Size effects in the magnetic susceptibility of metals

S. S. Nedorezov
Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences
(Submitted June 1, 1972)
Zh. Eksp. Teor. Fiz. **64**, 624-633 (February 1973)

The magnetic susceptibility χ_d of a metallic plate of thickness L is studied in a magnetic field H. The smooth part of the H dependence of χ_d is considered. It is shown that the difference between χ_d and the susceptibility of the massive sample χ_V is maximal in the range of fields $H \sim H_c$ (H_c is the field strength for which the magnetic length $l_H = \sqrt{\hbar c/eH} = L$. A formula is derived for the surface magnetic susceptibility of electrons with an arbitrary dispersion law, for strong fields $H \gg H_c$. In weak fields, $H \ll H_c \lambda\!\!\!^{-}_F/L$ ($\lambda\!\!\!^{-}_F$ is the electron Fermi wavelength), quantum size oscillations of the susceptibility are studied as a function of L.

1. INTRODUCTION

In magnetic fields H, when the Larmor radius of the electron trajectory is larger than or of the order of the characteristic size of the sample, it is necessary, in the calculation of the magnetic susceptibility, to take the presence of the boundary surface into account. In the present work, the effect of the sample boundary on the smooth part of the dependence of the magnetic susceptibility χ on H is investigated. A metal plate of thickness L is placed a parallel magnetic field. The susceptibility χ can be analyzed sufficiently completely in the range of "weak"

$$H \ll \lambda\!\!\!^{-}_F H_c / L \qquad (1)$$

and "strong"

$$H \gg H_c \qquad (2)$$

magnetic fields. Here $\lambda\!\!\!^{-}_F = \hbar\sqrt{2m\zeta}$ is the Fermi wavelength of the electron, $H_c = \hbar c/eL^2$ is the field strength for which the characteristic magnetic length $l_H = \sqrt{\hbar c/eH}$, which determines the region of localization of the wave function of the electron in the magnetic field, is equal to the thickness of the plate L, and e is the absolute value of the electron charge.

In weak fields (1), this problem was first considered by Papapetrou.[1] The boundary of the metal was approximated by an infinitely high potential wall. According to the results of this work, the diamagnetic susceptibility of the plate χ_d is identical with the Landau diamagnetic susceptibility[2] $\chi_{\varkappa}$ of a massive sample. Later on, the calculations of Papapetrou were refined by Friedman[3] (for a very detailed discussion of researches on this problem, see[3]), who found that χ_d differs from $\chi_{\varkappa}$ by a certain numerical coefficient which is independent of the plate thickness. The physical reason for the appearance of the difference between χ_d and $\chi_{\varkappa}$ has not been made clear, the more so that in the case of the parabolic approximation of the surface potential[3-5] the susceptibility of the plate $\chi_d = \chi_{\varkappa}$. In the present work, it is shown that the difference between χ_d and $\chi_{\varkappa}$ is due to the existence of quantum size oscillations of the magnetic susceptibility with change in L, which take place at temperatures $T \lesssim \Delta\epsilon_L$, where $\Delta\epsilon_L$ is the separation of the quantum size energy levels of the electron. The relative magnitude of the oscillations is ~1 at $T \ll \Delta\epsilon_L$. In the range of temperatures $T \gg \Delta\epsilon_L$ the diamagnetic susceptibility of the plate χ_d is identical with the Landau diamagnetic susceptibility, accurate to $\sim \lambda\!\!\!^{-}_F/L$.

In the weak field region (1), the energy spectrum of the electrons, which determines the magnetic susceptibility of the plate, has a relatively simple form and can be obtained in the form of an expansion in powers of H. In very strong fields $H \gtrsim H_c$, the energy spectrum of the electrons in the plate is very complicated.[6] Along with the Landau magnetic levels, there are magnetic surface levels, which are due to electrons skipping along the surface of the metal (see Fig. 1a). The experimental discovery (see[7]) of magnetic surface levels has stimulated the study of the effect of these levels on the thermodynamic and kinetic characteristics of the metals. As shown in[8], the contribution of the magnetic surface levels to the magnetization of metals turns out to be unimportant. The calculations carried out below show that the magnetic susceptibility turns out to be insensitive to the presence of the magnetic surface levels. Those electrons are important here[9] whose trajectories touch the boundary of the metallic surface (see Fig. 1b). In the strong field region (2), these electrons determine the characteristic dependence of χ on the magnetic field. The results obtained in[9] for the magnetic susceptibility are generalized in the present work to the case of electrons with an arbitrary dispersion law. The calculations are carried out in the approximation of specular reflection of the electrons from the surface of the sample. The specularity condition is sufficiently effective, since the principal contribution to the considered effect is made by electrons which have a sufficiently long wavelength in the direction of motion of the electron along the normal to the surface of the metal.

