Decay of 2S-states of the hydrogen atom in a magnetic field

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The 2S-2P transitions induced in hydrogen atoms located in a magnetic field by collisions with ions are considered. The cross section of this process and the decay probability of the 2S state due to motion of the atoms in the magnetic field are calculated. It is shown that both the cross section and the probability for four values of the magnetic field strength corresponding to intersections of the hydrogen-atom terms with principal quantum number n = 2 have sharp peaks. This phenomenon can be used to obtain polarized beams of hydrogen atoms in the 2S state.

INTRODUCTION

As we know, the metastable 2S state of the hydrogen atom has an extraordinarily long (on the atomic scale) lifetime relative to transition to the 1s state ($\sim 1/7$ sec). If, therefore some external factor makes the 2S → 2P transition possible (the 2P state goes over to the 1S state in a time of 10^{-9} sec), then the breakdown of the 2S state takes place from just this process. Such a process might be, for example, collision of the hydrogen atom with an ion (the cross section of such a process is especially large). The cross sections of these transitions were calculated in the work of Dubovik, Satarov and the author.[1] It was shown that if the collision takes place in the presence of a magnetic field, then the cross section changes. It has sharp peaks for four values of the magnetic field strength, at which intersections of one of the 2P terms with one of the 2S terms take place. In the presence of a magnetic field, it is also necessary to take into account the fact that there is also a Lorentz electric field present in the motion of the atom in its coordinate system. This field changes the state of the atom so that it now has a small admixture of the 2P state. Therefore, it is capable of going over to the 1s state by emitting a single photon. The probabilities of such a transition also have peaks at the same values of the magnetic field strength. The aim of the present paper is to study of all these phenomena.

1. TRANSITION CROSS SECTIONS FOR COLLISIONS OF A HYDROGEN ATOM WITH AN ION

1. We place the coordinate origin at the center of the atom. The equation for the electron wave function has the form

$$\hat{\mathscr{H}}(t)\Psi(\mathbf{r},t)=i\frac{\partial}{\partial t}\Psi(\mathbf{r},t).$$

Here and in what follows, e = m = ħ = 1,

$$\hat{\mathscr{H}}(t)=\hat{\mathscr{H}}_0+\hat{V}(t),$$

$$\hat{\mathscr{H}}_0=-{}^1/{}_2\Delta-r^{-1}+\hat{\mathscr{H}}',\quad V(t)=|\mathbf{r}-\mathbf{R}(t)|^{-1},$$

R and r are the radius vectors of the passing particle and the electron, respectively, and $\hat{\mathscr{H}}'$ is the operator of the spin-orbital interaction, the Lamb shift, and the interaction of the atom with the magnetic field.

Expanding $\hat{V}(t)$ in powers of $1/R$, we keep only those terms which correspond to the dipole approximation:

$$\hat{V}(t)=\frac{\hat{\mathbf{d}}\mathbf{R}(t)}{R^3(t)}. \quad (1.1)$$

We shall assume the trajectory of the passing particle to be a straight line (i.e., the momentum transfer in the collision is small at impact parameters that make an important contribution to the cross section both due to interaction of the atom with this particle and also due to the action of the magnetic field on it. We introduce the two coordinate systems xyz and x′y′z′. Their origins are the same. In the second system of coordinates, the z′ axis is directed along the velocity of the incident particle, the x′ axis also lies in the scattering plane, and the y′ axis is perpendicular to it. In the first coordinate system, the z axis is directed along the magnetic field and the y axis is the line of intersection of the planes x, y and x′, y′.

We calculate the probability $W(v, \rho, \varphi, \vartheta)$ of transition of the atom to the 2p state. Here v is the velocity of the incident particle, ρ the impact parameter, φ the angle between the y and y′ axes, and ϑ the angle between the z and z′ axes. Then the cross section of this transition will obviously be equal to

$$\sigma(v,\vartheta)=\int_0^\infty\int_0^{2\pi}W(v;\rho;\varphi;\vartheta)\,d\varphi\,\rho\,d\rho. \quad (1.2)$$

We shall calculate the following cross sections:

$$\sigma_\parallel(v)=\sigma(v,0)$$

(the flux of the incident particles is parallel to the direction of the magnetic field),

$$\sigma_\perp(v)=\sigma(v,\pi/2)$$

(the flux is perpendicular to this direction) and

$$\sigma_{\mathrm{iso}}(v)=\frac{1}{2}\int_0^\pi\sigma(v,\vartheta)\,d\cos\vartheta$$

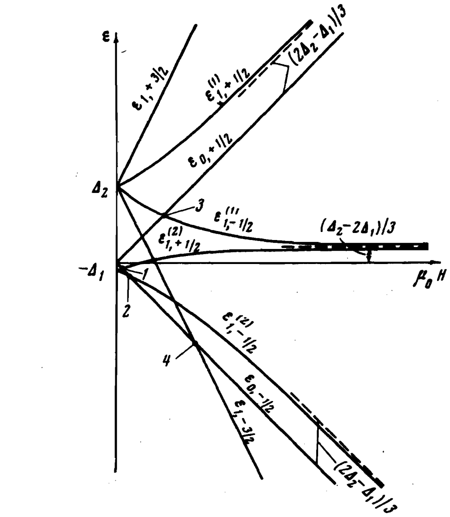
(the averaged cross section for isotropic distribution of the velocity vectors of the incident particles).

