Spontaneous positron production in collisions between heavy nuclei

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It is shown that in collisions between a completely stripped ("bare") nucleus (Z_2) and the atoms of a heavy target (Z_1) vacancies are formed in the K shell of the united atom, providing $Z_2 \geq Z_1$. Therefore such collisions, as well as those between two bare nuclei, will result in spontaneous production of electron-positron pairs in a static Coulomb field of the colliding nuclei if the nuclei approach each other at a distance $R < R_c$ (R_c is the critical distance at which the ground level of the discrete spectrum becomes as low as the boundary of the lower continuum). In this case one or two electrons fill the vacancies in the K shell of the united atom and the positrons escape to infinity. This circumstance should make feasible the experimental observation of spontaneous production of positrons in an ordinary heavy target by a bare nuclear beam. The kinematics of the process is considered and the threshold behavior of the positron production cross section and positron energy spectrum are considered. An estimate of background effects shows that they should not impede observation of the phenomenon. An experimental study of quasistatic positron production from vacuum due to lowering of the level to the lower continuum should be of interest from the viewpoint of verification of quantum electrodynamics in strong fields.

1. INTRODUCTION

In recent years, definite progress was made in the understanding of the phenomena that occur in the Coulomb field of nuclei with $Z > Z_c$, where Z_c is the critical charge of the nucleus[1], i.e., that value of Z at which the ground level of the discrete spectrum $1s_{1/2}$ drops to the boundary of the lower continuum $E = -mc^2$. The formation of a nucleus with charge $Z > Z_c$ and an unfilled K shell should lead to the production of two e^+e^- pairs, with the electrons landing on the K shell and the positrons passing through the Coulomb barrier and going off to infinity (see[2-8]). An experimental confirmation of these predictions is of interest from the point of view of verifying quantum electrodynamics in the region of superstrong fields, where perturbation theory with respect to the external field is no longer applicable.

Numerical calculation[2,7] yields $Z_c = 170$ for a spherical nucleus with radius $r_0 = 10$ F, which is far from the presently-known heavy elements. This forces us to seek new ways of verifying the theory.

As noted in[8], the situation with the supercritical charge can be reached by collision of two fully stripped ("bare") nuclei with $Z_1 + Z_2 > Z_c$, if the shortest approach distance $R_{min} < \hbar/mc$. It is feasible in principle[1)] to obtain at present a beam of fully stripped nuclei, but the experimental difficulties with colliding beams are too large. In the present paper we show that the effect of spontaneous positron production takes place also when only one of the colliding nuclei is bare (and the other has a normal electron shell). This uncovers a possibility of performing the experiment with an ordinary heavy target.

The kinematics of Coulomb collisions of nuclei and e^+ production is considered in Sec. 2. Section 3 is devoted to a comparison of the terms of the quasimolecule (Z_1, Z_2, e) at small and large R. It is shown that when a bare nucleus approaches a normal atom, the K-shell of the combined atom remains unfilled in many cases, as a result of which the "bare" level drops to the lower continuum at $R < R_c$. According to[3,4], this should lead to a spontaneous production of two positrons. In Sec. 4, the cross section for positron production and the positron energy spectrum are calculated. Section 5 is devoted to a discussion of the background effects that lead to production of e^+ by other mechanisms. Section 6 contains a brief discussion of the results and remarks concerning certain theoretical papers recently published on this subject. The Appendix contains the mathematical details of the calculations.

A system of units is used with $\hbar = c = m = 1$, where m is the electron mass, $\alpha = e^2/\hbar c = 1/137$, and $\zeta = (Z_1 + Z_2)\alpha$. The distance R between nuclei is measured in units of the Compton wavelength of the electron $\hbar/mc = 386$ F.

The present paper is the development of ideas advanced in an earlier note[10].

2. KINEMATICS

By virtue of the large masses of the nuclei, their motion can be regarded classically. Let Z_1 and M_1 be the charge and mass of the target nucleus, Z_2 and M_2 the same quantities for the incident nucleus. The closest-approach distance in frontal collision of the nuclei is equal to

$$R_0 = Z_1 Z_2 e^2 (1 + M_2/M_1)/E, \tag{2.1}$$

where $E = M_2 v^2/2$ is the kinetic energy of the incident nucleus. In the general case the closest-approach distance is (ρ is the impact parameter)

$$R_{min} = \tfrac{1}{2} R_0 \{1 + [1 + (2\rho/R_0)^2]^{1/2}\}. \tag{2.2}$$

The trajectories of the nuclei are conveniently specified in parametric form[11]

$$R = \tfrac{1}{2} R_0 (1 + e \operatorname{ch} \xi), \quad t = \tfrac{1}{2} \tau_0 (\xi + e \operatorname{sh} \xi),$$
$$v(R) = v\left(1 - \frac{R_0}{R}\right)^{1/2} = v\left(\frac{e \operatorname{ch} \xi - 1}{e \operatorname{ch} \xi + 1}\right)^{1/2}. \tag{2.3}$$

Here R is the distance between nuclei, v is the relative velocity, e is the eccentricity of the hyperbola, and τ_0 is the characteristic time

$$\tau_0 = \frac{R_0}{v} = C \zeta^{-1/2} R_0^{3/2}, \tag{2.4}$$

where

$$C = \left[\frac{A_1 A_2 (Z_1 + Z_2)}{2(A_1 + A_2) Z_1 Z_2} \frac{m_N}{m_e}\right]^{1/2} \approx 50.$$

If we measure R_0 in units of $\hbar/mc \approx 390$ F, then formula (2.4) yields τ_0 in units of $\hbar/mc^2 = 1.3 \times 10^{-21}$ sec.

Measurement of the scattering angle θ yields all the quantities that determine the trajectories of the nuclei[2)]

$$R_{min} = {}^1\!/_2 R_0[1 + \operatorname{cosec}(\theta/2)], \quad \rho = {}^1\!/_2 R_0 \operatorname{ctg}(\theta/2), \quad e = [1 + (2\rho/R_0)^2]^{1/2} = \operatorname{cosec}(\theta/2), \tag{2.5}$$

and makes it possible to select the events of interest to us, those with $R_{min} < R_c$. The relative velocity of the nuclei is equal to v at infinity and reaches the lowest value at $I = R_{min}$:

$$v_{min} = v \operatorname{tg}\frac{\pi - \theta}{4} = \frac{vp}{1 + (1 + p^2)^{1/2}}; \tag{2.6}$$

here $p = 2\rho/R_0 = \cot(\theta/2)$. At $Z_1 = Z_2 = Z$ we have

$$\beta = \frac{v}{c} = \frac{2Ze}{(Am_N c^2 R_0)^{1/2}} \approx 0.02\left(\frac{\zeta}{R_0}\right)^{1/2}, \tag{2.7}$$

where $\zeta = 2Z/137$ and $A = 2.5Z$. For uranium nuclei $\zeta = 1.34$; therefore $\beta = 0.025$ at $R_0 = 1$ and $\beta = 0.07$ at $R_0 = 0.1$. The motion of the nuclei is thus nonrelativistic; at the same time, a K-shell electron has a velocity of the order of c at large Z. It can therefore be assumed that the level energy and the wave functions of the electrons on the K shell follow adiabatically the variation of R during the course of the approach of the nuclei, and this facilitates the solution of the problem.

We introduce the characteristic frequency $\omega_c = 1/\tau_0$, corresponding to collision. The Fourier components of the trajectory x_ω and y_ω in the Coulomb repulsion field practically do not contain frequencies $\omega \gg \omega_c$. Therefore the spectrum of the bremsstrahlung that is produced upon hyperbolic motion of the charges Z_1 and Z_2 is exponentially cut off at frequencies $\omega \lesssim \omega_c$. This causes the background process of e^+e^- pair production as a result of bremsstrahlung to be negligibly small. Numerically we have

$$\nu = \omega_c/2m_e \approx 0.01\zeta^{1/2}R_0^{-3/2} \tag{2.8}$$

(for uranium nuclei $\nu = 0.012$ at $R_0 = 1$ and $\nu = 0.37$ at $R_0 = 0.1 = 40$ F).