2. WEAK FIELDS

In sufficiently weak magnetic fields (1), in the calculation of the energy spectrum $E_n(p_x, p_z, H)$ of the conduction electrons in a plate, the magnetic field can be considered as a small perturbation in comparison with $\Delta\epsilon_L$. The corresponding analysis of the energy spectrum has been carried out in[1] and in more detail in[3]. As a result, we have for E_n in the case of a quadratic isotropic dispersion law

$$E_n(p_x, p_z, H) = \frac{p_x^2 + p_z^2}{2m} + \frac{\pi^2 n^2 \hbar^2}{2mL^2} - \frac{p_x}{2m}\frac{LeH}{c} + \frac{1}{2m}\left(\frac{LeH}{c}\right)^2\left[\frac{1}{3} - \frac{1}{2\pi^2 n^2} + \left(\frac{Lp_x}{2\pi\hbar}\right)^2\left(\frac{1}{3n^2} - \frac{5}{\pi^2 n^4}\right)\right] + O(H^3), \qquad n = 1, 2, 3, 4\ldots \qquad (3)$$

(the magnetic field H is directed along the z axis, the normal to the plate is along the y axis). The presence of electron spin leads to a splitting of the energy levels which, in the case of a weak spin-orbit interaction, can easily be taken into account:

$$E_{n\sigma}(p_x, p_z, H) = E_n(p_x, p_z, H) + (-1)^\sigma \frac{\hbar e H}{2m_s c}, \quad \sigma = 1, 2, \tag{4}$$

where m_s is a certain spin mass which in the general case is different from the effective mass m.

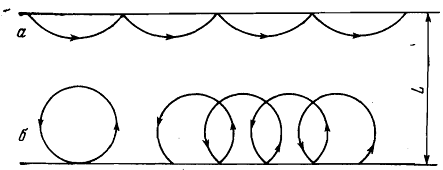

FIG. 1

According to the well known formulas of statistical physics, the thermodynamic potential Ω is determined from the following expression:

$$\Omega = -\frac{VT}{L(2\pi\hbar)^2} \sum_{\sigma=1}^{2} \sum_{n=1}^{\infty} \iint dp_z\, dp_x \ln\left[1 + \exp\frac{\zeta - E_{n\sigma}(p_x, p_z, H)}{T}\right], \tag{5}$$

where V is the volume of the plate. By calculating Ω, we can find the corresponding values for the other thermodynamic quantities. For the magnetic susceptibility χ, we have

$$\chi = -V^{-1}\partial^2\Omega / \partial H^2. \tag{6}$$

At sufficiently low temperatures

$$TL / \zeta\lambda_F \ll 1 \tag{7}$$

we get the following relation for the thermodynamic potential from (5):

$$\Omega(H) = \Omega(0) + \frac{Ve^2H^2}{12\pi^2\hbar c^2}\sqrt{\frac{\zeta}{2m}}\left\{\Phi_d\left(\frac{L\sqrt{2m\zeta}}{\pi\hbar}\right) - 3\left(\frac{m}{m_s}\right)^2 \Phi_p\left(\frac{L\sqrt{2m\zeta}}{\pi\hbar}\right)\right\} \tag{8}$$

where

$$\Phi_p(x) = [x] / x, \tag{9}$$

$$\Phi_d(x) = \frac{x}{8}\left\{[x]\left(2\pi^2 + \frac{9}{x^2}\right) + (\pi^2x^2 + 6)\sum_{n=1}^{[x]} \frac{1}{n^2} - 15x^2 \sum_{n=1}^{[x]} \frac{1}{n^4} - \frac{\pi^2}{2x^2}[x](1 + [x])(1 + 2[x])\right\}, \tag{10}$$

[x] is the largest integer in x.

