0)^2}\qquad \Phi(t)\equiv\int\limits_0^t\exp(i\varphi^2)\,d\varphi,$$

and z_* is the coordinate of the point at which the local phase velocity of the wave is equal to the beam velocity ($V_{ph}(z_*) = V_0$).

As seen from (7), the oscillation minimum of the beam-excited field corresponds to the vicinity of the point $z = z_*$, where $|\Delta\zeta|^2 \ll 1$. The most interesting applications arise from the dependence of the beam energy losses

$$W\equiv\int\limits_{z_*-l}^{z_*+l}dz\,I_b E_{\parallel}$$

on the plasma density gradient. Using (7) we obtain the following expression for this quantity:

$$W(\mathscr{L})=W(\infty)\begin{cases}1 & |\varepsilon'|k_\perp l^2\ll 4,\\ 4/k_\perp l^2|\varepsilon'|, & |\varepsilon'|k_\perp l^2\gg 4,\end{cases} \qquad (8)$$

where $W(\infty)$ represents the losses in a homogeneous resonator, and $\mathscr{L} \equiv \omega L/V_0$.

Thus the presence of inhomogeneity in the plasma only causes a decrease in the losses when the density gradient or the length of the interaction region is large enough ($|\epsilon'|k_\perp l^2 > 4$); in a relatively short system (at low density gradients: $|\epsilon'|k_\perp l^2 < 4$) the inhomogeneity does not affect the losses. This result explains qualitatively the experimentally observed[7,9] increase in the collective losses of the modulated beam as the degree of plasma inhomogeneity decreases.

We consider next the effect of inhomogeneity on the efficiency of amplification of regular charge-density waves in the plasma. This self-consistent problem requires simultaneous solution of the complete system of equations (2a) and (2b).

Expressing the beam current $\tilde{I}_b$ in terms of the field $\mathscr{E}_{\parallel}$ from (2b)

$$I_b=-ik_0\frac{\omega_b^2}{\omega^2}\int\limits^{z}dz'\int\limits^{z'}dz''\mathscr{E}_{\parallel}(z'')\exp\left[\frac{i\omega}{V_0}(z-z'')\right] \qquad (9)$$

and substituting (9) in (2a), we obtain the following fourth-order differential equation for the amplitude $A(\xi) \equiv \mathscr{E}_{\parallel}(\xi)\exp(-i\xi)$ of the field $\mathscr{E}_{\parallel}(z)$:

$$A^{(4)}+(4\varepsilon'+2i\varepsilon)A^{(3)}+(6i\varepsilon+\mu-\alpha^2-\varepsilon)A^{(2)}$$
$$+2(i\mu-\varepsilon')A'=\mu A,$$
$$\xi=\frac{\omega z}{V_0},\qquad \alpha=\frac{\lambda_n V_0}{\omega_p a},\qquad \mu=\frac{\omega_b^2}{\omega^2}=\frac{4\pi n_0 e^2}{m_0\gamma^3\omega^2}. \qquad (10)$$

We assume that the plasma density inhomogeneity is not too large ($\mathscr{L} \gg 1$), so that efficient energy exchange between the beam and field is assured by Cerenkov amplification of the slow charge-density wave.[3)] In this case analytic solutions for Eq. (10) can be obtained in the following limiting cases:

1. A large linear plasma density and not too small density gradients

$$\mu\gg\alpha^2,\quad \alpha^2\mathscr{L}\ll 1. \qquad (11a)$$

2. Low linear plasma density and small gradients

$$\mu\ll\alpha^2,\quad \alpha^2\mathscr{L}\gg 1. \qquad (11b)$$

In the first case, as $\alpha \to 0$, the longitudinal field component of the quasitransverse wave is small, and consequently this wave is not coupled with the beam charge-density waves. Integration of Eq. (10) gives the following expressions for the amplitudes of the beam waves:

$$\sqrt{\varepsilon}A(z)=\begin{cases}B_1H_1^{(1)}\left[\frac{2}{|\varepsilon'|}(\mu\varepsilon)^{1/2}\right]+C_1H_1^{(2)}\left[\frac{2}{|\varepsilon'|}(\mu\varepsilon)^{1/2}\right], & \operatorname{Re}\varepsilon>0; \quad (12a)\\ B_2I_1\left[\frac{2}{|\varepsilon'|}(-\mu\varepsilon)^{1/2}\right]+C_2K_1\left[\frac{2}{|\varepsilon'|}(-\mu\varepsilon)^{1/2}\right], & \operatorname{Re}\varepsilon<0. \quad (12b)\end{cases}$$

