Electric conductivity of ferromagnets with a domain structure

Yu. V. Zakharov, Yu. I. Man'kov, and R. G. Khlebopros

Physics Institute, Siberian Division, USSR Academy of Sciences
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The effect of an inhomogeneous internal magnetic field of ferromagnets with a domain structure on their conductivity is determined. The change in the conductivity with respect to the homogeneous case is determined in the mean free time approximation at low temperatures. An anisotropy of conductivity is observed in a plane perpendicular to the easy axis of the ferromagnet. The influence of domain structure on conductivity is related to the presence of a group of electrons near the domain wall which execute periodic movement in the field of two neighboring domains, the frequency differing from the cyclotron value.

The effect of the magnetic subsystem in ferromagnets on their conductivity is due to an internal magnetic field $\mathbf{B}(\mathbf{r}) = 4\pi\mathbf{M}(\mathbf{r})$ and to collective excitations of this subsystem. In a single-domain ferromagnet, the dependence of the conductivity on the magnetic subsystem has been studied quite thoroughly. In a many-domain ferromagnet, as compared with a uniformly magnetized one, there appear in the spin-wave spectrum new branches[1,2], whose influence on the conductivity has been investigated by Turov and Voloshinskiĭ[3] and by Volkenshteĭn and Dyakina[4]. Furthermore, in a many-domain ferromagnet the internal magnetic field becomes nonuniform. The number of electrons that experience the nonuniformity of the magnetic field is at low temperatures proportional to the ratio of the mean radius R of the cyclotron precession of a conduction electron in the magnetic field to the domain width D, and consequently the addition to the resistivity because of the nonuniformity of the field will have this same order of magnitude. Usually $R/D = 1$ to 10^{-2}; therefore it seems of interest to investigate the effect of the nonuniform internal magnetic field, in a ferromagnetic specimen with domain structure, upon its conductivity.

It is known[5] that the conductivity can be written in the form

$$\sigma_{ik}(\omega) = e^2 \int_0^\infty dt \int_\Gamma d\Gamma(r_k, f)\, v_i(t)\, e^{-i\omega t}. \qquad (1)$$

Here (a, b) are Poisson brackets, $f = f(\mathbf{r}, \mathbf{p})$ is the equilibrium distribution function, Γ is the phase space, $d\Gamma = 2(2\pi\hbar)^{-3} d\mathbf{p}\, d\mathbf{r}$, $v_i(t)$ is the electron velocity, which is determined from the equation

$$\dot{v}_i(t) = (v_i, \mathscr{H}); \qquad (2)$$

$\mathscr{H}$ is the Hamiltonian corresponding to the equilibrium distribution function.

We take account of relaxation in the conduction-electron system in the form of a factor $e^{-t/\tau}$; τ is the free-path time of the electrons. We consider the system of electrons uniform. Then the distribution function depends only on the electron energy ϵ, and $(r_k, f) = v_k \partial f/\partial\epsilon$. We assume that the dispersion law is quadratic and isotropic; in this case the Hamiltonian can be written in the form

$$\mathscr{H} = \frac{1}{2m}\left[\mathbf{P} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right]^2, \qquad (3)$$

where $\mathbf{P}$ is the canonical momentum and $\mathbf{A}$ is the vector potential. We suppose that the electric field in the specimen is uniform, $E(t) = E_0 \cos\omega t$. The requirement of uniformity in the case of a nondegenerate gas of conduction electrons is satisfied up to high frequencies ω of the external field; in particular, this requirement is satisfied near the cyclotron resonance frequency[6]. For a degenerate gas, the electric field in the specimen will be uniform only at small ω, and in this case we shall consider the static conductivity.

