Giant quantum oscillations of Rayleigh sound wave absorption in metals

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Giant quantum oscillations of the electron absorption of surface sound waves in metals located in magnetic fields perpendicular or parallel to the sample surface are investigated theoretically. It is shown that in a normal magnetic field the absorption experiences sharp oscillations despite the absence of the law of conservation of the electron momentum projection on the **H** vector in the field of an inhomogeneous sound wave. The amplitude and shape of the absorption line are found as depending on the frequency, temperature, and mean free path.

It is known that the absorption[1] and dispersion of the sound velocity[2] in metals at low temperatures undergo giant quantum oscillations in a magnetic field. This phenomenon is due to the quantization of the velocity of electrons located on the Fermi surface in the direction of the magnetic field **H**. The resonance maxima of the absorption and velocity of the sound arise at those values of H at which the projection of one of these velocities on the sound wave vector **k** becomes equal to the sound velocity s.

A characteristic feature of Rayleigh (surface) sound waves is that the component of the wave vector of these oscillations that is normal to the separation boundary does not have a defined value. Because of this fact, the effective value $\nu + \kappa |v_x|$ enters in the absorption of the Rayleigh waves in place of the ordinary collision frequency of electrons with the scatterers ν. Here κ is the damping decrement of the surface wave along the normal Ox, and **v** is the velocity of the electrons.[3] The study of the possibility of existence of the phenomenon of giant quantum oscillations of the Rayleigh sound wave is therefore of interest.

The present paper is devoted to the study of the features of Rayleigh wave absorption by conduction electrons of a metal in a quantizing magnetic field. The effect of scattering of the electrons and the temperature broadening of the Fermi level on the amplitude and shape of the resonance absorption lines are considered for different orientations of the vector **H** relative to the surface of the sample.

1. GENERAL RELATIONS

In this section we derive the general quantum-mechanical expression for the coefficient of electronic absorption of Rayleigh sound Γ in the case in which the vector **H** is oriented along the inner normal to the surface of separation. The metallic half-space ($x > 0$) is assumed to be acoustically isotropic. For simplicity, we limit ourselves to a consideration of an isotropic and quadratic dispersion law for the conduction electrons. Generalization to the case of an anisotropic spectrum of the electrons does not present great difficulty (see[4]).

1. The coefficient Γ is determined by the energy absorbed by the electrons in a unit time, which we denote by Q:

$$\Gamma = \overline{Q} / 2WL_yL_z. \qquad (1.1)$$

Here the bar denotes averaging over the period of oscillation, L_y and L_z are the dimensions of the crystal along the y and z axes, W is the energy density in the Rayleigh wave, averaged over the period and referred to unit area of the interface,

$$W = |u_x^l(0)|^2 A\rho_L\omega^2 k^{-1}, \quad A(\xi) = \xi^4 \frac{8 - 16\xi^2 + 11\xi^4 - 2\xi^6}{(1-\xi^2)^{1/2}(2-\xi^2)^6}; \qquad (1.2)$$

the parameter ξ depends in known fashion[5] on the ratio of the transverse (s_t) and longitudinal (s_l) sound velocities, ρ_L is the density of the crystal, ω is the frequency, **k** is the two-dimensional wave vector with components k_y and k_z, and $u_x^l(0)$ is the normal component of the displacement vector of the longitudinal mode (the superscript l) on the surface x = 0.

According to[4], the quantum-mechanical expression for the electronic absorption coefficient Γ at finite temperature and with account of scattering, has the following form:

$$\Gamma = \int_0^\infty dE \frac{f_0(E) - f_0(E + \hbar\omega)}{\hbar\omega} \Gamma(E), \qquad (1.3)$$

where $f_0(E) = \{1 + \exp[(E - \epsilon_F)/T]\}^{-1}$ is the Fermi distribution function, ϵ_F the Fermi energy, and $\Gamma(E)$ is the coefficient of sound absorption by electrons with given energy E:

$$\Gamma(E) = \frac{\pi\hbar\omega^2}{4WL_yL_z} \sum_{a,b} |\langle a|\hat{U}|b\rangle|^2 D(E - E_b) D(E - E_a + \hbar\omega). \qquad (1.4)$$

The summation in (1.4) is carried out over the quantum numbers a and b of the electron states in the magnetic field, E_a is the energy of the electron in the state $|a\rangle$, $\hat{U} = \Lambda_{ik} u_{ik}(\hat{\mathbf{r}})$ is the deformation interaction operator of the electron with the wave, Λ_{ik} is the deformation potential tensor, u_{ik} is the elastic deformation tensor, and by repeated vector indices we mean summation from 1 to 3. The function

$$D(E) = \frac{1}{2\pi} \frac{\hbar\nu}{E^2 + (\hbar\nu/2)^2} \qquad (1.5)$$

is the imaginary part of the single-particle Green's function of the electrons with account of their scattering by the short-range potential of the impurity. The product of two D functions in Eq. (1.4) arises as a result of factorization of the two-particle Green's function of the electrons. According to[6], replacement of the two-particle Green's function by the product of single-particle functions is valid under the conditions of strong spatial dispersion

$$\varkappa l = \varkappa v / \nu \gg 1 \tag{1.6}$$

and for all energies E with the exception of a small region of values

$$\Delta < 1/N, \tag{1.7}$$

where

$$E = (N + {}^1/_2 + \Delta)\hbar\Omega, \quad N = [E/\hbar\Omega - {}^1/_2], \tag{1.8}$$