For $x \gg 1$, it follows from (10) that

$$\Phi_d(x) = 1 - \frac{9}{16x} - \Phi_d^{\text{osc}}(x) + O\left(\frac{1}{x^2}\right), \tag{11}$$

where

$$\Phi_d^{\text{osc}}(x) = \frac{\pi^2}{2}\left(\{x\}^2 - \{x\} + \frac{1}{6}\right) \tag{12}$$

is the oscillating part of the function $\Phi_d(x)$, $\{x\}$ is a fractional part of the number x.

The second component in (8) describes the Landau diamagnetism of electrons in the plate, while the third term gives the Pauli paramagnetism. Formulas (8)–(10) are effective in the case $L/\lambda_F \sim 1$, when the condition (7) is satisfied comparatively easily, for example, for semi-metallic films. Under the condition

$$L / \lambda_F \gg 1, \qquad T / \zeta \ll 1 \tag{13}$$

we obtain the following expression for the thermodynamic potential Ω:

$$\Omega(H) = \Omega(0) + \Omega_d(H) + \Omega_p(H), \tag{14}$$

where

$$\Omega_d(H) = \frac{Ve^2H^2}{12\pi^2\hbar c^2}\sqrt{\frac{\zeta}{2m}}\left[1 - \frac{9\pi}{16}\frac{\lambda_F}{L} - \frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{k^2}\cos\left(\frac{2kL}{\lambda_F}\right)\Psi\left(\frac{\pi kTL}{\zeta\lambda_F}\right)\right], \tag{15}$$

$$\Omega_p(H) = -\frac{Ve^2H^2}{4\pi^2\hbar c^2}\left(\frac{m}{m_s}\right)^2\sqrt{\frac{\zeta}{2m}}\left[1 - \frac{\pi}{2}\frac{\lambda_F}{L} + \frac{\lambda_F}{L}\sum_{k=1}^{\infty}\frac{1}{k}\sin\left(\frac{2kL}{\lambda_F}\right)\Psi\left(\frac{\pi kTL}{\zeta\lambda_F}\right)\right], \tag{16}$$

$$\Psi(x) = x / \operatorname{sh} x.$$

The magnetic susceptibility is then easily found: $\chi = -V^{-1}\partial^2\Omega/\partial H^2 = \chi_d + \chi_p$. In the range of small temperatures (7), we have

$$\chi_d = \chi_{\mathcal{L}}\Phi_d(L/\pi\lambda_F), \tag{17}$$

$$\chi_p = \chi_{\mathcal{P}}\Phi_p(L/\pi\lambda_F). \tag{17'}$$

Here

$$\chi_{\mathcal{L}} = -\frac{e^2}{6\pi^2\hbar c^2}\sqrt{\frac{\zeta}{2m}} \tag{18}$$

is the Landau diamagnetic susceptibility,

$$\chi_{\mathcal{P}} = \frac{e^2}{2\pi^2\hbar c^2}\left(\frac{m}{m_s}\right)^2\sqrt{\frac{\zeta}{2m}} \tag{18'}$$

is the Pauli paramagnetic susceptibility in the case of a massive sample; Φ_d and Φ_F are determined by the formulas (9) and (10).

The function $\Phi_d(x)$ is positive, has minimal values of the order of 0.01–0.1 at the points x = n (n = 1, 2, 3, ...), and its derivative Φ_d' undergoes at these points a finite jump:

$$\Delta\Phi_d'(n) = \pi^2 - 6 / n^2. \tag{19}$$

At the points $x = n + 1/2$ we have $\Phi_d \sim 1$ (see Fig. 2). At thicknesses $L = \pi n \lambda_F$ the diamagnetic susceptibility of the plate is $|\chi_d| \ll |\chi_{\mathcal{L}}|$. With changing L, the susceptibility χ_p oscillates with a period

$$\Delta L = \pi\lambda_F. \tag{20}$$

At $L/\lambda_F \gg 1$, the oscillating part of χ_d is easily separated from the smooth part (see (11, 12)), and the relative value of the oscillations is ~1.