Even under the assumptions made, solution of equation (2) in the general case is difficult. But in the case of a single-domain ferromagnet, the magnetic field is uniform ($\mathbf{B}(\mathbf{r}) \equiv \mathbf{B}$, $\mathbf{A} = (0, Bx, 0)$), and this equation has a simple solution. If the z axis is parallel to $\mathbf{B}$, then

$$v_x(t) = v_\perp \sin(\Omega t + \varphi), \quad v_y(t) = v_\perp \cos(\Omega t + \varphi), \quad v_z(t) = \text{const.} \qquad (4)$$

Here $\Omega = |e|B/mc$ is the cyclotron frequency; m is the effective mass of a conduction electron; $v_\perp^2 = v_x^2 + v_y^2$; and φ is the phase response.

In the case of a ferromagnet with domain structure, the magnetic field $\mathbf{B}(\mathbf{r})$ is significantly nonuniform. We shall consider a single 180-degree domain wall. Its width δ can usually be neglected in comparison with the cyclotron radius. If we now choose the x axis perpendicular to the plane of the wall, the y axis in the plane of the wall, and the z axis parallel to the easy axis of the ferromagnet, then the magnetic field in the specimen is $B(x) = B\,\mathrm{sign}\,x$, $\mathbf{A} = (0, B|x|, 0)$. In this case also, equation (2) can be solved. In each domain, $x > 0$ and $x < 0$, we get a solution analogous to (4). On joining the solutions of equation (2) for adjoining domains at the point $x = 0$ for electrons crossing the domain wall[7], we get the components of velocity $v_i^{(W)}$ in the form of a piecemeal function. The expansion of $v_i^{(W)}$ as a Fourier series has the form

$$v_x^{(w)}(t) = v_\perp \frac{2\cos\alpha}{\alpha} \sum_{n=0}^{\infty} \frac{(-1)^n}{1 - (\pi/2\alpha)^2 (2n+1)^2} \times \sin\left[(2n+1)\frac{\pi}{2\alpha}(\Omega t + \varphi)\right], \qquad (5)$$

$$v_y^{(w)}(t) = v_\perp \frac{\sin\alpha}{\alpha}\left\{1 + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{1 - (\pi/\alpha)^2 n^2} \cos\left[n\frac{\pi}{\alpha}(\Omega t + \varphi)\right]\right\},$$

$$v_z^{(w)}(t) = \text{const.}$$

The components $v_{x,y}^{(W)}$ of electrons bound to the walls are periodic functions with a period $T = 4\alpha/\Omega$ different from the period of cyclotron precession in a uniform magnetic field. The parameter α is determined by the value of the momentum of the electron at the point

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$x = 0$ ($P_y = P_\perp \cos\alpha$) and is equal to half the arc that the electron traverses in the uniform magnetic field of a domain between two successive passages through the wall.

Thus in this problem two types of electrons are to be considered; their motion in the specimen is described by the expressions (4) and (5). If the electrons that cross the domain wall and exhibit a new periodicity of motion occupy a certain volume Γ_1 of the phase space, then we get for the conductivity of a ferromagnet with domain structure

$$\sigma_{ik} = -e^2 \int_0^\infty dt \frac{\partial f}{\partial \varepsilon} \exp\left[-t\left(\frac{1}{\tau} + i\omega\right)\right] \times \left\{ \int_{\Gamma_1} d\Gamma\, v_i^{(w)}(t) v_k^{(w)}(0) + \int_{\Gamma - \Gamma_1} d\Gamma\, v_i(t) v_k(0) \right\}. \quad (6)$$

Conduction electrons in a magnetic field can be characterized by two quantities of the dimension of time. These are the period of the motion and the relaxation time. If an electron, between two successive collisions, does not once pass through the wall and consequently does not exhibit the new periodicity of motion, then there is no reason to assign it to the electrons of Γ_1. The belonging of an electron to Γ_1 is determined by the inequality $\tau > T = 4\alpha/\Omega$. Hence it is possible to determine a limiting value for α: $\alpha_0 = \tau\Omega/4$. Since the parameter α characterizes the trajectory of an electron bound to the domain wall, it can be stated that electrons for which $0 \le \alpha \le \alpha_0$ belong to Γ_1. For $\tau\Omega \ge 4\pi$, the parameter α_0 is equal to its limiting value π.

