Induced two-photon synchrotron radiation and Compton scattering in a magnetic field

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Compton scattering of photons by electrons in a magnetic field and also stimulated two-photon synchrotron radiation when one of the photons is induced by an external electromagnetic wave are considered. The possibility of self-polarization of the electron spin as a result of Compton scattering in a magnetic field is demonstrated. The correction to the probability of synchrotron radiation due to absorption and emission of photons of the external electromagnetic wave by electrons is calculated. It is demonstrated that if this correction is taken into account in the case of low photon energies, the infrared divergence in a magnetic field is compensated in the second order in perturbation theory.

1. INTRODUCTION

In connection with the intensive use of synchrotron radiation in experiments[1], and also in view of the recently attained possibility of verifying the quantum theory of synchrotron radiation in experiments on scattering of high-energy electrons by a "magnetic target" with a strong magnetic field[2], interest in quantum processes that occur in an external field has increased. The simplest processes of first-order perturbation theory are spontaneous emission (for example, synchrotron radiation, the features of which are still being studied[3-5]) and single-photon production of electron-positron pairs[6]. With increasing electron energy and with increasing magnetic field intensity, second-order processes become significant in the presence of a strong electromagnetic wave. Recent investigations have been devoted to such second-order processes as Compton scattering and two-photon pair production in crossed fields ($\mathbf{E} \perp \mathbf{H}$, $|\mathbf{E}| = |\mathbf{H}|$)[7], and pair production by a particle in a magnetic field[8].

As is well known, the motion of a relativistic electron ($E \gg mc^2$) in an external constant and homogeneous magnetic field of not too large intensity ($H \ll H_0 = m^2c^3/e_0\hbar = 4.41 \times 10^{13}$ G) has a quasiclassical character. This circumstance makes it possible, on the one hand, to calculate approximately the matrix elements of the synchrotron radiation (see[3], also[9], p. 244), and on the other hand it permits the processes in the magnetic field to be described by means of probability formulas calculated for the case of a constant crossed field (see[10], and also[9], p. 468).

In the present paper we consider the solutions of the Dirac equation for an electron moving in a constant and homogeneous magnetic field and irradiated by an external electromagnetic wave. These solutions, expressed in the quasiclassical approximation, are used subsequently to calculate the matrix elements of photon emission by a relativistic electron in a given superposition of external fields. Expansion in terms of the intensity of the incident wave makes it possible to consider processes of second order in perturbation theory with two real photons, one of which is identical with the photons of the incident wave. The presence of an external magnetic field makes possible, besides Compton scattering, also another second-order process, namely induced two-photon synchrotron radiation, in which one of the emitted photons is induced by the incident wave. The probability of these processes is considered with allowance for the spin and polarization characteristics, and also as a function of the magnetic field and of the frequency of the incident wave. In the analysis of transitions with electron spin-flip, we demonstrate the possibility of self-polarization of the electrons as a result of Compton scattering in the magnetic field. At a low wave frequency, the maximum of the spectrum of the emitted photons overlaps the usual synchrotron radiation. It is shown that the resultant infrared divergence is offset by a correction to the synchrotron radiation; this correction is proportional to the intensity of the incident wave.

2. QUASICLASSICAL SOLUTION OF THE DIRAC EQUATION

We choose the coordinate system and the potential gauge A^μ, which specifies the constant magnetic field $\mathbf{H} \parallel z$ in the field of a plane electromagnetic wave propagating along the magnetic field, in the following manner (we use a system of units with $c = \hbar = 1$, a metric $(+---)$, $a^\mu = \{a^0, \mathbf{a}\}$,

$$= \{a^0, \mathbf{a}\},\ a_\mu = \{a^0, -\mathbf{a}\},\ \mu = 0, 1, 2, 3,\ a^1 = a_x,\ a^2 = a_y,\ a^3 = a_z):$$

$$A^\mu = A_H{}^\mu + A_W{}^\mu, \tag{1}$$

$$A_H{}^\mu = \{0, 0, xH, 0\}, \quad A_W{}^\mu = \{0, -A_0 \cos\omega\tau, A_0 g \sin\omega\tau, 0\},$$

where $g = \pm 1$ characterizes the direction of the circular polarization of the wave, while $\tau = t - z$, and $\omega\tau = k^\mu x_\mu$.

