Surface resistance of a cadmium plate in a magnetic field

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The dependence of the surface resistance of a cadmium plate on the strength of a magnetic field directed along the normal to the surface is investigated theoretically and experimentally. The spatial distribution of a radiofrequency field within a metal is analyzed theoretically on the basis of a paraboloidal Fermi-surface model which permits one to carry out all calculations in a closed form. It is shown that the wave field in a metal is the sum of a damping component, which describes the normal skin effect in a magnetic field, and of a doppleron field which is damped over distances of the order of the carrier mean free path. It is found that in diffuse reflection of the carriers from the surface the amplitudes of both components possess a root singularity near the doppleron threshold and the surface resistance of a massive sample exhibits a kink at this point. A knownledge of the spatial distribution of the alternating field permits one to determine the variation of the surface resistance of a thick plate due to transmission of a doppleron through it. The dependence of the resistance oscillation amplitude on magnetic field strength and carrier mean free path derived here is in agreement with direct measurements of the plate resistance.

1. We have previously investigated the Doppler shifted cyclotron resonance (DSCR) in cadmium[1-3]. Oscillations of the surface resistance of a cadmium plate in a magnetic field normal to the surface were discovered and studied (see[1]), and it was shown that these oscillations are due to the excitation of electromagnetic waves in the specimen as a result of the DSCR of the electrons at the limiting point of the "lens." This wave was called an electronic doppleron. Apart from these long-period high-amplitude oscillations, we later discovered short-period low-amplitude oscillations[2], which are due to the excitation of a hole doppleron as a result of the DSCR of the "monster" holes. The range of magnetic field strengths in which dopplerons exist is bounded from below, the threshold field for a hole doppleron being lower than that for an electronic one. Both waves are circularly polarized; the field of an electronic (hole) doppleron rotates in the same direction as the electrons (holes), i.e., it is negatively (positively) polarized. In our previous work[3] we used a circularly polarized rf field to excite the polarons and thus were able to distinguish between the resistance oscillations associated, respectively, with electronic and hole dopplerons. We also investigated the effect of dopplerons on the dependence of the surface resistance of a massive crystal on the magnetic field strength[3]. A simple model was considered, for which the Fermi surface is paraboloidal and all the calculations can be carried through in closed form. It was found that the dependence of the surface resistance $R_\pm$ on the magnetic field strength exhibits a singularity near the corresponding doppleron threshold, a root singularity if the carriers are specularly reflected at the metal surface[4], and a flex point if they are diffusely reflected[3].

The work reported here is a direct continuation of our earlier studies[3]; it was undertaken to investigate the depth distribution of the rf field in the crystal, since if one knows this distribution one can calculate the surface resistance of the plate as a function of the magnetic field strength. The treatment shows that above the doppleron threshold the wave field is the sum of two components: a damped component corresponding to the normal skin effect in a magnetic field, and a doppleron field, which damps out in a distance of the order of a carrier mean free path. Calculations of the resistance $R_\pm$ of the plate as a function of the field strength H are in agreement with direct measurements.

2. We consider a semi-infinite crystal whose surface z = 0 is normal to its hexagonal axis and to a uniform magnetic field **H** ($\mathbf{H} \parallel C_6 \parallel z$), and let us assume that an external electromagnetic wave of frequency ω is normally incident on the metal surface. The depth distribution of the field within the metal ($z > 0$) is determined by Maxwell's equations, which, on eliminating the alternating magnetic field, neglecting the displacement current, and assuming the exponential time dependence $\mathbf{E}, \mathbf{j} \sim e^{i\omega t}$ for the electric field **E** and the current density **j**, take the form

$$\frac{d^2E_\alpha(z)}{dz^2} = -\frac{4\pi i\omega}{c^2} j_\alpha(z), \quad \alpha = x, y. \tag{1}$$