Positron production is possible when the nuclei approach each other slowly if $R < R_c$; either R_c is the "critical" distance between the nuclei, at which the lower level of the discrete spectrum of the quasimolecule (Z_1, Z_2, e) crosses the boundary $\epsilon = -1$. The value of R_c depends on the charges of the nuclei and can be obtained by solving the relativistic problem of two (immobile) centers. We shall henceforth use the values of R_c obtained in[12].

From the condition $R_0 = R_c$ we determine the minimum energy of the incident nucleus E_t necessary for e^+ production. For a system consisting of two identical nuclei[3)] we have

$$E_t = 35\zeta^2/R_c \tag{2.9}$$

(R_c is taken in units of $\hbar/m_e c$, and E is MeV). For example, when two uranium nuclei collide $E_t = 550$ MeV, which corresponds to ~ 2.5 MeV/nucleon. The condition $R_{min} < R_c$ determines the region of scattering angles $\theta_1 < \theta \leq \pi$, in which positron production is possible:

$$\theta_1 = 2\arcsin\frac{1}{2\eta - 1}, \quad \eta = \frac{E}{E_t} = \frac{R_c}{R_0} \tag{2.10}$$

(see Fig. 1). In the language of impact parameters, the corresponding limitation on the trajectory takes the form

$$0 \leqslant \rho < \rho_1, \quad \rho_1 = [R_c(R_c - R_0)]^{1/2} = R_c[1 - 1/\eta]^{1/2}.$$

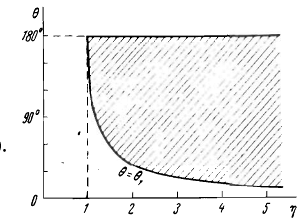

FIG. 1. Scattering-angle region in which spontaneous positron production is kinematically possible (shown shaded).

This determines the upper limit for the positron production cross section:

$$\sigma < 2\pi\rho_1^2 = 2\pi R_c^2(1 - 1/\eta). \tag{2.11}$$

The upper limit (2.11) would be reached if the Coulomb field of the nuclei would produce effectively two positrons in each collision with $R_{min} < R_c$. We shall show later (Sec. 4) that under optimal conditions the value of σ is smaller by three or four orders of magnitude than the estimate (2.11).

We define the nuclear collision time τ_{col} as the time during which $R < R_c$. For example, for frontal collision we obtain from (2.3)

$$\tau_{col} = 2\tau_0\{[\eta(\eta - 1)]^{1/2} + \ln[\eta^{1/2} + (\eta - 1)^{1/2}]\}. \tag{2.12}$$

At $\eta \gtrsim 2$, the quantity τ_{col} is larger by one order of magnitude than τ_0. For uranium nuclei $\tau_0 = 2 \times 10^{-21}$ sec, and $\eta_{col} = 10^{-20}$ sec at $\eta = 2$. At the same time, the motion of the electron along the K orbit is characterized by a period $T = \bar{r}/\bar{v} \approx 5 \times 10^{-22}$ sec (at $Z = Z_c$ we have[2] $\bar{r} = 0.3\hbar/mc$ and $\bar{v} \approx c$). Thus, $\tau_{col} \gg T$, by virtue of which the processes that occur when the level crosses the boundary $\epsilon = -1$ can be considered in the adiabatic approximation (with respect to the velocities of the nuclei). In this case the probability of spontaneous e^+ production on a trajectory with given ρ is

$$w_1(\rho) = \int \gamma(R(t))\,dt = 2\int_{R_{min}}^{R_c} \gamma(R)\left(\frac{\partial t}{\partial R}\right)_\rho dR, \tag{2.13}$$

where $\gamma/2$ is the imaginary part of the energy of the quasistationary level $1s_{1/2}$, which goes off at $R < R_c$ into the lower continuum. The quantity $\gamma = \gamma(R)$ gives the probability of the spontaneous production of a positron per unit time at fixed distance between nuclei R. The total e^+ production cross section is

$$\sigma = 2\pi\int_0^{\rho_1} w_1(\rho)\rho\,d\rho.$$

After interchanging the order of integration, the internal integral with respect to ρ is calculated with the aid of the parametrization (2.3):

$$\sigma = \frac{4\pi}{v}\int_{R_0}^{R_c} \gamma(R)R^{3/2}(R - R_0)^{1/2}\,dR. \tag{2.14}$$

We have not used yet any assumption concerning the form of the function $\gamma(R)$. We consider now the case when the excess of the total charge of the nuclei over Z_c is small

$$\delta = (Z_1 + Z_2 - Z_c)/Z_c \ll 1. \tag{2.15}$$

This occurs[4)] in the practically important region $Z = 90-100$. Then $R_c \ll 1$, and the other characteristic length, namely the radius $\bar{r}$ of the bound state at $\epsilon = -1$, remains of the order of unity (see[2]). Therefore γ depends mainly on the ratio R/R_c:

$$\gamma = \gamma(R/R_c) \quad \text{as} \quad \delta \to 0. \tag{2.16}$$

Substituting (2.16) in (2.14) we find that the cross section can be factorized

$$\sigma(E, Z) = \sigma_0 f(\eta); \qquad (2.17)$$

here

$$\sigma_0 = \sigma_0(Z) = R_c^2 \tau_c = C\zeta^{-1/2} R_c^{7/2} \qquad (2.18)$$

(the time τ_c is measured in units of $\hbar/m_e c^2 = 1.3 \times 10^{-21}$ sec, and for the critical distance R_c it is determined from the formula (2.4)), while $f(\eta)$ is a universal function of the ratio $\eta = E/E_2$:

$$f(\eta) = \frac{4\pi}{\eta} \int_{1/\eta} \gamma(x) x^{3/2} \times (\eta x - 1)^{1/2} dx. \qquad (2.19)$$

The quantity σ_0 (see Fig. 2) has the dimension cm^2 and depends strongly on Z (we note that the relation $\tau_c \sim R_c$ corresponds to Kepler's third law). It is therefore convenient to increase the charges of the colliding nuclei. For example, on going from U + U to Cf + Cf collisions, σ_0 increases by more than five times. The cross section will be discussed in greater detail in Sec. 4.

We proceed to the energy spectrum of the positrons. It can be shown[13] that the quasistationary level is narrow ($\gamma \ll \epsilon_0 = -\mathrm{Re}\,\epsilon$) even at $\epsilon_0 \gtrsim 1$. Therefore at a given R practically all the positrons are emitted with one and the same kinetic energy $T = \epsilon_0(R) - 1$. At an incident-nucleus energy $E > E_t$, the kinetic energy e^+ can lie in the interval

$$0 < T < T_{max}, \quad T_{max} = \varepsilon_0(R_0) - 1. \qquad (2.20)$$

The distribution of e^+ over the energies is obtained with the aid of (2.14):

$$\frac{d\sigma}{dT} = AR^{3/2}(R - R_0)^{1/2} \gamma(R) \left(\frac{d\varepsilon_0}{dR}\right)^{-1/2}. \qquad (2.21)$$

Here A is a normalization constant, and the variable R must be expressed in terms of T in accordance with the equation $\epsilon_0(R) = 1 + T$. Thus, the cross section for spontaneous production of e^+ and their momentum spectrum are determined completely if one knows the position of the quasidiscrete level in the lower continuum $\epsilon(R) = -\epsilon_0 + i\gamma/2$ as a function of the distance $R < R_c$.

In the collisions considered by us, the recoil nucleus acquires an appreciable energy:

$$E' = \mu E \sin^2(\theta/2), \quad \mu = 4A_1A_2/(A_1 + A_2)^2 \approx 1.$$

Registration of the scattered nucleus, the recoil nucleus, and the positron makes it possible to reconstruct the kinematics completely.

