Static surface resistance of zinc in a magnetic field

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The dependence of resistance on transverse and longitudinal magnetic fields $\rho^d(H)$ is investigated for zinc whiskers at $T = 4.2^\circ$K. The thickness d of the samples, which were in the form of filaments and plates, varied from 0.5 to 10 μ, and this allowed measurements to be carried out under conditions of a strong size effect, $d \ll l^\infty$, where l^∞ is the conduction-electron mean free path. For both longitudinal and transverse fields the $\rho^d(H)$ curves for samples of different thicknesses are similar to each other, and, in terms of the coordinates $\rho^d d$ and Hd, they reduce to a single curve. It is inferred that electron scattering at the surface plays the dominant role in the resistance of whiskers in a magnetic field. In the region of weak magnetic fields in which $d < r$ (r is the radius of the electron orbit), the experimental results do not agree with the theoretical calculations. The unusual field dependence of the resistance $\rho^d(H) \propto H^{2/3}$ observed in these fields is ascribed to specular reflection of electrons from the surface. In the region of strong transverse magnetic fields, where $r < d$, good qualitative agreement with the theory of the static skin effect is obtained. In this region, the experimental $\rho^d(H)$ curves can be represented as the sum of two terms, one of which is linear and the other quadratic in the field. The linear term corresponds to specular reflection of electrons from the surface, while the quadratic term corresponds to diffuse scattering. It is postulated that the ratio of these terms should allow the determination of the specular reflection coefficient for the given sample.

I. INTRODUCTION

The idea that the boundaries of a sample influence the kinetic properties of a metal (the size effect) was put forward by Thomson as far back as in 1901[1]. Since then a large number of experimental and theoretical investigations devoted to diverse size effects on resistance in the absence of a magnetic field have been carried out. This subject is reviewed by Brändli and Olsen in[2].

The boundaries of a sample can introduce important changes into the classical galvanomagnetic properties of a metal. The question of the influence of size on the galvanomagnetic properties when electrons are diffusely scattered at the surface has been considered by a number of authors. Chambers[3] has obtained the dependence for the longitudinal, while Sarginson and MacDonald[4] and Sondheimer[5] have obtained the dependence for the transverse resistance of wires and plates in a magnetic field. Common to these investigations is the conclusion that the contribution to the resistance of the electrons colliding with the surface decreases as the magnetic-field strength increases. An exception is the case when the sample is in the form of a plate and the magnetic field H is perpendicular to its plane: the magnetic field does not diminish the size effect.

The first experimental investigations of this sort date back to the beginning of the fifties. Analysis of the results shows these experiments to be in good qualitative agreement with theory. The main results of this comparison can also be found in Brändli and Olsen's review article[2]. We only add that an experimental proof of the influence of size on the galvanomagnetic properties is provided by the experiments on thin plates in a transverse magnetic field, proposed and carried out on bismuth samples by Borovik and Lazarev[6]. The case when the magnetic field is perpendicular to the surface of the plate is fundamentally different from the case when the field is parallel to the surface; therefore, the resistance must possess an additional anisotropy, which was observed in[6]. These results were confirmed by Forsvoll and Holwech on aluminum samples[7], and by Panchenko and Lutsishin on tungsten samples[8].

The theory of the galvanomagnetic size effects was further developed in the papers[9-12] by Azbel' and Peschanskiĭ. The basic idea of these investigations amounts to the fact that in a strong magnetic field, when the radius r of an electron orbit is much smaller than the sample thickness d, the electrons colliding with the surface possess a higher mobility than the electrons in the interior of the metal. This increases the conductivity in a surface layer of thickness of the order of r; therefore, the electric current is concentrated near the surface. The phenomenon was called the "static skin effect"[9].

The galvanomagnetic properties of thin sample, $d \ll l^\infty$ (where l^∞ is the conduction-electron mean free path in a thick sample), were considered in[10-12] in the entire range of magnetic fields from the point of view of the static skin effect. Since we shall henceforth be interested in this situation, we briefly discuss the main conclusions of these investigations below.

1. The static skin effect exists only in those cases when there is an unrestricted growth of the bulk (or volume) resistance in a magnetic field: $\rho^\infty \propto H^2$. The cause of this growth is immaterial—equality of the numbers of electrons and holes, open trajectories, etc.

2. In a strong magnetic field, in which $r \ll d$ (the field is parallel to the surface of the plate), the effective conduction-electron mean free path in a surface layer of thickness of the order of r is equal to r for the case of diffuse reflection (the specularity coefficient $P = 0$) and l^∞ for the case of specular reflection ($P = 1$); the relative number of electrons colliding with the surface is equal to 2r/d. Thus, the "surface" conductivities for diffuse and specular reflections are respectively equal to

$$\sigma_{\mathrm{dif}}^{s} \approx 2\sigma_0^{\infty} r^2/l^{\infty} d, \quad \sigma_{\mathrm{sp}}^{s} \approx 2\sigma_0^{\infty} r/d$$

(σ_0^{∞} is the bulk conductivity in the absence of a field).