In the general case, the relation between **j** and **E** is nonlocal and is described by the Reuter-Sondheimer equation[5]:

$$j_\alpha(z) = \rho \int_{-\infty}^{+\infty} dz' K_{\alpha\beta}(z-z')E_\beta(z') + (1-\rho)\int_0^\infty dz' K_{\alpha\beta}(z-z')E_\beta(z'), \tag{2}$$

$$K_{\alpha\beta}(z) = \int_{-\infty}^{+\infty} dk\, \sigma_{\alpha\beta}(k) e^{ikz}, \tag{3}$$

where $\sigma_{\alpha\beta}(k)$ is the nonlocal conductivity tensor in the k-representation, summation over the repeated tensor indices β in (2) is understood, and ρ is the fraction of the carriers that are specularly reflected at the metal surface. The first (second) term on the right in (2) represents the current due to the carriers that are specularly (diffusely) reflected. Most of the carriers travel at large angles to the surface, and these will be diffusely reflected; moreover, the behavior of the surface resistance of cadmium near the doppleron threshold also indicates diffuse reflection of the carriers[3]. We shall therefore consider only diffuse reflection and shall set $\rho = 0$ in what follows.

On introducing the vector components

$$A_\pm = A_x \pm iA_y, \tag{4}$$

appropriate for treating circularly polarized waves, the expression for the current takes the form

$$j_\pm(z)=\int_{-\infty}^{+\infty}dk\int_0^\infty dz'\,e^{ik(z-z')}\sigma_\pm(k)E_\pm(z'),\tag{5}$$

$$\sigma_\pm=\sigma_{xx}\pm i\sigma_{yx}.\tag{6}$$

In discussing the depth distribution of the rf field in the metal we shall use the same model as before[3], in which the Fermi surfaces for the electrons and holes have the shape of a double convex parabolic lens with a cylindrical equatorial band. The nonlocal conductivity $\sigma_\pm$ is given by[3]

$$\sigma_-(k,\mathbf{H})=\sigma_+(k,-\mathbf{H})=$$
$$\frac{Nec}{iH}\left\{\frac{1}{1+i\gamma_1}-\frac{1}{1+i\gamma_2}+\frac{\alpha_1k^2}{(\varkappa_1^2-k^2)(1+i\gamma_1)}-\frac{\alpha_2k^2}{(\varkappa_2^2-k^2)(1+i\gamma_2)}\right\},\tag{7}$$

in which

$$\varkappa_j=\frac{e_jH}{cp_j}(1+i\gamma_j)=\frac{e_jH}{cp_j}+\frac{i}{l_j},\quad \gamma_j=\frac{cm_j}{e_jH}(\nu_j-i\omega),$$
$$l_j=p_j/m_j\nu_j,\ e_1=-e_2=-e\ (j=1,2),\tag{8}$$

the subscripts 1 and 2 refer to electrons and holes, respectively, N is the carrier concentration ($N_1 = N_2 = N$), e is the magnitude of the electron charge, m_j is the cyclotron mass, p_1 and p_2 are parameters having the dimensions of momentum, which characterize the curvatures at the reference points of the paraboloids for the electrons and holes, α_j is the fraction of the j-type carriers within the corresponding parabolic bowl, ν_j is the carrier-lattice collision frequency, and l_j is the mean free path. The first and second terms in the braces in Eq. (7) represent the local parts of the electron and hole conductivities, while the third and fourth are nonlocal terms describing the DSCR of the electrons and holes.

Let us consider the electric field component E_-. We substitute expression (7) for $\sigma_-(k, H)$ in (5), integrate with respect to k, and substitute the resulting expression for $j_-(z)$ into Eq. (1). This gives the integro-differential equation

$$\frac{d^2E_-(z)}{dz^2}=-k_H^2\left\{\left(\frac{1-\alpha_1}{1+i\gamma_1}-\frac{1-\alpha_2}{1+i\gamma_2}\right)E_-(z)\right.$$
$$-\frac{i\alpha_1\varkappa_1}{2(1+i\gamma_1)}\int_0^\infty\exp(i\varkappa_1|z-z'|)E_-(z')\,dz'\tag{9}$$
$$\left.+\frac{i\alpha_2\varkappa_2}{2(1+i\gamma_2)}\int_0^\infty\exp(i\varkappa_2|z-z'|)E_-(z')\,dz'\right\},$$

$$k_H^2=4\pi\omega Ne/cH,\tag{10}$$

which is similar in form to the equation that one obtains when discussing the effect of space dispersion on the reflection coefficient of a dielectric and the properties of surface polaritons[6].

