Vortex energy in helium II

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The energy of a vortex is measured at various temperatures by the technique of accumulating small energy losses in multiple production of a known number of vortices.

INTRODUCTION

The quantized vortices that are formed in helium II have according to Onsager[1] and Feynman[2] a circulation and an energy per unit length given by

$$\Gamma = n\Gamma_0, \quad \Gamma_0 = 2\pi\hbar/m \approx 10^{-3}\ \text{cm}^2/\text{sec} \tag{1}$$

$$E = n\varepsilon, \quad \varepsilon = \pi\rho_s \frac{\hbar^2}{m^2} \ln\frac{b}{a_0} \sim 10^{-7}\ \text{erg/cm} \tag{2}$$

Here n is an integer, Γ_0 the circulation quantum, m the mass of the helium atom, ρ_S the density of the superfluid component, and b and $a_0 \sim 3\times 10^{-8}$ cm the effective radii of the vortex (i.e., the region of the liquid encompassed by the rotation around the vortex filament) and its core.

The circulation quantization and relation (1) have been confirmed by direct experiments.[3] So far as the quantity ϵ is concerned, it has not yet been measured directly (because of its smallness), although there is indirect evidence of the validity of the expression (2), obtained from resonance experiments, in which there was measured the elastic constant

$$\nu_s = \varepsilon/\rho_s\Gamma, \tag{3}$$

that enters in the dispersion law of vortex waves:[4]

$$k_\pm^2 \approx \mp(\Omega \pm 2\omega_0)/\nu_s$$

(here Ω is the frequency of the oscillations and ω_0 is the angular frequency of rotation). The results of these experiments, carried out by Hall,[4] Andronikashvili and the author[5] and by Nadirashvili and the author[6], are given below.

In this paper we describe an experiment in which we have succeeded in making a direct estimate of the energy of the quantized vortex, and also its dependence on the temperature of the liquid.

MEASUREMENT METHOD AND RESULTS

In carrying out the measurements, the small energy per unit length of the vortex was increased gradually by the multiple production of a number of vortices of the same length in a ring gap which executed oscillations with supercritical amplitudes. Two vessels of annealed copper were used (see Fig. 1). The walls of the vessels were carefully polished. In the first of them (I), a ring gap of the dimensions shown in Fig. 1 was made from the inner surface of the shell 1 and the polished outer surface of an aluminum cylinder located coaxially within it. In the second vessel (II), an aluminum cylinder of smaller diameter was set up along with two additional thin (~0.1 mm) rings 3, located coaxially relative to the external and internal cylinders and dividing the space between them into three ring gaps of the same thickness, but with different mean diameters. The vessels were in thermal contact with the helium bath through aluminum fingers 2.

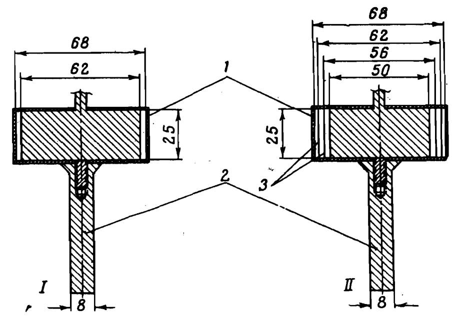

FIG. 1. Schematic diagram of the apparatus: I—with a single ring gap; II—with three coaxial ring gaps (the dimensions are in millimeters).

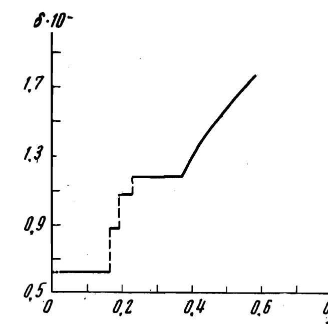

FIG. 2. Amplitude dependence of the logarithmic damping decrement of the oscillations for apparatus II.

