Dynamic suppression of the maxima of interference bremsstrahlung emitted by ultrafast electrons in a single crystal

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Coherent bremsstrahlung in a single crystal is investigated in the case when perturbation theory cannot be used to describe the interaction between fast charged particles and the potential of a crystal lattice. An appreciable weakening of the maxima of interference bremsstrahlung produced by ultrafast electrons is observed when the limits of perturbation theory are surpassed by means of the eikonal approximation.

1. The interference character of bremsstrahlung of relativistic charged particles was predicted theoretically by Ferretti[1] and Ter-Mikaelyan[2], who used the Weizsäcker-Williams method, and by Überall[3] with the aid of perturbation theory. Subsequently, there were a number of experimental investigations[4-7] of the influence of the crystal structure on the bremsstrahlung. Nonetheless, the growth of the bremsstrahlung intensity at the maximum, predicted by perturbation theory, was not observed[12]. In addition, only qualitative agreement was obtained[5-7] with the results of the theory[1-3], with systematic deviations.

Moreover it is noted in the paper by Ferretti[8] that the intensity at the maximum of the interference bremsstrahlung, obtained with the aid of perturbation theory[1-3], increases without limit with increasing energy. There is still no satisfactory explanation of the indicated discrepancies between the theoretical and experimental results. It was shown earlier[9-10], however, that the Born approximation is not valid for the description of the elastic scattering of fast charged particles in single crystals. It is therefore necessary to consider coherent radiation in crystal without the use of the Born approximation, and to do so more accurately than done in the Weizsäcker- Williams method. In this case one should expect the dynamic contraction of the coherence length in the interaction of fast charged particles in a single crystal, predicted by us in[9-10], should lead to the appearance of maxima of interference bremsstrahlung.

2. We consider the matrix element of the bremsstrahlung of a relativistic electron in a single crystal

$$M_{1\to 2} = -\frac{2\pi i e}{(2\omega)^{1/2}} \int \psi_2^*(y)\, \mathbf{e}\hat{\alpha} e^{ik_\nu y} \psi_1(y)\, d^4y, \tag{1}$$

where $\hbar = c = 1$; $\psi_{1,2}$ are the wave functions in the initial and final states with allowance for the interaction with the single-crystal atoms; $\mathbf{e}$ is a unit polarization vector of the emitted photon of energy ω; $\hat{\alpha}$ are Dirac matrices.

No perturbation theory is used in Eq. (1), in the description of the interaction of the charged particle with the atoms of the crystal, but the interaction with the radiation field is regarded as a perturbation, i.e., we neglect the processes of simultaneous emission of two or more photons. The wave functions of an ultrafast electron in the initial and final states is conveniently written by using the eikonal approximation[11]. In particular, following Schiff[11], we have for ψ_1

$$\psi_1(\mathbf{r},t) = \exp\left\{i\mathbf{p}_1\mathbf{r} - iE_1 t - \frac{i}{\beta}\int_0^\infty U\left(\mathbf{r} - \frac{\mathbf{p}_1}{p_1}s\right)ds\right\}$$
$$+\int \exp\left\{ip_1|\mathbf{r}-\mathbf{r}'| - \frac{i}{\beta}\int_0^{|\mathbf{r}-\mathbf{r}'|} U\left(\mathbf{r}' + \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}s\right)ds\right\}$$
$$\times \exp\left\{i\mathbf{p}_1\mathbf{r}' - iE_1 t - \frac{i}{\beta}\int_0^\infty U\left(\mathbf{r}' - \frac{\mathbf{p}_1}{p_1}s\right)ds\right\}\frac{d\mathbf{r}'}{4\pi|\mathbf{r}-\mathbf{r}'|}, \tag{2}$$

where

$$U(\mathbf{r}) = \sum_a U_0(\mathbf{r} - \mathbf{R}_a)$$

and the summation over $\mathbf{R}_a$ is carried out over all the points of the crystallographic lattice.