3. Surface scattering does not change the quadratic dependence of the resistance of thin wires on strong transverse magnetic field, but this resistance is less than the "bulk" resistance: $\rho^d(H) = \rho^{\infty}(H)d/l^{\infty}$ for diffuse and $\rho^d(H) = \rho^{\infty}(H)(d/l^{\infty})^2$ for specular reflections ($\rho^{\infty}(H)$ is the resistance of a thick sample in the field H).

4. As for wires, surface scattering (diffuse, as well as specular) does not lead to a qualitatively new transverse-field dependence for the resistance of plate samples: as before, $\rho^d(H) \propto H^2$ for any relative orientation of the field and the normal to the surface. An exception is the case of specular scattering for a field parallel to the surface, when a linear growth of the resistance should be observed: $\rho^d(H) \propto H$.

5. The size effect does not exist for wires and plates in a strong longitudinal magnetic field in which $r \ll d$.

6. The quality of the surface ceases to play an important role in weak magnetic fields in which $r > d$. The smallest characteristic length is the sample thickness d. The electric field is uniform over the interior of the sample. The role of the magnetic field amounts largely to decreasing the influence of electron scattering at the surface. Different cases of the behavior of the resistance of wires and plates in a weak magnetic field are shown in Fig. 1, which also shows the most important cases for a strong magnetic field.

There are no investigations at present the results of which will confirm the main predictions of the theory of the static skin effect that pertain to the resistance of metals. Such attempts have, however, been made. Thus, in the work[13] the authors, Bogod et al., expected to see a variation of the exponent n in the expression $\rho(H) \propto H^n$ as the shape, thickness, and degree of perfection of the surfaces of the bismuth samples were varied. Such a variation was not observed, the exponent n remaining equal to 1.65 in all cases[1)]. In another work[8] likewise no variation of the exponent n was observed for different directions of the field H relative to the surfaces of tungsten plates. It was always equal to 2. But since the shape effect was very clearly observed and attested to the existence of the size effect, Panchenko and Lutsishin concluded that electron reflection from the surface is a totally diffuse reflection. However, the final conclusion about this can be drawn only from the study of the sample-thickness dependence of resistance in a magnetic field.

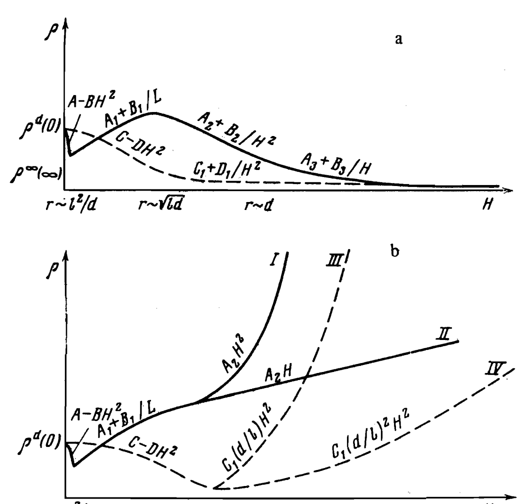

FIG. 1. Theoretical dependences of resistance on magnetic field for thin ($d \ll l^{\infty}$) samples: the dashed curves are for filaments and the continuous curves are for plates. $L \equiv \ln(H_0/H)$. H_0 is the field for which $r = l^{\infty}$; a—longitudinal field, $\mathbf{H} \parallel \mathbf{J}$. b—Transverse field, $\mathbf{H} \perp \mathbf{J}$: I—the reflection is diffuse or specular, but the field $\mathbf{H}$ is not parallel to the surface of the plate; II—specular reflection, the field $\mathbf{H}$ is parallel to the plate surface; III—diffuse reflection; IV—specular reflection.

In view of the uncertainty about the results of the experiments on the static skin effect, it seems to us that under the experimental conditions, when $d \approx l^{\infty}$, it is apparently not possible to detect the manifestation of the static skin effect because of the shunting action of the bulk resistance. More promising for this purpose will be measurements under conditions of strong size effect: $d \ll l^{\infty}$. Indeed, choosing the least favorable condition—totally diffuse reflection—for the bulk and surface conductivities, we can respectively write: $\sigma^V \approx \sigma_0^{\infty}(r/l^{\infty})^2$, $\sigma^S \approx \sigma_0^{\infty} r^2/dl^{\infty}$, from which $\sigma^S/\sigma^V \approx l^{\infty}/d$. Thus, for $l^{\infty} \gg d$, the bulk part of the conductivity can be neglected. The resistance of the sample will be determined by only the electrons in a surface layer of thickness of the order of r.

The present experiment was set up with the object of detecting the basic laws which govern the behavior of resistance in a magnetic field and which have been predicted by the theory of the static skin effect for thin samples. The first experiments in this direction have been discussed previously[15].

II. SAMPLES AND MEASUREMENT PROCEDURE

As the experimental object, we chose filamentary crystals (whiskers) of zinc, which satisfy all the requirements necessary for a successful detection of the static skin effect: the number of electrons is equal to the number of holes, the thickness of the whiskers is much smaller than the electron mean free path ($d \ll l^{\infty}$), ($d \ll l^{\infty}$), and a large specularity coefficient ($P \approx 0.5$). Furthermore, since zinc is a "good" metal, the interpretation of the results obtained should not be bound with such additional complications as the diffusive size effect and the variation of the number of electrons in the magnetic field.