2. For further calculations, we need the states of the hydrogen atom with n = 2 in a magnetic field. We write down these expressions (for the derivation, see the book by Bethe and Salpeter,[2] p. 333, and also[7]):

$$\begin{gathered}|0,0,\pm{}^1/{}_2\rangle,\quad|1,\pm1,\pm{}^1/{}_2\rangle,\\ a_+|1,+1,-{}^1/{}_2\rangle+b_+|1,0,+{}^1/{}_2\rangle,\\ b_+|1,+1,-{}^1/{}_2\rangle-a_+|1,0,+{}^1/{}_2\rangle,\\ a_-|1,0,-{}^1/{}_2\rangle+b_-|1,-1,+{}^1/{}_2\rangle,\\ b_-|1,0,-{}^1/{}_2\rangle-a_-|1,-1,+{}^1/{}_2\rangle.\end{gathered} \quad (1.3)$$

Here $|l, m, s_z\rangle$ is the state of the hydrogen atom with orbital quantum number l, magnetic quantum number m, and spin projection S_z on the direction of the magnetic field.

The first two of these states are $2S_{1/2}$, the second, $2P_{3/2}$, with the projection of the total electron angular momentum in the direction of the magnetic field J_z



Terms of a hydrogen atom with n = 2 in a magnetic field.

$= \pm^3/_2$; the remaining four states have the projections of the angular momentum $J_Z = \pm^1/_2$, and the angular momentum itself is defined only as $H \to 0$ (H is the magnetic field strength). The coefficients $a_\pm$ and $b_\pm$ are equal to

$$a_\pm{}^2 = \frac{1}{2}\left(1 - \frac{\xi \pm {}^1/_3}{\Xi_\pm}\right), \quad b_\pm{}^2 = \frac{1}{2}\left(1 + \frac{\xi \pm {}^1/_3}{\Xi_\pm}\right); \tag{1.4}$$

$$\Xi_\pm = \sqrt{1 \pm \frac{2}{3}\xi + \xi^2},$$
$$\xi = \mu_0 H / (\Delta_1 + \Delta_2) = H / 8700 \text{ [G]}. \tag{1.5}$$

Here μ_0 is the Bohr magneton, Δ_1 the Lamb shift, and Δ_2 the separation of the levels $2S_{1/2}$ and $2P_{3/2}$ in the absence of a magnetic field. The energies of these states are equal to (they are measured from the $2S_{1/2}$ level in the absence of a magnetic field)

$$\varepsilon_{0,\pm^1/_2} = \pm\Delta_2(1+\delta)\xi; \quad \varepsilon_{1,\pm^3/_2} = \Delta_2[1 \pm 2(1+\delta)\xi];$$
$$\varepsilon^{(1,2)}_{1,+^1/_2} = {}^1/_2\Delta_2[1 - \delta + (1+\delta)(\xi \pm \Xi_+)], \tag{1.6}$$
$$\varepsilon^{(1,2)}_{1,-^1/_2} = {}^1/_2\Delta_2[1 - \delta + (1+\delta)(-\xi \pm \Xi_-)],$$

where

$$\delta = \Delta_1 / \Delta_2. \tag{1.7}$$

Here ϵ_{l,J_Z} is the energy of the state with orbital number l and projection of the total electron angular momentum in the direction of the magnetic field J_Z. The terms with a plus sign before the radicand are represented by $\epsilon^{(1)}_{1,\pm 1/2}$, and those with a minus sign by $\epsilon^{(2)}_{1,\pm 1/2}$. The path of the terms as a function of the magnetic field strength is shown in the drawing.

3. If we do not take into account the removal of the level degeneracy, it can be shown that the cross section diverges logarithmically. If

$$\rho \ll \rho_1 = v / \Delta, \tag{1.8}$$

where Δ is the largest of the separations $(\epsilon^{(i)}_{1,J_Z} - \epsilon_{0,J_Z})$; we do not have to take the removal of the level degeneracy into account. Given satisfaction of the condition

$$\Delta' \gg d / \rho^2 \quad (\rho \gg \rho_2 = \sqrt{d/\Delta'}), \tag{1.9}$$

where Δ' is the smallest of these separations, perturbation theory is valid.

The idea of calculation of the transition cross section consists in the calculation of the probabilities $W_1(v;\rho;\varphi;\vartheta)$ and $W_2(v;\rho;\varphi;\vartheta)$ under the assumptions (1.8) and (1.9), with subsequent matching of the obtained solutions in the region of impact parameters ρ_0, where these two conditions overlap:

$$\rho_2 \ll \rho_0 \ll \rho_1.$$

Here Eq. (1.2) takes the form

$$\sigma(v,\vartheta) = \sigma_1(v,\vartheta,\rho_0) + \sigma_2(v,\vartheta,\rho_0);$$
$$\sigma_1(v,\vartheta,\rho_0) = \int_0^{\rho_0}\int_0^{2\pi} W_1(v,\rho,\varphi,\vartheta)\,d\varphi\,\rho\,d\rho;$$
$$\sigma_2(v,\vartheta,\rho_0) = \int_{\rho_0}^{\infty}\int_0^{2\pi} W_2(v,\rho,\varphi,\vartheta)\,d\varphi\,\rho\,d\rho,$$

where $W_1(v,\rho,\varphi,\vartheta)$ is the transition probability without account of level splitting, calculated on the basis of the exact solution of the Schrodinger equation; $W_2(v,\rho,\varphi,\vartheta)$ is the transition probability calculated from perturbation theory with account of level splitting. We further introduce the notation

$$\sigma_{i\,\mathrm{iso}} = \frac{1}{2}\int_0^\pi \sigma_i(v,\vartheta,\rho_0)\,d\cos\vartheta,$$

where $i = 1, 2$.