The paramagnetic susceptibility χ_p also oscillates with change of L [with period (20)]. The amplitude of the oscillations $\sim\lambda_F/L$. At $L = \pi n \lambda_F$ the value of χ_p experiences a jump

$$\Delta\chi_p = n^{-1}\chi_{\mathcal{P}}. \tag{21}$$

With increase in the temperature, the condition (7) ceases to be satisfied and the dependence of the amplitude of the oscillations of χ on the temperature becomes important. In the general case, the corresponding formulas for χ follow directly from (6), (14)–(16). For not too low temperatures,

$$\pi TL / \zeta\lambda_F \gg 1, \tag{22}$$

we then have

$$\chi_d = \chi_{\mathcal{L}}\left(1 - \frac{9\pi}{16}\frac{\lambda_F}{L} - \frac{\pi TL}{\zeta\lambda_F}\exp\left(-\frac{\pi TL}{\zeta\lambda_F}\right)\cos\frac{2L}{\lambda_F}\right), \tag{23}$$

and, similarly, for the paramagnetic susceptibility,

$$\chi_p = \chi_{\mathcal{P}}\left(1 - \frac{\pi}{2}\frac{\lambda_F}{L} + \frac{2\pi T}{\zeta}\exp\left(-\frac{\pi TL}{\zeta\lambda_F}\right)\sin\frac{2L}{\lambda_F}\right). \tag{24}$$

Thus, in the limit of weak magnetic fields (1), the diamagnetic susceptibility χ_d, for not too thin plates ($\lambda_F/L \ll 1$) and for temperatures (22), coincide with the Landau diamagnetic susceptibility $\chi_{\mathcal{L}}$ of the massive sample with accuracy to within a quantity $\sim\lambda_F/L$. In the region of lower temperatures, an oscillatory dependence of the susceptibility χ_d on the thickness L develops. In contrast with the comparatively small oscillations of the paramagnetic susceptibility χ_p, the oscillations of χ_d are appreciable.

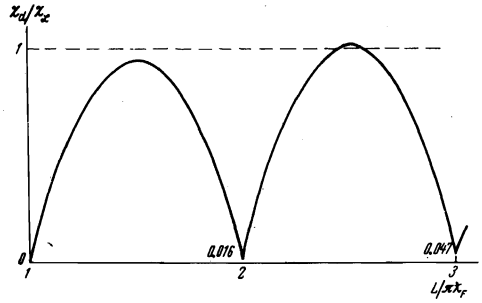

FIG. 2

The considered size oscillations of the magnetic susceptibility χ are a manifestation of the quantum size effect, due to the quantization of the energy of the electron in a sufficiently thin plate. For a number of metals, the existence of quantum size energy levels of the electrons has been demonstrated experimentally. In the case of semimetals, this has been done in[10,11]. (The possibility of observation of quantum size effects was shown in[12] and more detailed information on this problem can be found in the review[13].)

We apply the foregoing analysis of the magnetic susceptibility to conduction electrons with a quadratic dispersion law, a procedure valid near the edge of the band, for small electron groups, and so on. The essential characteristics of the considered effects are generalized directly to the case of an arbitrary dispersion law $\mathscr{E}(p)$.

In weak magnetic fields (1) most interest attaches to the quantum size oscillations of the magnetic susceptibility, the observation of which is possible in sufficiently thin plates. In correspondence with the general theory of oscillatory phenomena,[12] using the formulas of size quantization given in[14] in the case of an arbitrary dispersion law, we obtain for the period ΔL of the oscillations of the susceptibility χ:

$$\Delta L = 2\pi\hbar / d_e, \tag{25}$$

where d_e is the length of the extremal chord of the Fermi surface in the direction of the normal to the film. The amplitude of the oscillations for not too low temperatures (22) is proportional to $\exp(-2\pi^2 T/\Delta\epsilon_L)$, where $\Delta\epsilon_L$ is the separation of the quantum size energy levels. Here,

$$\Delta\varepsilon_L = 2\pi\hbar / L(v_{y1}^{-1} + |v_{y2}|^{-1}), \tag{26}$$

v_{yi} is the projection of the velocity of the electron $v = \partial\mathscr{E}/\partial p$ on the normal to the film at the points of intersection of the Fermi surface with the extremal chord. As in the case of other quantum size effects,[15] we can determine, from the measurement of the magnetic susceptibility of the films in the weak field limit (1), the extremal chord d_e and the velocity of the electron at points of intersection of the chord with the Fermi surface.