The samples were grown from the gaseous phase by a method suggested by Coleman and Sears[16]. The initial purity of the material for growing the crystals was characterized by the resistance ratio $\rho(300°)/\rho(4.2°) = 10^4$, which corresponds to $l^{\infty}(4.2°K) \approx 300\ \mu$. The electrical mounting of the samples was accomplished by the clamping-contact method[17]. The distance between the potential contacts was 400–500 μ. The sample thickness was determined to within 20% from the resistance at room temperature, where the size effect could be neglected. Then, in the case of filamentary whiskers $d = \sqrt{S}$ (S is the cross-section area of the whisker), while for plate whiskers $d = S/b$, where b is the plate's width[2)], which is determined under a microscope. The thicknesses of the samples ranged from 0.5 to 10 μ, i.e., the strong size effect condition, $l^{\infty} \gg d$, was fulfilled. Under this condition the resistance of the samples obeys the following dependences[18]: for filaments

$$\rho^d = \rho^\infty + \frac{1-P}{1+P}\frac{l^\infty\rho^\infty}{d} \qquad (1a)$$

and for plates

$$\rho^d = \rho^\infty + \frac{4}{3}\frac{1-P}{1+P}\frac{l^\infty\rho^\infty}{d}\frac{1}{\lg l^\infty/d}. \qquad (1b)$$

It has been previously[19] shown that the specularity coefficient for zinc whiskers is roughly equal to 0.6 for filaments and 0.75 for plates. Since the growth conditions and the purity of the initial metals in our case were the same as in[19], we selected for the measurements only those samples whose resistance satisfied the dependences (1) with P values equal to 0.6 and 0.75, and with a mean value of the product $\rho^\infty l^\infty = 1.8 \times 10^{-11}\,\Omega \times \mathrm{cm}^2$[20] 3). In this case we might hope that the samples did not contain perceptible quantities of defects that changed the value of the resistance.

The orientation of the whiskers could be judged on the basis of the indirect data on the anisotropy of the resistance in a magnetic field by comparing this anisotropy with the anisotropy for thick samples (see below).

As a magnetic-field source, we used two separate superconducting solenoids. One of them had an axial cross section 75 × 75 mm and an operating channel of diameter 26 mm. Its critical field was 60 kOe, and the remanent field at the center of the solenoid was about 50 Oe. The second solenoid, which was designed for the production of weak magnetic fields of up to 3 kOe, was a two-ply solenoid of length 30 cm and diameter 28 mm. Its remanent field at the center did not exceed 1 Oe.

The potential difference across the contacts of the samples was measured with the aid of the F-18 instrument connected to an automatic x-y recorder. All the measurement curves were obtained at a temperature of 4.2°K by automatic recording.

III. RESULTS OF THE MEASUREMENTS

1. Orientation of the Sample Axes

As is well known, the axes of whisker samples of zinc are grown along the crystallographic directions $[\bar{1}2\bar{1}0]$, $[\bar{1}2\bar{1}3]$, $[\bar{1}2\bar{1}2]$, and $[\bar{1}2\bar{1}1]$[21,22] (for the orientation $[\bar{1}2\bar{1}0]$, the samples have, as a rule, the form of plates). We tried accordingly to detect four types of anisotropy in the resistance $\rho^d_H(\theta)$ of whiskers in a constant magnetic field (θ is the angle of rotation of the field H about the axis of the whisker). The measurements were performed in a field ~50 kOe, which satisfied the strong-field conditions $r \ll d$ for practically all the samples. It turned out that the dependences $\rho^d_H(\theta)$ can in fact be split into four types (Fig. 2). Of the four, type IV occurs extremely rarely (only twice in our experiments). Comparison with the dependences $\rho^\infty_H(\theta)$ for thick samples4) whose axes were directed approximately along the four crystallographic directions indicated above allows us to make a judgement about the orientation of the axes of a whisker from the type of dependence $\rho^d_H(\theta)$.

It became clear in the course of the measurements that the results for the samples with the orientations $[\bar{1}2\bar{1}3]$ and $[\bar{1}2\bar{1}2]$ (types II and III) do not have qualitative and practically distinguishable quantitative differences5), and, therefore, the results we shall present below will be largely for the $[\bar{1}2\bar{1}3]$ (filament) and $[\bar{1}2\bar{1}0]$ (plate) samples. The measurements amounted to obtaining the dependences $\rho^d(H)$ for the longitudinal ($\mathbf{H} \parallel \mathbf{J}$) and transverse ($\mathbf{H} \perp \mathbf{J}$) orientations of the field H relative to the measuring current J. In the latter case the directions of the magnetic field for which the dependences $\rho^d(H)$ were most thoroughly studied are marked by the numbers 1–3 in Fig. 2.

