Oscillation damping of the free surface of a superfluid liquid

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A complete set of equations describing the surface of a superfluid liquid at low temperatures is found by employing the kinetic equation for the distribution function of impurity surface excitations.[2] The oscillation spectrum of the surface is investigated by taking damping into account. It is shown that low-frequency second sound [1] is ordinary sound in a two-dimensional Fermi fluid if the impurities are strongly degenerate; at high frequencies, surface second sound goes over to zero sound. Capillary waves with a wavelength $\lambda \ll l$ (l is the excitation mean free path) experience Landau damping. Waves with $\lambda \gtrsim l$ are damped because of the presence of surface viscosity.

INTRODUCTION

Andreev and the author of[1] have previously obtained a system of equations of motion of the surface of a superfluid liquid in the case when the mean free path of the surface excitation l is much larger than the distance λ over which an appreciable change takes place in all the surface quantities. As is well known, surface impurity levels of He^3 exist in a solution of He^3 in He^4[2,3]. At sufficiently low temperatures, the impurities become degenerate and their mean free path begins to increase like T^{-2} with increasing temperature. Under such conditions, the equations of surface hydrodynamics derived in[1] no longer hold. In this case the boundary condition for the bulk equations is the kinetic equation for the distribution function of the surface impurities.

The damping of the oscillations of the free surface of a superfluid liquid (capillary waves) due to the presence of viscosity of the bulk normal component become seceedingly small with decreasing temperature. Under such conditions, the principal role is assumed by damping due to the surface normal component.

1. HAMILTONIAN OF SURFACE IMPURITY EXCITATIONS

When a particle moves on a curved moving surface, if the particle mean free path is much larger than the characteristic dimensions of the problem (say the wavelength of the surface oscillations), the Hamiltonian depends explicitly on the shape of the surface and on its velocity. We assume that the surface deviates little from a plane and the rate of its change is small in comparison with the particle velocity. We shall show that in this case the corrections to the Hamiltonian are quadratic in the displacement of the surface and in its velocity.

Let $\zeta(x, y, t) = z$ by the equation of the surface. Then the equations of particle motions are

$$m\ddot{x}_\alpha = \lambda\zeta_\alpha, \quad m\ddot{z} = -\lambda,$$

where m is the particle mass, x_α is the component of the two-dimensional vector $\mathbf{r}$, $\zeta_\alpha = \partial\zeta/\partial x_\alpha$, and λ is a Lagrange multiplier. Eliminating λ and z, we obtain

$$(\delta_{\alpha\beta} + \zeta_\alpha\zeta_\beta)\ddot{x}_\beta + 2\dot{\zeta}_\alpha\dot{\zeta}_\beta\dot{x}_\beta + \zeta_\alpha\zeta_\beta\dot{x}_\beta\dot{x}_\gamma + \ddot{\zeta}\zeta_\alpha = 0.$$

Thus, accurate to terms of first order in ζ, the equation takes the form $\ddot{\mathbf{x}} = 0$. In this approximation, the Hamiltonian is equal to

$$H = p^2/2m.$$

We determine now the energy and momentum of the excitation in terms of the corresponding surface quantities. We follow the procedure used by Khalatnikov for the bulk case[4]. We note that to find the surface quantities, the surface function ζ must be determined exactly. Just as in[1], we define the surface by means of the condition that there be no surface mass, i.e., if the liquid occupies the half-space $z = \zeta(x_\alpha, t)$ under the surface, then

$$\int ds \int_{-\infty}^{\zeta(x_\alpha, t)} \rho dz = M,$$

where ρ is the bulk density of the liquid, M is the total mass of the liquid, and ds is an element of area of the plane (x, y).

Let E_S' and $\mathbf{i}'$ be the energy and momentum per unit surface in a system moving with the velocity $\mathbf{v}_S$ of the superfluid motion. Then, in a system at rest, the energy E_S and the momentum i are given by

$$E_s = E_s' + \mathbf{i}'\mathbf{v}_s, \tag{1}$$

$$\mathbf{i} = \mathbf{i}'. \tag{2}$$

The momentum $\mathbf{i}$ is the sum of the momentum of motion of the superfluid component with density ν_{sf} and excitation momentum $\int \mathbf{p} n_p d\tau$ ($\mathbf{p}$ is the excitation momentum, n_p is the distribution function, $d\tau$ is the element of phase space):

$$\mathbf{i} = \nu_{sf}\mathbf{v}_s + \int \mathbf{p} n_p d\tau. \tag{3}$$

Inasmuch as in our definition of the surface

$$\nu_{sf} = -\int m_3 n_p d\tau$$

(m_3 is the mass of the Fermi particle He^3), it follows from a comparison of (2) and (3) that

$$\mathbf{i} = \int (\mathbf{p} - m_3\mathbf{v}_s) n_p d\tau = \int \mathbf{p} n_p d\tau, \quad \mathbf{p} \equiv \mathbf{p} + m_3\mathbf{v}_s. \tag{4}$$

