Mechanism of emission of coherent phonons from a point junction

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A simple model is proposed which describes the generation of coherent phonons at the point junction between two metallic films located at the site of tunnel-structure breakdown. The generation mechanism is due to the Vavilov-Cerenkov effect for electrons in the metal when the drift velocity exceeds the velocity of sound. The length of the region of interaction between the electrons and sound wave does not significantly exceed the wave length of sound and hence interference occurs between sound waves excited at various points within the junction and leaving the interaction region. This results in an oscillating dependence of the phonon emission intensity on the electron drift velocity and consequently in oscillations of the junction conductivity.

As is well known[1], when the electron drift velocity v exceeds the speed of sound s in a metal, the electron flow becomes strongly decelerated as the result of coherent phonons along the entire path of the beam. The mechanism of this radiation is due to the Cerenkov effect[2], and in this case the charged-particle velocity should be taken to mean the drift velocity v of the electrons rather than the Fermi velocity $v_F \gg v$. If the electron free path is much larger than the radiation wavelength, then the wave vectors are arranged along the generators of a cone with apex angle $\theta = \cos^{-1}(s/v)$. If the region of interaction between the electrons and the sound-wave field is finite, the wave-vector cone of the emitted waves has a finite thickness. In addition, there appears a unique interference effect, which has been observed experimentally earlier[3] and is manifest in oscillations of the conductivity of a point contact between two metals. We propose here a simple theoretical model explaining this effect.

The point contact produced by short-circuiting a film tunnel structure can be represented in the form of an opening of radius r in a thin dielectric partition between two metals (Fig. 1a). The length of the channel joining the two metals is equal to the thickness L_1 of the dielectric and is usually smaller than its width. If a potential difference V is applied between the metals, then the electric field E will be essentially inhomogeneous in the regions of radius r adjacent to the contact channel from the left and from the right. Inside the channel, the field is practically homogeneous and has a maximum value of the order of V/r. The current density is also maximal inside the channel and falls off rapidly to the left of the entry to the channel and to the right of the emergence from the channel.

We consider the generation of coherent phonons within a channel of length L when electrons move through the channel with supersonic drift velocities. Since the threshold value of the angle θ is zero, we calculate the amplitude of the sound wave emerging from the interaction region x = L (Fig. 1b), assuming $\cos\theta_n = 1$. The number n = 0, 1, 2, ... corresponds to the n-th threshold value of the drift velocity, which will be determined later on. The phase of the sound wave excited at the point x inside the channel and arriving at the point x = L will lag the phase of the wave excited at the point x = L by an amount

$$\Delta\varphi(x) = \omega(L-x)\left(\frac{1}{s}-\frac{1}{v}\right), \tag{1}$$

where ω is the frequency of the sound.

Consequently, an elementary wave excited at the point x will have the following form on emerging from the contact:

$$dA(x,t) = A\cos[\omega t - \Delta\varphi(x)]\,dx/L, \tag{2}$$

where A is the amplitude of the wave and is proportional to the intensity of the electron-phonon interaction. The resultant oscillation at the point x = L is obtained by summing all the partial waves (2):

$$A_\Sigma\cos(\omega t-\varphi) = \frac{A}{L}\int_0^L \cos\left[\omega t - \omega(L-x)\left(\frac{1}{s}-\frac{1}{v}\right)\right]dx, \tag{3}$$

from which we can easily obtain[1)]

$$A_\Sigma = A\frac{\sin\varphi}{\varphi}, \tag{4}$$

$$\varphi = \omega L(v-s)/2sv. \tag{5}$$

The intensity I of the generated sound is proportional to A_Σ^2:

$$I = I_0\frac{\sin^2\varphi}{\varphi^2}. \tag{6}$$

The intensity has zero value whenever $\varphi = n\pi$, n = 1, 2, ..., corresponding to threshold-velocity values

$$v_n = s(1-ns/\nu L)^{-1} \approx s(1+ns/\nu L) \tag{7}$$

($\omega = 2\pi\nu$). The last expression is valid at small $ns/\nu L$.