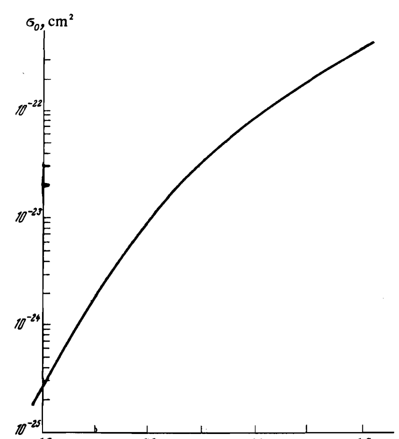

FIG. 2. Dependence of σ_0 on the charge of the nuclei (at $Z_1 = Z_2 = Z$, $\zeta = 2Z/137$).

3. COMPARISON OF THE TERMS OF THE QUASIMOLECULE (Z_1, Z_2, e) FOR LARGE AND SMALL DISTANCES R]

Using the adiabaticity of the approach of the nuclei, let us determine the unified-atom (UA) states into which the electrons go over from their initial positron on the K orbit of the target nucleus.

We start with the case $Z_1 = Z_2 = Z$. Let Σ_g and Σ_u be the wave functions of an electron in the field of two Coulomb centers (see, e.g.,[14]). A system of two K-electrons (without allowance for the Coulomb interaction between them) is described by the wave functions

$$\begin{aligned} \psi_1 &= \Sigma_g(1)\Sigma_g(2), \quad \psi_2 = \Sigma_u(1)\Sigma_u(2), \\ \psi_3 &= 2^{-1/2}\{\Sigma_g(1)\Sigma_u(2) + \Sigma_u(1)\Sigma_g(2)\}, \\ \psi_4 &= 2^{-1/2}\{\Sigma_g(1)\Sigma_u(2) - \Sigma_u(1)\Sigma_g(2)\}, \end{aligned} \qquad (3.1)$$

of which the first three pertain to a total spin S = 0, and ψ_4 pertains to S = 1. As $R \to 0$, they go over into the following states of the UA:

$$\psi_1 \to (1s)^2, \quad \psi_2 \to (2p\sigma)^2, \quad \psi_3 \to {}^1(1s, 2p\sigma), \quad \psi_4 \to {}^3(1s, 2p\sigma). \qquad (3.2)$$

At $R \gg 1$ we have

$$\Sigma_{g,u} = [2(1 \pm K)]^{-1/2}\{\varphi_a \pm \varphi_b\}, \qquad (3.3)$$

where $\varphi_a(i)$ and $\varphi_b(i)$ are the wave functions of the i-th electron on the K-orbit of the nucleus a or b, and $K = \int \varphi_a \varphi_b d^3r$ is the overlap integral. As $R \to \infty$, the integral K vanishes exponentially. The lower level has in this case fourfold degeneracy. This degeneracy is lifted when account is taken of the interaction between the electrons $V = e^2/r_{12}$. We note that the function Σ_g is not altered by the interchange of the nuclei $a \leftrightarrow b$, while Σ_u reverses sign. It follows therefore that $V_{13} = V_{23} = 0$; the only nonzero off-diagonal matrix element is V_{12}:

$$V_{12} = \int \psi_1(1) \frac{e^2}{r_{12}} \psi_2(2) dv_1 dv_2 = \alpha \int \rho(1) r_{12}^{-1} \rho(2) dv_1 dv_2, \qquad (3.4)$$

where $\rho(i) = \Sigma_g(i)\Sigma_u(i)$. Therefore the states ψ_3 and ψ_4 are separated in the secular equation, and as $R \to \infty$ we have

$$\begin{aligned} \psi_3 &\to 2^{-1/2}\{\varphi_a(1)\varphi_a(2) - \varphi_b(1)\varphi_b(2)\}, \quad E_3 \to 2E_Z + I, \\ \psi_4 &\to 2^{-1/2}\{-\varphi_a(1)\varphi_b(2) + \varphi_b(1)\varphi_a(2)\}, \quad E_4 \to 2E_Z. \end{aligned} \qquad (3.5)$$

Here $E_Z = [1 - (Z\alpha)^2]^{1/2}$ is the energy of the K electron in the atom Z, and I is the energy of the interacting two K-electrons in the atom Z:

$$I = \alpha \int \varphi_a^2(1) r_{12}^{-1} \varphi_a^2(2) dv_1 dv_2. \qquad (3.6)$$

In the nonrelativistic limit, $I = (5/8)Z\alpha^2$. Calculation with exact relativistic functions shows (see the Appendix) that although I increases together with Z, it remains of the order of α at $Z\alpha = 1$, thereby justifying the use of perturbation theory.

When the interaction between the electrons is taken into account, a superposition of the states ψ_1 and ψ_2 occurs. As $R \to \infty$, the corresponding wave functions (which we designate χ_1 and χ_2) are given by

$$\begin{aligned} \chi_1 &= 2^{-1/2}(\psi_1 - \psi_2) = 2^{-1/2}\{\varphi_a(1)\varphi_b(2) + \varphi_b(1)\varphi_a(2)\}, \quad E_1(\infty) = 2E_Z; \\ \chi_2 &= 2^{-1/2}(\psi_1 + \psi_2) = 2^{-1/2}\{\varphi_a(1)\varphi_a(2) + \varphi_b(1)\varphi_b(2)\}. \quad E_2(\infty) = 2E_Z + I. \end{aligned} \qquad (3.7)$$

At finite R we have

$$\chi_{1,2} = a\psi_1 \mp b\psi_2, \qquad (3.8)$$

$$a = \left\{\frac{1}{2}\left[1 + \frac{\Delta}{(\Delta^2 + V_{12}^2)^{1/2}}\right]\right\}^{1/2}, \quad b = \left\{\frac{1}{2}\left[1 - \frac{\Delta}{(\Delta^2 + V_{12}^2)^{1/2}}\right]\right\}^{1/2}.$$

The eigenfunctions of the Hamiltonian $\mathscr{H} = H_1 + H_2 + V$ are the states χ_1, χ_2, χ_3, and χ_4, the energies of which are

$$E_{1,2} = \varepsilon_0 \mp [\Delta^2 + V_{12}^2]^{1/2},$$
$$E_3 = \varepsilon_0 + V_{33} - {}^1/_2(V_{11} + V_{22}), \qquad E_4 = \varepsilon_0 + V_{44} - {}^1/_2(V_{11} + V_{22}). \tag{3.9}$$

Here

$$\varepsilon_0 = \varepsilon_g + \varepsilon_u + {}^1/_2(V_{11} + V_{22}), \qquad \Delta = \varepsilon_g - \varepsilon_u + {}^1/_2(V_{11} - V_{22}),$$

and $\epsilon_g(R)$ and $\epsilon_u(R)$ are the energies of the single-electron levels Σ_g and Σ_u at a fixed distance R between the nuclei. As $R \to \infty$, using (3.3), we get

$$V_{11} = V_{22} = {}^1/_2(I + Q), \quad V_{12} = {}^1/_2(I - Q), \quad V_{33} = I, \quad V_{44} = Q.$$

Here I is the Coulomb correction (3.6), and Q is the energy of interaction between electrons of different nuclei:

$$Q = \alpha \int \varphi_a^2(1) r_{12}^{-1} \varphi_b^2(2)\, dv_1\, dv_2 \approx \frac{\alpha}{R}, \quad R \gg 1.$$

Hence $\Delta = 0$ as $R \to \infty$, $a = b = 2^{-1/2}$, and

$$E_1 = E_4 = \varepsilon_0 - (I - Q)/2, \qquad E_2 = E_3 = \varepsilon_0 + (I - Q)/2. \tag{3.10}$$

This determines the arrangement of the terms at $R = \infty$. The approximate variation of the terms with changing R is shown in Fig. 3.

As follows from (3.5) and (3.8), as $R \to \infty$ we get

$$\varphi_a(1)\varphi_a(2) = 2^{-1/2}(\chi_2 + \psi_3). \tag{3.11}$$

Therefore the initial state, at which both electrons are in one nucleus, goes over as $R \to 0$ into the states $^1(1s, 2p\sigma)$ in $(2p\sigma)^2$ of the UA. In other words, when the nuclei approach each other adiabatically, there is a probability 1/2 of formation of one vacant places in the K shell of the UA and a probability 1/2 of formation of two vacancies.