The existence of an overlapping region requires the satisfaction of the inequalities

$$v \gg \sqrt{\Delta/m} \tag{1.10}$$

(in the usual units). This inequality will be considered more exactly in what follows.

The probability $W_1(v,\rho,\varphi,\vartheta)$ does not depend on the level splitting by the magnetic field. Therefore, it is obtained in the same way as it was calculated in[1]. The corresponding cross sections are

$$\sigma_{1\parallel}(v,\rho_0) = \sigma_{1\perp}(v,\rho_0) = \sigma_{1\,\mathrm{iso}}(v,\rho_0) = \frac{18\pi}{v^2}\Big[2\ln\frac{2\rho_0^2 v^2}{9}$$
$$\times(\pi^2-2)\,\mathrm{Ci}(\pi) - (\pi^2+1)\,\mathrm{Ci}(2\pi) - \ln 2\pi - 2 - C^*\Big],$$

where

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos t}{t}\,dt,$$

$C^* = 0.577$ is Euler's constant.

4. The transition proability $W_2(v,\rho,\varphi,\vartheta)$ is determined from perturbation theory with account of level splitting. After simple calculations, by integrating over the angles φ and ϑ and over the impact parameter ρ, we get

$$\sigma_{2k} = \frac{72\pi}{v^2}\ln\left(\frac{2v}{\gamma\rho_0\Delta_2}\frac{1}{F_k(H)}\right)$$

Here $\gamma = e^{0*}$, the index k takes on the "values" $\parallel$, $\perp$, and "iso," and F_k is determined by the formula

$$\ln F_k(H) = \alpha_k[(1+\mathscr{P})(\ln\delta^+_{+^3/_2} + b_-{}^2\ln|\delta^{+(1)}_{-^1/_2}| + a_-{}^2\ln|\delta^{+(2)}_{-^1/_2}|)$$
$$+(1-\mathscr{P})(\ln|\delta^-_{-^3/_2}| + a_+{}^2\ln\delta^{-(1)}_{+^1/_2} + b_+{}^2\ln|\delta^{-(2)}_{+^1/_2}|)]$$
$$+\beta_k[(1+\mathscr{P})(b_+{}^2\ln\delta^{+(1)}_{+^1/_2} + a_+{}^2\ln|\delta^{+(2)}_{+^1/_2}|)$$
$$+(1-\mathscr{P})(a_-{}^2\ln\delta^{-(1)}_{-^1/_2} + b_-{}^2\ln|\delta^{-(2)}_{-^1/_2}|)]. \tag{1.11}$$

The initial spin polarization of the hydrogen atoms in the 2S state along the direction of the magnetic field is denoted by $\mathscr{P}$, and

$$\delta^{\pm(i)}_{J_z} = \frac{1}{\Delta_2}(\varepsilon^{(i)}_{1,J_z} - \varepsilon_{0,\pm^1/_2}), \quad i = 1, 2.$$

The coefficients α_k and β_k are equal to

$$\alpha_{\parallel} = {}^1/_4,\ \alpha_{\perp} = {}^1/_8,\ \alpha_{\text{iso}} = {}^1/_6,$$
$$\beta_{\parallel} = 0,\ \beta_{\perp} = {}^1/_4,\ \beta_{\text{iso}} = {}^1/_6.$$

The desired quantities

$$\sigma_k(v) = \sigma_{1k}(v, \rho_0) + \sigma_{2k}(v, \rho_0)$$

are equal to (in the usual units)

$$\sigma_k(v) = 72\pi\left(\frac{\hbar}{mv}\right)^2\left(\ln\frac{mv^2}{\Delta_2 F_k(H)} - 1{,}53\right), \tag{1.12}$$

where m is the mass of the electron and v the velocity of the particle. The answer, is, as it should be, independent of ρ_0.

5. We write out the values of $\delta_{J_z}^{\pm(i)}$, using the formulas (1.6):

$$\delta^{\pm}_{\pm^3/_2} = 1 \pm (1+\delta)\xi,$$
$$\delta^{+(1,2)}_{+^1/_2} = {}^1/_2[1-\delta+(1+\delta)(-\xi\pm\Xi_+)],$$
$$\delta^{-(1,2)}_{-^1/_2} = {}^1/_2[1-\delta+(1+\delta)(\xi\pm\Xi_-)], \tag{1.13}$$
$$\delta^{-(1,2)}_{+^1/_2} = {}^1/_2[1-\delta+(1+\delta)(3\xi\pm\Xi_+)],$$
$$\delta^{+(1,2)}_{-^1/_2} = {}^1/_2[1-\delta+(1+\delta)(-3\xi\pm\Xi_-)].$$

The plus sign in front of Ξ corresponds to i = 1, the minus to i = 2. Here ξ is determined by Eq. (1.5), and δ by (1.7).

