Fine structure of cyclotron-phonon resonance lines

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It is shown that under certain conditions the cyclotron-phonon resonance lines should have a serial structure, due to the existence of long-lived quasi-bound electron-phonon states.

INTRODUCTION

Because of the interaction of the conduction electrons with the optical phonons in a strong magnetic field, cyclotron-phonon resonance (CPR) is observed when the transition of an electron between two Landau levels n and n′ upon absorption of a light quantum is accompanied by the creation of an optical phonon. The absorption coefficient for such a process has been calculated by perturbation theory[1] and it has been shown that, because of the singularity in the density of states at the bottom of each Landau band, absorption peaks appear at the frequencies $\omega = E_n - E_{n'} + \omega_0$, where E_n is the location of the bottom of the Landau band of number n and ω_0 is the frequency of the optical phonon. Such peaks have been observed in InSb[2-5] and in the alloy $Hg_{1-x}Cd_xTe$.[6]

Perturbation theory[1,3] corresponds to the representation that in the initial state there is an electron in the Landau band n′, and in the final state there are an electron in the band n and a phonon, but the two do not interact. In addition, it was shown recently that the electron in the band n = 0 and the phonon can form a series of bound states.[7-9] If such states exist also at $n \neq 0$, then the "interaction in the final state" can lead to a fine structure of the CPR line, and this is the subject of the present paper.

We restrict ourselves to low temperatures and concentrations, when all electrons are located in the lowest band n′ = 0 near its bottom k = 0. Then the region of the energy spectrum near $\epsilon = E_n + \omega_0$ is responsible for the CPR (energy is measured from the bottom of the band n = 0). In just this region we can expect the appearance of bound states, because the principal role in the binding is played by the electron states near the bottom of the band.[8,9] On the other hand, since the momentum of the photon is negligibly small, the final states of the electron-phonon with a total longitudinal (along H) momentum $p = k + q_{\parallel} = 0$ become important (q is the momentum of the phonon). Such states form a continuum at $\epsilon > \omega_0$ for noninteracting particles. At $n \neq 0$ the spectrum region of interest to us begins with this continuum and therefore strictly stationary bound states cannot exist—they dissociate. This is illustrated in Fig. 1, where two Landau bands n = 0 and n = 1 are shown, as well as the "hypothetical" bound state below $E_1 + \omega_0$. Such a state decays with the release of a phonon; the electron is then in the band n = 0 with a momentum k_0 offset by the phonon momentum $q_{\parallel} = -k_0$.

The question of the reality of the bound state is answered by comparing the binding energy W and the decay width Γ. The value of W is determined fundamentally by the density of states at the bottom of the band, which differs little for various n. It is therefore natural to estimate W by using the results for n = 0[8,9]; this gives $W \sim \alpha^2\omega_0$ for $\omega_c \sim \omega_0$, where α is the dimensionless constant of the electron-phonon interaction and ω_c is the cyclotron frequency. The estimate of Γ is essentially different for $E_n > \omega_0$ and $E_n < \omega_0$. In the first case, the electron states at the bottom of the band n, which play an important role in the binding, are decaying with a width of the order of $\alpha\omega_0$, which also determines the width of the bound state $\Gamma \sim \alpha\omega_0$. In the second case, only electronic states lying above ω_0, i.e., far from the bottom of the band, decay with a width of the order of $\alpha\omega_0$. The participation of these states in the binding is small (of the order of α^2) and therefore $\Gamma \sim \alpha^3\omega_0$. Thus the quasistationary bound states near $E_n + \omega_0$ exist only if $E_n < \omega_0$. In this case the structure of the CPR line should be observed. Such a structure has been calculated for n = 0,[15] when strictly stationary states exist; however, observation in the region $\omega = \omega_0$ is greatly complicated by lattice effects. One must therefore expect the observation of quasibound states for $n \neq 0$ in the CPR region to be experimentally simpler. To be sure, this calls for materials with a larger binding force than used in experiments on CPR. Thus, in n-InSb we have $\alpha \approx 0.02$, which gives $W \approx 10^{-2}$ meV at $H \approx 40$ kOe. This quantity lies beyond the limits of experimental possibilities. However, in n-CdTe, we can expect $W \approx 3$ meV at H = 180 kOe,[9] which is completely measurable even when the estimate is exaggerated by one order of magnitude.

It is curious to note that the quasistationary bound states near $E_n + \omega_0$ for $E_n < \omega_0$ possess a much smaller width than the polaron states with the same energy.