To solve Eq. (9) we operate on it with the differential operator

$$\left(\frac{d^2}{dz^2}+\varkappa_1^2\right)\left(\frac{d^2}{dz^2}+\varkappa_2^2\right),\tag{11}$$

This converts it to a sixth order ordinary differential equation with constant coefficients, whose characteristic equation (obtained by substituting ik for d/dz) can be written in the form

$$k^2=k_H^2\left\{\frac{1}{1+i\gamma_1}-\frac{1}{1+i\gamma_2}+\frac{\alpha_1k^2}{(\varkappa_1^2-k^2)(1+i\gamma_1)}-\frac{\alpha_2k^2}{(\varkappa_2^2-k^2)(1+i\gamma_2)}\right\}$$
$$=\frac{4\pi i\omega}{c^2}\sigma_-(k,H).\tag{12}$$

It is not difficult to see that (12) is the dispersion equation for an electromagnetic wave with negative circular polarization. This equation is bicubic so that Eq. (9) has a solution that damps out as $z\to\infty$, in the form of a linear combination of three exponentials:

$$E_-(z)=E_-(0)\sum_{s=0,1,2}a_s\exp(ik_sz),\tag{13}$$

$$\sum_s a_s=1,\tag{14}$$

where k_0, k_1, and k_2 are the roots of the dispersion equation (12) that have positive imaginary parts. Approximate formulas for the k_s are derived in[3]. To evaluate the a_s we substitute (13) into Eq. (9) and integrate with respect to z′. In addition to terms in $\exp(ik_sz)$, the resulting equation contains terms in $\exp(i\varkappa_1z)$ and $\exp(i\varkappa_2z)$ on the right, and since there are no such terms on the left, the coefficients of these exponentials must vanish. This gives two conditions, which, together with Eq. (14), determine the a_s. The result is

$$a_0=\frac{(\varkappa_1-k_0)(\varkappa_2-k_0)(k_2-k_1)}{k_0^2(k_2-k_1)+k_1^2(k_0-k_2)+k_2^2(k_1-k_0)},\tag{15}$$

a_1 and a_2 being obtained by cyclic permutation of the subscripts s = 0, 1, 2 that label the roots of the dispersion equation. Thus, the depth distribution of the field in the metal is fully determined by the roots k_s and the quantities $\varkappa_1$ and $\varkappa_2$.

The roots k_s and their dependence on the magnetic field strength H are discussed in[3]. The general expressions for the k_s are very cumbersome, so here we give approximate formulas that are valid throughout the region in which the electronic doppleron exists, except for a neighborhood of the threshold:

$$k_0\approx(1+i)\left[2\pi\omega N(m_1\nu_1+m_2\nu_2)H^{-2}\right]^{1/2},\tag{16}$$

$$k_1\approx\varkappa_1[1-\xi]^{1/2},\quad k_2\approx[\varkappa_2^2+\alpha_2k_H^2]^{1/2},\tag{17}$$

$$\xi=\frac{\alpha_1k_H^2}{\varkappa_1^2(1+i\gamma_1)}\left(1+\frac{\alpha_2k_H^2}{\varkappa_2^2}\right)^{-1}.\tag{18}$$

The doppleron threshold corresponds to the condition $\operatorname{Re}\xi=1$; Eqs. (16) and (17) are valid for fields for which $\operatorname{Re}\xi<1$.