The values of $a_{\pm}^2$ and $b_{\pm}^2$ are determined by Eqs. (1.4). It follows from the condition of the positiveness of the cross section $\sigma_k(v)$ that

$$\ln(mv^2/\Delta_2 F_k(H)) > 1{,}53.$$

From this the meaning of the inequality (1.10) is seen more precisely:

$$v \gg [\Delta_2 F_k(H)/m]^{1/2}. \tag{1.10'}$$

6. We now obtain expressions for the cross sections $\sigma_k(v)$ for small and large magnetic fields. We shall keep three terms each in the power series expansions in ξ and $1/\xi$. For small fields ($\mu_0 H \ll \Delta_1 + \Delta_2$) we have

$$\sigma_k(v) = 72\pi\left(\frac{\hbar}{mv}\right)^2\left(\ln\frac{mv^2}{\Delta_2\delta^{1/2}} - 1.53 - C_{1k}{}^0\mathscr{P}\xi + C_{2k}{}^0\xi^2\right). \tag{1.14}$$

For high fields ($\mu_0 H \gg \Delta_1 + \Delta_2$), we get

$$\sigma_k(v) = 72\pi\left(\frac{\hbar}{mv}\right)^2\left(\ln\frac{mv^2}{\Delta_2 F_{\infty k}(H)} - 1.53 - \mathscr{P}\frac{C_{1k}{}^\infty}{\xi} + \frac{C_{2k}{}^\infty}{\xi^2}\right). \tag{1.15}$$

Here

$$F_{\infty\parallel}(H) = (1+\delta)\xi;$$
$$F_{\infty\perp}(H) = [{}^1/_3(1+\delta)(2-\delta)\xi]^{1/2},$$
$$F_{\infty\text{iso}}(H) = [{}^1/_3(1+\delta)^2(2-\delta)\xi^2]^{1/3},$$

and the coefficients C_{ik}^0 and C_{ik}^∞ are expressed in terms of δ.

The zero-order terms in (1.14) are identical with the results of[1] which was to have been expected. It is interesting that the linear terms of all the expressions depend linearly on the initial polarization of the hydrogen atoms, vanishing together with them, while the quadratic terms generally do not depend on the polarization. For small magnetic fields, moreover, this is obvious. Actually, the contributions to the term linear in the field from states with oppositely oriented spins mutually cancel one another, while the sum of the squares of the coefficients for these states is equal to unity.

For small magnetic fields, the condition (1.10′) yields ($\mu_0 H \ll \Delta_1 + \Delta_2$)

$$v \gg (\Delta_1\Delta_2^2/m^3)^{1/6} = 1.91\cdot10^5 \text{ cm/sec},$$

For high magnetic fields ($\mu_0 H \gg \Delta_1 + \Delta_2$)

$$v \gg (\mu_0 H/m)^{1/2} \gg 3\cdot10^5 \text{ cm/sec for } \sigma_{\parallel}(v);$$
$$v \gg [(2\Delta_2-\Delta_1)\mu_0 H/3m^2]^{1/4} \gg 2{,}5\cdot10^5 \text{ cm/sec for } \sigma_{\perp}(v).$$
$$v \gg [(2\Delta_2-\Delta_1)(\mu_0 H)^2/3m^3]^{1/6} \gg 3\cdot10^5 \text{ cm/sec for } \sigma_{\text{iso}}(v).$$

Using the known values Δ_1 = 1057.77 MHz, Δ_2 = 10968.6 MHz, whence δ = 0.0964 (see, for example, the work of Gatto[3]), we find the values of C_{ik}^0 and C_{ik}^∞ for all three cross sections. They are tabulated below:

C:	$C_{1k}{}^0$	$C_{2k}{}^0$	$C_{1k}{}^\infty$	$C_{2k}{}^\infty$
$\sigma_{\parallel}(v)$:	5,915	42.5	0.330	0,145
$\sigma_{\perp}(v)$:	4.62	27.7	0,353	$-(11.4+1.09\ln\xi)$
$\sigma_{\text{iso}}(v)$:	5.00	32.4	0,346	$-(5.58+0.415\ln\xi)$

2. BEHAVIOR OF THE CROSS SECTIONS NEAR THE INTERSECTION POINTS OF THE TERMS

1. Seven points of intersection of terms of the level with n = 2 are seen in the figure. However, three of them are not of interest to us, since dipole electric transitions between the terms intersecting at these points are lacking. Therefore the energy differences of these terms do not enter into any of the functions $F_k(H)$. Terms between which there are such transitions intersect at the four points marked on the drawing by numbers. The energy differences of these terms enter into the quantities $F_k(H)$, and the latter vanish at these points, while the cross sections have logarithmic poles.