The momentum and the excitation energy in the system where $\mathbf{v}_S = 0$ are determined by the variational derivatives

$$\mathbf{p} = \frac{\delta \mathbf{i}}{\delta n_p}, \quad \varepsilon_p = \frac{\delta E_s'}{\delta n_p}.$$

We note that our definition of the surface, by means of the condition that there be no total surface mass, determines the excitation energy completely. Whereas in the case of a volume the state of the system is described by three independent variables, namely the density ρ (or the density ρ_S of the superfluid component), the velocity $\mathbf{v}_S$, and the distribution function n_p, while the excitation

energy can be determined in two ways (by varying the energy of the volume element at constant $\mathbf{v}_s$ and ρ or at constant $\mathbf{v}_s$ and ρ_s), the only constant that can be determined by variation on a surface defined in the manner indicated above is $\mathbf{v}_s$.

From (1) we get

$$H_\nu = \frac{\delta E_s}{\delta n_\nu} = \varepsilon_p + \mathbf{p}\mathbf{v}_s.$$

Just as in the case of the volume system, ϵ_p is a functional of the excitation density

$$\varepsilon_p = \varepsilon_{0p} + \int f(\mathbf{p}, \mathbf{p}')\,\delta n_{\nu'} d\tau'.$$

The equilibrium distribution function of the surface impurities is obviously the Fermi function. The surface normal density $\nu_n = -\nu_{sf}$ is defined as the coefficient of proportionality between $\mathbf{i}$ and $\mathbf{v}_n - \mathbf{v}_s$, where $\mathbf{v}_n$ is the normal (not superfluid) velocity). Calculations analogous to those performed by Khalatnikov[4] lead to the result

$$\nu_n = m^* N / (1 + {}^1/_2 F_1),$$

where N is the number of impurities on the surface, and F_n is defined by the formula

$$F(\varphi) = f(\varphi)\left(\frac{\partial \tau}{\partial \varepsilon}\right)_{\varepsilon=\varepsilon_F} = \sum F_n \cos n\varphi.$$

2. CONSERVATION LAWS FOR THE NUMBER OF PARTICLES, MOMENTUM, AND ENERGY OF THE SURFACE

We write down the kinetic equation for the distribution function of the surface impurities n_ν. We consider a solution weak enough to be able to neglect collisions with volume impurities. Since the surface density of the impurities, for a solution with concentration $c \approx 10^{-10}$, becomes atomic already at $T = 0.1°K$[1], there is a sufficiently wide range of applicability for the approximation under consideration:

$$\frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_\alpha}\frac{\partial H_\nu}{\partial p_\alpha} - \frac{\partial n_\nu}{\partial p_\alpha}\frac{\partial H_\nu}{\partial x_\alpha} = J(n_\nu), \tag{5}$$

where J(n) is the collision integral.

We obtain the hydrodynamic fluxes in an approximation linear in ζ. The conservation laws are written in the same form as in[1]:

$$\frac{\partial \nu_n}{\partial t} + \frac{\partial}{\partial x_\alpha} f_\alpha = 0, \tag{6}$$

$$\frac{\partial i_\alpha}{\partial x_\alpha} = \rho v_{si} n_i - \rho\dot{\zeta}, \tag{7}$$

$$\frac{\partial i_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}\pi_{\alpha\beta} = \Pi_{\alpha i} n_i - \rho v_{s\alpha}\dot{\zeta}, \tag{8}$$

$$\frac{\partial E_s}{\partial t} + \frac{\partial}{\partial x_\alpha}\theta_\alpha = Q_i n_i - E_V\dot{\zeta}, \tag{9}$$

where n_i is a vector normal to the surface with components $\{-\zeta_\alpha, 1\}$, f_α is the flux density of the surface impurities, $\pi_{\alpha\beta}$ and $\Pi_{\alpha i}$ are the momentum flux densities per unit surface and per unit volume, respectively (the subscript i runs through the values x, y, and z); $E_V(E_S)$ and $Q_i(\theta_\alpha)$ are the volume (surface) element energy and volume (surface) energy flux density. Our task is to find the unknown fluxes $\pi_{\alpha\beta}$, f_α, and θ_α.