The ring gaps were filled with He II through an opening of 0.5 mm diameter in the cap. An estimate showed that about ~0.02 cm³ of liquid flowed per hour through such an opening from the vessel along the film, which corresponds to a negligible decrease, ~0.01 mm, in the level of He II in the gap. To fill the ring gap with liquid, we used a movable beaker which could be moved vertically in the dewar by means of drawbars controlled from the outside. The liquid helium was drawn from the helium bath by the beaker and the ring gap was filled by upward displacement of the beaker with the helium. The beaker was then lowered, so that the vessel was in the helium vapor and the aluminum finger 2 was in the liquid. The vessel was reliably insulated from external thermal radiation by means of polished copper screens that made contact with the helium.

The vessel was suspended by phosphor bronze wire of diameter 100 μ and length $l \sim 100$ cm. The logarithmic damping decrement of the coaxial torsional vibrations of the vessel was measured. The period of oscillation was 154 sec and did not vary with the temperature of the liquid. The selection of a long period of oscilla-

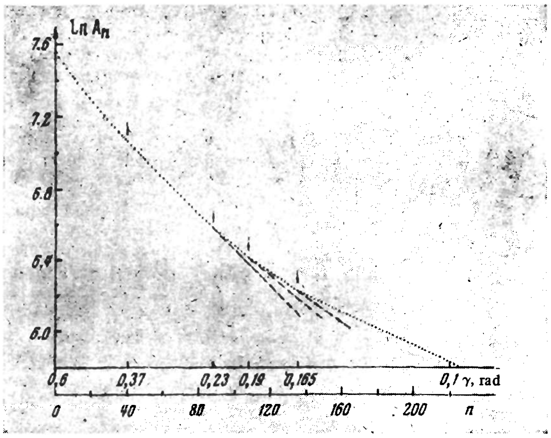

FIG. 3. Dependence of the half-swing A_n of the oscillations of the light spot on the number n of the half-period; φ—amplitude of the oscillations of the apparatus. The points are the experimental data. The arrows indicate the ends of the straight-line portions corresponding to the cuts of constant decrement in Fig. 2; the dotted line indicates the slope of these portions.

tion was dictated by the following considerations: it was known from the second-sound experiments of Hall and Vinen,[7] and also those of Andronikashvili et al.[8] that the equilibrium amplitude of the second sound was established within ~40—70 sec after the onset of motion of the radial resonator with supercritical velocity. This amplitude corresponded to scattering by the number of vortex filaments that is in equilibrium at the given velocity. About the same time is required for the decay of the vortices upon cessation of motion of the resonator. The width of the radial resonator (which is a doubly-connected annular volume) in the experiments described above amounted to ~1.2 cm. Taking these data into consideration, and also the results of[9], in which it was found that the relaxation time of vortex production decreases in proportion to the decrease in the width of the gap, one can conclude that in the case of the annular gap of width 3 mm used in the present work, an equilibrium number of vortices should develop after the time of a half-period of oscillation (~77 sec) in motion with supercritical velocity. Conversely, if the velocity of the ring gap is subcritical, the vortices should decay within this time. Kiknadze and Mamaladze (private communication) arrived at this conclusion on the basis of a theoretical analysis of this problem.

It is just this feature of our setup—the multiple production of vortices in the oscillations—that allows us to measure their total energy and to determine the energy of a single vortex.

In the doubly-connected region, the first series of vortices, according to Fetter,[10] is formed at the angular velocities

$$\omega_0 = \frac{2\hbar}{md^2}\ln\frac{d}{a},$$

where d is the width of the gap. This formula, which is valid for $R/d \gg 1$ (where R is the mean radius of the ring gap) gives, for d = 0.3 cm

$$\omega_0 \sim 10^{-2}\ \text{rad/sec}$$

In our experiments the critical velocities for both vessels vary in the range

$$5.6\cdot10^{-3} \div 1{,}02\cdot10^{-2}\ \text{rad/sec}$$

and turn out to be in fairly good agreement with the theoretical formula.