The solution of the Dirac equation for an electron moving in a given superposition of fields (1), is

$$\Psi_{\Omega p_y p_\perp}(\mathbf{r}, t) = N \exp[i(yp_y + s - \Omega\nu) + i\alpha\xi + \beta\partial/\partial\xi]$$
$$\times \left[\begin{pmatrix} C_1 \\ i\gamma_2 \\ C_3 \\ -i\gamma_2 \end{pmatrix} \varphi_1(\xi) + \begin{pmatrix} \gamma_1 \\ iC_2 \\ \gamma_1 \\ iC_4 \end{pmatrix} \varphi_2(\xi)\right], \tag{2}$$

where

$$\gamma_1 = -{}^1/_2 i\eta(C_2 + C_4)e^{ig\omega\tau}, \quad \gamma_2 = -{}^1/_2 i\eta(C_1 - C_3)e^{-ig\omega\tau},$$

C_i are spin coefficients that do not depend on τ, while the functions $\varphi_{1,2}$ are solutions of second-order equations for the electron in a magnetic field[10,12]

$$\left(\frac{1}{e_0H}\frac{d^2}{d\xi^2} + 2p_\perp\xi - e_0H\xi^2 \mp 1\right)\varphi_{1,2} = 0. \tag{3}$$

We have introduced here the variables

$$\nu = {}^1/_2(t + z), \quad \xi = x + (p_y + p_\perp)/e_0H \tag{4}$$

and the notation

$$\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \frac{\eta}{\omega}\begin{pmatrix} ge_0H\cos\omega\tau \\ \sin\omega\tau\end{pmatrix}, \quad \eta = \frac{e_0A_0}{\Omega + ge_0H/\omega},$$
$$s = -\frac{p_\perp{}^2 + \mathcal{M}^2}{2\Omega}\tau - \frac{p_\perp}{e_0H}\alpha, \quad \mathcal{M}^2 = m^2 + e_0A_0\Omega\eta. \tag{5}$$

For unique determination of the spin coefficients, C_i, we stipulate that the function (2) in the absence of an electromagnetic wave be the eigenfunction of a spin operator that is an integral of the motion in a homogeneous magnetic field[3]:

$$\hat{M} = \frac{e_0 H}{2m}\left(\sigma_z + \frac{1}{m}\rho_2[\sigma\hat{\mathbf{P}}]_z\right), \quad \hat{\mathbf{P}} = -i\nabla + e_0\mathbf{A}. \qquad (6)^*$$

In the case of an arbitrary constant electromagnetic field $F_{\mu\nu}$, the operator $\hat{M}$ can be expressed in invariant form either in terms of the tensor operator $M_{\mu\nu}$ or in terms of the spin vector operator S_μ (for the explicit form of the operators $M_{\mu\nu}$ and S_μ see[3]):

$$\hat{M} = \frac{e_0}{2m^2}F^{\mu\nu}M_{\mu\nu} = \frac{e_0}{2m^3}F^{*\mu\nu}\hat{p}_\mu S_\nu, \qquad F_{\mu\nu}^* = {}^1/_2 e_{\mu\nu\lambda\rho}F^{\lambda\rho}, \quad \hat{p}_\mu = -i\partial/\partial x^\mu. \qquad (7)$$

This enables us to write down subsequently the probability of the processes in invariant form, with allowance for the spin states, and in the case of mixed spin states the eigenvalues of the operator $\hat{M}$

$$M = \zeta(H/2H_0)\sqrt{m^2 + p_\perp^2} \quad (\zeta = \pm 1), \qquad (8)$$

should be replaced by the invariant

$$M' = \frac{e_0}{2m^2}F^{*\mu\nu}p_\mu a_\nu = \frac{1}{2}\frac{H}{H_0}\left(\zeta_z E - p_z\frac{(\zeta\mathbf{p})}{E+m}\right), \qquad (9)$$

where a^ν is the known polarization four-vector[9] and $\boldsymbol{\zeta}$ is double the average value of the spin vector in the rest system.