The root k_0, being essentially complex, is determined by carrier scattering and describes the normal skin effect in the magnetic field; the skin depth is proportional to H. The root k_1 is the wave vector for the electronic doppleron; it approaches $\varkappa_1$ at small ξ, so that the doppleron is damped out in a distance of the order of the electron mean free path. The root k_2, which exceeds $\varkappa_2$ in magnitude, describes a hole doppleron with negative polarization. The fact that k_2 is almost real, i.e., that the hole doppleron is weakly damped, is due to the presence of a region of the Fermi surface on which all the holes have the same mean displacement. Such a group of holes give rise to a pole singularity in the nonlocal conductivity and causes Im k_2 to be small. If there is no such constant hole displacement region of the Fermi surface, however, k_2 will be essentially complex because of the cyclotron absorption of the wave by the holes, and the negatively polarized hole doppleron will be strongly damped. We shall therefore neglect this component of the electric field in what follows.

Dropping the last term from Eq. (13) and neglecting small quantities of the order of k_0/k_2 and k_1/k_2 in Eq. (15), we obtain

$$E_-(z)=E_-(0)\left\{\frac{\varkappa_1-k_0}{k_1-k_0}e^{ik_0z}+\frac{\varkappa_1-k_1}{k_0-k_1}e^{ik_1z}\right\}. \qquad (19)$$

Thus, the electric field in the metal is the sum of two essentially different components. The first component describes the skin effect; it damps out in a distance proportional to H, and its relative amplitude a_0 tends to unity as $\xi \to 0$. The second component is the field of an electronic doppleron; in strong fields its amplitude is proportional to ξ, and it damps out in a distance of the order of an electron mean free path. Near the doppleron threshold, both components increase in amplitude because of the decrease of $|k_1 - k_0|$, and both damp out in shorter distances.

3. Having determined the depth distribution of the field component E_- in the metal, we can calculate the surface impedance $Z_-^{(d)}$ of a cadmium plate whose thickness d is greater than or of the order of the electron mean free path l_1. If the field is symmetrically excited from both sides of the plate, the surface impedance can be expressed in the form[7]

$$Z_-^{(d)}=\frac{8\pi i\omega}{c^2}\frac{E_-(0)-E_-(d)}{E_-'(0)}, \qquad (20)$$

where E_-' is the gradient of the electric field in the direction of the inward normal to the surface, and the term containing $E_-(d)$ is due to the penetration of the field through the plate.

Substituting expression (19) with z = d into Eq. (20) and making use of the relation

$$E_-'(0)=i(k_0a_0+k_1a_1+k_2a_2)E_-(0), \qquad (21)$$

we find

$$Z_-^{(d)}=\frac{8\pi\omega}{c^2(k_0+k_1+k_2-\varkappa_1-\varkappa_2)}\left\{1-\left(\frac{\varkappa_1-k_0}{k_1-k_0}\exp(ik_0d)+\frac{\varkappa_1-k_1}{k_0-k_1}\exp(ik_1d)\right)\right\}. \qquad (22)$$

The first term in the braces (i.e., unity) corresponds to the impedance of a thick specimen[3], and the second term describes its variations resulting from the penetration of the wave field through the plate (it must be emphasized that formula (22) is valid only when the second term is considerably smaller than the first). It has been shown[3] that the surface resistance of a thick specimen has a flex point at the doppleron threshold, where k_1 decreases sharply. Scattering of the carriers tends to smooth out the flex, whose derivative becomes infinite in the limit $\nu_j \to 0$.

In addition to its monotonic growth with increasing H, $Z_-^{(d)}$ executes oscillations described by the term in $\exp(ik_1d)$ and due to the change in the phase of the doppleron on traversing the plate. These oscillations are present above the doppleron threshold in the region $\xi < 1$, where k_1 is mainly real; their relative amplitude is