In the calculation of the absorption, we shall first assume that the Fermi energy ϵ_F and the temperature T are less than W and Γ; this allows us to use the diagram technique at T = 0, as in[10]. The effect of a finite temperature will then be discussed and it will be shown that the condition $T \ll W, \Gamma$ is not obligatory for observation of the CPR line structure.

1. ABSORPTION AT ZERO TEMPERATURE

The absorption coefficient κ can be represented by the polarization operator Π in the form

$$\varkappa^{\pm}(\omega) = -\mathrm{Im}\, 4\pi c\omega^{-1}\Pi^{\pm}(\omega), \tag{1.1}$$

where the plus and minus signs correspond to right and left polarizations of the light. According to[10],

$$\Pi^{\pm}(\omega) = N[P^{\pm}(0, 0, \omega) + P^{\mp}(0, 0, -\omega)], \tag{1.2}$$

where N is the concentration of electrons, and $-iP(\epsilon p, \omega)$ is a sum of diagrams determined in[10]. When $\omega \approx E_n + \omega_0$, the principal contribution to P is

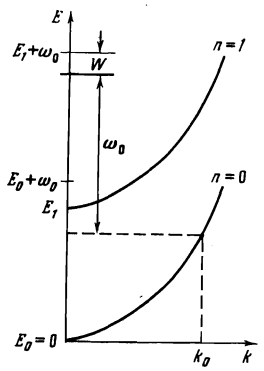

FIG. 1

made by diagrams with dangerous cuts through a single phonon line and a single electron line pertaining to the band n. Diagrams without dangerous cuts make a smoothly varying contribution to the absorption in the vicinity of the CPR. This contribution is identical with the contribution of the free electron, accurate to within α. This contribution will henceforth be disregarded.

The contribution from diagrams with a single dangerous cut can be represented in the form:[10]

$$P^{\pm}(\varepsilon p,\omega)^{\mathrm{I}}=\left(\frac{e}{mca}\right)^2\tilde{M}_n(\omega+\varepsilon)\frac{1}{[\omega_0+(n\mp1)\omega_c]^2}\frac{1}{n!}\int_0^\infty dt t^{n+1}e^{-t}\Phi(t). \tag{1.3}$$

Here $a=(c/eH)^{1/2}$ is the magnetic length, Φ the dimensionless form factor of the electron-phonon interaction,[9] $t=\frac{1}{2}(q_\perp a)^2$ and

$$\tilde{M}_n(\varepsilon)=\tilde{\alpha}\omega_0 v_0\int_{-\infty}^{\infty}\frac{dk}{2\pi}G_n(\varepsilon-\omega_0,k),$$
$$\tilde{\alpha}=\frac{1}{2}\alpha\frac{\omega_c}{\omega_0},\quad \frac{1}{2}mv_0^2=\omega_0, \tag{1.4}$$

where G is the Green's function of the electron.

The contribution from diagrams with more than one dangerous cut can be expressed in terms of the amplitude of the electron-phonon scattering Σ:[9,10]

$$P^{\pm}(\varepsilon p,\omega)^{\mathrm{II}}=\left(\frac{e}{mca}\right)^2[\tilde{M}_n(\varepsilon+\omega)]^2\frac{1}{[\omega_0+(n\mp1)\omega_c]^2}$$
$$\times\frac{1}{n!}\int_0^\infty dt\int_0^\infty dt'(tt')^{(n+1)/2}e^{-(t+t')/2}[\Phi(t)\Phi(t')]^{1/2}\Sigma_{-n\pm1}(\varepsilon+\omega,p;tt'). \tag{1.5}$$

Here the Fourier component Σ_l of the expansion of Σ in the difference of the azimuthal angles of the vectors $q_\perp$ and $q'_\perp$ enter in.[9] This Fourier component satisfies the integral equation

$$\Sigma_l(\varepsilon p;t,t')=\square_l(\varepsilon p;t,t')+\tilde{M}_n(\varepsilon)\int_0^\infty d\bar{t}\square_l(\varepsilon p;t,\bar{t})\Sigma_l(\varepsilon p;\bar{t},t') \tag{1.6}$$

with a kernel $\square$ that represents the sum of the diagrams entering into Σ and which has no dangerous cuts.