We can consider similarly the case of unequal charges. Let $Z_1 < Z_2$. It then follows from the formulas of[15] that as $R \to 0$ the electrons go over into the 2p state of the UA, that is, its K shell is completely free. This result is explained qualitatively by the fact that in the system (Z_1, Z_2, e) the state with the bare nucleus Z_2 and with an electron on the K orbit of the nucleus Z_1 is excited with respect to the state with the bare nucleus Z_1 and an electron in Z_2. To the contrary, when $Z_1 > Z_2$, the K orbit of the UA is completely filled. Thus, in the adiabatic approximation we have

$$\frac{\tilde{\sigma}(E; Z_1, Z_2)}{\sigma(E; Z_1, Z_2)} = \begin{cases} 1 & \text{if} \quad Z_1 < Z_2 \\ {}^3/_4 & \text{if} \quad Z_1 = Z_2. \\ 0 & \text{if} \quad Z_1 > Z_2 \end{cases} \tag{3.12}$$

Here σ is the cross section for positron production for bare nuclei Z_1 and Z_2, and $\tilde{\sigma}$ is the same cross section but when one of the nuclei (Z_2) is bare and the other has a normal K shell.

The corrections to (3.12) are determined by the parameter

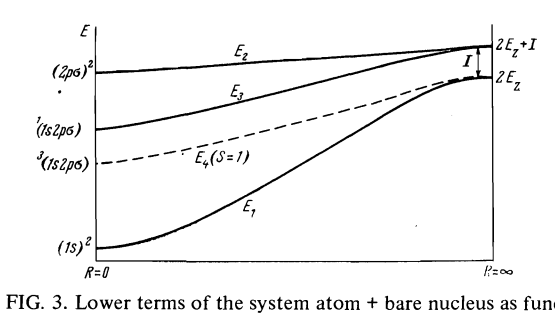

FIG. 3. Lower terms of the system atom + bare nucleus as functions of the internuclear distance R (at $Z_1 = Z_2$).

$$\nu = \Delta E(R_0)\tau_0' \sim c/v \gg 1,$$

where $\Delta E = E(2p) - E(1s) \sim m_e c^2$, $\tau_0' \sim \bar{r}/v$, and $\bar{r}$ is the radius of the K orbit. The probability of transitions with violation of the adiabaticity depends exponentially on ν: $w \sim e^{-\nu}$. This is connected with the fact that the lower terms in the two-center problem do not interact (see the Appendix, and also[14]), and therefore the point of transition lies in the complex plane[16] at $t = i\tau_0'$.

It follows from analogous considerations that when the atom Z_1 approaches the ion Z_2, many vacancies are produced not only in the K shell of the UA, but also in the higher shells. This conclusion is confirmed experimentally by observing the characteristic γ lines in the collisions of iodine ions with targets made of gold, thorium, and uranium. These lines are interpreted[17] as the result of radiative transitions to vacancies in the M and L shells of the UA.

4. PRODUCTION CROSS SECTION AND ENERGY SPECTRUM OF POSITRONS

When the distance between the nuclei is $R < R_c$, the 1s level goes off to the lower continuum and is transformed into a quasistationary state with energy ϵ:

$$\varepsilon = -(|\varepsilon_0| - i\gamma/2), \quad \varepsilon_0 < -1. \tag{4.1}$$

The determination of $\epsilon_0(R)$ and $\gamma(R)$ in the two-center problem calls for very cumbersome calculations, since the variables in the Dirac equation are not separated in any of the orthogonal coordinate systems. However, the threshold behavior of the cross section is relatively easy to determine. In the vicinity of the point R_c, the real part of the energy of the level $\epsilon_0(R) = \mathrm{Re}\,\epsilon$ is expanded in an asymptotic series:

$$\mathrm{Re}\,\varepsilon = -1 + \sum_{n=1}^{\infty} \beta_n \left(\frac{R - R_c}{R_c}\right)^n. \tag{4.2}$$

As $R \to R_c$, we can confine ourselves to the first term of the series, the coefficient of which (β_1) will henceforth be designated simply β. As to $\gamma(R)$, it vanishes exponentially as $R \to R_c$; this is explained by the Coulomb barrier in the effective potential $U(r)$ for slow positrons (the same as in the case of the spherical nucleus[3]). In this case $\beta = \beta(Z_1, Z_2)$ is expressed in terms of the wave function of the level lying at the edge of the lower continuum

$$\beta = \int \psi_0^+ R \frac{\partial V}{\partial R} \psi_0 d^3r,$$
$$V(\mathbf{r}) = -\alpha\left(\frac{Z_1}{r_1} + \frac{Z_2}{r_2}\right). \tag{4.3}$$

To determine $\psi_0(\mathbf{r})$ it is necessary to solve the Dirac equation with energy $\epsilon = -1$

$$(\boldsymbol{\sigma}\mathbf{p})\varphi_1 = -V\varphi_2, \quad (\boldsymbol{\sigma}\mathbf{p})\varphi_2 = -(V+2)\varphi_1, \quad \psi_0 = \begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix}, \tag{4.4}$$

that is, it is necessary to find the value of $R = R_c$ at which

$$\psi_0 \propto (\xi^2 - \eta^2)^{-\nu}, \quad \nu = 1 - (1 - \zeta^2/4)^{1/2} \tag{4.5}$$

near the nuclei, and as $r \to \infty$

$$\psi_0 \propto \exp\{-(8\zeta r)^{1/2}\} \tag{4.6}$$

($Z_1 = Z_2 = Z$, $\rho = 2Z\alpha$; ξ and η are elliptic coordinates in the two-center problem). We shall show that $\gamma = 2\,\mathrm{Im}\,\epsilon$ is completely determined at the threshold $R \to R_c$ if ψ_0 and β are known.

We consider first the scalar case. The Klein-Gordon equation is mathematically equivalent to the Schrödinger equation with effective energy E and effective potential U:

$$E = {}^1/_2(\varepsilon^2 - 1) = {}^1/_2 k^2, \quad U = \varepsilon V - {}^1/_2 V^2. \tag{4.7}$$

For the two-center problem at $r \gg R$ and ϵ close to -1 we have

$$U(r) = \zeta/r - \zeta^2/2r^2. \tag{4.8}$$

As $k \to 0$, the turning point $r_0 = 2\zeta k^{-2}$ lies far from the nucleus (Fig. 4), and the wave function of the quasidiscrete level takes the form[5)]

$$\chi_k(r) = iNp^{-1/2}\exp\left\{i\left(\int_{r_0}^{r} p\,dr - \frac{\pi}{4}\right)\right\}, \quad r > r_0,$$
$$\chi_k(r) = N|p|^{-1/2}\exp\left\{\int_{r}^{r_0} |p|\,dr\right\}, \quad r < r_0, \tag{4.9}$$

where $p(r) = k(1 - r_0/r)^{1/2}$ if $r \gg 1$. From this we get at $r > r_0$

$$\chi_k(r) = Ne^{i\pi/4}k^{-1/2}\left(1 - \frac{r_0}{r}\right)^{-1/4}$$
$$\times \exp\left\{i\left[k(r(r - r_0))^{1/2} - \frac{2\zeta}{k}\operatorname{Arth}\left(\frac{r - r_0}{r}\right)^{1/2}\right]\right\}. \tag{4.10}$$

In particular, as $r \to \infty$ the function χ_k has an asymptotic form corresponding to a diverging wave:

$$\chi_k(r) \approx Nk^{-1/2}\exp\{i(kr - \zeta k^{-1}\ln r + \text{const})\}.$$

The probability of e^+ production is equal to the flux of the outgoing particles at infinity, i.e.,

$$\gamma = |N|^2. \tag{4.11}$$

In the region $1 \ll r \ll r_0$, the potential is $U(r) \gg E$, and therefore $\chi_k(r)$ coincides here with the wave function $\chi_0(r)$ of the 1s level in the critical point. At $r \gg R$ we have

$$\chi_0(r) = (4\pi)^{1/2} r\psi_0(r) \approx C_0 r^{1/4}\exp\{-(8\zeta r)^{1/2}\} \tag{4.12}$$

(the constant C_0 is obtained by normalizing χ_0^2 to unity).