$$(1-\varkappa_1/k_1)\exp(-k_1''d), \qquad (23)$$

where $k_1'' = \mathrm{Im}\,k_1$. The argument $k_1''d$ of the exponential is large near the doppleron threshold and decreases monotonically with increasing field strength, approaching the value d/l_1. The coefficient of the exponential in (23) is large compared with unity near the threshold and tends to zero with ξ in strong fields. The amplitude of the oscillations therefore has a maximum not far from the threshold, where the argument $k_1''d$ of the exponential has already fallen off considerably but the coefficient is still comparable with unity. Curve 1 of Fig. 1 shows the H dependence of the surface resistance $R_- = \mathrm{Re}\,Z_-^{(d)}$ at f = 1 MHz as calculated for the following parameter values: $N = 5\times10^{21}$ cm^{-3}, $p_1 = 4p_2 = 1.5\,\hbar/\text{Å}$, $\alpha_1 = 0.4$, and $\alpha_2 = 0.8$. The oscillations of the $R_-(H)$ curve are better seen in Fig. 2, where we have plotted the derivative dR_-/dH for d = 0.5 mm and for l_j equal to 0.5 and 0.25. On comparing the two curves we see that increasing the electron mean free path increases the amplitude of the oscillations and shifts its maximum toward the threshold. This shift is due to an increase in the importance of the part played by the singularity in the coefficient of the exponential in (23).

4. Now let us consider the depth distribution of the electric field component E_+. The equation for E_+ is derived in the same way as Eq. (9) for E_-. Its solution is

$$E_+(z)=E_+(0)\{A_0e^{iK_0z}+A_1e^{iK_1z}+A_2e^{iK_2z}\}, \qquad (24)$$

where K_0, K_1, and K_2 are the roots of the dispersion equation

$$k^2c^2=4\pi i\omega\sigma_+(k,H), \qquad (25)$$

that have positive imaginary parts[3], and the coefficients A_s are obtained from the expressions for the a_s by the substitution

$$k_s\to K_s,\quad s=0,1,2,$$
$$\varkappa_j\to\bar\varkappa_j(H)\equiv\varkappa_j(-H),\quad j=1,2. \qquad (26)$$

The root K_0, like k_0 in the case of negative polarization, corresponds to the normal skin effect in the magnetic field. The other two roots behave asymptotically in strong fields as

$$K_1=[\bar\varkappa_1^2+\alpha_1k_H^2]^{1/2},\quad K_2=-[\bar\varkappa_2^2-\alpha_2k_H^2]^{1/2}. \qquad (27)$$

The root K_1, which corresponds to a positively polarized electronic doppleron, exceeds $\bar\varkappa_1$ in magnitude, i.e., it lies in the region of electronic cyclotron absorption. The

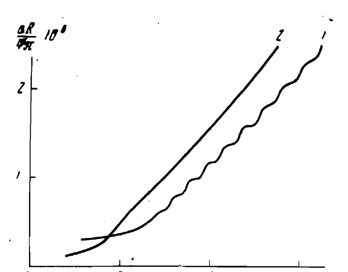

FIG. 1. Theoretical $R_\pm(H)$ curves calculated for f = 1 MHz, d = 0.5 mm, and $l_1 = 4l_2$ = 0.5 mm: 1) $R_-(H)$, negative polarization; 2) $R_+(H)$, positive polarization.

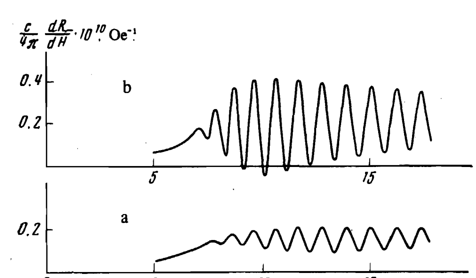

FIG. 2. Theoretical dR_-/dH curves calculated for f = 1 MHz, d = 0.5 mm, and a) l_1 = 0.25 mm; b) l_1 = 0.5 mm.

fact that we find this root to be almost real is due to our use of a model that does not include cyclotron damping. For the real metal, K_1 would have a large imaginary part and the term $A_1 \exp(iK_1 z)$ would damp out rapidly with distance. Hence we can drop this term from Eq. (24).

The modulus $|K_2|$ of the remaining root, which is the wave number of the hole doppleron, lies in the region $k < \kappa_2$ where there is no hole cyclotron absorption. Although there is electronic cyclotron absorption in this region, it falls off rapidly with increasing $k - \bar{\kappa}_1$, and for $k \approx \kappa_2$ it is very small[2]. The positively polarized hole doppleron is therefore weakly damped. Thus, the field E_+ within the metal is the sum of a damped component corresponding to the normal skin effect and a hole doppleron field that damps out in a distance of the order of a hole mean free path.