The threshold velocities begin with the sound velocity s, and initially the intervals between them are

$$\Delta v = s^2/\nu L. \tag{8}$$

Every time that v increases, the phase φ changes by π, a new Cerenkov-radiation cone is produced and is imbedded in the preceding one (Fig. 1c), and at this instant the apex angle of the m-th cone is equal to

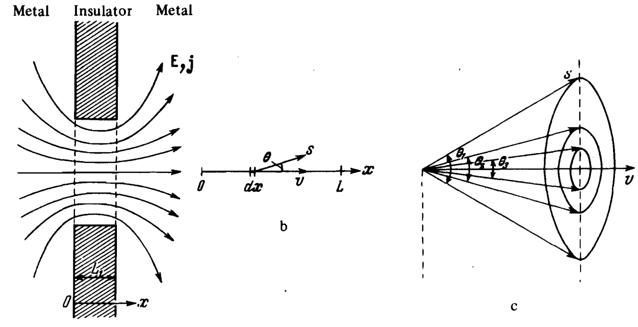

FIG. 1

$$\theta_m = \arccos[1-(n-m)s/vL], \quad m<n. \tag{9}$$

Whenever the drift velocity reaches a new threshold, a new energy-dissipation channel is produced, and at the given current it leads to an increase in the voltage and consequently in the resistance of the contact (Fig. 2). Each value of ω_q corresponds to a definite value of the velocity s_q. It is easily seen from (7) that the experimentally measured value of v_0 yields s_q, and the oscillation period is $q = \omega/s$. In the general case of an arbitrary phonon spectrum, it is necessary to take into account all the frequencies and to integrate with respect to frequency. Then, however, by virtue of the rapid decrease of the Cerenkov-radiation energy with increasing wavelength ($W \sim \lambda^{-3}$)[2], the effect will be maximal for the short-wave photons, which are thus favored.

We can simplify (7)–(9) by assuming that the density of the phonon states has sharp maxima at definite energies ω_k. It is precisely in this case that one can observe sufficiently distinct oscillations of the conductivity of point contacts[3]. Let the phonon spectrum consist of two peaks ω_1 and ω_2 corresponding to the limiting energies for the transverse and longitudinal phonons (see, for example, the phonon spectrum of lead[5]), corresponding to sound velocities $s_1 = s_{TA}$ and $s_2 = s_{LA}$. Then

$$s_1/\omega_1 \approx s_2/\omega_2 \approx a/\pi$$

and

$$v_{n,k} = s_k(1-2na/L)^{-1} \approx s_k(1+2na/L); \quad k=1,2, \quad n=0,1,2,\ldots \tag{10}$$

Consequently, we obtain two series of oscillations, and the number of oscillations in each series does not exceed $L/2a - 1$. If we neglect the small change of the dynamic resistance of the contact in the investigated range of voltages, then we can establish a simple connection between the drift velocity and the voltage on the contact $v \approx eV/p_F$, and then the period of the oscillations with respect to voltage is $\sim \hbar s_k/L$.

Let us compare the consequences of the proposed model with the experimental data[3]. Figure 3 shows the current-voltage characteristic (CVC) (curve a) and its second derivative (curve b) for a point contact produced when a Pb-PbO-Pb tunnel junction is short-circuited. Up to the voltage $V_1 = 5.3$ mV and the current $I_1 = 14$ mA, the CVC is linear, and its second derivative is equal to zero. The threshold drift velocity v_{TA}, which is equal to

$$v_{TA} = \frac{j_1}{en} = \frac{I_1}{enS} = \frac{V_1}{en(\rho l)} \cong 10^5 \text{ cm/sec}, \tag{11}$$

(n is the electron density in Pb, and ρl is the product of the resistivity by the mean free path) corresponds to the velocity s_{TA} of the transverse sound in Pb.

As shown earlier[6], in the nonlinear region $R(V) = A + CV$, and the CVC of the point junction can be approximate by the expression

$$V(I) = \frac{AI}{1-CI}, \quad CI<1, \tag{12}$$

where A and C are constants, with C proportional to the intensity of the electron-phonon interaction. To first order, the derivatives of the CVC are equal to