On substituting (4) and (5) into (6), transforming to variables of integration ϵ, φ, and α, and integrating, we get for a nondegenerate gas of conduction electrons, $f(\epsilon) = \exp\{-(\epsilon - \mu)/k_0T_0\}$,

$$\mathrm{Re}\,\sigma_{xx}(\omega) = \sigma_0\{(1 - LF)G(\omega, \Omega) + {}^1\!/_2 LU(\alpha_0, \omega)\}. \quad (7)$$

Here $\sigma_0 = ne^2\tau/m$, n is the density of the electron gas, and

$$L = \frac{12R_T}{\sqrt{\pi}D}, \quad R_T = \frac{c}{eB}(2mk_0T_0)^{1/2}, \quad F = {}^1\!/_8(\sin\alpha_0 - \alpha_0\cos\alpha_0),$$

$$G(\omega, \Omega) = \frac{1 + \tau^2(\Omega^2 + \omega^2)}{[1 + \tau^2(\Omega^2 - \omega^2)]^2 + 4\tau^2\omega^2},$$

$$U(\alpha_0, \omega) = \sum_{n=0}^{\infty} \int_0^{\alpha_0} \frac{d\alpha \cos^2\alpha \sin\alpha}{\alpha[1 - (\pi/2\alpha)^2(2n+1)^2]^2} G(\omega, \omega_1), \quad (8)$$

$$\omega_1 = \Omega(2n+1)\pi/2\alpha.$$

The expression for σ_{yy} has a form analogous to (7) but contains instead of the function U

$$V(\alpha_0, \omega) = \frac{0{,}5}{1 + \tau^2\omega^2} \int_0^{\alpha_0} \frac{\sin^3\alpha}{\alpha} d\alpha + \sum_{n=1}^{\infty} \int_0^{\alpha_0} \frac{d\alpha \sin^3\alpha}{[1 - (\pi/\alpha)^2 n^2]^2} G(\omega, \omega_2), \quad (9)$$

$$\omega_2 = \pi\Omega n/\alpha.$$

The expression (7) applies to ferromagnets with a plane-parallel domain structure; the density of walls is taken into account by the factor 1/D in the parameter L. The product LF is proportional to the number of electrons bound to a wall and exhibiting the new period of motion. Consequently the first term in (7) is the conductivity of electrons that are not bound to a domain wall; the second term gives the contribution of electrons that exhibit the new period of motion. The dependence of the components of the conductivity tensor on frequency ω is shown graphically in Fig. 1. The height of

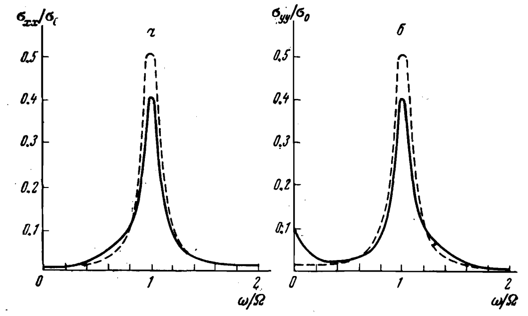

FIG. 1. Dependence of the conductivity of a ferromagnet with domain structure on the frequency of the external field (solid line). Dotted line: uniformly magnetized ferromagnet. The electric field is applied in the plane perpendicular to the easy axis of the ferromagnet: a—perpendicular to the plane of the wall; b—in the plane of the wall ($R_T/D = 0.1$; $\tau\Omega = 10$).

the resistance peak is basically determined by the first term in (7). Consequently, if one knows the parameters of the domain structure, one can determine the number of electrons in Γ_1 from the amount of absorption at cyclotron resonance.