For bismuth films of thickness $L \sim 10^{-5}$ cm, the condition (1) is equivalent to $H \ll 10^3$ Oe. At sufficiently low temperatures $T \ll \Delta\epsilon_L$ and thickness $L \approx 2\pi\hbar n/d_e$, where $n = 1, 2, 3, \ldots$, the diamagnetic susceptibility of the film can be different from the susceptibility of the massive sample by a factor of 10—100.

3. MAGNETIC SURFACE LEVELS

The contribution of the magnetic surface levels to the thermodynamic quantities has been studied in a number of works. As was shown in[4], in the calculation of thermodynamic quantities, it is necessary to take into account the deviations of the exact values of the magnetic surface levels E_n from their classical values (a more detailed discussion of the literature on this problem can be found in[8]). As a consequence of the non-analytic dependence of E_n on H (see the formulas for E_n cited below), one could in principle have expected a similar dependence of the thermodynamic potential Ω on H:

$$\Omega(H) = \Omega(0) + a_1 H^{2/3} + a_2 H^{4/3} + \ldots, \tag{27}$$

which would have led to a strong surface magnetization of the metals. It was shown in[8] that $a_1 = 0$. The next term in the expansion (27) is calculated in the present work; this could turn out to be significant. For this purpose, we must first analyze the energy spectrum of the electrons in the plate with the formulas for E_n obtained in[6,16].

In the case of a quadratic electron dispersion $\mathscr{E} = p^2/2m$, the energy spectrum $E_n(p_x, p_y, H)$ of the conduction electrons in a plate in a parallel magnetic field H is determined by the solution of the equation

$$(\hbar eH/c)^2 \psi''(\eta) + (P^2 - \eta^2)\psi(\eta) = 0 \tag{28}$$

with the boundary condition

$$\psi(-p_x) = \psi(LeHc^{-1} - p_x) = 0, \tag{29}$$

where $P = \sqrt{2mE - p_z^2}$.

In the quasiclassical approximation, we seek a solution of the equation in the form

$$\psi(\eta) = \exp\left\{\frac{ic}{\hbar eH}\int \sigma(\eta)\,d\eta\right\}.$$

Here

$$\sigma(\eta) = \sigma_0 + \frac{\hbar eH}{c}\sigma_1 + \left(\frac{\hbar eH}{c}\right)^2 \sigma_2 + \ldots,$$

where

$$\sigma_0^2 = P^2 - \eta^2, \qquad \sigma_1 = \frac{i}{2}\frac{d}{d\eta}\ln|\sigma_0|, \qquad \sigma_2 = \frac{i\sigma_1' - \sigma_1^2}{2\sigma_0}.$$

Matching the quasiclassical wave functions in the well-known way,[17] and using the boundary condition (29), we get as a result the quantum condition

$$S(E, p_x, p_z, H) = 2\pi\hbar eHc^{-1}(n + \gamma), \quad n = 0, 1, 2, 3, 4, \ldots, \tag{30}$$

where the function S and the parameter γ are determined by the following equations. In the interval $P < p_x < LeHc^{-1} - P$, the electron trajectory is inside the plate, $S = \pi p^2$ and $\gamma = 1/2$, so that from (30) we obtain

$$E_n = \frac{\hbar eH}{mc}\left(n + \frac{1}{2}\right) + \frac{p_z^2}{2m}$$

which are the magnetic Landau levels. In the range $-P < p_x < P$, $p_x < LeHc^{-1} - P$, and also for $p_x > P$, $LeHc^{-1} - P < p_x < LeHc^{-1} + P$, the electron is reflected from one of the boundary surfaces of the plate, $\gamma = 3/4$ and we have for S, respectively,

$$S = P^2[1/2\pi - f(-p_x/P)] \tag{31}$$

and

$$S = P^2\left[\frac{\pi}{2} + f\left(\frac{LeH}{cP} - \frac{p_x}{P}\right)\right], \tag{31'}$$

where

$$f(x) = \arcsin x + x\sqrt{1 - x^2} + \frac{1}{12}\left(\frac{\hbar eH}{cP^2}\right)^2\left[\frac{5x}{(1 - x^2)^{3/2}} + \frac{1}{x\sqrt{1 - x^2}} - \frac{\sqrt{1 - x^2}}{x}\right]. \tag{32}$$