$$\frac{dV}{dI} \approx A[1+2(CI)+3(CI)^2], \tag{13}$$

$$\frac{d^2V}{dI^2} \approx 2AC[1+3(CI)]. \tag{14}$$

As soon as the second threshold value of the drift

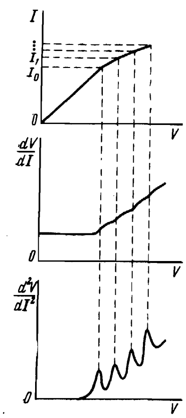

FIG. 2

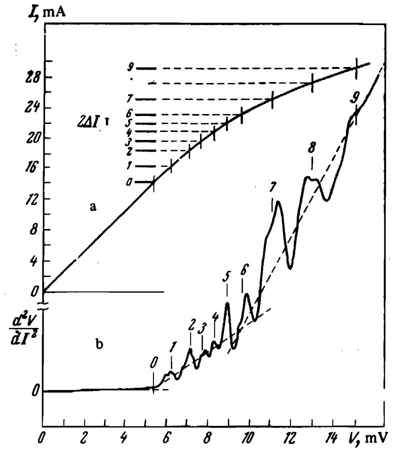

FIG. 3

FIG. 2. Schematic form of the current-voltage characteristic and of its first and second derivatives.

FIG. 3. a–Current-voltage characteristic of Pb-Pb contact. The contact diameter is 2r = 1100 Å, and the film thickness is 1500 Å. The first threshold value of I_1 corresponds to zero level, the second I_2 coincides with the position of the sixth maximum on the I axis. The horizontal lines indicate the positions of the maxima on the I axis. b–Second-harmonic voltage of the modulating signal as a function of the contact voltage V. The total sweep along the ordinate axis corresponds to $V_2 = 7.5$ μV; $2\Delta I = 2i$ is double the oscillation amplitude; T = 1.5°K, and H = 10 kOe is used to suppress the superconductivity of Pb.

velocity v_{LA} is reached, corresponding to the current $I_2 = 23.5$ mA and the voltage $V_2 = 9.7$ mV in Fig. 3, the monotonic component of d^2V/dI^2 experiences a kink corresponding to a jump-like increase of the coefficient C. The second threshold value of the drift velocity is approximately equal to the velocity of the longitudinal sound s_{LA} in Pb. The $v_2(V)$ oscillations dealt with in the present paper are superimposed on the background described above, and become observable only at sufficiently low temperature $kT \ll \Delta V$, where ΔV is the period of the oscillations with respect to the voltage.

The most prominent feature of the effect is its threshold character. According to the proposed model there should be no oscillations at $v < v_{TA}$, and a new series of oscillations with period v_{LA}/v_{TA} times larger than the preceding period should appear when $v > v_{LA}$. This is precisely what is observed in experiment (Fig. 3). The presence of the threshold allows us to reject certain other possible mechanisms of phonon generation, for example bremsstrahlung, although its presence can apparently likewise be observed in more sensitive experiments. The average period of the oscillation current in the first series (1–6 in Fig. 3; $v_{TA} < v < v_{LA}$) is equal to $\Delta I_1 = 1.16$ mA, and in the second series (6–9) we have $\Delta I_2 = 2$ mA. In accordance with formula (10), their ratio is equal to the ratio of the threshold velocities, $\Delta I_2/\Delta I_1 \approx v_{LA}/v_{TA} = 1.7$.

Let us determine the effective length L over which phonon interference takes place. It follows from (10) that

$$L = 2a\frac{I_1}{\Delta I_1} = 2a\frac{I_2}{\Delta I_2} = 125\text{Å} \tag{15}$$

for the considered contact (Fig. 3), where a is the lattice constant, equal to 5 Å for Pb. The obtained value of L is of the same order of magnitude as the thickness of the dielectric layer (~50 Å), and exceeds the latter by a factor 2.5. This is not surprising, since the emission of coherent phonons in a real contact (Fig. 1a) occurs also in regions adjacent to the left and to the right of the channel joining the two metals. We note that L is smaller by one order of magnitude than the characteristic dimensions of the junction, namely, of the junction diameter and of the metal-film thickness, and also of the mean free path l.

To estimate the amplitude of the effect, we introduce an expression for the second-harmonic voltage of the modulating signal, which is usually measured in the experiment (Fig. 3b). We assume that the CVC consists of two parts, a monotonic component $V(\mathscr{I})$ and a small oscillating increment $V_{osc}\sin(\Omega\mathscr{I})$, where V_{osc} is the amplitude of the oscillations on the CVC, and $\Omega = 2\pi/\Delta I$ is the "frequency," which is inversely proportional to the oscillation period ΔI. Since the resistance of the point constant is much smaller than the internal resistance of the source, what is usually given in a real measuring circuit is the current $\mathscr{I}$:

$$\mathscr{I} = I + i \sin \omega t, \tag{16}$$

and one measures the voltage on the sample:

$$\mathscr{V}(\mathscr{I}) = V(I + i \sin \omega t) + V_{osc}\sin[\Omega(I + i \sin \omega t)]. \tag{17}$$

Here I is the dc component of the current through the sample, and i and ω are respectively the amplitude and frequency of the modulating signal.