Since $A_2 \approx 1 - \bar{\kappa}_2/K_2$ above the doppleron threshold, the impedance of the plate for positive polarization can be written in the form

$$Z_+^{(d)} \approx \frac{8\pi\omega}{c^2(K_0+K_1+K_2-\bar{\varkappa}_1-\bar{\varkappa}_2)}\left\{1-\frac{\bar{\varkappa}_2}{K_2}e^{iK_0 d}-\left(1-\frac{\bar{\varkappa}_2}{K_2}\right)e^{iK_2 d}\right\}. \quad (28)$$

This formula is similar to Eq. (22) for $Z_-^{(d)}$. However, the mean free path is considerably shorter for holes than for electrons, so the amplitude of the $Z_+^{(d)}$ oscillations, which are due to passage of a hole doppleron, should be much smaller than that of the $Z_-^{(d)}$ oscillations described by Eq. (22). This conclusion is in good agreement with experimental data[2]. Curve 2 of Fig. 1 shows the H dependence of R_+ for f = 1 MHz and l_2 = 0.125 mm. The surface resistance depends weakly on the magnetic field strength for H < 4 kOe. The R_+(H) curve has a flex point near the hole doppleron threshold (H ≈ 4 kOe) and rises monotonically as H increases further. We note that the R_+(H) curve is almost a straight line above the threshold, but that a linear extrapolation of this part of the curve does not pass through the origin. This means that the observed linear rise of R_+ is not associated with the normal skin effect, for the surface resistivity associated with that effect would be proportional to H. The normal skin effect is found at considerably higher magnetic field strengths, where $K_1 - \bar{\kappa}_1$ is small compared with K_0. The linear rise of R_+ is associated with the existence of a hole doppleron.

5. We made an experimental study of the H dependence of the surface resistance $R_\pm$ and its derivative $dR_\pm/dH$ for a single-crystal cadmium plate. The method used to measure dR/dH has been described elsewhere[1,8]. To measure the surface resistance R we mounted the specimen within the tank coil of a constant-sensitivity autodyne detector[8], which is so designed that the tank-circuit Q is automatically held constant. The manner in which the device is used to measure the surface resistance of the specimen can be understood by reference to the equivalent circuit of the oscillator tank circuit (Fig. 3). In the figure, L and C represent the circuit inductance and capacitance, R_1 is a resistance representing the circuit losses with the specimen absent, and R_2 is a resistance representing the losses in the specimen. The specimen losses depend on the magnetic field strength, so R_2 is a variable. R_3 is the variable resistance introduced by the oscillator feedback. The device operates in such a way that R_3 is determined by the relation $R_1 + R_2 + R_3$ = const, so that $\Delta R_2 = -\Delta R_3$. Thus, by measuring R_3, we can follow the variations of

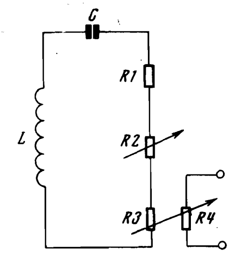

FIG. 3

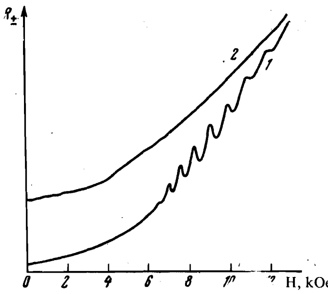

FIG. 4

FIG. 3. Equivalent circuit for the autodyne-detector tank.

FIG. 4. Experimental $R_\pm$(H) curves recorded with the same gain at 1.6 K for f = 1 MHz and d = 0.57 mm: 1) R_-(H), negative polarization; 2) R_+(H), positive polarization.

R_2, and hence those of the surface resistance of the specimen, in the varying magnetic field.

To measure the surface resistance R of the specimen without introducing further losses into the tank circuit we used an additional resistor R_4 identical to R_3 and varying with it. In our apparatus, R_3 and R_4 were two specially selected type FSK-1 photoresistors illuminated by the same light source and connected in parallel to the tank circuit. Then by measuring R_4 we could follow the variations of R_3, and hence those of R.