The damping of $\chi_0(r)$ as $r \to \infty$ is due to the Coulomb barrier in the effective potential (4.8). In the region $1 \ll r < r_0$ we have

$$\chi_k(r) = Nk^{-1/2}\left(\frac{r_0}{r} - 1\right)^{-1/4}\exp\left\{\frac{2\zeta}{k}\left[\operatorname{arctg}\left(\frac{r_0}{r} - 1\right)^{1/2} - \left(\frac{r}{r_0}\left(1 - \frac{r}{r_0}\right)\right)^{1/2}\right]\right\}$$

By matching χ_k with χ_0 at $1 \ll r \ll r_0$ we establish the connection between the constants N and C_0. As a result,

$$\gamma = (2\zeta)^{1/2}|C_0|^2\exp\{-2\pi\zeta/k\}. \tag{4.13}$$

The calculations for spin 1/2 are analogous. The asymptotic form of the bound level at the critical point takes now the form ($r \gg R$)

$$G = C_1(\zeta/2r)^{1/4}\exp\{-(8\zeta r)^{1/2}\},$$
$$F = -C_1(2r/\zeta)^{1/4}\exp\{-(8\zeta r)^{1/2}\}, \tag{4.14}$$

where G and F are the radial functions for the upper (φ_1) and lower (φ_2) spinors. As $r \to \infty$ we have

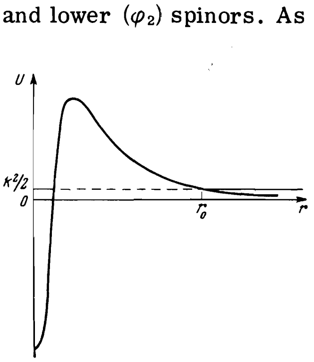

FIG. 4. Effective potential in the Coulomb problem for a level close to $\epsilon = -1$.

$$G = N\left(\frac{\varepsilon + 1}{2k\varepsilon}\right)^{1/2} e^{i\psi}, \quad F = -iN\left(\frac{\varepsilon - 1}{2k\varepsilon}\right)^{1/2} e^{i\psi},$$
$$\psi = kr - \frac{\zeta}{k}\ln r.$$

The main contribution to the flux of the outgoing particles at $\epsilon \to -1$ is made by the lower component F. The effective potential is obtained for it with the aid of the substitution $\chi = (1 - \epsilon + V)^{-1/2}F$:

$$U = \varepsilon V - \frac{1}{2}V^2 + \frac{\varkappa(\varkappa - 1)}{2r^2} - \frac{V''}{4W} + \frac{3}{8}\left(\frac{V'}{W}\right)^2 - \frac{\varkappa V'}{2rW},$$
$$W = 1 - \varepsilon + V. \tag{4.15}$$

At $V = -\zeta/r$, $\epsilon = -1$, and $\kappa = -1$ (1s ground level) we have

$$U(r) = \zeta/r - (\zeta^2 - 3/4)/2r^2, \quad r \gg 1. \tag{4.16}$$

At $\zeta \gtrsim 1$ one can apply the quasiclassical approximation to the potential U(r). The succeeding operations do not differ from those in the scalar case; we present the final formula:

$$\gamma(k) = 2|C_1|^2\exp\{-2\pi\zeta/k\}, \quad \zeta = (Z_1 + Z_2)/137 \tag{4.17}$$

(k is the positron momentum). We note that although the upper component G vanishes at $r \to \infty$, it is appreciable at $r \sim 1$. In particular, its contribution to the normalization condition for $\psi_0(\mathbf{r})$ (and hence to the constant C_1 and to the pre-exponential factor in $\gamma(k)$) is of the same order as the contribution of F. From (4.2) and (4.17) we obtain γ as a function of $\rho = R/R_c$:

$$\gamma = 2|C_1|^2\exp(-b/\sqrt{1 - \rho}), \quad b = \pi\zeta\sqrt{2/\beta}. \tag{4.18}$$

Substituting (4.18) in (2.19), we obtain $f(\eta)$ near the thres- threshold ($\eta = E/E_t$):

$$f(\eta) = f_0(\eta - 1)^{9/4}\exp\{-b/(\eta - 1)^{1/2}\} \quad \text{if} \quad \eta > 1,$$
$$f_0 = 4|C_1|^2(2\beta/\zeta^2)^{3/4}. \tag{4.19}$$

Thus, $\gamma(k)$ and $f(\eta)$ are determined by two constants (β and C_1), and to determine these it suffices to know the wave function of the 1s level at the critical point: $\epsilon = -1$, $R = R_c$.

In the calculation of the energy spectrum of the produced positrons we make use of the adiabaticity in the velocity of the nuclei and the condition $\gamma(R) \ll 1$. By virtue of these conditions, at a given distance $R < R_c$, almost all the positrons are emitted with one and the same kinetic energy $T = k^2/2 = \beta(R_c - R)/R_c$. The distribution of e^+ over the energies is given by

$$d\sigma/dT = A(T_m - T)^{1/2}e^{-a/\sqrt{T}}, \quad 0 < T < T_m, \tag{4.20}$$

where $a = \pi\zeta\sqrt{2} \approx 4.5\zeta$, $T_m = \beta(1 - E_t/E)$. This distribution differs from the Breit-Wigner distribution. The maximum ds/dT is obtained at an energy $T = T_0$ close to the upper end of the spectrum:

$$T_0 = T_m - a^{-1}T_m^{3/2} \quad \text{if} \quad T_m \ll m_e c^2, \tag{4.21}$$

and the distribution with respect to T is narrow: $\Delta T/T_0 \sim \sqrt{T_m} \ll 1$. With increasing E, the maximum of the probability $d\sigma/dT$ shifts towards smaller T, and the width of the spectrum increases.

We note the following with respect to the angular distribution of e^+. In the region $r \gg R$ we have

$$V(\mathbf{r}) = -\frac{\zeta}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = -\frac{\zeta}{r}\left[1 + \left(\frac{R}{2r}\right)^2 P_2(\cos\theta) + \ldots\right]$$

and therefore the Coulomb barrier in $U(\mathbf{r})$ deviates little from spherical symmetry. Since the sub-barrier region extends at $E \to E_t$ all the way to $r \sim k^{-2} \gg 1$, the angular distribution at the positron production threshold is isotropic.

We now present a numerical estimate of the cross section σ for spontaneous positron production. In the case of low supercriticality ($\delta \ll 1$) the constants C_1 and β can be obtained in the following manner. The solution of the Dirac equation with $\epsilon = -1$ and $\kappa = -1$ in a field $V(r) = -\zeta/r$ takes the form

$$G = cK_{i\nu}(z), \quad F = -\frac{cz}{2\zeta}\operatorname{Re}\left\{\left(1-\frac{2i}{\nu}\right)K_{1+i\nu}(z)\right\}, \tag{4.22}$$

where $\nu = 2(\zeta^2 - 1)^{1/2}$, $z = (8\zeta R)^{1/2}$, $\zeta > 1$. In the two-center problem, the wave function at $r \gg R$ coincides with (4.22), and the region $r \lesssim R$ makes a small contribution to the normalization integral if $R \ll 1$ (we recall that the maximum of $G^2 + F^2$ lies at $r \sim \bar{r} \sim 1$; see [2]). Therefore in the calculation of the normalization constant c we use expression (4.22) down to $r = 0$, and obtain

$$C_1 = \frac{c}{2}\left(\frac{\pi}{\zeta}\right)^{1/2} = \left\{\frac{\zeta^2}{1+{}^2/_3\zeta^2}\frac{\operatorname{sh}\pi\nu}{\nu}\right\}^{1/2}, \tag{4.23}$$

$$\gamma(k) = \gamma_0 \exp\left\{-2\pi\left[\frac{\zeta}{k} - (\zeta^2-1)^{1/2}\right]\right\}, \tag{4.24}$$

where

$$\gamma_0 = \frac{3\zeta^2(1-e^{-2\pi\nu})}{(2\zeta^2+3)\nu} = \begin{cases} 6\pi/5 & \text{if} \quad \zeta = 1 \\ 3/4\zeta & \text{if} \quad \zeta \gg 1 \end{cases}.$$