Thus, for the coefficients C_i corresponding to spin orientation along the magnetic field direction ($\zeta = 1$) and in the opposite direction ($\zeta = -1$) we have[3]

$$C_{1,3} = \frac{1}{2\sqrt{2}}\sqrt{1 + \zeta\frac{m}{E_\perp}}\left(\sqrt{1 + \frac{p_z}{E}} \pm \zeta\sqrt{1 - \frac{p_z}{E}}\right),$$

$$C_{2,4} = \frac{-\zeta}{2\sqrt{2}}\sqrt{1 - \zeta\frac{m}{E_\perp}}\left(\zeta\sqrt{1 - \frac{p_z}{E}} \mp \sqrt{1 + \frac{p_z}{E}}\right); \qquad (10)$$

$$C_1^2 + C_2^2 + C_3^2 + C_4^2 = 1, \quad E = (E_\perp^2 + \Omega^2)/2\Omega,$$

$$p_z = (E_\perp^2 - \Omega^2)/2\Omega, \quad E_\perp = \sqrt{m^2 + p_\perp^2}.$$

With the aid of (2) and (10) we easily obtain for the mean energy of an electron moving in a constant homogeneous magnetic field and in the field of a plane electromagnetic wave

$$\langle\hat{E}\rangle = \langle\hat{p}_z\rangle + \Omega = E + \frac{\eta^2}{2}\left[\Omega + 2\zeta g\omega\frac{m}{E_\perp}\left(1 - \frac{p_z}{E}\right)\right]. \qquad (11)$$

It is known that a relativistic electron moving in a constant and homogeneous magnetic field radiates only on a very small section of its quasiclassical orbit (see, for example,[9]). It is therefore expedient to use from now on the following approximation of the solutions of (3):

$$\varphi_{1,2}(\xi) = \Phi[-\lambda(\xi \mp 1/2p_\perp)], \quad \lambda = (2e_0 H p_\perp)^{1/3}, \qquad (12)$$

where Φ is the Airy function

$$\Phi(x) = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\infty} d\psi \exp\{ix\psi + i\psi^3/3\}. \qquad (13)$$

This approximation is valid in the region $\xi^2 \ll R^2$ ($R = p_\perp/e_0H$ is the radius of the quasiclassical orbit of the electron) if the condition $p_\perp \gg \sqrt{e_0 H}$ is satisfied. Taking (12) into account, we obtain the value of the normalization coefficient

$$N = 2(\lambda/L_0 L_y L_z(2 + \eta^2))^{1/2} \qquad (14)$$

by normalizing to unity in the interval $-\infty < x < \infty$, $-L_y/2 \le y \le L_y/2$, $-L_z/2 \le z \le L_z/2$, where we have introduced the finite "volume" L_0 of the variable ψ in the integral representation of the Airy function (13).

3. PHOTON EMISSION PROBABILITY

The total probability of photon emission by an electron in a nonstationary external field (1) is given by

$$\mathscr{P} = \frac{1}{(2\pi)^3}\sum_{n'}\int\frac{d^3l}{2l_0}|M_{n'n}|^2. \qquad (15)$$

Here

$$M_{n'n} = e_0\sqrt{4\pi}\int d^4x \exp(il^\mu x_\mu)\Psi_{n'}^+(e^\nu\alpha_\nu)\Psi_n, \quad \alpha^\nu = \{I, \alpha\}, \qquad (16)$$