2. Longitudinal Magnetic Field

Only those mounted samples which were not bent and appeared straight under a microscope were selected for the measurements. As the measurements showed, a small bend of the sample (a depth of curvature of a few microns in the length of the sample) leads to an appreciable distortion of the results, especially in the region of strong fields.

To find in the magnetic field the position of the sample that corresponded to the condition $\mathbf{J} \parallel \mathbf{H}$, the sample holder was mounted with the sample on a two-coordinate rotatable device. The $\mathbf{J} \parallel \mathbf{H}$ position was fixed by the minimum value of the resistance in the static field.

Figure 3 shows typical dependences of the resistance

$$\Delta\rho^d(H)/\rho^d(0) = (\rho^d(H) - \rho^d(0))/\rho^d(0)$$

on the magnitude of the magnetic field for filaments and plates. Their behavior can be described in the following manner: starting from the smallest values of the mag-

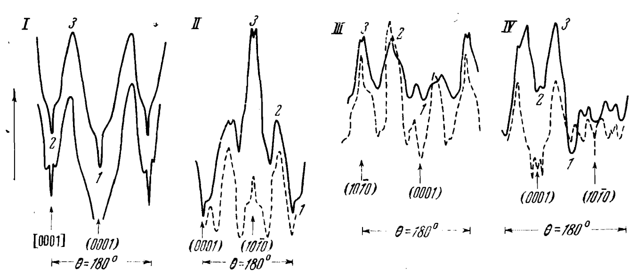

FIG. 2. Anisotropy of the resistance of Zn whiskers in a field H = 50 kOe: the continuous curves are for whiskers and the broken curves are for thick samples. Type I (plates), the upper curve is for the case when the axis of the sample was perpendicular to **H**; the lower curve, when the axis was displaced by 5° in the (H, J) plane and H was perpendicular to the surface of the plate. The characteristic change at the point 2 indicates that the orientation of the axis of the sample was $[\bar{1}2\bar{1}0]$ [23]. Type II: the orientation of the thick sample was $[\bar{1}2\bar{1}3]$. Type III: the orientation of the thick sample was $[\bar{1}2\bar{1}2]$. Type IV: the orientation of the thick sample was $[\bar{1}2\bar{1}1]$.

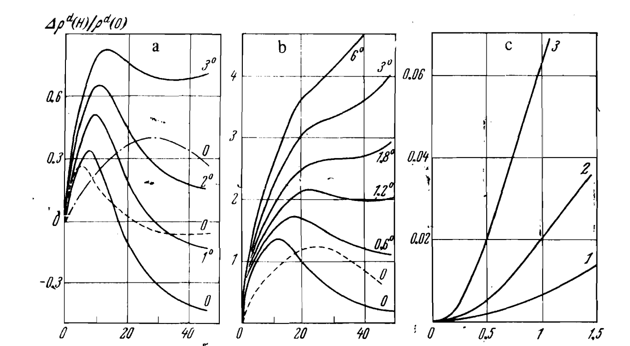

FIG. 3. Dependence of resistance on longitudinal magnetic field. a—Filaments: d = 7.5μ for the broken curve, d = 5.5μ for the continuous curves, d = 1.6μ for the dot-dash curve. b—Plates: d = 1.6μ, b = 120μ for the continuous curves; d = 0.5μ, b = 45μ for the broken curve. (The values of the angle between the axis of the sample and the direction of the magnetic field are indicated on the curves in Figs. a and b). c—Filaments: 1—d = 1.4μ, 2—d = 3.3μ, 3—d = 7.5μ.

netic field, the resistance increases according to the law $\rho^d(H) \propto H^{1.5}$ (notice that the same law is also observed for thick zinc samples[24]). We studied the initial region of the magnetic field only qualitatively. It was established that the thicker the sample the smaller the value of H bounding this region (Fig. 3c). For samples whose thickness $\sim 1\ \mu$, this region is bounded by a field ~1000 Oe for filaments and 400 Oe for plates.

At larger values of the magnetic field the resistance satisfies the dependence $\rho^d(H) \propto H^n$, where the exponent $n = 0.65 \pm 0.05$ for plates and $n \approx 1$ for filaments. In the field $H = H_{max}$ the resistance attains its maximum value ρ^d_{max}, after which it decreases, tending in fields $H > 5H_{max}$ to a constant value equal in order of magnitude to the resistance of a whisker in the absence of a field. In this region of fields, the results are not completely reproducible from sample to sample. We link this circumstance to the strong dependence of the resistance on the angle between the current and the field (see Fig. 3). A small camber in the samples therefore always distorts the true behavior of $\rho^d(H)$ for the longitudinal field. The position of the maximum is less sensitive to the angle between the field and the axis of the sample.

Figure 4 shows the dependence of the characteristic quantities H_{max} and $\Delta\rho^d_{max} = \rho^d(H_{max}) - \rho^d(0)$ on the inverse sample thickness. Within the limits of the errors in the determination of the thickness, these dependences can be considered to be linear.