It is easily seen that at small gradients Eq. (12b) reduces to the well-known result for the longitudinal wave amplitude buildup in a homogeneous plasma. At higher values of the density gradient, the total gain in the same interaction interval decreases (at sufficiently high beam density the argument of the exponential is proportional to the square root of the coordinate):

$$\mathscr{K}(\infty,l)\equiv\ln\frac{|A(\infty,l)|}{|A(\infty,-l)|}=\frac{2\omega l}{V_0}\sqrt{\frac{\mu}{|\varepsilon(\bar{z})|}},\qquad |\varepsilon(\bar{z})|\gg|\varepsilon' l|; \quad (13a)$$

$$\mathscr{K}(\mathscr{L},l)\equiv\ln\frac{|A(\mathscr{L},l)|}{|A(\mathscr{L},-l)|}=2\left(\frac{\mu}{|\varepsilon'|}\frac{2\omega l}{V_0}\right)^{1/2},\qquad |\varepsilon(\bar{z})|\ll|\varepsilon' l|, \quad (13b)$$

where $\bar{z}$ is the coordinate of the center of the interaction region ($\bar{z} - l < z < \bar{z} + l$). For inhomogeneities strong enough that the inequality $\mu l L\omega^2/V_0^2 < 1$ is satisfied, the amplification of the charge-density wave ceases.

We note that the result (13a) for a homogeneous plasma with growing density follows from the first term of (12b) in the region $z \gg z_0$, while for decaying density it follows from the second term of the same formula at $z \ll z_0$; in the region Re $\epsilon > 0$ the amplified wave goes over into a slow charge-density wave in both cases.

The difference between the analytic dependences of the beam-amplified slow longitudinal wave amplitude on the coordinate at different directions of the gradient (amplification anisotropy) can be attributed to the difference in the conditions of radiation from the beam electrons and of the reaction of this radiation on the beam. In fact, as shown in[15], the hydrodynamic instability of the beam in a homogeneous plasma is based on the Cerenkov radiation[4)], the reaction of which to the motion of the beam particles is amplified by the coherence of the elementary radiators within the bunches formed by the modulating signal and between these bunches. The total field induced by the radiation of the beam particle turns out to be proportional to the local value of the amplitude of the modulating field and to the local intensity of spontaneous particle radiation (i.e. to the plasma density at the given point in space). Therefore, at positive density gradients, when both effects (spatial growth of the amplitude of the amplified field and increase in the intensity of the spontaneous radiation) are mutually reinforcing, the resultant losses of beam energy to radiation, which determine the amplification efficiency, are larger than in the case of decaying plasma density.[5)]

The magnitude of the amplification anisotropy effect is determined by the ratio of the amplification factors.

$$R = \frac{\mathscr{K}_+(\mathscr{L}, l)}{\mathscr{K}_-(\mathscr{L}, l)} = \ln\left[\frac{I_1(k_+^+)}{I_1(k_-^+)}\sqrt{\frac{k_-^+}{k_+^+}}\right] \Big/ \ln\left[\frac{K_1(k_+^-)}{K_1(k_-^-)}\sqrt{\frac{k_-^-}{k_+^-}}\right] > 1,$$

$$k_\pm^+ = \frac{2}{|\varepsilon'|}[\mu|\varepsilon(\bar{z}_+ \pm l)|]^{1/2}, \qquad k_\pm^- = \frac{2}{|\varepsilon'|}[\mu|\varepsilon(\bar{z}_- \pm l)|]^{1/2}. \tag{13c}$$

Here $\bar{z}_\pm$ are the coordinates of the centers of the amplification regions for positive and negative density gradients, respectively: $\epsilon(\bar{z}_+) = \epsilon(\bar{z}_-)$. It is easily seen that the degree of amplification anisotropy increases when the density gradient increases and the interaction region approaches the plasma resonance point; the amplification anisotropy effect is small in the region where the WKB method applies. At relatively low values of the linear plasma density, when the conditions of (11b) are satisfied, the plasma density region of greatest interest is the one in which the phase velocity of the quasitransverse wave in the waveguide without the beam is close to the beam velocity V_0. In this region the gain for a slow charge-density wave reaches a maximum proportional to the cube root of the linear beam density. Recognizing that the plasma parameters vary slowly, we can find this coefficient by the WKB method[6)]:

$$A(\xi) = A_0 \exp\left[i\int^{\xi} \varkappa(\xi')\,d\xi'\right],$$
$$\alpha^2 + (\varepsilon - \mu\varkappa^{-2})[(1+\varkappa)^2 - \beta^2] = 0. \tag{14}$$