It is easy to obtain the result that these intersections occur at the following values of the magnetic field strength:

$$H_1 = {}^1/_{12}[5\delta - 4 + (16 + 32\delta + 25\delta^2)^{1/2}]\Delta_2/\mu_0 = 572 \text{ G},$$
$$H_2 = \frac{3\delta}{2-\delta}\frac{\Delta_2}{\mu_0} = 1184 \text{ G},$$
$$H_3 = {}^1/_{12}[4 - 5\delta + (16 + 32\delta + 25\delta^2)^{1/2}]\Delta_2/\mu_0 = 5165 \text{ G},$$
$$H_4 = \Delta_2/\mu_0 = 7835 \text{ G}.$$

The quantity $\delta^{-(2)}_{+1/2}$ vanishes at point 1, the quantity $\delta^{(2)}_{-1/2}$ at point 2, $\delta^{+(1)}_{-1/2}$ at point 3, and $\delta^{-(1)}_{-1/2}$ at point 4.

2. It is easy to obtain the asymptotic form of the cross sections near the singular points from Eqs. (1.11)–(1.13). However, we shall at first rewrite them somewhat differently. Taking into account that in the reference system connected with the hydrogen atom, the kinetic energy of the incident particle $\mathscr{E} = Mv^2/2$, where M is its mass, we get (assuming that the incident particle is a hydrogen ion):

$$\sigma_k(\mathscr{E}) = \frac{A}{\mathscr{E}}\left(\ln\frac{\mathscr{E}}{\Delta_2 F_k(H)} - 8.57\right),$$

where $A = 1.58\times10^{-10}$ eV-cm^2.

It is evident that the functions $F_k(H)$ have the following form near singular points:

$$F_k(H) = c|\Delta H|^b,\ \Delta H = H - H_{\text{cr}}$$

(H_{cr} is the critical value of the magnetic field strength), and therefore the asymptotic form of the cross sections will be

$$\sigma_k(\mathscr{E}) = (A/\mathscr{E})[\ln\mathscr{E} + a - b\ln|\Delta H|].$$

Here a and b are constants which depend on what cross section is considered and near what point, and also on the initial polarization $\mathscr{P}$ of the hydrogen atom (the constant a also depends on the units in which the energy

Table I

H, G	$\sigma_{\parallel}(\mathscr{E})$		$\sigma_{\perp}(\mathscr{E})$		$\sigma_{iso}(\mathscr{E})$	
	a	b	a	b	a	b
$H_1=572$	$3{,}42-1{,}27\mathscr{P}$	$0{,}174(1-\mathscr{P})$	$3.10-0{,}61\mathscr{P}$	$0{,}087(1-\mathscr{P})$	$3{,}21-0{,}83\mathscr{P}$	$0{,}116(1-\mathscr{P})$
$H_2=1184$	$2.25-0.27\mathscr{P}$	0	$2.97-0.985\mathscr{P}$	$0.1(1-\mathscr{P})$	$2.76-0{,}71\mathscr{P}$	$0.0665(1-\mathscr{P})$
$H_3=5165$	$3.32+0.81\mathscr{P}$	$0.159(1+\mathscr{P})$	$2.785+0.315\mathscr{P}$	$0.08(1+\mathscr{P})$	$2{,}94+0.55\mathscr{P}$	$0.106(1+\mathscr{P})$
$H_4=7835$	$4.12-2.24\mathscr{P}$	$0.25\ (1-\mathscr{P})$	$2.91-1.27\mathscr{P}$	$0.125(1-\mathscr{P})$	$3{,}22-1{,}59\mathscr{P}$	$(1-\mathscr{P})/6$

of the incident particle $\mathscr{E}$ and the magnetic field strength H are measured). The values of a and b for all the singular points and the three cross sections considered by us are given in Table I (the values of a are given for $\mathscr{E}$ expressed in electron volts and H in gauss).

Attention is called to two circumstances in this table. First, the coefficient b is proportional to $1 - \mathscr{P}$ near the singular points 1, 2, 4, and to $1 + \mathscr{P}$ near point 3. This circumstance is explained by the fact that at singular points 1, 2, 4 one of the 2P terms intersects the 2S term which corresponds to the state with spin projection on the z axis equal to $s_z = -\frac{1}{2}$, while at point 3 the correspondence is with $+\frac{1}{2}$. Second, at each singular point, all the cross sections become infinite except the cross section $\sigma_{\parallel}(\mathscr{E})$ at singular point 2. The reason for this is as follows. At singular points 1, 3, 4, the intersecting terms have different projections of the total electron angular momentum onto the z axis, and those at point 2 have like projections. Therefore, as is easily seen from (1.1) and (1.3), transitions between terms which intersect at singular points 1, 3, 4 are brought about by an electric field perpendicular to the magnetic field, and those between terms intersecting at point 2 by a parallel electric field. But at the time of closest approach of the ion and hydrogen atom for the cross section $\sigma_{\parallel}(\mathscr{E})$ this field is perpendicular to the magnetic field, and for the cross sections $\sigma_{\perp}(\mathscr{E})$ and $\sigma_{iso}(\mathscr{E})$ it can be directed differently.