3. Transverse Magnetic Field

It has been established that for any direction of the magnetic field, and for both filaments and plates of different thicknesses, the dependences $\rho^d(H)$ have qualitatively the same behavior. For one and the same direction of the field, the $\rho^d(H)$ curves for samples of different thickness can always be made to coincide by a proportionate change of scale. A typical form of these dependences is shown in Fig. 5. They have the following distinctive features. In the initial region of the magnetic field, the resistance increases according to the law $\rho^d(H) \propto H^{1.5}$ (special measurements we carried out showed that this same law is characteristic of thick zinc samples in the initial region of the field). The law $\rho^d(H) \propto H^{1.5}$ is observed in roughly the same magnetic-field region as for the case of the longitudinal field. Then it is replaced by another dependence: $\rho^d(H) \propto H^{0.65} \pm 0.05$. We emphasize that in contrast to the case of the longitudinal field, this law is also characteristic of filaments. In a field $H > H_1$, the law $\rho^d(H) \propto H^{0.65\pm0.05}$ goes over into a linear dependence $\rho^d(H) \propto (H - H_1)$, which extends to the field H_2 (this region can be considered as a transition region). In fields $H > H_2$ the dependence $\rho^d(H)$ can be considered as consisting of two parts, one of which is linear and the other quadratic: $\rho^d(H) \propto B_1(H - H_2) + B_2(H - H_2)^2$ (B_1 and B_2 are constants).

The above-described behavior of the resistance is especially well noticeable in Fig. 5 for those directions of the magnetic field for which the angular dependences $\rho^d_H(\theta)$ have minima (the directions marked by the number 1). Notice also the characteristic detail of the $\rho^d(H)$ curves for the filaments (Fig. 5a). The curves for the directions of the maxima and minima of the dependence $\rho^d_H(\theta)$ (the points 1 and 3) intersect at some value $H = H_{inv}$. It can be seen from this that in the field $H = H_{inv}$ the anisotropy in the resistance changes sign (inversion).

Averaging over all the investigated samples shows that the characteristic fields H_2 and H_1 are related to within 20% by the relation $H_2 \approx 2.5H_1$. The field H_2 increases as the sample thickness is decreased. The product H_2d for the various samples remains roughly a constant quantity. As an illustration, we present in Fig. 6 the dependence of the field H_2 on the inverse

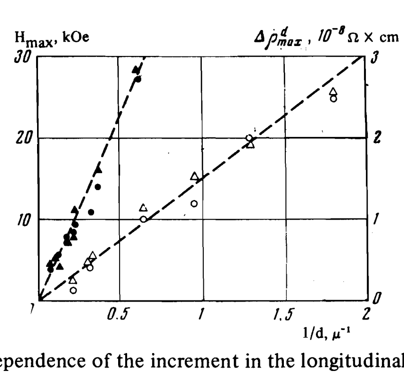

FIG. 4. Dependence of the increment in the longitudinal resistance at the peak of the $\rho^d(H)$ curve, and in the magnitude H_{max} of the field at this point, on the inverse thickness of the sample. Filaments: ▲ stands for $2\Delta\rho^d_{max}$; ●—H_{max}. Plates: △ stands for $\Delta\rho^d_{max}$; ○—H_{max}.

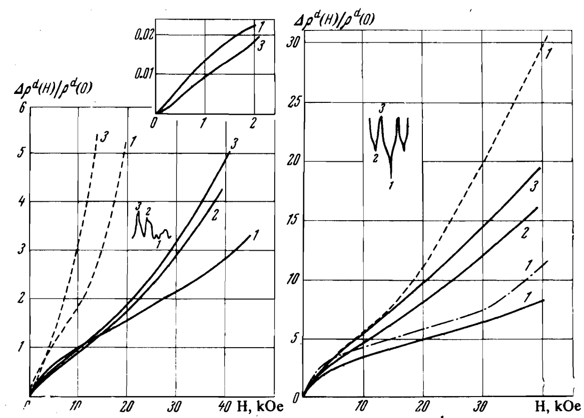

FIG. 5. Dependence of the resistance of samples on the transverse magnetic field. The numbers on the curves correspond to the peaks in the $\rho^d_H(\theta)$ curve also shown here (see also Fig. 2). a—Filaments: $d = 1.6\mu$ for the continuous curves and $d = 5.8\mu$ for the broken curves; in the insert, a sample with $d = 3.3\mu$. b—Plates: $d = 0.5\mu$, $b = 45\mu$ for the continuous curves; $d = 1.6\mu$, $b = 120\mu$ for the dot-dash curve; $d = 2.7\mu$, $b = 220\mu$ for the broken curve.

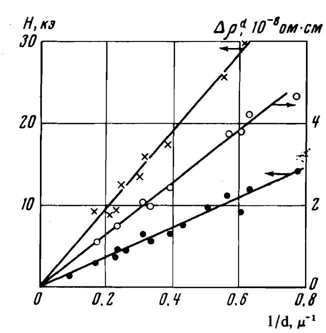

FIG. 6

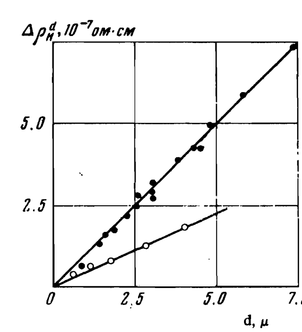

FIG. 7

FIG. 6. Dependence on the inverse thickness: ×—the values of H_2, ●—H_{inv}, ○—$\Delta\rho^d = \rho^d(H_{inv}) - \rho^d(0)$.