Ψ_n and $\Psi_{n'}$ are the wave functions of the initial and final states of the electron in the external field (1) and e^μ are unit four-vectors of the linear polarization of the photon:

$$e^\mu(1) = \frac{F^{\mu\nu}l_\nu}{\sqrt{-(F^{\nu\rho}l_\rho)^2}}, \quad e^\mu(2) = \frac{F^{*\mu\nu}l_\nu}{\sqrt{-(F^{\nu\rho}l_\rho)^2}}. \qquad (17)$$

The covariant definition (17), using the constant external field tensor $F_{\mu\nu}$, makes it possible from now on to express the probabilities of the processes in an explicitly invariant form, with allowance for the polarization of the emitted photon. Following a gauge transformation of the vectors (17), we obtain the well-known three-dimensionally-transverse σ and π components of the polarization[3]:

$$e^\mu(1) = \{0, \mathbf{e}(\sigma)\}, \ \mathbf{e}(\sigma) = (\sin\varphi, -\cos\varphi, 0),$$
$$e^\mu(2) = \{0, \mathbf{e}(\pi)\}, \ \mathbf{e}(\pi) = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta), \qquad (18)$$

where θ and φ are the spherical angles of the photon momentum vector $\mathbf{l}$.

In the calculation of the matrix elements (16), integration with respect to y and ν yields the conservation laws

$$p_y - p_y' = l_y, \quad \Omega - \Omega' = l_0 - l_z, \qquad (19)$$

while integration with respect to x, with allowance for the quasiclassical approximation (12), leads to integrals of the type

$$\int_{-\infty}^{\infty} dx \exp(-iax)\Phi[-\lambda'(x + b_i')]\Phi[-\lambda(x + b_k)] = \frac{\sqrt{\pi}\exp(i\Omega_{ik})}{(\lambda^3 - \lambda'^3)^{1/3}}\Phi(z_{ik}), \qquad (20)$$

where

$$a = l_1 - \alpha + \alpha', \quad b_{1,2} = \frac{p_y + p_\perp}{e_0 H} \mp \frac{1}{2p_\perp} + \beta \quad (i, k = 1, 2),$$

$$z_{ik} = \frac{\lambda\lambda'}{(\lambda^3 - \lambda'^3)^{1/3}}\left(b_k - b_i' - \frac{a^2}{\lambda^3 - \lambda'^3}\right),$$

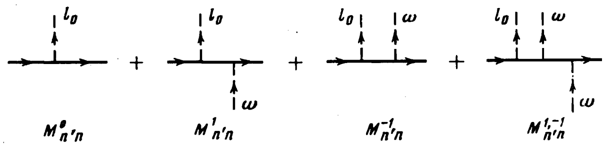
$$\Omega_{ik} = -\frac{a^3}{3}\frac{\lambda^3 + \lambda'^3}{(\lambda^3 - \lambda'^3)^2} + a\frac{\lambda^3 b_k - \lambda'^3 b_i'}{\lambda^3 - \lambda'^3}.$$

The presently attained laser intensities correspond to small values of the wave parameter $\gamma = e_0 A_0/m \lesssim 10^{-3}$, and for x-rays the values of γ are much smaller. Bearing in mind just this realistic case, we carry out in (16) an expansion in the small parameter $\gamma \ll 1$, corresponding to taking the incident wave into account by perturbation theory.

Then, after integrating with respect to the variable $\tau = t - z$, the matrix element $M_{n'n}$ can be represented in the form of the series

$$M_{n'n} = M_{n'n}^0 + \gamma M_{n'n}^1 + \gamma M_{n'n}^{-1} + \gamma^2 M_{n'n}^{1,-1} + \ldots, \qquad (21)$$

which is shown schematically in the figure (each diagram stands for a set of diagrams that differ in permutation of the vertices). The term $M_{n'n}^0$ corresponds to the conservation law $E' = E - l_0$, which is a characteristic of spontaneous single-proton emission (synchrotron radiation). The term $\gamma^2 M_{n'n}^{1,-1}$ describes the correc-



tion of order γ^2 to the synchrotron radiation, due to absorption and emission of an external-wave photon ω, with the same conservation law $E' = E - l_0$. The two other terms linear in γ correspond to the conservation laws $E - E' = l_0$ and $E - E = l_0 + \omega$. The first of these describes the Compton scattering of the photon ω by an electron in the magnetic field, and the second describes the emission of a photon l_0, accompanied by induced emission of a photon ω, i.e., "induced-two-photon synchrotron radiation."