N is the integral and Δ the fractional part of the quantity $E/\hbar\Omega - \frac{1}{2}$. Here $\Omega = eH/mc$ is the cyclotron frequency, e the absolute value of the electronic charge, c the velocity of light, and v the Fermi velocity of the electrons.

The functions D in (1.4) describe the collision broadening of the energy levels of the electrons in the magnetic field and the degree of violation of the law of energy conservation for absorption of the quantum $\hbar\omega$. We note that in the case (1.6) the collision frequency ν in (1.5) describes only the "drift" of the electrons and is expressed in terms of the total scattering cross section from the impurity.

2. We proceed to the calculation of the matrix elements of the deformation interaction of electrons with the Rayleigh sound wave. This calculation can be carried out directly only in the case of specular reflection of the electrons from the surface of the metal. It will be shown below that the quantum effect of interest to us is due to electrons with a small value of their velocity projection v_x. For these electrons, the scattering by rough surfaces is close to specular.[7] The contribution of the rest of the electrons to Γ is nonresonant and is essentially identical with the classical absorption Γ_{cl} which was calculated in[3]. In other words, the nonresonant part of the absorption, found below for specular reflection, must be replaced in the general case by the classical absorption with account of nonspecular reflection of electrons from the surface.

The wave function and the energy levels of the electron in a metal for specular reflection from the boundary have the form

$$|a\rangle \equiv |n_a, \sqrt{2m\varepsilon_a}, p_{za}\rangle = (\gamma L_z)^{-1/2}$$
$$\times \sin\left(\frac{\sqrt{2m\varepsilon_a}}{\hbar}x\right)\exp\left(i\frac{p_{za}}{\hbar}z\right)\Phi_{na}\left(\frac{y}{\gamma}+\frac{\gamma p_{za}}{\hbar}\right), \tag{1.9}$$
$$E_a = (n_a + {}^1/_2)\hbar\Omega + \varepsilon_a,$$

where $\epsilon_a = p_x^2/2m$ is the longitudinal kinetic energy of the electrons; p its momentum; $\Phi_n(y) = e^{-y^2/2}H_n(y)$ the Hermite function; $H_n(y)$ the Hermite polynomial, normalized to unity. The vector potential $\mathbf{A}_0$ of the constant magnetic field has the gauge $A_{0x} = A_{0y} = 0$, $A_{0z} = Hy$; $\gamma = (\hbar c/eH)^{1/2}$ is the magnetic length.

In the interaction Hamiltonian $\Lambda_{ik}u_{ik}$ there enters the

$$\mathbf{u}(\mathbf{r}) = \sum_\alpha \mathbf{u}^\alpha(0)\exp(-\varkappa_\alpha x + i\mathbf{kr}),$$

displacement vector in which the index α takes on the values l and t, corresponding to longitudinal and transverse sound, and

$$\varkappa_\alpha = (k^2 - \omega^2/s_\alpha^2)^{1/2} \tag{1.10}$$

is the damping decrement of the wave of given type. Consequently, the matrix element can be written in the form

$$\langle a|\hat{U}|b\rangle = \Lambda_{ik}\sum_\alpha u_{ik}^\alpha(0)\langle a|\exp(i\mathbf{kr} - \varkappa_\alpha x)|b\rangle,$$

$$\langle a|\exp(i\mathbf{kr} - \varkappa_\alpha x)|b\rangle = \delta_{p_{za}, p_{zb}+\hbar k_z}(n_a|\exp(ik_y y)|n_b)\int_0^\infty dx\exp(-\varkappa_\alpha x)\cdot$$
$$\times \sin\left(\frac{\sqrt{2m\varepsilon_a}}{\hbar}x\right)\sin\left(\frac{\sqrt{2m\varepsilon_b}}{\hbar}x\right), \tag{1.11}$$

where $\delta_{pz, p'z}$ is the Kronecker delta, equal to 0 or 1. The integral over x is easily calculated, but the answer fills a great deal of space and we shall not write it down. The matrix element of $\exp ik_y y$ is determined by the formula

$$|(n_a|\exp(ik_y y)|n_b)| = M_{n_a n_b}\left[\frac{(\gamma k)^2}{2}\right], \quad M_{nm}(\tau) = \tau^{(n-m)/2}L_m^{n-m}(\tau)e^{-\tau/2}, \tag{1.12}$$

$L_m^{n-m}(\tau)$ is the Laguerre polynomial, normalized to unity.