FIG. 7. Dependence of the increment in the resistance $\Delta\rho^d = \rho^d(H) - \rho^d(0)$ in a field H = 40 kOe on sample thickness: ●—filaments, ○—plates.

thickness of the filaments for the direction $H \parallel [001]$. The same figure shows the dependences $H_{inv}(1/d)$ and $\Delta\rho^d_{inv}(1/d)$ ($\Delta\rho^d_{inv}(1/d) = \rho^d(H_{inv}) - \rho^d(0)$ is the increment in the resistance at the point $H = H_{inv}$). These dependences are nearly linear.

To establish the dependence of the resistance on sample thickness for a fixed value of the magnetic field, we selected the curves for the directions corresponding to the maxima of the resistance (the point 3 on the $\rho^d_H(\theta)$ curves), since only in this case could we choose the fixed value of the field to be much larger than H_2. This allowed us to carry out measurements far from the various transition regions of the dependence $\rho^d(H)$. The values of $\Delta\rho^d(H = \text{const})$ were chosen for the field $H = 40$ kOe, which is roughly three times larger than H_2 for microns-thick samples. It can be seen in Fig. 7 that for both filaments and plates the resistance in a strong magnetic field is proportional to the sample thickness.

4. Generalization of the Results

The obtained dependences of the resistance on magnetic field for samples of different thicknesses are similar to each other in both the longitudinal and transverse magnetic field cases. Therefore, these dependences can be represented in the form of generalized curves in the coordinates Hd and $\Delta\rho^d d$ (or in the coordinates Hd and $\Delta\rho^d(H)/\rho^d(0)$, since for the resistance, it is convenient to introduce instead of d the factor $1/\rho^d(0)$, which is proportional to d[6]). Such curves for filaments and plates in a longitudinal field (Fig. 8)

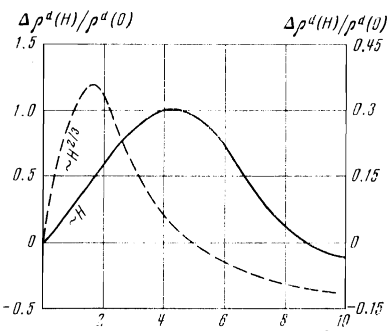

FIG. 8. Generalized curves for the dependence of the resistance on longitudinal magnetic field: the broken curve is for plates (scale on the left) and the continuous curve is for filaments (scale on the right).

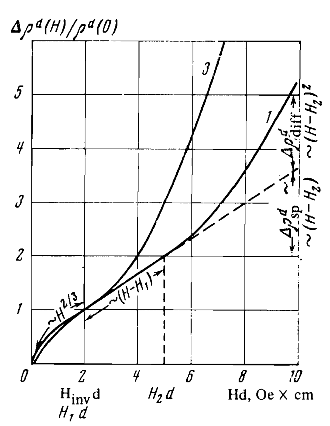

FIG. 9. Generalized curves for the dependence of the resistance of filaments on the transverse magnetic field. The quantities H_1 and H_2 pertain to the direction 1 (see Fig. 2).

and for filaments in a transverse field (Fig. 9) were constructed on the basis of the averaged values of the products of sample thicknesses and the characteristic fields (H_1d, H_2d, etc.) and the averaged values of the ratios $\xi = \Delta\rho^d(H)/\rho^d(0)$ at the characteristic points of the field ($\xi(H_1)$, $\xi(H_2)$, etc.). The generalized curves agree with the experimental curves to within 20%. An exception is the region $H > H_{max}$ for the longitudinal field, where the agreement is worse.

The good reproducibility of the results in the transverse field allows an analytical description of the universal curve in the various magnetic-field regions (with the exception of the insignificant initial region, which is not considered below):

$$\begin{aligned} \xi(H) &= A_1(Hd)^{2/3}, \quad H < H_1, \\ \xi(H) &= \xi(H_1) + A_2(H - H_1)d, \quad H_1 < H < H_2, \\ \xi(H) &= \xi(H_2) + A_2(H - H_2)d + A_3(H - H_2)^2 d^2, \quad H > H_2; \end{aligned} \tag{2}$$

the constants $A_{1,2,3}$ depend only on the direction of the magnetic field. The expression (2) is applicable for both filaments and plates.