We consider the case when the electron described by the quasiclassical function (12) moves in a magnetic field of not too large an intensity $H \ll H_0$ and has an ultrarelativistic transverse momentum $p_\perp \gg m$ (for simplicity we put $p_3 = 0$). Let the frequency of the incident electromagnetic wave lie in the range $e_0H/p_\perp \ll \omega \ll m$. Then the total probability of the considered processes will depend only on two invariant parameters:

$$\chi = \frac{H}{H_0}\frac{p_\perp}{m} = \frac{e_0}{m^3}\sqrt{-(F^{\mu\nu}P_\nu)^2},\quad \varkappa = \frac{2\omega\Omega}{m^2} = \frac{2}{m^2}k^\mu P_\mu, \tag{22}$$

and it follows from the conditions indicated above that

$$\chi \gg f,\quad \varkappa \gg f,\quad f = \frac{e_0}{m^2}\left(\frac{1}{2}F^{\mu\nu}F_{\mu\nu}\right)^{1/2} = \frac{H}{H_0} \ll 1. \tag{23}$$

Taking this into consideration and carrying out the corresponding expansions in the matrix elements, we find that the probability is maximal at

$$\left(\theta - \frac{\pi}{2}\right)^2 \sim \left(\varphi + \frac{\pi}{2}\right)^2 \sim \left(\frac{m}{p_\perp}\right)^2. \tag{24}$$

The main contribution to the formation of the Airy function in the right-hand side of (20) is made by the region near the points of the stationary phase in its integral representation. Therefore, owing to the independence of z_{ik} of φ, we obtain the relation $d\varphi = (2e_0H/\lambda^2)d\psi$. Thus, the integral with respect to the angle φ in the probability (15) turns out to be proportional to the volume L_0 of the phase ψ, which is cancelled by L_0 in the normalization (14). After integrating with respect to the angle θ, we obtain the spectral distribution with respect to the invariant variable $u = l_0/(p_\perp - l_0) = (\chi - \chi')/\chi'$ for the probability per unit time, which can be represented accurate to terms $\sim\gamma^2$ in the form

$$dw = dw_0 + dw_0' + dw_1 + dw_{-1}, \tag{25}$$

where dw_0 coincides exactly with the known result (see[3]) for the synchrotron-radiation probability, dw_0' is the correction of order γ^2 to dw_0 and is due to the term $2\mathrm{Re}(M^{0*}_{n'n}M^{1,-1}_{n'n})$ in the square of the matrix element (21), and dw_1 and dw_{-1} describe respectively the Compton scattering and the two-photon induced emission in the magnetic field

$$\begin{bmatrix} dw_\sigma \\ dw_\pi \end{bmatrix}_{1,-1} = \gamma^2 \frac{e_0^2}{8\sqrt{\pi}}\frac{m^2}{E}\left(\frac{\chi}{u}\right)^{1/3}\left(\frac{2u}{\varkappa}\right)^2 \frac{du}{(1+u)^3}\left\{\frac{1-\zeta\zeta'}{2}u^2 \right.$$
$$\left.\times\begin{bmatrix} F_4 \\ F_1 \mp F_2 + \zeta F_3\end{bmatrix} + \frac{1+\zeta\zeta'}{2}(2+u)^2 \begin{bmatrix} F_1 - \dfrac{u}{2+u}\left(\dfrac{u}{2+u}F_2 + \zeta F_3\right) \\ F_4 \end{bmatrix}\right\} \tag{26}$$