The exponential factor in (4.24) coincides exactly with the penetrability of the barrier for the effective potential $U(r) = \zeta/r - (\zeta^2 - 1)/2r^2$, which is obtained from (4.16) by adding the Langer correction $1/8r^2$ (the latter, as is well known [16,18], greatly improves the accuracy of the WKB method at small r). The pre-exponential factor γ_0 at $Z = 90-100$ is a sluggish function of Z and assumes values close to 1/2 (see Fig. 5). By examining the course of the $1s_{1/2}$ level near the boundary $\epsilon = -1$ we can also obtain β:

$$\beta = {}^3/_5, \quad b = \pi\zeta\sqrt{{}^{10}/_3} = 5.7\zeta \tag{4.25}$$

(the calculation method is similar to that in [3]). Substituting γ_0, β, and b in (4.19), we arrive at estimates for the cross section for the production of positrons σ as a function of $\eta = E/E_p$ and the charge of the colliding nuclei, as given in the table. Although σ is exponentially small at the threshold, it increases rapidly with increasing E. Comparison with (2.11) shows that at $\eta \sim 2$ the production of e^+ occurs in one out of several thousand nuclear collisions in which distances $R < R_c$ are reached.

To observe the spontaneous production of e^+ it is convenient to increase the charges of the colliding nuclei. Indeed, this (i) greatly increases the factor σ_0 in (2.17) and (ii) increases the critical distance [12] R_c, as the

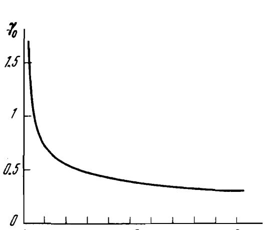

FIG. 5. Pre-exponential factor γ_0 in formula (4.24) as a function of $\zeta = 2Z/137$.

Nuclei	Z	ζ	$\sigma_0 \cdot 10^{24}$, cm^2	σ, cm^2		
				$\eta = 1,25$	$\eta = 1,5$	$\eta = 2$
U + U	92	1.34	30	$3 \cdot 10^{-31}$	10^{-28}	$\sim 10^{-26}$
Cf + Cf	98	1.43	160	$6 \cdot 10^{-31}$	$3 \cdot 10^{-28}$	$\sim 5 \cdot 10^{-26}$

result of which the experiments can be performed in a wider range of energies

$$1 < \eta = \frac{E}{E_t} < \frac{R_c}{2r_0 + \Delta R}. \tag{4.26}$$

Here r_0 is the radius of the nucleus and $\Delta R = 4-5$ F. The upper limit of η corresponds to the start of the nuclear processes. The values of ΔR for each pair of nuclei can be determined experimentally by observing the deviations of the cross section for elastic scattering from the Rutherford cross section [19].

5. BACKGROUND EVENTS

We consider several side effects that can complicate the experimental observation of positron production via the quasistatic mechanism discussed here.

1. When heavy particles collide, e^+e^- pairs can be produced directly by the alternating electric field of the nuclei. Under the condition $\zeta \gg V/c$, the cross section of this process can be estimated by using the quasiclassical approximation. The effective bremsstrahlung cross section in Coulomb collisions is equal to

$$\frac{d\sigma_\gamma}{d\omega} = B\alpha R_0^2\left(\frac{v}{c}\right)^2 \exp\left(-\frac{\pi\omega}{\omega_c}\right), \tag{5.1}$$

where $\omega_c = v/R_0$ and B is a numerical factor.

$$B = \frac{4\pi}{3\sqrt{3}}\left(\frac{Z_1A_2 - Z_2A_1}{A_1 + A_2}\right)^2.$$

Taking part in the production of the electron-positron pairs are the Fourier components of the electric field with frequency $\omega > 2m_e$. Multiplying (5.1) by the coefficient of pair conversion of the dipole radiation $\beta_{E1} = (2\alpha/3\pi)(\ln 2\omega - 5/3)$ and integrating over the frequency region $\omega > 2m_e$, we obtain the estimate [20]

$$\sigma^{(0)}(\gamma \to e^+e^-) = B'\alpha^2R_0^2\left(\frac{v}{c}\right)^2\frac{\omega_c}{m_e}\exp\left(-2\pi\frac{m_e}{\omega_c}\right),$$
$$B' = \frac{\ln 2}{9\pi\sqrt{2}}\left(Z_1\frac{A_2}{A_1} - Z_2\right)^2 \sim 10^{-2}. \tag{5.2}$$

This expression, however, is valid only when $Z \ll 137$. In our case it is necessary to take into account also the fact that the probability of production of slow positrons is strongly suppressed owing to the Coulomb barrier. Introducing in the integration over the bremsstrahlung-photon spectrum (5.1) an additional factor $\exp[-2\pi Ze^2/\hbar v']$, where v' is the positron velocity, we obtain

$$\sigma(\gamma \to e^+e^-) < \sigma^{(0)}(\gamma \to e^+e^-)\exp\{-3\pi\zeta^{2/3}(m/\omega_c)^{1/3}\}. \tag{5.3}$$

Numerically, at $R_0 = 0.1$ and $\zeta = 1.34$ we have $\sigma(\gamma \to e^+e^-) < 10^{-40}$ cm^2, i.e., this process can be neglected. The smallness of the cross section σ is due to the fact that the motion of the nuclei is nonrelativistic. For identical nuclei ($Z_1/A_1 = Z_2/A_2$), the cross section σ decreases by a few more orders of magnitude, for in this case there is no dipole radiation and $B = B' = 0$.

2. More important is the production of e^+e^- pairs by pair conversion in transitions resulting from Coulomb excitation of nuclei. The pair-conversion probability depends little on Z and at energies on the order of several MeV it amounts to $\approx 10^{-3}$ of the radiative-transition probability [21]. The Coulomb-excitation cross section [22,23] is equal to

$$\sigma_C(EL) = \eta^2R_0^{2(1-L)}\frac{B(EL)}{(Z_1e)^2}f_L(\xi), \tag{5.4}$$

where $\eta = Z_1Z_2e^2/\hbar v$, L is the multipole moment of the transition,

$$\xi = \frac{Z_1 Z_2 e^2}{\hbar}\left(\frac{1}{v_i} - \frac{1}{v_f}\right) = \eta \frac{\Delta E}{2E}\left(1 + \frac{M_2}{M_1}\right) = \frac{\tau_0 \Delta E}{2}, \quad (5.5)$$

$\Delta E > 2m_e c^2$ is the excitation energy of the nucleus, and τ_0 is defined in (2.4). At $\xi > 1$ we have $f_L(\xi) \propto e^{-2\pi\xi}$. For U + U collisions, $\xi = 1.5$ at the threshold ($\Delta E = 2m_e c^2$) and $f_2(\xi) \approx 10^{-4}$. At L = 2 and Z ~ 100, assuming by way of estimate $B(E2) = e^2 Q_0^2$ (where $Q_0 \sim 10^{-23}$ cm² is the quadrupole moment of the nucleus), we get $\sigma_c(E2) \sim 10^{-25}$ cm². The cross section for e^+ production in Coulomb excitation is $\sim 10^{-28}$ cm², which is comparable with the cross section for spontaneous production of positrons (and exceeds it at the threshold). In principle, however, processes with $e^\pm$ pair production are experimentally distinguishable from the process of interest to us, in which only positrons are emitted at infinity[6)].