IV. DISCUSSION OF RESULTS

Let us compare the experimental results with the results of the theoretical papers[10-12]. Unfortunately, the comparison is complicated by the fact that for zinc, which has a complicated Fermi surface, it is not possible to use a single radius for the electron orbits in the magnetic field. However, we can, on the basis of the volume and the dimensions of the Fermi surface of zinc, introduce for a qualitative comparison some mean radius r_m, for which the product r_mH is equal to a few units of Oe-cm. Below, by r we shall mean the quantity r_m (for the alkali metals, for example, $rH \approx 8$ Oe-cm). Then, the fields $H \ll H_{max}$ and H_2 should be considered as weak fields, i.e., $d \ll r$. Comparing the curves shown in Figs. 1a and 8, we can see that for the longitudinal field in the region $r > d$, there is no agreement between theory and experiment: instead of the expected decrease of the resistance in a wide range of magnetic fields (for plates, beginning from fields with $r < \sqrt{ld}$), a significant growth was observed.

We can arrive at similar conclusions for filaments in the case of the transverse field (Figs. 1b and 9). For plates in fields with $r > d$, the theory predicts a gradual increase of the resistance, $\rho^d(H) \propto A + B/ln(H_0/H)$ (Fig. 1b), which also does not agree with the experimentally observable strong dependence $\rho^d(H) \propto H^{0.65}$ (Fig. 9).

The reason for such a disagreement between theory and experiment lies, in our opinion, in the underestimation of the role of the nature of electron reflection from the surface. Evidently, the presence of electrons undergoing specular reflection from the surface substantially changes the field-dependence picture for the resistance in the diffuse-scattering case even in the region of weak fields where $r > d$. There do not exist at present theoretical papers in which this circumstance could have been taken into account.

At the same time the physical ideas which form the basis of the existing theory of the static skin effect allow a satisfactory explanation of the results obtained in the region of strong transverse magnetic fields where $r < d$. For the strong longitudinal field, the question remains open. It is not clear what causes such a sharp quantitative discrepancy in the value of the limit to which the resistance tends in high fields (the resistance tends to a value close to $\rho^d(0)$, and not to $\rho^\infty(\infty)$, as the theory predicts; a similar result has been obtained in [7]. It seems to us that to explain this only by the inexact fulfilment of the condition $J \parallel H$ is not possible. Evidently, for thin samples ($d \ll l^\infty$), and in strong longitudinal magnetic fields, scattering at the surface is decisive.

Below, we shall discuss only the results for the transverse field. To begin with, let us discuss two experimental facts which are vital for the physical interpretation of the results obtained.

1) The similarity law which has been established for the dependences $\rho^d(H)$, and in which the sample thickness d plays the role of a similarity coefficient, confirms the viewpoint that when $l^d \gg d$ the dominant role in electron scattering in a magnetic field is played by the surface of the sample. Then, as for the bulk resistance, we can, in analogy with the Köhler rule, write for the surface resistance in a magnetic field

$$[\rho^s(H) - \rho^s(0)] / \rho^s(0) = F(H / \rho^s(0)),$$

where the universal function F is given by the expression (2).

2) Zinc whiskers have a large specularity coefficient at T = 4.2°K. We can conclude from this that the probability of specular reflection is close to unity for up to large angles of impact on the surface.

The subsequent qualitative arguments are based on these two circumstances.

In the region of small values of the magnetic field ($r \gg d$) the electrons colliding with the surface can be divided into two groups. The electrons in the first group collide with and are specularly reflected (owing to the small impact angle) from any one surface, while those in the second group collide with two surfaces (in the main, diffuse scattering). The motion of the electrons of the first group in the "hopping" orbits is connected with the effective mean free path, which is equal to l^∞. However, the contribution of the electrons of this group to the total conductivity of the sample is small, since their relative number in weak fields is small[7)]. Therefore, the conductivity of a thin sample in weak fields is largely determined by the electrons of the second group, whose effective mean free path is in order of magnitude equal to the sample thickness (as in the magnetic-field free case). Thus, in a magnetic field in which $r \gg d$, the boundaries of the sample only lead to a limitation of the mean free path l^∞, while in the remaining region of magnetic fields everything must proceed as in the bulk metal: $\rho^d(H) \propto (d/r)^n$, where n is a typical exponent for the bulk metal: n = 1.5.

As the magnetic field increases, the role of the electrons of the first group in the total conductivity increases, while that of the second group of electrons decreases. We ascribe this unusual law of increase of the resistance, $\rho^d(H) \propto H^{0.65} \approx H^{2/3}$, to the electrons moving in the "hopping" orbits. Fal'kovskiĭ's paper[26], in which it was shown that the density of quantum surface states varies in a magnetic field as $H^{2/3}$, attests to this.

When the maximum angle of impact on the surface for the electrons of the first group becomes large and attains a certain value α_0 in the magnetic field, the number of quantum surface levels attains its maximum value and subsequently remains constant. The value α_0 of the angle can be compared to the field H_1 starting from which the law $\rho^d(H) \propto H^{0.65}$ is replaced by a linear growth of the resistance. Starting from this moment, the "specular" electron conductivity is a constant quantity (the number and mean free path l^∞ of the electrons remain constant). At the same time the number of the "diffuse" electrons interacting with only one surface of the sample begins to increase (at the expense of the electrons of the second group); when this happens, both the number and the mean free path of the electrons then vary. The transition region of the linear growth of the resistance in the field range from H_1 to H_2 should be linked to these processes.