Giant quantum oscillations of the absorption of the wave take place in strong magnetic fields, when the length of the sound wave is large in comparison with the characteristic size of the electron orbit $R = v/\Omega$:

$$kR \ll 1. \tag{1.13}$$

In this case, the matrix $M_{n_a n_b}$ can be replaced by unity, i.e., only the components with $n_a = n_b = n$ should remain in the sum over n_a and n_b. This result is obtained formally from the asymptotic form of (1.12) under the condition that the argument M_{nn} is smaller than the reciprocal value of the index $n \lesssim \epsilon_F/\hbar\Omega$. In other words, the giant quantum oscillations are due to electronic transitions without change in the magnetic quantum number n.

3. Taking into consideration (1.17) and (1.13), we can simplify the expression (1.4) for $\Gamma(E)$. We take into account the law of conservation of the z component of the momentum and substitute in (1.4) the explicit expression for the magnetic element (1.11). Then

$$\Gamma(E) = 2\mathcal{T}\hbar\Omega\frac{(\hbar k)^2}{m}\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{\mathrm{Re}(B_\alpha B_\beta^*)}{\varkappa_\beta^2 - \varkappa_\alpha^2}\sum_{n=0}^\infty\int_0^\infty\int_0^\infty d\varepsilon_a d\varepsilon_b$$
$$\times\frac{D[E-(n+{}^1/_2)\hbar\Omega - \varepsilon_b]D[E-(n+{}^1/_2)\hbar\Omega - \varepsilon_a + \hbar\omega]}{(\varepsilon_a - \varepsilon_b)^2 + (\sqrt{\varepsilon_a}+\sqrt{\varepsilon_b})^2(\hbar\varkappa_\alpha)^2/2m} \tag{1.14}$$

where

$$\mathcal{T} = \frac{3}{8}\zeta\frac{N_e\varepsilon_F k}{\rho_L v}, \quad \zeta = \left(\frac{\Lambda}{\varepsilon_F}\right)^2, \quad B_\alpha = \frac{\Lambda_{ik}u_{ik}^\alpha(0)}{ku_x^l(0)\varepsilon_F(\zeta A)^{1/2}}. \tag{1.15}$$

The quantity $\mathcal{T}$ is of the order of the coefficient of collisionless absorption of volume sound, ζ is a dimensionless parameter of the electron-phonon interaction; the quantity Λ, which is of the order of ϵ_F, represents the characteristic value of the deformation potential. The parameters B_α characterize the contribution to the interaction of the longitudinal and transverse modes of the Rayleigh wave with the conduction electrons. Summation over α and β eliminates the apparent singularity at $\kappa_\alpha = \kappa_\beta$. In the derivation of Eq. (1.14) we assumed the tensor Λ_{ik} to be independent of p and, furthermore we replaced the smooth functions of the energy by their values at $E = \epsilon_F$.

The most significant difference between Eq. (1.14) and the corresponding absorption coefficient of volume sound is that instead of a single integration p over the longitudinal energy, there is a double integral over ϵ_a and ϵ_b. In the absorption of volume waves, the law of conservation of the x component of the electron momentum ($\mathbf{H} \parallel Ox$) is satisfied, as a consequence of which one integration over p_x is removed by the corresponding δ function. In the case of the Rayleigh wave, there is no such conservation law, because of the fact that the x component of the wave vector does not have a definite value. Therefore, in place of the delta function in (1.14) there appears a denominator which describes the nonconservation of the quantity p_x in the absorption of a quantum of the inhomogeneous Rayleigh wave. The characteristic parameter which describes the diffusion of the delta function is the quantity $\hbar\kappa_\alpha|v_x|$, which represents the indeterminacy of the longitudinal (relative to the vector **H**) energy of the electron in the field of the Rayleigh wave. We note that in the classical case[3] collision broadening and indeterminacy of the longitudinal energy enter additively through the effective collision frequency $\nu_{eff} = \nu + \kappa_\alpha|v_x|$. In the quantum case, these scattering mechanisms are separated. To be precise, the collisions smear out the conservation law and the energy levels themselves, while the spatial inhomogeneity of the field of the wave smears out the conservation law of the normal component of the longitudinal momentum (or of the longitudinal energy).

2. ABSORPTION AT ABSOLUTE ZERO TEMPERATURE

We investigate first the quantum features of the absorption of Rayleigh waves in the very low-temperature region, when $T \ll \hbar(\omega + \nu)$ and the thermal diffuseness of the Fermi level cannot be taken into account. Two limiting cases can be distinguished here—the lower and upper frequencies—depending on the relation between ω and ν.