Substitution of (2) (and of an analogous expression for $\bar{\psi}_2^*$) in (1) enables us to calculate the matrix element of the bremsstrahlung in explicit form. Since the characteristic scattering angles m/E that leads to the radiation are much larger than the characteristic angles $(pR_{at})^{-1}$ for elastic scattering, the matrix element of the bremsstrahlung reduces ultimately to the usual matrix element of second-order perturbation theory, multiplied by the eikonal phase factors corresponding to the change of phase of the particle in the initial, intermediate virtual, and final states. However, at entrance angles close to the position of the maximum in the spectrum of the interference bremsstrahlung[12], the electron moves parallel to the crystallographic axis only in the intermediate virtual state, and this enables us to neglect the changes of the phase of the wave function in the initial and final states.

Using the foregoing, we obtain by simple calculations the following expression for the intensity of the bremsstrahlung in the principal maximum

$$d\sigma_{max}^{\text{int}} = 2\frac{Z^2 e^6}{m^2}\frac{E-\omega}{E}\frac{d\omega}{\omega}\left(\frac{2\pi}{a}\right)^2$$
$$\times N\exp(-2\varkappa^2\bar{u}^2)\left[\frac{m^2\omega}{E(E-\omega)} + \frac{Ze^2}{a}\right]^{-1}, \tag{3}$$

where $\overline{u^2}$ is the square of the thermal vibrations of the lattice atoms.

The obtained bremsstrahlung intensity in the principal maximum is much smaller than the corresponding value obtained with the aid of perturbation theory[1-3]. Comparing expressions (3) with the results of[1-3], we find

$$d\sigma_{max}^{\text{int}}/d\sigma_{\text{maxPT}}^{\text{int}} = \frac{m^2\omega}{E(E-\omega)}\Big/\left[\frac{m^2\omega}{E(E-\omega)} + \frac{Ze^2}{a}\right]$$
$$= \left[1 + \frac{Ze^2}{a}\frac{E(E-\omega)}{m^2\omega}\right]^{-1}. \tag{4}$$

Relations (3)—(4) enable us to estimate the deviations from the predictions of perturbation theory. Panofsky and Saxena[4] investigated the bremsstrahlung of electrons with energy 575 MeV in single-crystal silicon. According to (3), the radiation intensity of photons with ω = 300 MeV in the principal maximum amounts to 0.88 of the value predicted by Überall[3], which agrees with experiment[4]. With increasing energy of the incident electrons, the obtained expression (3), unlike the predictions of perturbation theory, does not increase without limit, and tends, as expected, to a finite limit[8].

3. A simple estimate of the considered effect can be obtained with the aid of qualitative arguments. The bremsstrahlung of relativistic electrons comes into being in an effective region with a large longitudinal length $l \approx E(E-\omega)/m^2\omega$ and with transverse dimensions

$$m^{-1} \leqslant r_\perp \leqslant R_{at} \sim \varkappa^{-1} = (me^2Z^{1/3})^{-1}.$$

All the atoms located in the effective region take part in the process coherently, and the intensity of the bremsstrahlung in the single crystal increases in comparison with an amorphous body in proportion to the number of the crystal atoms in the effective region[1-3]

$$N_{eff} \sim l/a \sim E(E-\omega)/m^2\omega a. \tag{5}$$

The interaction of the incident particle with atoms located in the effective region can be interpreted as interactions with a potential characterized by an effective force $N_{eff}Z(e^2/\beta)$. With increasing energy, this quantity becomes larger than unity, which leads to violation of the applicability of perturbation theory to the description of the scattering by the atoms of the crystal[9-10]. By going outside the framework of perturbation theory and using the eikonal approximation[11], one obtains the following expression for N_{eff}[9-10]:

$$N_{eff} \sim \frac{l_{eff}}{a} \sim a^{-1}\left[\frac{m^2\omega}{E(E-\omega)} + \frac{Ze^2}{\beta}\right]^{-1}, \tag{6}$$

which coincides with the exact result (3).

The intensity of the bremsstrahlung at the minima remains practically unchanged. The reason is that usually, when bremsstrahlung by one atom is considered, perturbation theory can be used, and at the minimum the effective interaction region subtends over a very small number of crystal-lattice points.

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