We now derive the basic formula which we have used for the calculation of the energy of the vortex. As is well known, the logarithmic damping decrement δ of the oscillations is the ratio of the energy ΔE dissipated over a half-period of the oscillation to the total energy E of the oscillating system:

$$\delta = \Delta E / E.$$

In our case, ΔE consists of the dissipation of energy in viscous friction in the vapor and in the liquid, $E_{vis} \equiv \delta_{vis}E$, and of the energy E_v expended in formation of the vortices:

$$\delta = \delta_{vis} + E_v / E; \tag{4}$$

here

$$E_v = Nl\varepsilon, \tag{5}$$

N is the number of vortices and l their length. If we use the expressions

$$N = 2\pi m\varphi_c R^2 / T\hbar, \quad E = 2\pi^2 J\varphi_c^2 / T^2,$$

where φ_c is the critical amplitude of the oscillation, R the mean radius of the ring gap, I the moment of inertia of the oscillating system, and T the period of oscillation, we can get the following relation from (4) and (5):

$$\varepsilon = \pi\hbar J\varphi_c(\delta - \delta_{vis}) / mR^2Tl. \tag{6}$$

The experimentally obtained dependence of the logarithmic damping decrement on the amplitude of the oscillation for the second vessel is shown in Fig. 2. The initial dependence of the logarithm of the amplitude A_n of the oscillations of the light spot on the amplitude φ of the oscillations of the apparatus is shown in Fig. 3, which is a photograph of the set of the corresponding experimental points. It is seen from Fig. 3 that in certain intervals of variation of φ the logarithmic damping decrement is practically unchanged (Fig. 2). The three steps on the graph of Fig. 2 (after the amplitude-independent region, which extends to $\varphi_c^1 = 0.165$ rad) correspond to the critical amplitudes in the first, second, and third gaps. The damping in the amplitude-independent region is $\delta_{vis}^1 = 0.62 \times 10^{-2}$, the damping after the first step is $\delta = 0.88 \times 10^{-2}$.

The results of calculations with the use of Eq. (6), in which the experimental data are used (data similar to those given in Fig. 2), are shown in Fig. 4. The broken curve indicates the results of a calculation according to

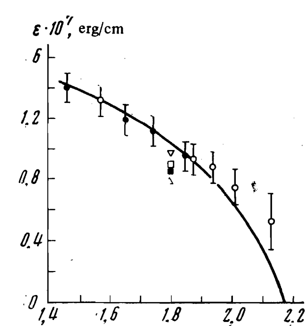

FIG. 4. Dependence of the vortex energy on the temperature: continuous curve—calculated from Eq. (2); O—results obtained with use of apparatus II; ●—results obtained in apparatus I. For T = 1.80°K, the results are given of a calculation of the vortex energy from resonance experiments: ∇, ■—from [4]; Δ—from [5]; □—from [6].

Eq. (2), in which the effective radius b was estimated as one-half the inter-vortex distance $2\pi R/N$, which gives $\ln(b/a_0) \approx 14$. As is known, the quantity b is actually not determined, but rather the value $\ln(b/a_0)$, which depends slightly on the various possible variants of estimate of b. For example, if we use for b the width or half-width of the ring gap d = 3 mm, then $\ln(b/a_0) \sim 16$. The values of ϵ calculated from data on the resonance experiments, are plotted in the same Fig. 4 for T = 1.80° K.

One can conclude from Fig. 4 that the Feynman formula generally give a valid determination of the vortex energy at all temperatures with the exception of the region abbutting the λ point: beginning with T ~1.9° K and above, the experimental results somewhat exceed the calculated values of the vortex energy.

It should be noted that the fact that the values obtained in our experiment exceed the theoretical ones is directly connected with the fact that the additional damping Δ of the disk oscillations in rotating He II above 1.9° K exceeds the temperature dependence expected at $\Delta \sim \rho_s$.[11] The additional damping of the oscillations of the disk in rotating He II is due to removal of kinetic energy of the disk oscillation by waves propagating along the vortices pinned to the disk. It is therefore proportional to the damping, i.e., to the energy.

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