1. Low frequencies ($\omega \ll \nu$). In this region, we can neglect the quantity $\hbar\omega$ in the argument of the second D function of Eq. (1.14). Furthermore, inasmuch as the width of the D functions significantly exceeds $\hbar\omega$, and the difference of the Fermi functions is different from zero over a small range of energies $\epsilon_F - \hbar\omega \le E \le \epsilon_F$, the integration over E reduces to multiplication by the energy quantum $\hbar\omega$ and the replacement of E by ϵ_F. In other words, the difference $f_0(E) - f_0(E + \hbar\omega)$ must be set equal to $\hbar\omega\delta(E - \epsilon_F)$. As a result, we obtain

$$\Gamma = 2\mathcal{T}\hbar\Omega\frac{(\hbar k)^2}{m}\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{\mathrm{Re}(B_\alpha B_\beta^*)}{\varkappa_\beta^2-\varkappa_\alpha^2}\sum_{n=0}^{\infty}\int_0^\infty d\varepsilon_a\int_0^\infty d\varepsilon_b$$
$$\times\frac{D[(N-n+\Delta)\hbar\Omega-\varepsilon_a]D[(N-n+\Delta]\hbar\Omega-\varepsilon_b]}{(\varepsilon_a-\varepsilon_b)^2+(\sqrt{\varepsilon_a}+\sqrt{\varepsilon_b})^2(\hbar\varkappa_a)^2/2m}. \quad (2.1)$$

where N and Δ now have the meaning of the integral and fractional parts of the quantity $\epsilon_F/\hbar\Omega - \frac{1}{2}$ (cf. (1.8)).

In the sum over n there is left only those components in which $n \le N + 1$. In the remaining terms of the sum ($n \le N + 1$) the arguments of the D functions never vanish and are of the order of $(n - N)\hbar\Omega \gg \hbar\nu$. Therefore, such components make a small contribution (significantly smaller than $(\nu/\Omega)^2$, see below) and can be neglected.

The double integral over ϵ_a and ϵ_b in (2.1) is conveniently represented in the following form (the substitutions $\epsilon_a = t^2$, $\epsilon_b = \tau^2$ are made):

$$I=\int_0^\infty d\varepsilon_a\int_0^\infty d\varepsilon_b\ldots=4\int_0^\infty dt\int_0^\infty d\tau\frac{t\tau}{(t+\tau)^2}$$
$$\times\frac{D[(N-n+\Delta)\hbar\Omega-t^2]D[(N-n+\Delta)\hbar\Omega-\tau^2]}{(t-\tau)^2+(\hbar\varkappa_a)^2/2m}. \quad (2.2)$$

First we consider the quantity I for $n \le N$. There are three "sharp" functions under the integral of (2.2). The D functions have maxima for $t = \tau = \eta$, where

$$\eta = [(N-n+\Delta)\hbar\Omega]^{1/2}.$$

The width of these maxima is of the order of $\hbar\nu/4\eta$. The function

$$d(t-\tau) = [(t-\tau)^2+(\hbar\varkappa_a)^2/2m]^{-1} \quad (2.3)$$

is maximal for $t = \tau$ and has the width $\hbar\kappa_a/(2m)^{1/2}$. We note that the width of the d-function is significantly smaller than the width of η for any values $n \le N$. Even for $n = N$ and for the minimal value $\Delta_{min} = N^{-1}$, the width of the d function is less than η by a factor $(\kappa_a R)^{-1}$. Inasmuch as the maxima of both D functions and the d function are located on the line $t = \tau$ in the (t, τ) plane, then all the smooth factors can be replaced by their values for $t = \tau$. The value of the integral I depends on the ratio of the characteristic widths of the D and d functions, and also on the value of n. Here we should in turn distinguish two limiting cases.

Let the width of the d function be the smallest parameter, i.e.,

$$\hbar\varkappa_a/(2m)^{1/2} \ll \eta \ll (2m)^{1/2}\nu/2\varkappa_a. \quad (2.4)$$

Then the d function can be replaced by $(\pi\sqrt{2m}/\hbar\kappa_\alpha)\delta(t - \tau)$. We then get for I

$$I=\frac{\pi(2m)^{1/2}}{\hbar\varkappa_a}\int_0^\infty dt D^2(\eta^2-t^2)$$
$$=\frac{\mathrm{Re}(\eta^2+i\hbar\nu/2)^{-1/2}-{}^1/_4\hbar\nu\,\mathrm{Im}(\eta^2+i\hbar\nu/2)^{-3/2}}{2\hbar^2\varkappa_a\nu}(2m)^{1/2}. \quad (2.5)$$

In this expression, we must choose that branch of the radicand $(\eta^2 + i\hbar\nu/2)^{1/2}$ which has a positive real part.