In the interval $LeHc^{-1} - P < p_x < P$, the electron is reflected from both boundary surfaces of the plate:

$$S = P^2\left[f\left(\frac{LeH}{cP} - \frac{p_x}{P}\right) - f\left(-\frac{p_x}{P}\right)\right], \qquad \gamma = 1. \tag{33}$$

Equations (30)—(33) describe in implicit form the energy spectrum $E_n(p_x, p_z, H)$ in the quasiclassical approximation $n \gg 1$, excluding the values of p_x for which $f \to \infty$. From (30) and (31) it is easy to find the values of the magnetic surface levels E_n for $n \gg 1$ for electrons skipping along the surface of the metal. The corresponding values of E_n, when $n \gtrsim 1$, are determined as a result of the approximation of the solution of Eq. (28) with boundary conditions (29), if we take the linear approximation of the potential as the zeroth approximation. Taking into account the deviation of the parabolic potential in (28) from its linear approximation, we get for E_n (from perturbation theory)

$$E_n(p_x, p_z, H) = \frac{p_x^2 + p_z^2}{2m} + s_n \frac{p_x^{2/3}}{2^{1/3} m}\left(\frac{\hbar eH}{c}\right)^{2/3} + s_n^2 \frac{2^{4/3}}{15} \frac{p_x^{-2/3}}{m}\left(\frac{\hbar eH}{c}\right)^{4/3}, \tag{34}$$

where $-s_n$ are the zeroes of the Airy function

$$\mathrm{Ai}(-s_n) = 0. \tag{35}$$

In comparison with the formulas of[16], we have taken into account higher order terms in the expansion in H in (34). For $n \gg 1$,

$$s_n \sim \left[\frac{3\pi}{2}\left(n + \frac{3}{4}\right)\right]^{2/3} + \frac{5}{48}\left[\frac{3\pi}{2}\left(n + \frac{3}{4}\right)\right]^{-4/3}. \tag{36}$$

In the calculation of the contribution of the magnetic surface levels to the thermodynamic quantities, it is convenient to write down the thermodynamic potential Ω in the following form:

$$\Omega = -\frac{2VT}{L(2\pi\hbar)^2}\iint dp_z\, dp_x \left\{\sum_{n=n_0}^{n_0-1} \ln\left(1 + \exp\frac{\zeta - E_n(p_x, p_z, H)}{T}\right) + \sum_{n=n_0}^{\infty} \ln\left(1 + \exp\frac{\zeta - E_n^{\text{quas}}(p_x, p_z, H)}{T}\right)\right\}, \tag{37}$$

where $E_n(p_x, p_z, H)$ is determined by Eq. (34) and E_n^{quas} is found from the quasiclassical quantization condition (30) for $\gamma = 3/4$. Using perturbation theory for the calculation of the first sum in (37) and the Poisson summation formula for calculation of the second sum, we get the result that the coefficient a_2 in the expansion (27) is proportional to

$$a_2 \propto \left[\sum_{n=0}\left(s_n^2 - \left[\frac{3\pi}{2}\left(n + \frac{3}{4}\right)\right]^{4/3} - \frac{5}{24}\left[\frac{3\pi}{2}\left(n + \frac{3}{4}\right)\right]^{-2/3}\right) + \frac{5}{24}\left(\frac{2}{3\pi}\right)^{2/3}\zeta\left(\frac{2}{3}, \frac{3}{4}\right) - \frac{\Gamma(1/3)}{6^{2/3}\pi}\sum_{k=1}^{\infty}\frac{1}{k^{7/3}}\cos\left(\frac{3\pi k}{2} - \frac{\pi}{6}\right)\right], \tag{38}$$

where $\zeta(s, c)$ is the generalized Riemann zeta function. The coefficient of proportionality in (38) is bounded. The first sum in (38) is calculated by the method put forward in[8]. Substituting the result for this sum in (38), we get

$$a_2 = 0. \tag{39}$$

Thus the contribution of the magnetic surface levels to the thermodynamic potential Ω and, correspondingly, to the magnetization and the susceptibility of metals, turns out to be insignificant.