Here

$$\begin{aligned} F_1 &= \left(\frac{3\chi}{2u} \mp 2\frac{\chi}{\varkappa}\right)\Phi - \frac{1}{4}\left[1 + 3\left(\frac{2\chi}{\varkappa}\right)^2\right]\left(\frac{\chi}{u}\right)^{1/3}\Phi' \\ &\quad + \frac{1}{4}\left(\frac{u}{\chi}\right)^{1/3}\left[1 - \left(\frac{2\chi}{\varkappa}\right)^2 \mp \frac{\varkappa}{u} + \frac{1}{2}\left(\frac{\varkappa}{u}\right)^2\right]\Phi_1, \\ F_2 &= \pm 2\frac{\chi}{\varkappa}\Phi + \left(\frac{\chi}{u}\right)^{1/3}\Phi' + \frac{1}{2}\left(\frac{u}{\chi}\right)^{1/3}\left[1 \mp \frac{\varkappa}{u} - 4\left(\frac{\chi}{\varkappa}\right)^2\right]\Phi_1, \\ F_3 &= \left[1 \mp \frac{1}{2}\frac{\varkappa}{u} + 4\left(\frac{\chi}{\varkappa}\right)^2\right]\Phi \pm 4\frac{\chi}{\varkappa}\left(\frac{\chi}{u}\right)^{1/3}\Phi', \\ F_4 &= \frac{1}{2}\frac{\chi}{u}\Phi + \frac{1}{4}\left(\frac{\chi}{u}\right)^{1/3}\left[1 - 4\left(\frac{\chi}{\varkappa}\right)^2\right]\Phi' \\ &\quad + \frac{1}{4}\left(\frac{u}{\chi}\right)^{1/3}\left[1 \mp \frac{\varkappa}{u} - 4\left(\frac{\chi}{\varkappa}\right)^2 + \frac{1}{2}\left(\frac{\varkappa}{u}\right)^2\right]\Phi_1, \\ \Phi &= \Phi(y_{1,-1}),\quad \Phi_1 = \int_{y_{1,-1}}^\infty \Phi(x)\,dx,\quad y_{1,-1} = \left(\frac{u}{\chi}\right)^{2/3}\left(1 \mp \frac{\varkappa}{u}\right). \end{aligned} \tag{27}$$

The index 1 and the upper signs correspond to Compton scattering in the magnetic field, while the index -1 and the lower signs correspond to induced two-photon synchrotron radiation.

We have separated in formula (26) the terms that are connected with transitions without ($\zeta = \zeta'$) and with ($\zeta = -\zeta'$) spin flip, and have also indicated the σ and π components of the photon linear polarization. Expressions (26) and (27), after averaging and summing over the polarizations in the initial and final states of the electrons and photons, agree with the results of[7], in which a constant crossed field was considered in place of a magnetic field[1)].

4. LIMITING VALUES OF THE PARAMETER $\chi/\varkappa$ AND DISCUSSION OF THE RESULTS

Expressions (26) and (27) contain only two parameters, χ and κ. Their ratio χ/κ characterizes the relative influence of the external magnetic field and of the wave on the considered processes. Let us examine the limiting values of χ/κ.

1) $\chi/\kappa \ll 1$. In this region, the functions $\Phi(y_1)$ and $\Phi_1(y_1)$ oscillate at $y_1 < 0$ ($u < \kappa$), and attenuate rapidly (exponentially) at $y_1 > 0$ ($u > \kappa$). Therefore the spectral distribution of the probability of the Compton scattering differs from the known Klein-Nishina distribution (see[9]) in that oscillations are superimposed on it in the region $\kappa > u$. The probability of two-photon synchrotron radiation w_{-1} is in this case exponentially small throughout, and in the absence of the field ($H \to 0$) it vanishes ($w_{-1} \to 0$).