3. PROBABILITY OF THE DECAY OF THE 2S STATE ASSOCIATED WITH MOTION OF THE ATOMS

1. When an atom moves in a magnetic field, an electric field appears in the coordinate frame bound to it, with the intensity $\mathbf{E} = \mathbf{H} \times \mathbf{u}/c$ (u is the velocity of the atom). Under the action of this field, the 2S state decays (see[2], p. 451). The decay probability per unit time is equal to (for an electric field of arbitrary origin)

$$\dot{W} = \left(\frac{2}{3}\right)^6 \frac{e^8}{m\hbar^2c^3}\left(\frac{E}{\Delta_2}\right)^2 [2f_{\parallel}(H)\cos^2\gamma + f_{\perp}(H)\sin^2\gamma] \qquad (3.1)$$

(c is the velocity of light, e the charge on the electron). Here γ denotes the angle between the directions of the electric and magnetic fields, and $f_{\parallel}(H)$ and $f_{\perp}(H)$ are the functions

$$f_{\parallel}(H) = (1+\mathscr{P})[(b_+/\delta^{+(1)}_{+1/2})^2 + (a_+/\delta^{+(2)}_{+1/2})^2] + (1-\mathscr{P})[(a_-/\delta^{-(1)}_{-1/2})^2 + (b_-/\delta^{-(2)}_{-1/2})^2], \qquad (3.2)$$

$$f_{\perp}(H) = (1+\mathscr{P})[(1/\delta^{+}_{+3/2})^2 + (b_-/\delta^{+(1)}_{-1/2})^2 + (a_-/\delta^{+(2)}_{-1/2})^2]$$
$$+ (1-\mathscr{P})[(1/\delta^{-}_{-3/2})^2 + (a_+/\delta^{-(1)}_{+1/2})^2 + (b_+/\delta^{-(2)}_{+1/2})^2]. \qquad (3.2')$$

2. It is easy to obtain the asymptotic form of the decay probability per unit time near the singular points from Eqs. (3.1), (3.2), and (3.2′). We have:

$$\dot{W} = BE^2\left[\left(a_{\parallel} + \frac{b_{\parallel}}{(\Delta H)^2}\right)\cos^2\gamma + \left(a_{\perp} + \frac{b_{\perp}}{(\Delta H)^2}\right)\sin^2\gamma\right].$$

Just as in Sec. 2, $\Delta H = H - H_{cr}$, where H_{cr} is the critical value of the magnetic field strength near which the decay probability is observed. The quantities $a_{\parallel,\perp}$ and $b_{\parallel,\perp}$ depend on the singular point near which the probability is considered and on the initial polarization $\mathscr{P}$ of the hydrogen atom. The coefficient B is equal to

$$B = 4.11 \cdot 10^6 \text{ (CGSE intensity units)}^{-2}\ \text{sec}^{-1}.$$

The values of $a_{\parallel}$, $b_{\parallel}$, $a_{\perp}$ and $b_{\perp}$ for all the critical points are given below:

H, g	$H_1 = 572$	$H_2 = 1184$	$H_3 = 5165$	$H_4 = 7835$
$a_{\parallel}$:	$5.57-4.65\mathscr{P}$	$0.26+0.20\mathscr{P}$	$0.75-0.55\mathscr{P}$	$0.58-0.34\mathscr{P}$
$b_{\parallel}$, g²:	0	$2.545\cdot10^6(1-\mathscr{P})$	0	0
$a_{\perp}$:	$1{,}215+0{,}98\mathscr{P}$	$5{,}41-4{,}46\mathscr{P}$	$1.03+0{,}94\mathscr{P}$	$0.37+0.24\mathscr{P}$
$b_{\perp}$, g²:	$1.65\cdot10^6(1-\mathscr{P})$	0	$1.27\cdot10^6(1+\mathscr{P})$	$5.81\cdot10^6(1-\mathscr{P})$

Just as for the cross sections, the divergent term is proportional to $1 - \mathscr{P}$ near singular points 1, 2, 4 and to $1 + \mathscr{P}$ near point 3. For parallel electric and magnetic fields, the divergence is observed at singular point 2, while for perpendicular fields it occurs at points 1, 3, 4.

3. At the point of intersection of the terms (which, naturally, would take place only in the absence of an electric field), it is no longer possible to consider the action of the electric field as a small perturbation. For determination of the path of the terms near this point and the states corresponding to them, it is necessary to solve the secular equation (see[4], p. 332). It is easy to see that for a critical value of the magnetic field, the states of the atom corresponding to the terms of interest to us have the form

$$(\psi_{2S} \pm \psi_{2P})/\sqrt{2},$$

where ψ_{2S} and ψ_{2P} are the wave functions of the states corresponding to these terms at some distance from the critical point. Therefore the decay probabilities of the 2S state at the critical points do not become infinite, but become equal to

$$\dot{W} = BE^2(a_{\parallel}\cos^2\gamma + a_{\perp}\sin^2\gamma) + (1 \pm \mathscr{P})C.$$

It is necessary to put a plus sign before $\mathscr{P}$ at critical point 3, and a minus sign at the points 1, 2, 4. The coefficient

$$C = \frac{C_{2P}}{4} = \frac{2^6}{3^8}\frac{me^{10}}{\hbar^6c^3} = 1.48\cdot10^8\ \text{sec}^{-1}$$

(C_{2P} is the rate of decay of the 2P state of the hydrogen atom).