3. The filling of the K orbit of the UA is possible through capture of target electrons by the "bare" nucleus. We consider a symmetrical resonant charge exchange ($Z_1 = Z_2$). The electron wave function, which goes over into (3.11) at $t \to -\infty$, has the following form at finite t:

$$\Psi(t) = \frac{1}{\sqrt{2}}\left[\chi_2 \exp\left\{-i\int_{-\infty}^{t} E_2\, dt\right\} + \psi_3 \exp\left\{-i\int_{-\infty}^{t} E_3\, dt\right\}\right].$$

As $t \to \infty$, taking (3.5) and (3.7), we obtain from this

$$\Psi(\infty) = \frac{e_2 + e_3}{2}\varphi_a(1)\varphi_a(2) + \frac{e_2 - e_3}{2}\varphi_b(1)\varphi_b(2), \quad (5.6)$$

where

$$e_{2,3} = \exp\left\{-i\int_{-\infty}^{\infty} E_{2,3}(R(t))\, dt\right\}.$$

The second term in (5.6) corresponds to charge exchange; its probability is

$$w(\rho) = \sin^2\left\{\frac{1}{2}\int_{-\infty}^{\infty} \Delta\varepsilon(R(t))\, dt\right\} \approx \sin^2\left\{\frac{1}{v}\int_{\rho}^{\infty} \frac{\Delta\varepsilon(R) R\, dR}{(R^2 - \rho^2)^{1/2}}\right\} \quad (5.7)$$

(ρ is the impact parameter, $\Delta\epsilon(R) = E_2(R) - E_3(R)$). At $R \gg 1$, the distribution of the terms $\Delta\epsilon$ decreases exponentially:

$$\Delta\varepsilon(R) = Q(R) e^{-Z\alpha R} \quad (5.8)$$

(see[25], where the pre-exponential factor Q(R) was also calculated; the explicit form of Q(R), however, is immaterial here). Since $v \ll c$, it follows that $w(\rho) \approx 1/2$ when $\rho < \rho_0$, and the probability $w(\rho)$ decreases rapidly at $\rho > \rho_0$. The value of ρ_0 is determined from the condition $\Delta\epsilon(\rho_0) \sim v/c$, i.e., $\rho_0 \sim \ln(c/v)$. The order of magnitude of the charge-exchange cross section is[7)]

$$\sigma_1 = \frac{\pi}{2}\rho_0^2 \sim \left(\frac{\hbar}{m_e c}\right)^2 \ln^2\frac{c}{v} \sim 10^{-20}\ \mathrm{cm}^2 \quad (5.9)$$

(at v/c ~ 0.1). The mean free path of the nuclei with respect to resonant charge exchange is $l_1 = (n\sigma_1)^{-1} \sim 10^{-3}$ cm ($n = 5 \times 10^{22}$ nuclei/cm³ for U).

More probable is charge exchange at large distances[28], which leads to landing of the electron on high orbits: $n \sim Z\alpha c/v = 10$–20. Although the charge-exchange cross section can reach here values $\sigma_2 = \pi a_0^2 \sim 10^{-16}$ cm², the K shell of the incident nucleus remains vacant and can be filled via radiative transitions from the states with $n \gg 1$.

4. To estimate the probabilities of the radiative transitions we use a formula pertaining to hydrogen-like atoms[8)]:

$$w_\gamma(np \to 1s) = Z^4 \frac{2^7 n(n-1)^{2n-2}}{9(n+1)^{2n+2}} \cdot 1.6 \cdot 10^{10}\ \mathrm{sec}^{-1}. \quad (5.10)$$

From this we have at $Z\alpha \sim 1$

$$w_\gamma(np \to 1s) = \begin{cases} 2 \cdot 10^{17}\ \mathrm{sec}^{-1} & n = 2 \\ 0.26 Z^4 \alpha^3 n^{-3} \omega_0, & n \gg 1 \end{cases}$$

($\omega_0 = me^4/\hbar^3$). The probability of the transition np → 1s in a collision between a bare nucleus and a target atom is $w_n = w_\gamma T$, where T is the time during which R(t) $< \bar{r}_n$, with $\bar{r}_n$ the average radius of the np state. For n = 2 the radius is $\bar{r} \sim \hbar/m_e c$, $T = 3 \times 10^{-20}$ sec (see formula (2.12) with $\eta = \bar{r}/R_0 \sim 10$) and $w \lesssim 0.01$. With increasing n, we get $w \propto n^{-3}$, $T = \bar{r}_n/v \sim n^2$, and w_n decreases like 1/n. Therefore the probability of filling the K shell of the UA through radiative transitions can amount to several per cent. This quantity can be determined experimentally by studying the yield of the characteristic γ quanta.

If the electron is captured at levels nl with $l \gg 1$, then, owing to the selection rule $\Delta l = \pm 1$, the direct transition to the 1s level is impossible, and a cascade is produced. Just as in the case of mesic atoms[31], the probability w_γ decreases in this case strongly because of the factor $(\Delta E)^3$, which is characteristic of electric dipole transitions.

5. The vacancies in the K shell can also be filled through Auger transitions, but their role decreases with increasing Z and becomes negligible in heavy atoms: $w_A/w_\gamma < 0.05$ at $Z \geq 80$ (see[21]).

The processes considered in subsections 3 and 4 necessitate the choice of as thin a target as possible ($\sim 10^{-5}$–10^{-6} cm). Nonetheless, all the background effects listed above (which lead either to production of $e^\pm$ by other mechanisms or to filling of the K shell of the bare nucleus by electrons) do not exclude a possibility of observing quasistatistical production of positrons in nuclear collisions.

6. CONCLUSION

Thus, the cross section for spontaneous e^+ production in slow collisions of heavy nuclei is $\sigma \sim 10^{-26}$–10^{-27} cm² at Z ~ 95 and E ~ 1 GeV. In the derivation of these figures we used two approximations: 1) adiabaticity with respect to the velocity with which the nuclei approach each other; 2) the small-supercriticality condition (2.15).

Satisfaction of the approximation (1) is guaranteed by the fact that $v < 0.1c$ at $R_0 \sim 40$ F (we emphasize that spontaneous e^+ production is a quasistatic process and, in principle, takes place at an arbitrarily slow approach of the nuclei if $R < R_c$). The use of the condition 2) leads to factorization of σ in the form (2.17) and to formula (4.19) for $f(\eta)$. The factor σ_0 in (2.17), which determines the cross section, depends strongly on R_c (i.e., on Z_1 and Z_2). In this paper we used for R_c an asymptotic formula derived in[12]. Although a number of arguments can be advanced in favor of this approximation[9)], an exact numerical calculation of R_c in the two-center problem is highly desirable. We wish to call attention to this problem, which calls for solving the Dirac equation at only one energy $\epsilon = -1$.

We note in conclusion that the question of spontaneous positron production at $Z > Z_c$ is discussed also in[33,34]. There, however, it was concluded that $\gamma \sim (Z - Z_c)^2$ at the threshold (furthermore, the perturbation of the wave functions of the lower continuum is neglected in[33], which is incorrect when $Z > Z_c$; see[4]). Such a dependence of the width of the quasistationary level γ on

$Z - Z_c$ contradicts the formula $\gamma = \gamma_0 \exp\{-b[Z_c/(Z - Z_c)]^{1/2}\}$ obtained in[2]; here γ_0 and b are constants of the order of unity and depends on the nuclear radius R. The exponential vanishing of γ as $Z \to Z_c$ follows from the existence of a Coulomb barrier for slow positrons. This question was analyzed in detail[2-4] by the effective-potential method, which offers the most natural way of understanding the physics of the phenomena near the boundary of the lower continuum.