In the solution of this equation for the case n = 0[9] it was sufficient to calculate $\tilde{M}$ with the aid of the free G, and to use for $\square$ the two simplest diagrams. Such an approximation is inadequate for $n\neq0$. Computing M as in[11], we find

$$\tilde{M}_n(\varepsilon)=-i\tilde{\alpha}\omega_0^{1/2}[\varepsilon-(E_n+\omega_0+\Delta E_n)+i0]^{-1/2},$$
$$\operatorname{Im}\tilde{M}_n(\varepsilon)<0, \tag{1.7}$$

where ΔE_n is the polaron shift of the bottom E_n of the Landau band. If $E_n>\omega_0$, then ΔE_n contains $\operatorname{Im}\Delta E_n\sim\tilde{\alpha}\omega_0$. This smears out the singularity of $\tilde{M}$, so that $\tilde{M}$ is bounded by a small quantity of the order $\tilde{\alpha}^{1/2}\omega_0$. Therefore the integral term in (1.6) is always small and we can assume $\Sigma=\square$. Substituting this in (1.5), we can prove that $P^{II}\ll P^{I}$. On the other hand, it turns out that P^{I} gives an absorption calculated in second order of perturbation theory with account of the finite lifetime of the intermediate state. The shape of the CPR line is determined by $\tilde{M}_n(\omega)$ and does not reveal any structure.

If $E_n<\omega_0$, then $\operatorname{Im}\Delta E_n=0$ and the singularity of $\tilde{M}$ is not smeared out, but is only shifted by a small quantity of the order of $\tilde{\alpha}\omega_0$, which can be disregarded. In this case, the integral term in (1.6) is significant near the singularity. Proceeding as in[10], we can obtain

$$\Pi^{\pm}(\omega)=N\left(\frac{e}{mca}\right)^2\frac{\omega_0}{[\omega_0+(n\pm1)\omega_c]^2}\sum_{r=0}^{\infty}\frac{|d_r^{\pm}|^2}{-\lambda(\omega)^{-1}+(\lambda_r^{\pm})^{-1}} \tag{1.8}$$

where

$$\lambda(\omega)=-\frac{1}{\omega_0}\tilde{M}_n(\omega),$$
$$d_r^{\pm}=\frac{1}{(n!)^{1/2}}\int_0^\infty dt\,e^{-t/2}t^{(n+1)/2}[\Phi(t)]^{1/2}\chi_r^{\pm}(t), \tag{1.9}$$

and $\chi_r^{\pm}(t)$ and $\lambda_r^{\pm}$ are the eigenfunctions and eigenvalues of the dimensionless kernel

$$L^{\pm}(t,t')=-\omega_0\square_{-n\pm1}(E_n+\omega_0,0;t,t'). \tag{1.10}$$

If we limit ourselves to the simplest diagrams for $\square$, then the kernel L is real and all the λ_r are real. $\Pi(\omega)$ has singularities for $\lambda_r>0$.[10] Analyzing the spectrum of L for $n\neq0$ in a way similar to what was done in[8,9] for n = 0, one can show that even in the case $n\neq0$ there exists an infinite sequence of positive λ_r.

In such an approximation for $\square$, the structure of the energy spectrum close to $E_n+\omega_0$ with $n\neq0$ does not differ qualitatively from the spectrum of the spectrum close to $E_0+\omega_0$ studied in[8,9]. In the region below the threshold $E_n+\omega_0$ there exists an infinite sequence of stationary bound states with binding energies $W\sim\alpha^2\omega_0$ at $\omega_c\sim\omega_0$. Therefore in this approximation the structure of the absorption spectrum near the peak of CPR with $n\neq0$ does not differ qualitatively from the structure obtained in[10] for n = 0. This means that the fundamental CPR peak, due to the creation of a free phonon (with oscillator strength $f\sim\alpha$) ought to be accompanied from the long-wave side by a series of delta-shaped satellites, due to the creation of a bound phonon (with oscillator strength $f\sim\alpha^2$). To calculate the width of nonstationary bound states in reality, one should take into account in $\square$ diagrams of higher order $\delta\square$, shown in Fig. 2. One can take them into account by regarding the additional terms in the kernel of the integral equation (1.6) as a small perturbation. As a result, it was found that in (1.8) one should add the following correction to the eigenvalue $(\lambda_r^{\pm})^{-1}$:

$$\delta(\lambda_r^{\pm})^{-1}=\omega_0\int_0^\infty dt\int_0^\infty dt'\chi_r^{\pm}(t)\delta\square_{-n\pm1}(E_n+\omega_0,0;t,t')\chi_r^{\pm}(t'). \tag{1.11}$$