Under the conditions of the experiment, a matter of great interest is the dependence of the total gain at a given length and that of the cutoff conditions on the degree of plasma inhomogeneity. To elucidate these matters we carry out the integration in (14), using the fact that the parameter μ/α^2 is small. Thus, we find:

$$\mathscr{K}_+(\mathscr{L}, z) = \frac{3\sqrt{3}}{2^{2/3}}\left(\frac{\mu^2}{\alpha^4}\right)^{1/3} \frac{\omega\alpha^2}{V_0|\varepsilon'|}\int_{x_0}^{x(z)} F(x')\,dx';$$
$$F(x) \equiv [x^3 + 1/2 + \sqrt{x^3 + 1/4}]^{1/3} - [x^3 + 1/2 - \sqrt{x^2 + 1/4}]^{1/3},$$
$$x_0 = -2^{-2/3}; \quad x(z) \equiv |\varepsilon'|(z - z_*)/2^{2/3}3(\mu\alpha^4)^{1/3}, \tag{15}$$

while $z_* \equiv z_0 - \alpha^2/\epsilon'$ is the coordinate of the synchronism point. It is clear from this expression that the presence of inhomogeneity has a telling effect on the amplification everywhere except in the immediate vicinity of the synchronism point where $|X| < 1$; and so, even at $x \sim 3$ the local gain is smaller by a factor 2.5 than its maximum value for an homogeneous plasma, while the integral gain is smaller by a factor 1.5. The condition for absence of attenuation of the instability can be represented in the following form:

$$1 \ll \frac{\omega l}{V_0}\left(\frac{\mu}{\alpha^2}\right)^{1/3} \ll (\mu\alpha)^{2/3}\mathscr{L}.$$

The left-hand side of this inequality ensures exponential growth of the amplitude, while the right-hand side means that it is possible to ignore the effect of plasma inhomogeneity on the amplification in the resonance region. At higher values of ω this condition is violated, and this is apparently the reason for the relatively low intensity of short-wave excitation in the plasma-beam experiments.

The condition for the applicability of the WKB method proves to be most rigid near the boundary of the amplification region $(\epsilon + \alpha^2 + (\mu\alpha^4)^{1/3} = 0)$, where the difference $\Delta\kappa$ between the wave numbers of the slow beam wave and the quasitransverse wave is proportional to a small parameter: $\Delta\kappa \sim (\nu/\omega)^{1/2}\mu^{1/6}$. In this region the amplification is resistive: the beam wave builds up with an increment of Im $\kappa \sim \frac{1}{2}\Delta\kappa$ and the quasitransverse wave decays with the same decrement. As $\nu \to 0$ the difference between these waves vanishes, meaning a possibility of their mutual transformation, which can be significant when the plasma density is decaying.

In the general case, when the problem lacks the small parameters for estimating the dependence of the amplification and transformation efficiencies on the degree of plasma inhomogeneity, a numerical solution of the initial system of equations (2) is necessary. A digital computer was used for this purpose. The values chosen for the inhomogeneity parameters ($\mathscr{L}$), the linear plasma density (α), the beam density (μ) and the beam velocity (β) values were close to the experimental ones: $\mu = 10^{-2}$, $\beta = 0.2$, $\alpha = 1.4$. Here the main difficulty is that of choosing the right boundary conditions to guarantee that the slow charge-density wave amplification effect that we are interested in emerges in its pure form. In the electrodynamics of transparent media the problem of uniqueness is solved with the Sommerfeld radiation conditions. An analogous condition is necessary in our problem to isolate the amplified wave in the non-equilibrium plasma-beam system. Such isolation is accomplished relatively easily in the coordinate region where the WKB method applicability conditions are satisfied. If the plasma density in that region is not too high, then the difference in the wave numbers of the fast charge-density wave and the slow one is finite (no degeneracy), and so the field pattern of the slow wave is of the form

$$\mathscr{E}_\parallel = -i\mu(\varepsilon + \alpha^2)^{-1}\tilde{N}, \quad \mathscr{E}_\perp = i\alpha\mathscr{E}_\parallel,$$
$$\tilde{V} = i\left(\frac{\mu}{\varepsilon + \alpha^2}\right)^{1/2}\tilde{N}, \tag{16}$$

where $\tilde{N}$ and $\tilde{V}$ are the modulation indices of the beam density and the beam velocity respectively. Assuming that $\tilde{N}$ is known, we find the remaining values of the initial amplitudes from (16) and substitute them in the right-hand side of the first-order equations (2) in the capacity of initial conditions.