4. SURFACE MAGNETIC SUSCEPTIBILITY

In the region of strong[1)] magnetic fields (2), an appreciable contribution to the magnetic susceptibility is made, as shown in[9], by electrons whose trajectories touch the surface of the metal (Fig. 1b). In the case of a quadratic isotropic dispersion law, $\mathscr{E} = p^2/2m$, we have for the magnetic susceptibility of the plate χ_d, according to[9],

$$\chi_d = \chi_{\mathscr{L}} + \frac{2\beta}{L}\left(\frac{e}{c}\right)^{3/2}\sqrt{\frac{\zeta}{2m\hbar}}\, H^{-1/2}, \tag{40}$$

$$\beta = \frac{3}{8\pi^4}\left[\frac{2^{3/2} - 1}{4}\zeta\left(\frac{5}{2}\right) + \pi^{1/6}\left(\frac{2}{3}\right)^{1/3}\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{5}{6}\right) \cdot \sum_{k=1}^{\infty}\frac{1}{k^{5/2}}\sin\left(\frac{\pi k}{2} + \frac{\pi}{12}\right)\right] = 0{,}78 \cdot 10^{-2}. \tag{41}$$

In the weak field limit $H \ll H_c$, the susceptibility of the plate χ_d, neglecting quantities $\sim\lambda_F/L$, is identical with $\chi_{\mathscr{L}}$ (see Sec. 2). With increase in the field, upon reaching values of $H \gg H_c$, the susceptibility χ_d also tends to $\chi_{\mathscr{L}}$, revealing here the characteristic dependence on H. Thus, one should expect a maximum deviation of the smooth part of the magnetic susceptibility of the plate χ_d from the Landau diamagnetic susceptibility $\chi_{\mathscr{L}}$ of the massive sample in the range of fields $H \sim H_c$ (see Fig. 3). For a plate of thickness $L \sim 10^{-4}$ cm, the field is $H_c \sim 10$ Oe.

The surface magnetic susceptibility, which is described by the formula (40), can be directly generalized to the case of conduction electrons with an arbitrary dispersion law $\mathscr{E}(p)$. In contrast with the case of magnetic surface levels, one can use the quasiclassical approximation for the energy spectrum in this case[6]. Summing (5) according to the Poisson formula, we get for Ω:

$$\Omega(H) = \Omega(0) + \frac{V}{2\pi^3\hbar^2 L}\sum_{l=1}^{\infty}\frac{1}{l}\int d\varepsilon \iint dp_z dp_x \times \frac{\sin[2\pi l\gamma - (lc/\hbar eH)S(\varepsilon, p_z, p_x)]}{1 + \exp[(\varepsilon - \zeta)/T]}, \tag{42}$$

where $S(\epsilon, p_z, p_x)$ is the area of the intersection of the constant energy surface $\mathscr{E}(p) = \epsilon$ with the plane $p_z = \text{const}$ bounded by the straight lines p_x and $p_x + LeH/c$.

Separating quantities proportional to $H^{3/2}$ from (42) and differentiating with respect to H, we obtain the following expression for the magnetic susceptibility of the plate:

$$\chi_d = \chi_V + \frac{\beta}{2L}\left(\frac{e}{c}\right)^{3/2}(\hbar H)^{-1/2}\sum_j \int_0^{\zeta} K_j^{3/4}(\varepsilon)\, k_j^{-1/2}(\varepsilon, \mathrm{N})\, d\varepsilon. \tag{43}$$

Here $K(\epsilon)$ is the Gaussian curvature of the constant energy surface $\mathscr{E}(p) = \epsilon$ at the j-th limiting point with the normal parallel to H (see Fig. 4), $k_j(\epsilon, N)$ is the curvature of the normal section perpendicular to N at the j-th point (N is perpendicular to the plate), χ_V is the volume part of the susceptibility, which does not depend on the magnetic field and the thickness of the plate. For a quadratic dispersion law $p^2/2m = \epsilon$, we have $K_j = (2m\epsilon)^{-1}$ and $k_j = (2m\epsilon)^{-1/2}$, so that (43) naturally coincides with (40) in our case.