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APPENDIX

We consider the matrix element of the Coulomb interaction (3.6). At $Z < \alpha^{-1} = 137$ we can use the wave functions of the Dirac equation[29] in the field of a point-like charge Ze. For the $1s_{1/2}$ level, the Coulomb correction takes the form

$$I = 2\alpha \int_0^\infty \frac{dr}{r} \rho(r) \int_0^r dr' \rho(r') = \frac{4\alpha z}{\Gamma^2(2\gamma + 1)} J(2\gamma), \qquad (A.1)$$

where $z = \zeta/2 = Z\alpha$, $\gamma = \sqrt{1 - z^2}$ and

$$J(\lambda) = \int_0^\infty dx\, e^{-x} x^{\lambda-1} \int_0^x dy\, e^{-y} y^\lambda. \qquad (A.2)$$

The integral of (A.2) converges when $\lambda > -1/2$; we calculated by using the formula[35]

$$\int_0^\infty e^{-x} x^{p-1} \gamma(q, x)\, dx = \frac{\Gamma(p+q)}{2^{p+q} q} F\left(1, p+q; q+1; \frac{1}{2}\right)$$

(here $\gamma(q, x)$ is the incomplete Γ-function), and also the identity

$$F\left(1, 4\gamma + 1, 2\gamma + 2; \frac{1}{2}\right) = \sqrt{\pi} \frac{(2\gamma + 1)\Gamma(2\gamma)}{\Gamma(2\gamma + {}^1/_2)} - 1 - \frac{1}{2\gamma}.$$

As a result we get

$$J(\lambda) = 2^{-2\lambda} \Gamma(2\lambda) \left[\frac{\sqrt{\pi}\,\Gamma(\lambda + 1)}{\Gamma(\lambda + {}^1/_2)} - 1\right]. \qquad (A.3)$$

Putting $I = Z\alpha^2 g(z)$, we get ultimately ($\gamma = (1 - z^2)^{1/2}$)

$$\gamma g(z) = 1 - \Gamma(2\gamma + {}^1/_2) / \sqrt{\pi}\,\Gamma(2\gamma + 1). \qquad (A.4)$$

With increasing z, the function g(z) increases monotonically, especially near z = 1 (see Fig. 6). At $z \ll 1$ (nonrelativistic region) we have

$$g(z) = {}^5/_8 + {}^3/_4(1 - \ln 2) z^2 + O(z^4), \qquad (A.5)$$

and as $z \to 1$

$$g(z) = c_0 - c_1(1 - z^2)^{1/2} + \ldots \qquad (A.6)$$

($c_0 = 4 \ln 2$). Although the effective charge increases when the nuclei come closer together, I remains of the same order as α and no qualitative changes take place in the arrangement of the levels.

Expression (3.6) takes into account only the "instantaneous" Coulomb interaction $V = e^2/r$ between the electrons. At $Z\alpha \sim 1$, the K electrons have a velocity $\sim c$, and it is therefore necessary to add to V correction terms to account for the delay (the Breit operator):

$$V' = -\frac{e^2}{2r}\{(\alpha_1\alpha_2) + (\alpha_1 \mathbf{n})(\alpha_2 \mathbf{n})\}, \quad \mathbf{n} = \mathbf{r}/r, \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2. \qquad (A.7)$$

It can be shown, however[29], that $\langle V' \rangle = 0$ for the K shell. Therefore allowance for V' does not change the result.

We consider the question of the term crossing. To simplify the notation, we use the symbols

$$(g^2, g^2) = \int \Sigma_g^2(1) \frac{e^2}{r_{12}} \Sigma_g^2(2)\, dv_1\, dv_2 \qquad (A.8)$$

etc. Then (see Sec. 3)

$$V_{11} = (g^2, g^2), \quad V_{22} = (u^2, u^2), \quad V_{12} = (gu, gu),$$
$$V_{33} - V_{12} = V_{44} + V_{12} = (g^2, u^2). \qquad (A.9)$$

We note that for an arbitrary function $f(\mathbf{r})$ we have

$$(f, f) = \int f(1) \frac{e^2}{r_{12}} f(2)\, dv_1\, dv_2 > 0,$$

since in electrostatics the energy of self-action of any charge distribution is always positive. Therefore V_{11}, V_{22}, and $V_{12} > 0$. Taking (3.9) into account, it follows therefore that $E_3 > E_4$. Furthermore, we have

$$E_2 - E_3 = (\Delta^2 + V_{12}^2)^{1/2} - V_{12} + {}^1/_2(g^2 - u^2, g^2 - u^2) > 0,$$
$$E_3 - E_1 = (\Delta^2 + V_{12}^2)^{1/2} - V_{12} + 2(gu, gu) - {}^1/_2(g^2 - u^2, g^2 - u^2). \qquad (A.10)$$

Therefore at any R for levels with spin S = 0 we have $E_1 < E_3 < E_2$. The absence of term crossing in the relativistic two-center problem follows also from the Wigner-Neumann theorem, since the valuables do not separate in this case (see[16], p. 334).

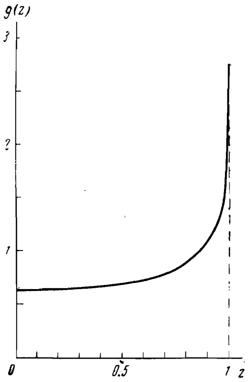

FIG. 6. The function g(z), which determines the dependence of the Coulomb splitting of the levels on the nuclear charge Z; here $z = Z\alpha$.

Note added in proof (4 June 1973). In a recent paper (M. Bawin and J. P. Lavine, Nuovo Cimento 15A, 38, 1973) it is stated that the spontaneous production of positrons $Z > Z_c$ is possible, thus contradicting our results. The authors base their conclusion on the fact that the wave function of a positron at rest vanishes identically in a Coulomb field. We note that the last statement is true only when $Z \neq Z_c$. Although $\gamma = 0$ at the critical point, this does not mean that the width γ remains equal to zero at $Z > Z_c$. In the single-particle theory, the dropping of the level into the lower continuum with increasing Z is inevitable, since the decrease of the energy is $\Delta\epsilon_0 = -\beta(Z - Z_c) < 0$.

1) For example, with the aid of the electron rings of a "Smokotron" type accelerator. [9]

2) Here θ is the scattering angle in the c.m.s.

3) We note that in order for two uranium nuclei to come close together to a distance $R_0 = 40$ F, an initial energy $E = 2Z^2e^2/R_0 \approx 600$ MeV is required. The height of the Coulomb barrier for the uranium nuclei amounts to $E_b = Z^2e^2/2r_0 = 800$ MeV at $r_0 = 1.2A^{1/3} = 7.5$ F. Since nuclear interactions begin at $E > 2E_b$, experiments on positron production can be performed in the energy interval $E_t < E < 2E_b$.

4) Thus, $\delta = 0.08$ for the uranium nuclei and $\delta = 0.15$ for the Cf + Cf collision. We consider only the lower level ls, since the values of Z_c for the remaining states are too large. For example, for the $2p_{1/2}$ level the charge is $Z_c = 185$, which already exceeds the summary charge of the two uranium nuclei.

[5]At $\zeta > 1$ the condition for the applicability of the classical approach to the Coulomb field [16] is always satisfied in the threshold region:

$$Z\alpha / \hbar v = \zeta(1 + k^{-2})^{1/2} \gg 1.$$

[6]In the case of pair conversion, emission of monochromatic positrons is also possible, wherein e^- is captured by one of the K-shell vacancies [24]. The probability of such a process is $w = w_1 w_3$, where $w_1 \sim 10^{-4}$ is the probability of conversion with production of a pair of the discrete spectrum, and w_3 is the probability of electron capture at an atomic level. For M1 and E2 transitions, $w \sim 10^{-6}-10^{-7}$ (see [24]). The most dangerous in the sense of the background are E1 nuclear transitions.

[7]This formula differs from the corresponding formula in theory of atomic collisions [26,27] in that the Bohr radius a_0 is replaced by the Compton wavelength of the electron.

[8]See e.g., [29]. Calculation with exact relativistic function [30] decreases the probability w_γ somewhat at large Z in comparison with (5.10).

[9]For example, the analogous asymptotic formula for Z_c in the case of a spherical nucleus is quite accurate [12,32].

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