It must be emphasized that in the case in question the coordinate dependence of the field amplitude is calcula-

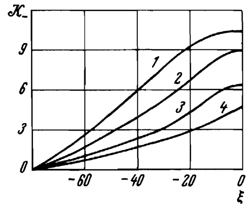

FIG. 1

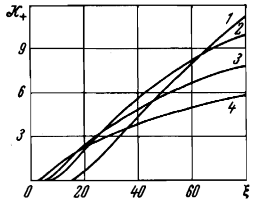

FIG. 2

FIG. 1. Gain $\mathcal{K}_-(\mathcal{L}, \xi)$ vs the coordinate ξ at $\alpha^2 = 0.5$; $\mu = \nu/\omega = 10^{-2}$. 1–$\mathcal{L}$ = 80; 2–$\mathcal{L}$ = 40; 3–$\mathcal{L}$ = 20; 4–$\mathcal{L}$ = 10.

FIG. 2. Gain $\mathcal{K}_+(\mathcal{L}, \xi)$ vs the coordinate ξ at $\alpha^2 = 0.5$; $\mu = \nu/\omega = 10^{-2}$. 1–$\mathcal{L}$ = 80; 2–$\mathcal{L}$ = 40; 3–$\mathcal{L}$ = 20; 4–$\mathcal{L}$ = 10.

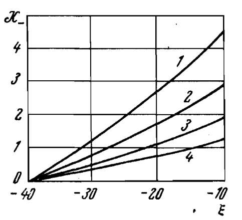

FIG. 3

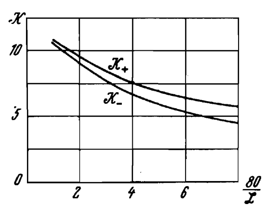

FIG. 4

FIG. 3. Gain $\mathcal{K}_-(\mathcal{L}, \xi)$ vs the coordinate ξ in the nonresonant region at $\alpha^2 = 0.25$; $\mu = \nu/\omega = 10^{-2}$. 1–$\mathcal{L}$ = 40; 2–$\mathcal{L}$ = 20; 3–$\mathcal{L}$ = 10; 4–$\mathcal{L}$ = 5.

FIG. 4. Total gain $\mathcal{K}_+(\mathcal{L}, l)$ over a given length ($2\omega l/V_0 = 80$) vs the magnitude and sign of the plasma density gradient; $\mu = \nu/\omega = 10^{-2}$, $\alpha^2 = 0.5$.

ted with any practical degree accuracy; only the initial conditions, which were derived by using the idea of the WKB method, are approximate. The corresponding relative error then decreases with the coordinate ξ, since the amplitude of the slow beam wave increases exponentially at the same time that the amplitudes of the remaining waves in this coordinate region (Re $\epsilon < 0$) are either falling off (fast beam wave) or building up at a lower exponential rate (quasitransverse waves).

The results of these numerical calculations are illustrated in Figs. 1–4. The first three show the dependence on the coordinate ξ_0 of the integral gains for the amplitude A_N of the variable beam density component ($\mathscr{K}(\mathscr{L}, \xi) \equiv \ln\{|A_N(\mathscr{L}, \xi)|/|A_N(\mathscr{L}, -l)|\}$), corresponding to different sets of plasma parameters. As expected, in the coordinate region where the amplification conditions are not satisfied there is no increase in amplitude (see Figs. 1 and 2); in the nonresonant region (far from the quasilongitudinal and quasitransverse wave synchronization region) the total gain at the given length ($\mathscr{K}_-(\mathscr{L}, l)$—Fig. 3) decreases in rough proportion to the square root of the characteristic inhomogeneity length $\mathscr{L}$. The plots in Fig. 4 show the dependence of the amplification anisotropy on the magnitude and the sign of the density gradient. Clearly, when the gradient is increased the degree of anisotropy grows, in full accord with the physical considerations mentioned above.