In strong magnetic fields, when there is room in the sample for the electron orbits to be wholly located in it ($r < d$), the relative number of electrons colliding with the surface varies. It is proportional to the ratio 2r/d. This subsidiary factor decreases the conductivity in the field. We associate the value H_2 of the field with the moment when 2r = d. Taking, as before, only the electrons colliding with a surface into account, we can assume that for $r < d$ these electrons have two scattering channels—specular (with probability equal to P) and diffuse (with probability 1 − P). To the first of these channels corresponds a linear growth of the resistance, to the second a quadratic growth.

Then, we can write

$$\rho^d(H) \sim P\rho^\infty \frac{d}{2r} + (1-P)\rho^\infty \frac{l^\infty d}{2r^2}. \tag{3}$$

This expression describes qualitatively well the experimental results in the region $H \gg H_2$.

Unfortunately, a quantitative comparison is not possible, since the expression (3) is approximate in nature and is correct for $r \ll d$, whereas the experimental results were obtained in fields when r and d were comparable in order of magnitude. Nevertheless, we wish to draw attention to the astonishing formal similarity between the expressions (2) and (3) for fields $H > H_2$. To show this, let us write the expression (3) differently, using the equality 2r = k/H (k is a constant):

$$\rho^d(H) \sim P\frac{d}{2r} + (1-P)\frac{l^\infty d}{2r^2} = P\frac{Hd}{k} + (1-P)\frac{2H^2 l^\infty d}{k^2}. \tag{4}$$

Let us also transform the expression (2) for fields $H > H_2$:

$$\rho^d(H) \sim A_2(H-H_2)d + A_3(H-H_2)^2 d^2 = A_2'\frac{hd}{k} + A_3'\frac{2h^2 l^d d}{k^2}, \tag{5}$$

where $h = H - H_2$ and l^d is the effective mean free path of electrons in the field h = 0 ($2l^d = d$). The physical meaning of the replacement of the quantities H and l^∞ in the expression (4) by h and l^d in the expression (5) is not entirely clear. On the formal side the matter reduces simply to a change of origin for the curve (3). If we succeed in justifying such a change, then there will arise the alluring possibility of determining the specularity coefficient P from measurements on one sample under conditions when $l^\infty \gg d$. Indeed, it can be seen from the expression (5) that, in analogy with the expression (4), the ratio of the quadratic part of the resistance $\Delta\rho^d_{\text{diff}}$ to the linear part $\Delta\rho^d_{\text{sp}}$ should be equal to

$$\Delta\rho^d_{\text{diff}}/\Delta\rho^d_{\text{sp}} = (1-P)(H-H_2)/PH_2.$$

Having analyzed the experimental curves, we deduced for P the mean value of 0.5, which agrees with the specularity coefficient for the investigated samples.

Strictly speaking, the analysis carried out should pertain only to the case of a plate in a field parallel to the surface. The experimental curves are however uniform, and do not reveal a qualitative difference in the dependence on sample shape and field orientation relative to the surface, and this does not agree with the deductions of the theory. It is possible this is connected with the fact that the investigated filamentary and plate whiskers are not, from the theoretical point of view, fundamentally different, since filamentary whiskers have a specular facet, while the width of the plates was less than the electron mean free path. Further, the static skin effect should manifest itself in the resistance only in the case when $\rho^{\infty}(H) \propto H^2$. For zinc, this condition is not fulfilled for $H \parallel [0001]$ (the direction 2 for the whisker plates)[23]. Nevertheless, there are in this case also no qualitative differences between the $\rho^d(H)$ curves and the curves for the other directions of the field.

An important clarification of the physical picture of the observed behavior of the surface resistance in a magnetic field can be obtained by varying the specularity coefficient of the samples. Only in this case can we provide a complete picture and an unambiguous interpretation of the experimental results of the present paper.

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1)The deviation of n from two is explained at present by the so-called diffusive size effect [14].

2)By plate we mean a sample for which the ratio of width to thickness is not less than 10.

3)The quantity $\rho_\infty l^\infty$ should be chosen in accordance with the orientation of the whisker. However, allowance for this circumstance leads in our case to an insignificant change in the coefficient P. Thus, for filaments and plates, which have been most thoroughly investigated, the coefficient P should respectively be equal to 0.5 and 0.8.

4)Despite the fact that the resistance of thin samples is completely determined by scattering at the surface, all the characteristics of the dependence $\rho_H^\infty(\theta)$ for thick samples should also manifest themselves in the case of the strong size effect. This proposition has been theoretically validated in Peschanskiĭ's paper [12].

5)This is explained by the fact that the orientations of the $[\bar{1}2\bar{1}3]$ and $[\bar{1}2\bar{1}2]$ axes were very close. The angle between them was $\sim 10°$.

6)The logarithmic factor in the expression (1) for $\rho^d(0)$ of plates is unimportant, considering our experimental error (20%).

7)This is connected, in particular, with the fact that as a result of the quantization of the motion of the electrons in the "hopping" orbits [25], a finite number N of such orbits (quantum surface levels) can be located in a sample of finite dimensions; the number N then decreases with decreasing magnetic field intensity.

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