4. We now turn to the motion of the atom in a magnetic field. The decay probability of the 2S state per unit time is

$$\dot{W} = \left(\frac{2}{3}\right)^6 \frac{e^8}{m\hbar^2c^3}\left(\frac{H}{\Delta_2}\right)^2\left(\frac{u}{c}\right)^2 \sin^2\alpha\, f_{\perp}(H),$$

where α is the angle between the velocity of the atom and the magnetic field. All the remaining formulas of this section are rewritten for this decay mechanism with the replacement of E by $Hc^{-1}u \sin\alpha$ and with account of the fact that $\gamma = \pi/2$, since $\mathbf{E} \perp \mathbf{H}$.

Assuming that the hydrogen atoms are in a state of thermal equilibrium and that their velocities have a

Maxwellian distribution, we obtain the averaged decay probability per unit time:

$$\dot{W}_{\mathrm{M}}=\frac{2^7}{3^8}\frac{e^8}{m\hbar^2c^5}\frac{kT}{M_{\mathrm{H}}}\left(\frac{H}{\Delta_2}\right)^2 f_\perp(H).$$

Here k is Boltzmann's constant, T the temperature and M_H the mass of the hydrogen atom. All the remaining formulas are obtained by the replacement of E^2 by $2H^2kT/c^2M_H$, with account of the fact that $\gamma = \pi/2$. Naturally, the decay probability per unit time associated with motion of the atom has singularities at singular points 1, 3, 4 but not at point 2.

5. In order to observe the effect of the transition $2S \to 2P$ due to collisions, it is necessary that the 2S state not break down too rapidly from motion of the hydrogen atoms in the magnetic field. The rate of such a breakdown will be equal to

$$\dot{W}_{\mathrm{M}} = 3.18\cdot 10^{-7} TH^2 f_\perp(H)$$

(here T is in °K and H in gauss).

At low magnetic fields ($H \ll 600$ G) this rate is

$$\dot{W}_{\mathrm{M}} = 4.65\cdot 10^{-5}\, TH^2 \ll 20T.$$

For high magnetic fields ($H \gg 9000$ G)

$$\dot{W}_{M} = 78.1T.$$

We find the decay rate of the 2S state of the hydrogen atom due to its motion in the magnetic field near the singular points:

$$\dot{W}_{\mathrm{M}} = KT/(\Delta H)^2.$$

The coefficient K depends on the singular point at which the decay probability is considered, and is equal to (in the units $G^2\text{-}deg^{-1}sec^{-1}$): $K_1 = 1.81 \times 10^6(1 - \mathscr{P})$; there is no singularity at singular point 2; $K_3 = 1.14 \times 10^6(1 + \mathscr{P})$; $K_4 = 1.22 \times 10^6(1 - \mathscr{P})$.

6. We now find the probability of an atom making the transition $2S \to 2P$ due to collision with an ion. If the ion belongs to a beam which passes through a gas containing hydrogen atoms in the state 2S, then this probability will be equal to

$$\dot{W}_{k\,\mathrm{coll}} = 36\pi\left(\frac{\hbar}{m}\right)^2\frac{MI}{\mathscr{E}}\left(\ln\frac{\mathscr{E}}{\Delta_2 F_k(H)} - 8.37\right),$$

where the index k denotes ∥ or ⊥, depending on the orientation of the beam relative to the magnetic field, I is the particle current density in the beam, E the energy of each of them, and M their mass.

If ions are present in the gas and in thermal equilibrium with it, then this probability is also easily calculated (see[5], p. 140). It is

$$\dot{W}_{\mathrm{iso}}^{\mathrm{coll}} = 18\left(\frac{\hbar}{m}\right)^2 n\eta(\pi M_{\mathrm{H}}/kT)^{1/2}\left(\ln\frac{kT}{\Delta_2 F_{\mathrm{iso}}(H)} - 9.77\right),$$

where n is the number density of atoms in the gas, the η-th part of which is ionized and M_H is the mass of the hydrogen atom.

7. The number of decays of the 2S state of the hydrogen atoms per unit time due to their collisions with the ions should exceed the number of such decays associated with the motion of the hydrogen atoms in the magnetic field. This leads to the conditions

$$I\ln\frac{\mathscr{E}}{\alpha F_k(H)} \gg K_{11}\mathscr{E}TH^2 f_\perp(H) \gg K_{12}TH^2 f_\perp(H)F_k(H) \tag{3.3}$$

(for the beam of ions) and

$$n\eta\ln\frac{T}{\beta F_{\mathrm{iso}}(H)} \gg K_{21}T^{3/2}H^2 f_\perp(H) \gg K_{22}H^2 f_\perp(H)F_{\mathrm{iso}}^{3/2}(H) \tag{3.4}$$

Table II

H, G	γ	a′	δ
$H_1 = 572$	$1{,}145\cdot 10^{16}(1-\mathscr{P})$	$7{,}55+0.83\mathscr{P}$	$3.88\cdot 10^{8}(1-\mathscr{P})$
$H_3 = 5165$	$7.21\cdot 10^{17}(1+\mathscr{P})$	$7.82-0{,}55\mathscr{P}$	$2{,}44\cdot 10^{10}(1+\mathscr{P})$
$H_4 = 7835$	$7{,}72\cdot 10^{18}(1-\mathscr{P})$	$7.54+1{,}59\mathscr{P}$	$2.615\cdot 10^{11}(1-\mathscr{P})$