A signal proportional to R was fed to one of the Y inputs of a dual automatic X-Y plotter[9], and a voltage proportional to dR/dH was fed to the other Y input. The signal from a Hall-effect pickup fastened to one of the pole pieces of the electromagnet was fed to the X input of the plotter. The two curves (R and dR/dH vs H) for a given circularly polarized rf field were plotted simultaneously.

Measurements were made on a cadmium single-crystal plate 0.57 mm thick at temperatures between 1.6 and 4.2° K and frequencies from 0.25 to 1.5 MHz. Experimental curves for the resistance of the plate at 0.6 MHz are shown in Fig. 4 as examples. R_- increases slowly with H in weak fields, but in stronger fields, in which the electronic doppleron exists (H > 5 kOe), it rises more rapidly, and on this otherwise smooth rise are superimposed oscillations due to the passage of the electronic doppleron through the plate. The amplitude of these oscillations increases rapidly with H near the threshold and then falls slowly as H increases further. These oscillations are fairly strong at their peak; their maximum amplitude amounts to ~5% of the total change in R_- over the interval 0 < H < 12 kOe. The R_+(H) plot (curve 2) has a sharper kink near the hole doppleron threshold at H ≈ 4 kOe. R_+ is almost independent of H below the threshold and rises linearly with H above the threshold. The flex in the R_+(H) curve near the hole doppleron threshold is much sharper than the flex in the R_-(H) curve near the electronic doppleron threshold. The damping of the doppleron near the threshold tends to smooth out the flex, so this difference would appear to indicate that the electronic doppleron is more strongly damped near the threshold than the hole doppleron. Actually, formulas (18) and (30) of[3] show that the imaginary part of the wave vector k_1 is proportional to $\gamma^{1/4}$ at $\xi = 1$, whereas the imaginary part of K_2 is proportional to $\gamma^{1/2}$. The attenuation distances for the electronic and hole dopplerons increase with increasing field strength and approach the mean free paths l_1 and l_2 of

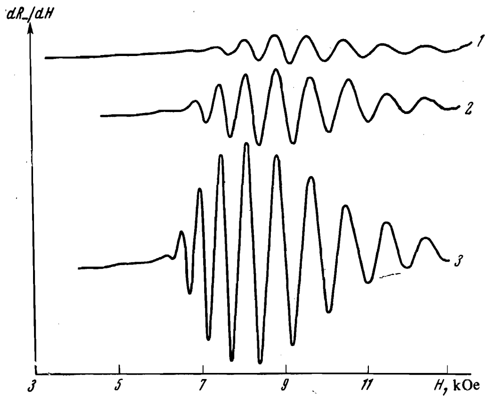

FIG. 5. Experimental dR_/dH curves recorded with the same gain at f = 1 MHz, d = 0.57 mm, and the following temperatures (K): 1) 4.2; 2) 3.55; 3) 1.7.

the corresponding carriers. Since $l_1 \approx 4 l_2$, the hole doppleron is much more strongly attenuated on traversing the plate than the electronic doppleron. That is why no oscillations can be seen on curve 2 of Fig. 4. It is possible to discern hole doppleron oscillations on the experimental dR_+/dH curves, thanks to the considerably higher sensitivity of the modulation method used in recording them.

Figure 5 shows experimental dR_-/dH curves recorded with the same gain at temperatures of 4.2, 3.55, and 1.7°K (curves 1, 2, and 3, respectively). As the temperature falls, the electron mean free path increases while the amplitude of the doppleron oscillations rises rapidly and its maximum shifts toward the weaker fields. Thus, the observed oscillations behave in accordance with the theoretical predictions. The theoretical curves (Fig. 2) show the amplitude of the doppleron oscillations falling less rapidly with increasing field strength than do the experimental curves (Fig. 5); this is due to our use of a model for which k_1 approaches κ_1 less rapidly than it does for real cadmium (see Fig. 9 of[3]).

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