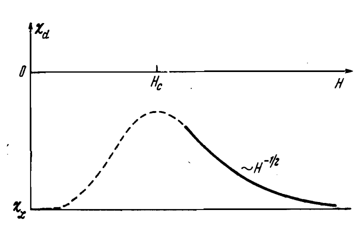

FIG. 3

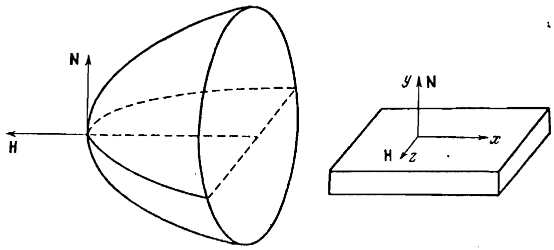

FIG. 4

The resultant formulas (40) and (43) describe the contribution of the metal surface to the Landau diamagnetism. In contrast with the paramagnetic susceptibility, for which the presence of the boundary of the metal leads to the appearance of additional components of the order λ_F/L (see[14]), in the case of diamagnetism the effect of the boundary turns out to be more important and determines the appearance of additional terms of the order of

$$|(\chi_d-\chi_v)/\chi_v| \sim (H_c/H)^{1/2}. \quad (44)$$

This characteristic dependence of χ_d on H allows us to separate relatively easily the contribution of the conduction electrons to the magnetic susceptibility of the plate. In the case of massive samples, this is a rather complicated problem.

In conclusion, I express my deep gratitude to I. M. Lifshitz for attention to the work and valued discussions, to M. I. Kaganov and V. G. Peshchanskiĭ for useful discussions.

[1)]The condition $\mu H/\zeta \ll 1$ is assumed to be satisfied, where μ is the Bohr magneton.

[1]A. Papapetrou, Z. Phys. 107, 387 (1937).
[2]L. D. Landau, Z. Phys. 64, 629 (1930).
[3]L. Friedman, Phys. Rev. 134, A336 (1964).
[4]I. M. Lifshitz and A. M. Kosevich, Dokl. Akad. Nauk SSSR 91, 795 (1953).
[5]D. Childers and P. Pincus, Phys. Rev. 177, 1036 (1969).
[6]A. M. Kosevich and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 743 (1955) [Soviet Phys.-JETP **2**, 646 (1956)].
[7]M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. 39, 212 (1960) [Soviet Phys.-JETP **12**, 152 (1961)].
[8]S. S. Nedorezov, Zh. Eksp. Teor. Fiz. 60, 1938 (1971) [Soviet Phys.-JETP 33, 1045 (1971)].
[9]S. S. Nedorezov, ZhETF Pis. Red. **14**, 597 (1971) [JETP Lett. 14, 415 (1971)].
[10]Yu. F. Ogrin, V. N. Lutskiĭ, and M. I. Elinson, ZhETF Pis. Red. 3, 114 (1966) [JETP Lett. **3**, 71 (1966)].
[11]Yu. F. Komnik, E. I. Bukhshtab, ZhETF Pis. Red **6**, 536 (1967) [JETP Lett **6**, 58 (1967)].
[12]I. M. Lifshitz and A. M. Kosevich, Izv. Akad. Nauk SSSR, ser. fiz. **19**, 395 (1955).
[13]B. A. Tavger and V. Ya. Demikhovskiĭ, Usp. Fiz. Nauk **96**, 61 (1968) [Soviet Phys.-Uspekhi **11**, 644 (1969)].
[14]S. S. Nedorezov, Zh. Eksp. Teor. Fiz. **51**, 868 (1966) [Soviet Phys.-JETP **24**, 578 (1967)].
S [15]I. O. Kulik, ZhETF Pis. Red. **6**, 652 (1967) [JETP Lett. **6**, 143 (1967)].
[16]T. W. Nee and R. E. Prange, Phys. Rev. Lett. **25A**, 582 (1967).
[17]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics) Fizmatgiz, 1963 [Addison-Wesley, 1965].

Translated by R. T. Beyer
69