From the expressions for σ_{xx} and σ_{yy} there follows an anisotropy of the conductivity in the plane perpendicular to the easy axis of the ferromagnet. This effect is wholly due to the domain structure. The anisotropy shows up most clearly in the static case ($\omega = 0$); then formulas (7)—(9) simplify:

$$G(0, \Omega) = (1 + \tau^2\Omega^2)^{-1},$$

and it follows that for $\tau\Omega \gg 1$

$$\sigma_{xx} \approx \sigma_0(\tau\Omega)^{-2}, \quad \sigma_{yy} \approx \sigma_0(R_T/D + 1/\tau^2\Omega^2).$$

The expression for σ_{xx} is the usual one for conductivity in a magnetic field. The factor $(\tau\Omega)^{-2}$ describes the localization of the conduction electrons by the magnetic field. The new part of the expression for σ_{yy} is determined by the first term in V, with the factor $(1 + \tau^2\omega^2)^{-1}$. Such a dependence of the conductivity on the frequency of the external field occurs in the absence of a magnetic field, that is when there is no localization of the electronic trajectories. Actually, it is clear from (5) that the mean value of $v_y^{(W)}$ over a period is different from zero, and this means absence of localization of the electrons in the y direction. The different nature of the motion of electrons bound to the wall in the x and in the y directions leads to the anisotropy of the conductivity.

Being alternately in domains with different orientations of the magnetization, the electrons experience during a period a certain mean field. In our case of domain structure, this field is zero. For this reason the Hall current of electrons from Γ_1 is zero. The Hall current of electrons that do not cross a wall will have different signs in adjoining domains. The conductivity in the z direction remains the same as in a single-domain specimen.

In the case of a degenerate gas of conduction electrons,

$$f(\varepsilon) = \left[1 + \exp\left(-\frac{\varepsilon - \mu}{k_0T_0}\right)\right]^{-1},$$

and as was mentioned above, we shall be interested in the static conductivity. For the difference $\Delta\sigma_{xx}$ between the conductivity $\sigma_{xx}^{(W)}$ of a specimen with domain structure and the conductivity of a uniformly magnetized

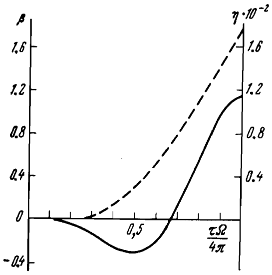

FIG. 2. The solid line shows the dependence of $\Delta\sigma_{xx}$, the dotted the dependence of $\Delta\sigma_{yy}$, on the free-path time of an electron. $\beta = D\Delta\sigma_{xx}/R_0\sigma_{xx}$, $\eta = D\Delta\sigma_{yy}/R_0\sigma_{yy}$.

specimen, we have

$$\Delta\sigma_{xx}/\sigma_{xx} = -LF + {}^1/_2 LU(\alpha_0, 0)(1+\tau^2\Omega^2). \tag{10}$$

Here $L = 9R_0/2D$, where R_0 is the cyclotron radius of electrons from the Fermi surface. The dependence of $\Delta\sigma_{xx}$ on τ for $0 < \tau\Omega < 4\pi$ is shown in Fig. 2 (solid line). The behavior of the curve can be understood by comparing $G(0, \Omega)$ and $G(0, \omega_1)$. At small τ ($\alpha_0 < \pi/2$), among the electrons bound to the wall will be only electrons with $\omega_1 > \Omega$; this indicates a larger localization of electrons by the magnetic field, as compared with the uniform case. This in turn must lead to a decrease of conductivity, that is to a negative value of $\Delta\sigma_{xx}$. With increase of τ, there appear in the phase volume Γ_1 electrons for which $\omega_1 < \Omega$ ($\alpha_0 > \pi/2$). Because of them, the conductivity of the specimen will increase, and $\Delta\sigma_{xx}$ will also increase. At $\alpha_0 \approx \pi$, when the number of electrons with $\omega_1 < \Omega$ becomes larger than the number of electrons with $\omega_1 > \Omega$, the value of $\Delta\sigma_{xx}$ turns positive.

The expression for $\Delta\sigma_{yy}/\sigma_{yy}$ can be obtained by replacement of $U(\alpha_0, 0)$ in (10) by the function $V(\alpha_0, 0)$, the first term in which basically determines the value of this ratio.