Multiplying (5) by m_3 and integrating with respect to $d\tau$, we obtain after simple transformations

$$\frac{\partial \nu_n}{\partial t} + \frac{\partial}{\partial x_\alpha}\left\langle m_3 n_\nu \frac{\partial H_\nu}{\partial p_\alpha}\right\rangle = 0.$$

(the angle brackets denote integration over phase space). Comparing these expressions with (6), we get

$$f_\alpha = \left\langle m_3 n_\nu \frac{\partial H_\nu}{\partial p_\alpha}\right\rangle.$$

We transform the right-hand side of (8) by using the "continuity" equation (7) and the explicit form of the flux $\Pi_{\alpha k}$[4]. We have

$$\Pi_{\alpha k} n_k - j_\alpha\dot{\zeta} = -P\zeta_\alpha + v_{s\alpha}\frac{\partial i_\beta}{\partial x_\beta},$$

where P is the pressure of the liquid. We write the condition for the absence of surface momentum normal to the surface (the consequence of the absence of surface mass) in the form $\mathbf{i}\cdot\mathbf{n} = 0$. Differentiating this expression with respect to time, and using the equations of surface hydrodynamics, we find that P = 0 in the zeroth order in ζ.

We multiply the kinetic equation (5) by p_α and integrate with respect to $d\tau$. After simple transformations we obtain

$$\frac{\partial i_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}\left\langle n_\nu p_\alpha \frac{\partial H_\nu}{\partial p_\beta}\right\rangle + \left\langle \frac{\partial H_\nu}{\partial x_\alpha} n_\nu\right\rangle = 0. \tag{10}$$

We transform the last term in the right-hand side of this expression in the following manner:

$$\left\langle \frac{\partial H_\nu}{\partial x_\alpha} n_\nu\right\rangle = \frac{\partial}{\partial x_\alpha}\langle n_\nu H_\nu\rangle - \frac{\partial E_s}{\partial x_\alpha} + \frac{\partial E_s}{\partial v_{si}}\frac{\partial v_{si}}{\partial x_\alpha}.$$

Using formula (1), the condition that the superfluid motion be potential, and the fact that the volume quantities are differentiated with respect to x_α not at a fixed value of z but at $z = \zeta(x_\alpha, t)$, we can express the last term of this formula in the form

$$\frac{\partial E_s}{\partial v_{si}}\frac{\partial v_{si}}{\partial x_\alpha} = i_\beta\frac{\partial v_{s\alpha}}{\partial x_\beta} + i_\beta\zeta_\alpha\frac{\partial v_{sz}}{\partial x_\alpha}.$$

With the aid of the continuity equation (7) we can show that $\partial v_{SZ}/\partial x_\alpha$ contains, in addition to terms linear in ζ, also terms containing the second derivative with respect to the coordinate. As noted earlier[1], such expressions should be neglected in the hydrodynamic approximation.

Thus, Eq. (10) takes the form

$$\frac{\partial i_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}\left[\left\langle p_\alpha n_\nu \frac{\partial H_\nu}{\partial p_\beta}\right\rangle - \delta_{\alpha\beta}(E_s - \langle n_\nu H_\nu\rangle)\right] = -i_\beta\frac{\partial v_{s\alpha}}{\partial x_\beta}.$$

Comparing this expression with Eq. (8) with its right-hand side transformed, we obtain

$$\pi_{\alpha\beta} = \left\langle p_\alpha n_\nu \frac{\partial H_\nu}{\partial p_\beta}\right\rangle + i_\beta v_{s\alpha} - \delta_{\alpha\beta}(E_s - \langle n_\nu H_\nu\rangle).$$

The quantity $E_S - n_\nu H_\nu$ has the meaning of surface tension.