In the other limiting case, the width of the D function is small in comparison with the location of its maximum:

$$\hbar\nu/2\eta \ll \eta. \quad (2.6)$$

Thanks to the condition (2.6), the arguments of the D functions can be located close to the maxima, and the lower limit in each of the integrals is replaced by $-\infty$. Then the integral I is calculated exactly:

$$I=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\frac{D(2\eta x)D(2\eta y)}{(x-y)^2+(\hbar\varkappa_a)^2/2m}=\frac{(2m)^{1/2}}{2\hbar^2\varkappa_a\nu\eta}\left(1+\eta\frac{2\varkappa_a}{\nu\sqrt{2m}}\right)^{-1}. \quad (2.7)$$

The regions of applicability of Eqs. (25) and (2.7) overlap in the interval

$$(\hbar\nu/2)^{1/2} \ll \eta \ll (2m)^{1/2}\nu/2\varkappa_a. \quad (2.8)$$

Therefore, it is not difficult to construct an excellent integration formula for the integral I, which gives the correct result in both limiting cases (2.4) and (2.6):

$$I=\frac{(2m)^{1/2}}{4\hbar\varkappa_a}\frac{\mathrm{Re}(\eta^2+i\hbar\nu/2)^{-1/2}-{}^1/_4\hbar\nu\,\mathrm{Im}(\eta^2+i\hbar\nu/2)^{-3/2}}{\hbar\nu/2+\hbar\varkappa_a\eta/(2m)^{1/2}}. \quad (2.9)$$

The criterion of validity of this expression is the condition (1.13), which is equivalent to the left inequality of (2.4). We note that the region of overlap (2.8) of the asymptotic expressions (2.5) and (2.7) is important only in the case in which

$$(\hbar\varkappa_a)^2/m \ll \hbar\nu. \quad (2.10)$$

If this requirement is violated then, by virtue of the left inequality of (2.4), the region of applicability of Eq. (2.5) vanishes and the integral I is described by the expression (2.7) inasmuch as the condition $\hbar\kappa_\alpha(2m)^{-1/2} \ll \eta$ will not reach the limitingly small values of η.

As has already been pointed out above, an important role is played in the sum over n in (2.1) by such components with $n = N + 1$ if the parameter Δ is close to unity. The emergence of this and subsequent terms in the sum over n is connected with the collision broadening of the electron states on the Fermi surface. For the integral

$$I=\int_0^\infty dt\int_0^\infty d\tau\frac{D[t^2+(1-\Delta)\hbar\Omega]D[\tau^2+(1-\Delta)\hbar\Omega]}{(t-\tau)^2+(\hbar\varkappa_\alpha)^2/2m},$$

which corresponds to the (N + 1)st term of the sum, the expression (2.5) is valid, in which the quantity $\eta^2-(1-\Delta)\hbar\Omega$ is negative. The asymptotic form (2.7) cannot be realized in this term of the sum, because the maxima of the D functions are located at $t=\tau=0$ and their width is always much greater than the width of the function (2.3).

It is seen from Eq. (2.9) that for small ν the quantity I has a singularity as $\eta\to 0$. This means that the components with n = N or n = N + 1 play a fundamental role in the sum (2.1), because they contain singularities at small values of Δ or $1-\Delta$. We separate these components and replace the sum of the remaining terms with n < N approximately by an integral over n. In the calculation of this integral one should use the expression (2.7), since the inequality (2.6) is always satisfied in the corresponding terms of the sum. The integrated contribution of all nonsingular components gives the monotonic part of the absorption coefficient $\Gamma_{\rm mon}$:

$$\Gamma_{\rm mon}=\mathscr{T}\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{\operatorname{Re}(B_\alpha B_\beta^*)}{\varkappa_\beta^2-\varkappa_\alpha^2}q_\alpha^2\int_0^{N-1}dn\frac{(N-n)^{-1/2}}{(N-n)^{1/2}+N^{1/2}/\varkappa_\alpha l},\quad q_\alpha=k/\varkappa_\alpha. \tag{2.11}$$

Calculating the integral, we obtain

$$\Gamma_{\rm mon}=\Gamma_{\rm cl}-\mathscr{T}\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{B_\alpha B_\beta^*}{\varkappa_\beta^2-\varkappa_\alpha^2}\left[q_\alpha^2\ln\left(1+\frac{\varkappa_\alpha l}{N^{1/2}}\right)-q_\beta^2\ln\left(1+\frac{\varkappa_\beta l}{N^{1/2}}\right)\right], \tag{2.11'}$$

where $\Gamma_{\rm cl}$ represents the classical absorption coefficient of low-frequency sound in a strong magnetic field:[3]

$$\Gamma_{\rm cl}=\mathscr{T}\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{B_\alpha B_\beta^*}{\varkappa_\beta^2-\varkappa_\alpha^2}(q_\alpha^2\ln\varkappa_\alpha l-q_\beta^2\ln\varkappa_\beta l).$$

The contribution to the absorption from the singular components with n = N and n = N + 1 describes the giant quantum oscillations of the quantity Γ. For small Δ, the quantum effect is due to the N-th term of the sum (2.1), which is equal to

$$\Gamma_N=\frac{1}{2}\mathscr{T}\sum_{\alpha,\beta}q_\alpha q_\beta B_\alpha B_\beta^*\frac{\Delta^{1/2}+N^{1/2}/(\varkappa_\alpha+\varkappa_\beta)l}{(\Delta^{1/2}+N^{1/2}/\varkappa_\alpha l)(\Delta^{1/2}+N^{1/2}/\varkappa_\beta l)}\times\left[\operatorname{Re}\left(\Delta+i\frac{\nu}{2\Omega}\right)^{-1/2}-\frac{\nu}{4\Omega}\operatorname{Im}\left(\Delta+i\frac{\nu}{2\Omega}\right)^{-3/2}\right]. \tag{2.12}$$