To be noted is the strong dependence of the conductivity of a many-domain specimen on the free-path time of an electron for $1 < \tau\Omega < 4\pi$. This is due to the fact that in this interval of variation of τ, the number of electrons in the phase region Γ_1 depends on the free-path time. With increase of τ (for $\tau\Omega > 4\pi$), the value of $\Delta\sigma_{xx}$ changes little, whereas

$$\Delta\sigma_{yy}/\sigma_{yy} \sim \tau^2\Omega^2.$$

On the basis of similar ideas about the effect of domain structure on the conductivity of ferromagnets, Man'kov attempted to explain qualitatively the experiments of Semenenko and Sudovtsev[9] on the magnetoresistance of ferromagnetic metals in weak magnetic fields at low temperature.

It is interesting to follow the change of conductivity with distance from the domain wall:

$$\frac{\Delta\sigma_{yy}(x)}{\sigma_{yy}} = \frac{2}{\pi} I(\gamma)\left[\frac{1}{\pi}V(\pi, 0)\tau^2\Omega^2 - 1\right],$$

$$I(\gamma) = \frac{2}{3}\gamma^{1/2}\left\{\Pi\left(-2k^2, k^2, \frac{\pi}{2}\right) + \left[\frac{17}{15}K(k) + \frac{32}{15}E(k)\right](k^4 - k^2 + 1)\right\},$$

$$k^2 = {}^1/_2(1 - {}^1/_2\gamma).$$

Here $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds, respectively; Π is the elliptic integral of the third kind; k is the modulus of the elliptic integrals; $\Delta\sigma_{yy}(x)$ is the difference between the conductivity of the specimen with domain structure and the conductivity of a specimen with uniform magnetization, at the point x; and $\gamma = x/R_0$. The function $I \approx 2$ at $x = 0$ and decreases monotonically to zero at $x = 2R_0$. The vanishing of I at this point means that for $x > 2R_0$ the conductivity becomes uniform and equal to the conductivity in a single-domain specimen. At large τ, therefore, the conductivity in the y direction is basically determined by the electrons in a layer of thickness $4R_0$. This, in essence, is analogous to the static skin effect[10].

We assume above that the domain-wall thickness was zero. Allowance for the structure of the wall gives a small correction to the conductivity, proportional to

$$\sigma_0\frac{\delta}{D}\left(\frac{\delta}{R_0}\right)^2(\tau\Omega)^{-2}, \quad \frac{\delta}{D}, \frac{\delta}{R_0} \sim 10^{-2} - 10^{-3}.$$

Thus one of the reasons for an effect of domain structure on the conductivity of ferromagnets may be the presence, at low temperature, of electrons that execute a periodic motion in the nonuniform magnetic field of two adjoining domains, with a frequency different from the cyclotron frequency. A consequence of this, in the case of a nondegenerate electron gas, is a change of the frequency dependence of the conductivity of a many-domain ferromagnet as compared with a uniformly magnetized one; in particular, the cyclotron resonance curve is deformed. The decrease in the height of the peak (increase in line-width) is proportional to the number of electrons that exhibit a new periodicity of motion, and it can be estimated from the ratio R_T/D. For $R_T/D = 0.1$, as in Fig. 1 the height of the line decreases by about 20%. Such a value of the ratio R_T/D occurs at 4°K in copper ferrite ($B = 1700$ G, $T_c = 728$°K) at specimen thickness 10^{-3} cm.

We remark that our results can be extended to ferromagnets, since, as was mentioned above, the conduction electrons in the specimen are in a magnetic-induction field. The static conductivity in the direction of the y axis is also proportional to the ratio R_T/D. The nonuniform internal magnetic field leads to an appreciable change of conductivity of many-domain metallic ferromagnets. Their resistivity for $1 < \tau\Omega < 4\pi$ depends strongly on the free-path time of the conduction electrons. The presence of domain structure leads to anisotropy of the conductivity in the plane perpendicular to the easy axis of the ferromagnet. Observation of effects due to domain structure is possible when there are a large number of electrons that exhibit a new peridocity of motion, and this is so only at large τ ($\tau\Omega > 1$).

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