We now obtain the energy conservation law. To this end, we multiply (5) by H_ν and integrate over the phase space:

$$\left\langle H_\nu \frac{\partial n_\nu}{\partial t}\right\rangle + \frac{\partial}{\partial x_\alpha}\left\langle n_\nu H_\nu \frac{\partial H_\nu}{\partial p_\alpha}\right\rangle = 0. \tag{11}$$

The derivative of the surface-element energy with respect to time is

$$\frac{\partial E_s}{\partial t} = \left\langle H_\nu \frac{\partial n_\nu}{\partial t}\right\rangle + \frac{\partial E_s}{\partial v_{si}}\frac{\partial v_{si}}{\partial t}.$$

We transform the last term on the left:

$$\frac{\partial E_s}{\partial v_{si}}\frac{\partial v_{si}}{\partial t}=i_\beta\frac{\partial v_{s\beta}}{\partial t}+i_\beta\zeta_\beta\frac{\partial v_{sz}}{\partial t}=i_\beta\frac{\partial}{\partial x_\beta}\left(\frac{\partial E_V}{\partial\rho}\right)-i_i\dot{\zeta}\frac{\partial v_{si}}{\partial z}.$$

We have used the bulk hydrodynamics equation[4] $\partial v_{Si}/\partial t = -\partial(\partial E_V/\partial\rho)/\partial x_i$, and also the fact that differentiation with respect to x_α and t is carried out at $z = \zeta(x_\alpha, t)$.

Substituting $\partial E_S/\partial t$ in (9), and transforming its right-hand side with the aid of (7) and with the aid of the explicit form[4] of Q_i, we obtain

$$\left\langle H_\nu\frac{\partial n_\nu}{\partial t}\right\rangle+\frac{\partial}{\partial x_\alpha}\left(\theta_\alpha-i_\alpha\frac{\partial E_V}{\partial\rho}\right)=\dot{\zeta}\left(-E_V+\frac{\partial E_V}{\partial\rho}\rho\right)-i_i\dot{\zeta}\frac{\partial v_{si}}{\partial z}. \tag{12}$$

The last term of this expression can be reduced with the aid of (7) to a form that contains only terms that are quadratic in ζ and contain second derivatives. At low temperatures

$$-E_V+(\partial E_V/\partial\rho)\rho=P.$$

Comparing (11) and (12) we get

$$\theta_\alpha=\left\langle n_\nu H_\nu\frac{\partial H_\nu}{\partial p_\alpha}\right\rangle+i_\alpha\frac{\partial E_V}{\partial\rho}.$$

3. FREE-SURFACE OSCILLATIONS OF A SUPERFLUID LIQUID

On the surface of a normal liquid there exists only one type of oscillation, namely capillary waves with spectrum $\omega^2 = (\alpha/\rho)k^3$, where ω and k are the frequency and wave number of the oscillations and α is the surface tension. In a superfluid liquid, at not too low temperatures, when the influence of the surface normal component can be neglected, there exist likewise capillary waves with damping coefficient $\gamma = 2\eta k^2/\rho$ (η is the viscosity of the volume normal component). At very low temperatures, when the influence of the volume excitations is negligibly small, there exist on the surface, as shown in[1], two types of oscillations, capillary waves and surface second sound. When the surface viscosity is taken into account, their spectrum is given by

$$\omega_1^2=\frac{\alpha}{\rho}k^3\left(1+\frac{k\nu_s}{\rho}\right)-i\omega\frac{\eta_s\alpha^2k^5}{\rho^3u^4}, \tag{13}$$

$$\omega_2^2=u^2k^2\left(1-\frac{\alpha\nu_sk^2}{\rho^2u^2}\right)-i\omega\frac{\eta_sk^2}{\nu_s}, \tag{14}$$

where ν_S is the density of the surface normal component and u is the velocity of the surface second sound:

$$u^2=-\frac{1}{\nu_s}\left\{\left(\frac{\partial\alpha}{\partial\nu_n}\right)_\sigma\nu_n+\left(\frac{\partial\alpha}{\partial\sigma}\right)_{\nu_n}\sigma\right\},$$

where σ is the surface entropy.

If the impurities on the surface are strongly degenerate and their mean free paths $l \gg a_0$, where a_0 is the atomic distance, then the spectrum of the waves with length $\lambda \sim l$ must be investigated by using boundary conditions in the kinetic-equation form. We assume that the deviations of the distribution function from equilibrium are small. The velocity v_S and the deviations ζ of the surface will be of the same order. We seek a solution in the form of small oscillations, i.e., their dependence on the coordinates and on the time is given by the expression $\exp\{ik_\alpha x_\alpha - i\omega t\}$. The linearized kinetic equation is

$$i(\omega-k_\alpha v_\alpha)n_1+ik_\alpha v_\alpha\frac{\partial n_0}{\partial\varepsilon}\left\{p_\beta v_{s\beta}+\int f(\mathbf{p},\mathbf{p}')n_1'\,d\tau'\right\}=J(n_1),$$

where v is the excitation velocity and n_1 is the deviation of the distribution function from the equilibrium value n_0.