This expression determines the shape of the absorption line from the low-field side with account of electron scattering. The other wing of the line is due to the (N + 1)st term of the sum:

$$\Gamma_{N+1}=\frac{1}{2}\mathscr{T}\frac{kl}{N^{1/2}}\left[\operatorname{Re}\left(\Delta-1+i\frac{\nu}{2\Omega}\right)^{-1/2}-\frac{\nu}{4\Omega}\operatorname{Im}\left(\Delta-1+i\frac{\nu}{2\Omega}\right)^{-3/2}\right]\times\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}. \tag{2.13}$$

We now consider the problem of the amplitude and shape of the lines of the quantum oscillations of the coefficient Γ. First of all, we note that the line shape is complicated by the fact that the absorption coefficient (2.12) is a sum of three like components with different coefficients and parameters, which depend on the character of the sound mode (parameters κ_α and κ_β). In what follows, we shall analyze the form of the individual components of the sum over α and β in (2.12).

As has been shown in the previous section, factorization of the two-particle Green's function is valid upon satisfaction of the condition $N\Delta>1$. Therefore the maximal value of Γ_N or Γ_{N+1} turns out to be of the order of

$$\mathscr{T}kl(1+N\nu/2\Omega)^{-1/2}. \tag{2.14}$$

The situation here is entirely analogous to that which exists in the case of quantum oscillations of volume sound.[4] Consequently, the necessary and sufficient condition that the maximal value of the singular components significantly exceed the value of the monotonic part, is

$$kl(1+N\nu/2\Omega)^{-1/2}\gg\mathscr{L}=|\ln(1/kl+N^{-1/2})|. \tag{2.15}$$

In the case of a not too strong magnetic field, when the parameter $\rho=(\hbar\Omega^2/\nu\epsilon_F)^{1/2}$ is small in comparison with unity, the amplitude of the singular part of the absorption is of the order of $\Gamma_{\rm mon}\rho kl/\mathscr{L}$. The parameter ρ represents the relative amplitude of the static quantum oscillations of the density of states at zero temperature.[8] It then follows that the oscillations of the absorption of the Rayleigh wave are $kl/\mathscr{L}$ times greater than the static ones. They are giant if $\rho kl\gg\mathscr{L}$. In the region of strong magnetic fields ($\rho^2\gg 1$), the inequality (2.15) transforms into $kl\gg\mathscr{L}$, which is practically identical with the condition of strong spatial inhomogeneity (1.6).

The shape of the individual component in (2.12) is very complicated, asymmetric and dependent on the quantities $N/(\kappa_\alpha l)^2$ and $\nu/2\Omega$. The relation between them does not contain the magnetic field and is determined by the already known parameter $\hbar\kappa_\alpha^2/m\nu$. Evidently, the greatest practical interest in the low-frequency case being considered ($\omega\ll\nu$) is represented by the range of frequencies bounded by the inequality (2.10). Here $\nu/2\Omega\ll N/(\kappa_\alpha/l)^2$ and the shape of the absorption line is described by the expression

$$\Gamma_N=\frac{1}{2}\mathscr{T}\sum_{\alpha,\beta}q_\alpha q_\beta B_\alpha B_\beta^*\frac{1}{\Delta^{1/2}}\frac{\Delta^{1/2}+N^{1/2}/(\varkappa_\alpha+\varkappa_\beta)l}{(\Delta^{1/2}+N^{1/2}/\varkappa_\alpha l)(\Delta^{1/2}+N^{1/2}/\varkappa_\beta l)}. \tag{2.16}$$

It is valid for $\Delta\gg\nu/2\Omega$, which corresponds to the right wing of the line in the scale of the inverse magnetic field. The absorption on the left wing is small and falls off more rapidly than in the case of (2.16):

$$\Gamma_{N+1}=\frac{5}{2^7}\mathscr{T}\left(\frac{\nu}{\Omega}\right)^2\frac{kR}{N^{1/2}(1-\Delta)^{7/2}}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta},\quad \nu/2\Omega\ll 1-\Delta\ll 1. \tag{2.17}$$

We note that if $1-\Delta$ in (2.17) is replaced by $n-N-\Delta$, we then obtain an expression for the degenerate terms of the sum (2.1) with $n>N+1$.