Usually $i \ll I$, but frequently $i \sim \Omega^{-1}$ (see Fig. 3a) and the expansion of (17) in a Taylor series, from which the proportionality of the second-harmonic voltage v_2 to the second derivative of the CVC follows, is not suitable in this case. We represent instead the oscillating factor in the second term of the sum (17) in the form

$$\sin[\Omega(I + i \sin \omega t)] = J_0(\Omega i)\sin \Omega I + \sum_{m=1}^{\infty} J_{2m}(\Omega i) 2 \sin \Omega I \cos 2m\omega t + \sum_{m=1} J_{2m-1}(\Omega i) 2 \cos \Omega I \sin(2m-1)\omega t, \tag{18}$$

with no limitations whatever imposed on the modulation amplitude. In (18), J_m is a Bessel function of order m.

It follows from (18) that the second-harmonic voltage

$$v_2 = V_{osc} J_2(\Omega i) 2\sin\Omega I \tag{19}$$

is by far not proportional to the second derivative of the CVC. Thus, for example, at $\Omega i = \alpha_{2,n}$ ($\alpha_{2,n}$ is the n-th root of the Bessel function of second order) we have $v_2 = 0$, even though the second derivative of the CVC differs from zero. At an arbitrary form of the CVC, it can be expanded in a Fourier series of its harmonic component, in each of which it is necessary to employ the transformation (18). As a result, when the modulation amplitude becomes comparable with the dimension of the inhomogeneity along the corresponding axis, the form of the observed $v_2(V)$ or $v_2(I)$ curves, which are usually called "tunnel spectra," will be significantly altered by relatively small variations of the modulating-signal amplitude.

This fact must be taken into account when using large amplitudes of the modulating signal in experiments. Only under the condition

$$i \ll \Omega_i^{-1} = \Delta I / 2\pi \quad \text{or} \quad \tilde{v} \ll \Omega_v^{-1} = \Delta V / 2\pi \tag{20}$$

is the second-harmonic amplitude proportional to the second derivative of the CVC. In (20), i and $\tilde{v}$ are the amplitudes of the modulating signals, while ΔI and ΔV are the characteristic dimensions of the CVC nonlinearity along the current and voltage axes, respectively.

Returning to an estimate of the amplitude of the effect observed in the experiment (Fig. 3), we make use of the fact that the amplitude of the second-harmonic voltage oscillations in the second series is $\lesssim 10^{-6}$ V, $\Omega i = (2\pi/\Delta I)(\Delta I/4) = 1.57$, and $J_2(1.57) \approx 0.25$. Consequently, the amplitude of the oscillating increment of the CVC is $V_{osc} \leq 2 \times 10^{-6}$ V and amounts to only $\sim 10^{-4}$ of the monotonic component of the voltage across the junction. The smallness of the effect is not surprising from the point of view of the proposed model. Indeed, the deviation of the CVC from a linear dependence and the associated kinks in the average line, shown dashed in Fig. 3b, are due mainly to the production of incoherent phonons in metallic films at distances $\sim l$ [3,6]. The oscillations, on the other hand, are due to interference of coherent phonons produced by electrons in the channel over a relatively short segment $L \ll l$, where the drift velocity can be regarded as approximately constant.

In connection with the last remark, let us make more precise the meaning of the concepts "drift velocity" v, used in the present paper. According to the universally accepted definition $v = j/en$. But usually the word "drift" is taken to mean the diffuse advance of a particle in the direction of the driving force, and the distances considered are much larger than the mean free path l. In the proposed model, to the contrary, everything occurs in a short segment of the path ($L \ll l$) between two collisions, and in this segment we have $\mathbf{v}$ approximately constant and the distribution function can be represented in the form $f(\epsilon + \mathbf{p}\cdot\mathbf{v})$, where $\epsilon(\mathbf{p})$ is the energy of the quasiparticles. In this situation, a more accurate designation for v would probably be not "drift velocity," but, for example, "loss rate."

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[1)] We omit a factor $\sin\theta$, which appears in the right-hand side of (4) when a more rigorous analysis is made, and corresponds to the fact that the radiation intensity near the threshold is low. Its absence does not influence the threshold velocities obtained below.

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