In the calculation of the satellite line width Γ, i.e., the actual widths of the bound states, $\operatorname{Im}\delta\square$ is important;

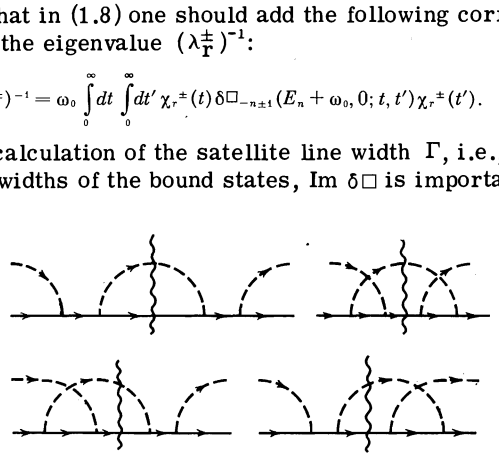
FIG. 2

in lowest order it is obtained from the diagrams of Fig. 2, by virtue of the indicated cuts in the phonon lines and the electron lines with Landau quantum number less than n. Explicit formulas for the widths are not of interest in the present stage of the experimental work; therefore, we shall show only that it is easy to obtain from (1.11) the estimate $\delta\lambda^{-1} \sim \alpha$ at $\omega_c \sim \omega_0$, whence it follows that $\Gamma \sim \alpha^3\omega_0$.

The CPR line structure is shown schematically in Fig. 3. It is of interest to note that bound states appear in the absorption with $l = -n \pm 1$. This equality can be written in the form of a "conservation law": $0 \pm 1 = n \pm l$, where the zero is the "quasimomentum" of the photon, n is the "quasimomentum" of the electron in the final state, and l is the "quasimomentum" of the phonon bound to the electron. For $n = 1$, we have $l = 0.2$. The states with $l = 0$ have the greatest binding energy; therefore CPR offers an additional convenience in finding the bound states.

2. ALLOWANCE FOR THE TEMPERATURE DISTRIBUTION IN THE INITIAL STATE

At a finite temperature the polarization operator can be calculated by means of the temperature Green's function and analytically continued to the real frequency axis. This procedure is easily carried out in general form if we can neglect the absorption of optical phonons and the population of all but the lowest Landau bands, i.e., at $\exp(-\omega_0/T) \ll 1$ and $\exp(-\omega_c/T) \ll 1$. We then find

$$\Pi^{\pm}(\omega) = N\int_{-\infty}^{\infty} dp\, f_T(p)\,[P^{\pm}(\varepsilon, p; \omega) + P^{\mp}(\varepsilon, p; -\omega)]_{\varepsilon=E_0(p)}, \qquad (2.1)$$

where f_T is the Boltzmann distribution (normalized to unity) in the lowest Landau band with a dispersion law $E_0(p)$. This formula is an obvious generalization of (1.2) with averaging over the initial state. The dependence of P on ϵ is determined by the dependence of Σ on ϵ, which enters through $\tilde{M}(\epsilon)$ near the singularity. On the other hand, the characteristic scale of the dependence of P on the p which enters explicitly is $p_0 = (2m\omega_0)^{1/2}$; at the same time, only $p \sim p_T = (2mT)^{1/2} \ll p_0$ is important in integration in (2.1). Therefore it is clear that the P which enters in (2.1) can be obtained from $P(0, 0; \omega)$ by the simple substitution $\lambda(\omega) \to \lambda(\omega + E_0(p))$. This allows us to use the results of the previous section and carry out the averaging over the equilibrium distribution.

If we neglect the width of the quasibound states Γ, then the averaging leads to the result that the delta-like peaks of the satellites are replaced by singularities of the type

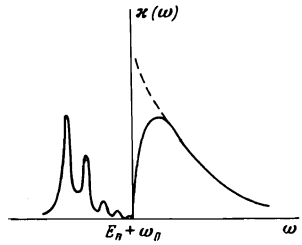

FIG. 3

$$\varkappa(\omega) \sim \exp\{-|\Delta\omega|/T\}\,|\Delta\omega|^{-1/2}, \quad \Delta\omega < 0,$$
$$\varkappa(\omega) \sim 0, \quad \Delta\omega > 0, \qquad (2.2)$$

where $\Delta\omega$ is the energy range of the quasibound state. Preservation of the singularity at finite temperatures indicates that the conditions $T \ll \omega_c, \omega_0$ are sufficient for the observation of the effect.

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