Integrating with respect to u, we obtain the Compton-scattering probability with allowance for the terms $\sim(\chi/\kappa)^2$ inclusive:

$$\begin{aligned} w_1 = \gamma^2\frac{e_0^2m^2}{8E}\Bigg\{&1 + \frac{16}{\varkappa} - \frac{1}{(1+\varkappa)^2} + 2\left(1 - \frac{4}{\varkappa} - \frac{8}{\varkappa^2}\right)\ln(1+\varkappa) \\ &- 4\zeta\frac{\chi}{\varkappa}\frac{\varkappa^2}{(1+\varkappa)^3} + \frac{32\varkappa}{3(1+\varkappa)^3}\left(\frac{\chi}{\varkappa}\right)^2\Bigg[1 + \frac{3\varkappa^2(2-\varkappa)}{4(1+\varkappa)} \\ &- \frac{\varkappa(4-\varkappa+\varkappa^2)}{8(1+\varkappa)^2} - 3\left(\frac{1+\varkappa}{\varkappa}\right)^3\left(\frac{\varkappa(2+\varkappa)}{1+\varkappa} - 2\ln(1+\varkappa)\right)\Bigg]\Bigg\}. \end{aligned} \tag{28}$$

The first term, which does not depend on χ, coincides with the known Klein-Nishina formula. The term linear in χ/κ vanishes after averaging over the initial orientation of the electrons spin ζ. We present now the probability of scattering with spin flip accurate to terms $\sim\chi/\kappa$:

$$w_1^{\zeta,-\zeta} = -\gamma^2 \frac{3e_0^2 m^2}{16E}\left[1 - \frac{4\varkappa+3}{3(1+\varkappa)^2} - \frac{2}{3}\ln(1+\varkappa) - \zeta\frac{4}{3}\frac{\chi}{\varkappa}\left(\frac{\varkappa}{1+\varkappa}\right)^3\right]. \quad (29)$$

We see that spin flip is more probable in the case of scattering in a state with $\zeta = 1(w_1^{1,-1} > w_1^{-1,1})$, i.e., with the initial spin orientation along the magnetic field. This conclusion is analogous to the corresponding result for transitions with spin flip in synchrotron radiation[3]. Thus, Compton scattering in a magnetic field, just like synchrotron radiation, leads to self-polarization of the electron spin.

2) $\chi/\kappa \gg 1$. The emission spectrum for both Compton scattering and two-photon synchrotron radiation is concentrated mainly in the region $u \sim \chi \gg \kappa$, and is therefore superimposed on the synchrotron radiation spectrum. In fact, on the basis of formula (26) we can easily show that at $\chi \gg \kappa$ we have, accurate to $(\chi/\kappa)^4$

$$dw_1 = dw_{-1} = 4\gamma^2\left(\frac{\chi}{\varkappa}\right)^4\left(\frac{u}{\chi}\right)^2 dw_0, \quad (30)$$

i.e., in this approximation the probabilities dw_1 of the Compton scattering and dw_{-1} of the two-photon emission coincide and turn out to be proportional to the probability of the synchrotron radiation dw_0.

It is easily seen that as $\kappa \to 0$, an infrared divergence appears in (30). Under these conditions, however, the really observable quantity is the summary probability (25). It can be shown that when all orders in χ/κ are taken into account in the expansion $dw = dw_0 + dw_0' + dw_1 + dw_{-1}$, the terms that diverge as $\kappa \to 0$ cancel each other, and this eliminates the infrared divergence. The summary probability then coincides with the synchrotron-radiation probability: $dw \approx dw_0$.

For photons with $\omega = 1.76$ eV of laser radiation propagating along a magnetic field of intensity $H = 4 \times 10^7$ G (the value planned in the experiments of[2,5]), we have $\chi/\kappa \approx 0.1$. When the wave propagates not along the field, but at a certain angle to it, the main conclusions of the present paper obviously remain in force. Therefore experimental observation of the influence of a superposition of a magnetic field and a wave field on the processes considered here is quite feasible.

In conclusion, the authors are deeply grateful to A. A. Sokolov for his constant interest in the work.

*$[\sigma P] = \sigma \times \hat{P}$.

1)There is a misprint in[7]: the number 3 in two places in the expression for v_3 (formula (13)) should be replaced by the number 2.

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