We introduce dimensionless variables in accordance with the formula

$$n_1=\frac{\partial n_0}{\partial\varepsilon}m^*v_F^2\nu(\cos\varphi),$$

where $v_F = (\partial\epsilon/\partial p)_{\epsilon=\epsilon_F}$, and m* is the effective mass of the excitation. The kinedic equation for the function ν is

$$(V-\cos\varphi)\nu+\cos\varphi\left[\cos\varphi\frac{v_{sz}}{v_F}-\langle F(\varphi-\varphi')\nu(\varphi')\rangle\right]=J(\nu), \tag{15}$$

where $V = \omega/kv_F$.

To satisfy the energy and momentum conservation laws, we write the collision integral in the form

$$J(\nu)=(\nu-\langle\nu\rangle-2\langle\nu\cos\varphi\rangle\cos\varphi)/\tau,$$

where τ is a certain effective time.

It is necessary to add to Eq. (15) the continuity equation and the equation reflecting the absence of a momentum normal to the surface ($\mathbf{i}\cdot\mathbf{n}=0$). In linearized form, these equations are

$$-i\frac{k_\alpha}{\rho}\int n_1p_\alpha\,d\tau=v_{sz}+i\omega\zeta,\qquad P=\alpha k^2\zeta. \tag{16}$$

The potential φ_S of the superfluid velocity satisfies the Laplace equation $\nabla^2\varphi_S = 0$. We write the solution of this equation in the form

$$\varphi_s=(-i/k)\exp\{ikx-i\omega t-kz\}v'.$$

From this we get

$$v_{sx}=v',\quad v_{sz}=iv',\quad P=-\rho\omega v'/k.$$

Thus, we have a system of three equations with three unknowns.

Let us consider for simplicity the case when the first two harmonics of F differ from zero, i.e.,

$$F=F_0+F_1\cos\varphi.$$

Solving the kinetic equation with respect to ν and calculating $\langle\nu\rangle = \nu_0$ and $\langle\nu\cos\varphi\rangle = \nu_1/2$ (ν_0 and ν_1 are the zeroth and first harmonics of ν), we obtain two equations for these quantities:

$$\begin{aligned}&\nu_0\left[-\left(1+\frac{1}{\sigma\xi}\right)+w\left(F_0-\frac{1}{\sigma\xi}\right)\right]+\nu_1\xi w\left(\frac{F_1}{2}-\frac{1}{\sigma\xi}\right)-\frac{v'}{2iv_F}\xi w=0,\\&-\nu_0\left(F_0-\frac{1}{\sigma\xi}\right)\xi w+\frac{\nu_1}{2}\left[\left(1+\frac{F_1}{2}\right)-\xi^2w\left(F_1-\frac{2}{\sigma\xi}\right)\right]\\&-\left(-\frac{1}{2}+\xi^2w\right)\frac{v'}{v_F}=0,\end{aligned} \tag{17}$$

where

$$\sigma=-i\tau kv_F,\quad \xi=(i\omega\tau-1)/i\tau kv_F,$$
$$w=\frac{1}{2\pi}\int_0^{2\pi}\frac{d\varphi}{\xi-\cos\varphi}. \tag{18}$$

Eliminating ζ from (16), we get

$$\frac{p_F^2k}{2\pi\hbar^2\rho}\nu_1=\left(1-\frac{\rho}{\alpha}\frac{\omega^2}{k^3}\right)v'. \tag{19}$$

We write down the determinant of the system (17) and (19):

$$\left(1-\frac{\rho}{\alpha}\frac{\omega^2}{k^3}\right)\left\{\left(1+\frac{1}{\sigma\xi}\right)\left(1+\frac{F_1}{2}\right)-w\left[\left(1+\frac{F_1}{2}\right)\left(F_0-\frac{1}{\sigma\xi}\right)\right.\right.$$

$$+\xi^2\left(1+\frac{1}{\sigma\xi}\right)\left(F_1-\frac{2}{\sigma\xi}\right)\Big]\Big\}-\frac{kp_F^3}{2\pi\hbar^2\rho}\Big\{\Big[-\left(1+\frac{1}{\sigma\xi}\right)$$
$$+w\left(F_0-\frac{1}{\sigma\xi}\right)\Big]\left(\frac{1}{2}-\xi^2w\right)+\xi^2w^2\left(F_0-\frac{1}{\sigma\xi}\right)\Big\}=0. \quad (20)$$