The characteristic width of the peak of the quantum oscillations depends on the value of the factor $N/(\kappa_\alpha l)^2$ in comparison with unity. If it is large, then the width of the peak $\delta\Delta$ turns out to be the same as in the static oscillations of the density of states, i.e., $\delta\Delta\sim\nu\Omega$. Here the quantity Γ_N has the form

$$\Gamma_N=\frac{1}{2}\mathscr{T}\frac{kl}{(N\Delta)^{1/2}}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}. \tag{2.18}$$

The amplitude of the oscillations in this case is small in comparison with $\Gamma_{\rm cl}$ by virtue of condition (2.10). When the opposite condition is satisfied,

$$N/(\varkappa_\alpha l)^2\ll 1, \tag{2.19}$$

the width of the quantum oscillation line $\delta\Delta \sim N/(\kappa_\alpha l)^2$, and Γ_N turns out to be much greater than the monotonic part Γ_{mon}. For $\Delta \gg \delta\Delta$ we have

$$\Gamma_N = \frac{\mathcal{T}}{2\Delta}\sum_{\alpha,\beta} q_\alpha q_\beta B_\alpha B_\beta^*. \tag{2.20}$$

We return our attention to the following circumstance. If we integrate the expression (2.12) for Γ_N over Δ from 0 to 1, we then obtain a result compensating the second component in (2.11′) in accuracy. This conclusion on the invariance of the integrated (over the magnetic field) absorption coefficient is valid for low-frequency Rayleigh sound to the same extent as for volume waves.[4] In other words, for giant oscillations, only the redistribution of electron damping in the scale of the magnetic field takes place: the narrow peak of the giant absorption arises from the corresponding decrease in the monotonic part.

At the end of the previous section, we spoke of the fact that the law of conservation of the x component of the electron momentum is not satisfied in the field of an inhomogeneous sound wave. However, in spite of the absence of an exact δ function corresponding to this conservation law, the effective collision frequency for "resonance" electrons $\nu + \kappa_\alpha|v_{xN}| = \nu + \kappa_\alpha v(\Delta/N)^{1/2}$, which enters into (2.7) is of the order of or practically identical with ν. This is connected with the fact that the electrons near the central cross section of the Fermi surface (with n = N or n = N + 1) possess a small drift velocity along the magnetic field and the corresponding diffusion of the d function (2.3) and its difference from the δ function are comparatively small. Herein lies the reason that the oscillations of the absorption of Rayleigh waves are giant, and the criteria of their existence (2.15) are practically identical with the condition for giant quantum oscillations of volume sound.[4]

2. High frequencies ($\omega \gg \nu$). It is here that the difference between the giant oscillations of Rayleigh sound and the similar effect in the damping of volume waves is most pronounced. Since there is no exact law of conservation of the x component of the electron momentum for the absorption of a quantum of inhomogeneous sound wave, the D functions in (1.14) can be replaced by δ functions. Actually, in the integral over the energy (1.3), the values of E from $\epsilon_F - \hbar\omega$ to ϵ_F are important. Therefore, one of the D functions in (1.14) in the region of high frequencies can always be regarded as a δ function. After this, the argument of the second D function turns out to be $\epsilon_b - \epsilon_a + \hbar\omega$. The characteristic value $\epsilon_b - \epsilon_a$, which is determined from the denominator of (1.14), is not less than the value $\hbar\kappa_\alpha(\hbar\omega/2m)^{1/2}$, i.e., the argument of the D function behaves as a δ function. Thus the integrals over ϵ_a and ϵ_b are easily computed and only the components with $n \le N$ (for $\omega \ll \Omega$) are left in the sum over n. As a result, we obtain

$$\Gamma = 2\mathcal{T}\frac{\Omega}{\omega}\sum_{\alpha,\beta} q_\alpha q_\beta B_\alpha B_\beta^* \sum_{n=0}^{N} G\left[\frac{\Omega}{\omega}(N-n+\Delta);\, g_\alpha, g_\beta\right]. \tag{2.21}$$

Here we have introduced the notation

$$G(x;g_\alpha,g_\beta) \equiv \int\limits_{x-\min(1,x)}^{x} dx' \frac{1+\mu(x')}{[1+g_\alpha+\mu(x')][1+g_\beta+\mu(x')]},$$
$$\mu(x) = 2x + 2x^{1/2}(1+x)^{1/2}, \quad g_\alpha = 2m\omega/\hbar\varkappa_\alpha^2. \tag{2.22}$$

In all the terms of the sum (2.21) except the last (n = N), the first argument of the function G is much greater than unity and the quantity G is approximately equal to the integrand for x′ = x ≫ 1, i.e.,

$$G(x;g_\alpha,g_\beta) = \frac{1+4x}{(1+g_\alpha+4x)(1+g_\beta+4x)}. \tag{2.23}$$

The sum of the nonspecial terms can now be computed explicitly. We put down the answer in unsymmetrized form in α and β:

$$\Gamma_{\text{mon}} = \mathcal{T}\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{\operatorname{Re}(B_\alpha B_\beta^*)}{\varkappa_\beta^2-\varkappa_\alpha^2}q_\alpha^2\left[\psi\left(N+\frac{\omega g_\alpha}{4\Omega}+1+\Delta\right) - \psi\left(\frac{\omega g_\alpha}{4\Omega}+1+\Delta\right)\right], \tag{2.24}$$

where $\psi(x)$ is the Euler function.