These numerical experiments also make it possible to estimate the efficiency of transformation of the beam-amplified slow charge-density wave into a quasitransverse wave in the plasma waveguide. As shown above, there is no transformation in a plasma with a large linear density ($\alpha^2\mathscr{L} \ll 1$) and in a weakly inhomogeneous plasma waveguide in the region where the WKB method is applicable ($\epsilon + \alpha^2 + (\mu\alpha^4)^{1/3} \neq 0$). To estimate the efficiency of this process near the boundary of the amplification region, where the quasiclassical approximation is inapplicable, the waveform of the pulse $\tilde{N}$ was analyzed simultaneously with the gain calculation. The results of this analysis show that in the vicinity of the transformation point the modulus of the transformation coefficient with respect to density does not exceed the error in the numerical experiment (3–4%) at a relatively weak inhomogeneity $\mathscr{L} \geq 20$. When the gradients for the linear density decay are larger it is not practical to distinguish the transformation effect from the background of the errors in the initial conditions, since both the gain and the size of the transformation region turn out to be too small. In addition, when the gradients are large it is necessary to take into account the field of the incident quasitransverse wave. The order of the transformation coefficient with respect to density T_N can be estimated from the results of [14] for a steplike density profile. In the limiting case $\mu \ll 1$ and $\beta_0^2|\epsilon_\pm| \ll \alpha^2$ we find

$$T_N = \frac{\mu\alpha^2(|\varepsilon_-| - |\varepsilon_+|)(\sqrt{|\varepsilon_-|} - \alpha)}{(|\varepsilon_+| - \alpha^2)(|\varepsilon_-| - \alpha^2)(\sqrt{|\varepsilon_+|} + \sqrt{|\varepsilon_-|})(\sqrt{|\varepsilon_+|} - \alpha)^2},$$

$$\varepsilon_+ \equiv \varepsilon(\xi > 0), \quad \varepsilon_- \equiv \varepsilon(\xi < 0). \tag{17}$$

For $\mu \sim 10^{-2}$ and $|\epsilon_+| \sim |\epsilon_-| \sim 1$ this formula gives values for T_N of the order of several per cent, depending on both the magnitude and the sign of the plasma density gradient (transformation anisotropy[7)]).

The main conclusions of the preceding discussion can be formulated as follows:

1. As expected, the presence of inhomogeneity decreases the effectiveness of beam-plasma interaction, viz., the losses of the modulated beam by excitation of quasitransverse waves in the plasma waveguide ($L < k_\perp l^2$) and the total gain for the density wave at a given interaction length decrease (see Figs. 1–3). When the gradients are large enough ($\omega_b^2 L l < V_0^2$), the amplification cuts off, as there is no exponential growth of the field amplitude (see curves 4 in Figs. 1–3).

2. Anisotropy of the amplification and of the transformation are observed in an inhomogeneous plasma—the gains and transform-coefficients depend on the sign of the plasma density gradient (see Fig. 4), because the beam particle radiation intensity in the elementary act of emitting Cerenkov and transition quanta, respectively, and their bunching efficiency by the radiation field (during amplification) all depend on the relative orientation of the directions of plasma density growth and beam velocity.

3. When the plasma density gradients are relatively small ($\mathscr{L} \geq 20$), the transformation of the beam-amplified quasilongitudinal wave into a quasitransverse wave in the plasma waveguide under the conditions of the given numerical experiment ($\mu \approx \nu/\omega \approx 10^{-2}$) turns out to be weak: the amplitude of the quasitransverse wave at the exit of the amplification region does not exceed three or four percent of the quasilongitudinal wave amplitude.

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[1)]The longitudinal plasma waves can be neglected when the condition: $\nu m_0 V_0^2 > \omega T$ is satisfied (see [11], secs. 20 and 27), where T is the plasma temperature.

[2)]The typical values of the experimental parameters for non-relativistic beams are such $[n_0 \approx (10^{-2} - 10^{-3})n_p,\ T \sim (10^{-3} - 10^{-4})m_0 V_0^2,\ \nu/\omega \approx 10^{-2} - 10^{-3}]$ that these inequalities can be satisfied.

[3)]In the case of strong inhomogeneity, a field can be excited by the transition radiation of the beam particles [10,12-14].

4)The connection between plasma-beam instabilities and the elementary Cerenkov radiation from a charge in the plasma was emphasized earlier (see [1,16,17]).

5)In the quasilinear case [3,4] the anisotropy effect is caused by removal of field energy by the beam particles as a result of their being accelerated by the field at positive values of the density gradient.

6)Plots of the coordinate dependence of the quasiclassical gains for several values of the inhomogeneity parameters are given in [5].

7)At present there is no lack of information regarding the transformation of the natural waves of a weakly inhomogeneous equilibrium plasma (see [11,18-20]); the anisotropy of particle transition radiation is pointed out, in particular, in [20].

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