(for a gas at thermal equilibrium and containing hydrogen atoms in the 2S state and hydrogen ions). The coefficients K_{ij}, α and β are given by

$$K_{11} = 2000\ \mathrm{cm^{-2}sec^{-1}\,eV^{-1}deg^{-1}G^{-2}},\ K_{12} = 300\ \mathrm{cm^{-2}} \times \mathrm{sec^{-1}deg^{-1}G^{-2}},\ K_{21} = 6\cdot 10^{-5}\cdot \mathrm{cm^{-2}deg^{-1/2}G^{-2}},\ K_{22} = 50\ \mathrm{cm^{-2}G^{-2}}$$

$$\alpha = 0{,}196\ \mathrm{ev},\ \beta = 9170\ \mathrm{deg}.$$

We obtain the conditions for satisfying the inequalities (3.3)—(3.4) near the singular points. It is easy to see that the inequality (3.3) is satisfied under the condition

$$I(\ln\mathscr{E} + a - b\ln|\Delta H|) \gg \gamma T\mathscr{E}/(\Delta H)^2,$$

and the inequality (3.4) for the condition

$$n\eta(\ln T - a' - b\ln|\Delta H|) \gg \delta T^{5/2}/(\Delta H)^2.$$

(T is measured in °K, ΔH in G, $\mathscr{E}$ in eV). The values of a and b are given in Table I. The values of a′, γ and δ are shown in Table II.

8. When the magnetic field strength takes on one of the critical values, the transition cross sections become infinite only in the case in which the corresponding terms really intersect. However, as is clear from what has been said above, such an intersection does not take place as a consequence of the motion of the atoms in the magnetic field. Therefore, the cross sections have definite finite values at the critical values of the magnetic field strength. But their calculation is not of interest since the transition cross sections of $2S \to 2P$ (and with them the number of collision transitions per unit time) increase near the critical values of the magnetic field strength as $-\ln|\Delta H|$, while the probability of breakdown of the 2S state due to motion of the atoms (and, accordingly, the number of decay atoms) increases as $(\Delta H)^{-2}$ i.e., much more rapidly. Therefore, the second effect is much more important than the first near the critical point and at it.

9. This does not apply for the critical point $H_2 = 1184$ G. The intersection of the terms at this point does not disappear due to motion of the atoms (the transition between these terms is brought about by an electric field parallel to the magnetic field, which does not arise through motion of the atoms). Therefore, the cross sections $\sigma_\perp(\mathscr{E})$ and $\sigma_{iso}(\mathscr{E})$ (and with them the corresponding number of transitions per unit time) are cut off only by the finite width of the level 2P. This width is equal to $\Gamma = 2^8me^{10}/3^8\hbar^5c^3$ and hence the cross sections are

$$\sigma_\perp(\mathscr{E}; H_2) = \frac{A}{\mathscr{E}}(\ln\mathscr{E} + 2.62 - 0.63\mathscr{P});$$

$$\sigma_{\mathrm{iso}}(\mathscr{E}; H_2) = \frac{A}{\mathscr{E}}(\ln\mathscr{E} + 2.53 - 0.48\mathscr{P}).$$

(here the energy $\mathscr{E}$ is measured in eV).

4. POSSIBILITY OF OBTAINING POLARIZED HYDROGEN ATOMS

If the magnetic field strength is close to one of the critical values or is equal to it, then, as is seen from the foregoing, the probability of decay of the 2S state of the hydrogen atom, due to both collisions with ions and its motion in the magnetic field, depends very strongly on the polarization $\mathscr{P}$. At the singular points 1, 2, and 4, it is large for $\mathscr{P} = -1$ and small for $\mathscr{P} = +1$; at the singular point 3, the situation is just the opposite. Therefore, the phenomena considered can be used for the polarization of hydrogen atoms. If a gas containing hydrogen atoms in the 2S state is irradiated by a beam of ions (or a certain number of ions is simply introduced into it) at an appropriate value of the magnetic field strength, then the states of the 2S hydrogen atoms with one of the two projections of the electron spin onto the direction of the magnetic field will "burn out" far more rapidly than those with the other projection. As a result, the gas is polarized. The ions can also be dispensed with. Breakdown of the 2S state of the hydrogen atoms with one of the two projections of the spin onto the direction of the magnetic field can also take place due to an external electric field or due to motion of the hydrogen atoms in the magnetic field.

One can also obtain polarized beams of hydrogen atoms in the 2S state. For this purpose, we need to pass the beam through a cloud of ions or to have it collide with a beam of ions at an appropriate value of the magnetic field. Once again, the ions can be done without. For this, we need to use an external electric field or to direct the beam at an angle to the direction of the magnetic field such that, in the coordinates moving with the beam, a Lorentzian electric field is produced and breaks down the 2S state of the hydrogen atoms with the projections of the electron spin onto the direction of the magnetic field that are not of use to us.

The method of obtaining polarized beams of hydrogen atoms proposed in this paper naturally extends the ideas of Zavoĭskiĭ [6], who first proposed the use of the Lamb shift for this purpose. In the Zavoĭskiĭ method, a longitudinal magnetic field of intensity equal to the first critical value is superposed on a beam of unpolarized hydrogen atoms in the 2S state, along with a transverse electric field.

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