We seek a solution $\omega \propto k$ for $\omega\tau \gg 1$; in this case $\sigma \to \infty$ and $\sigma\xi \to \infty$. Accurate to terms of order $(a_0k)^2$, we have

$$\left(1+\frac{F_1}{2}\right)-w\left[\left(1+\frac{F_1}{2}\right)+V^2F_1\right]=0. \quad (21)$$

This is indeed the equation for finding the velocity of zero sound in a two-dimensional Fermi liquid. Evaluating the integral (18), we get

$$w=V/(V^2-1)^{1/2}-1.$$

It should be noted that, in accordance with[5], there is attraction between the surface impurities. This means that there is no surface zero sound of this type.

It can be shown that two types of zero sound are possible in a two-dimensional Fermi liquid. Equation (21) determines the zero-sound velocity corresponding to a Fermi-circle deformation that is symmetrical with respect to the angle φ. Antisymmetrical zero sound is also possible, however. It is easy to show that the equations for the velocities of zero sounds of different types separate and that for the velocity of the second zero sound we obtain, using the indicated approximation of the function F, the following equation:

$$w=\frac{F_1-2}{2F_1(V^2-1)}.$$

In order for zero-sound of this type to exist, it is necessary to satisfy the condition $F_1 > 2$. There are no experimental data on this quantity.

When $\omega\tau \ll 1$ there exists a solution of (20) in the form

$$(\omega/kv_F)^2=\tfrac{1}{2}(1+F_0)(1+\tfrac{1}{2}F_1)-i\omega\tau(1+\tfrac{1}{2}F_1).$$

This is first sound in a two-dimensional Fermi liquid. Comparing the damping coefficient with the results of hydrodynamics, we obtain

$$\eta_s=\nu_s(1+F_1/2)u^2.$$

In addition to the indicated solutions, Eq. (20) has a solution corresponding to capillary waves. At $\lambda \sim l$ there exists, in addition to viscous damping, a damping mechanism connected with the motion of impurities with velocity equal to the phase velocity of the surface capillary wave. This is the well-known Landau damping. Since $\omega/kv_F \sim (a_0k)^{1/2} \ll 1$ in a capillary wave, we obtain, by calculating the integral (18),

$$w=-1-i\omega/kv_F.$$

Substituting this expression in (20), we obtain for the damping coefficient of capillary waves

$$\gamma=\frac{\alpha^2m^{*3}}{4\rho^4u^2\pi\hbar^2}k^4.$$

We see that the Landau damping is proportional to the fourth power of λ, i.e., to a lower power than the damping connected with surface viscosity.

Let us trace the damping of the capillary waves as the temperature is lowered in a solution of He^3 in He^4 with concentration $c = 10^{-10}$. The contribution of the impurities to the volume thermodynamic and kinematic quantities in such a solution is small in comparison with the phonon contribution all the way to $T = 10^{-4}$ °K. The damping connected with the presence of surface viscosity takes place in a temperature region in which the mean free path of the volume excitation is $l \gg \lambda$. At $T < 0.6$° K, the principal effect of the interaction of the volume excitations is the scattering of phonons by phonons[4]. The phonon mean free path becomes large. The damping in this region is due to collisions of the phonons with the surface. It is easy to show that the imaginary increment to the frequency is in this case $\gamma \sim \rho_{ph}sk/\rho$, where ρ_{ph} is the phonon part of the normal volume density of the helium and s is the speed of sound. With further decrease of temperature, we reach a region in which the volume normal component is small in comparison with the surface component. For frequencies $\omega \sim 10^7$ sec^{-1}, the damping connected with the surface viscosity becomes comparable with the damping connected with the collisions between the surface and the phonons at $T \sim 0.3$° K. The temperature dependence of the surface thermodynamic quantities was calculated in[1]. With further lowering of the temperature, the impurities on the surface become degenerate and their mean free paths begin to increase like T^{-2}. For temperatures

$$T/T_F \sim (kv_F/\omega_0)^{1/2}$$

(T_F is the Fermi temperature of the surface impurities and ω_0 is the atomic frequency), the impurity mean free path is of the order of λ. Under these conditions, the Landau damping becomes the principal damping mechanism.

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