The last term in the sum (2.21) describes the resonance singularity of the absorption

$$\Gamma_N = 2\mathcal{T}\frac{\Omega}{\omega}\sum_{\alpha,\beta} q_\alpha q_\beta B_\alpha B_\beta^* G\left(\frac{\Omega}{\omega}\Delta;\, g_\alpha, g_\beta\right). \tag{2.25}$$

The amplitude and the shape of the Rayleigh maxima depend essentially on the quantity g_α, which is equal to the ratio of the energy of the sound quantum to the characteristic "energy yield" $(\hbar\kappa_\alpha)^2/2m$, which can be both greater and less than unity. In the region of not too high frequencies, where

$$g_\alpha,\ g_\beta \gg 1, \tag{2.26}$$

we can use the formula (2.23) for $G(\Omega\Delta/\omega)$; this formula is valid for $\Delta > \omega/\Omega$. At lower values of Δ we must use the exact formula (2.22). The function $G(\Omega\Delta/\omega;\, g_\alpha, g_\beta)$ for this case is shown schematically in Fig. 1. Its maximum is located at $\Delta = \omega(g_\alpha g_\beta)^{1/2}/4\Omega$, and the value of G at the maximum is $(g_\alpha^{1/2} \pm g_\beta^{1/2})^{-2}$. At $\Delta = \omega/\Omega$, the function G has a kink, and the jump in the derivative with respect to Δ is equal to $-\Omega/\omega g_\alpha g_\beta$. The derivative at the left of the break is $5.83\,\Omega/\omega g_\alpha g_\beta$, i.e., the change in the angle of inclination at the point $\Delta = \omega/\Omega$ is small.

The value of the absorption coefficient at the maximum is much greater than the monotonic part if

$$\Omega/\omega g \sim \hbar\Omega/ms^2 \gg 1. \tag{2.27}$$

Here

$$\Gamma_{N\,max} \sim \Gamma_{\text{mon}}\frac{\hbar\Omega}{ms^2\ln N},$$
$$\Gamma_{\text{mon}} = {}^1/_2\mathcal{T}(\ln N + C)\sum_{\alpha,\beta} q_\alpha q_\beta B_\alpha B_\beta^*, \tag{2.28}$$

where $C \approx 0.577$ is Euler's constant. We recall that to determine the line shape and the exact value of the ab-

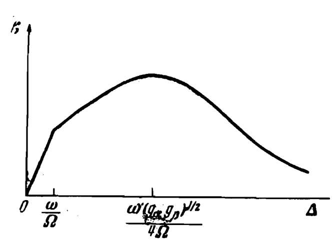

FIG. 1

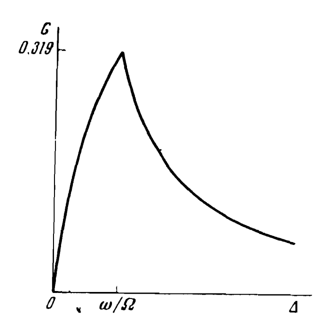

FIG. 2

FIG. 1. Schematic dependence of the function G from Eq. (2.25) on Δ in the region of not very high frequencies (2.26). The parameter Δ represents the fractional part of the quantity $(\epsilon_F/\hbar\Omega) - 1/2$.

FIG. 2. Dependence of the function G on Δ in the region of high frequencies (2.29).

sorption coefficient at the maximum we need to take into account all the terms of the sum (2.25) over the indices α and β of the various modes of oscillation. In the region of weak magnetic fields, when the inequality (2.27) is replaced by its opposite, the amplitude of the oscillations turns out to be a small quantity of the order of $(\hbar\Omega/ms^2)^2$.

On going to hypersonic frequencies, the parameter g_α decreases and the limiting case

$$g_\alpha,\ g_\beta \ll 1, \tag{2.29}$$

which is the reverse of (2.26), becomes important. Here one cannot take g_α and g_β into account in the definition (2.22) for the function G. Correspondingly, the small terms $\omega g_\alpha/4$ in the arguments of the functions ψ in Eq. (2.24) should be neglected. In this case, the amplitude of the oscillations reaches a maximum value of the order of Ω/ω:

$$\Gamma_{N\,max} = 0.638\,\mathcal{T}\frac{\Omega}{\omega}\sum_{\alpha,\beta} q_\alpha q_\beta B_\alpha B_\beta^*, \tag{2.30}$$

which corresponds to $\Delta = \omega/\Omega$. Figure 2 shows the schematic dependence of the function G on Δ in the case of (2.29). At the maximum, the jump in the derivative is equal to $-\Omega/\omega$ and the angle at the vertex of the kink is $(3 + 2\sqrt{2})^2\omega/\Omega(2 + \sqrt{2}) \approx 7\omega/\Omega$. The shape of the line is determined by the function $G(\Omega\Delta/\omega;\ 0,\ 0)$ (see (2.22)). We shall show that the presence of a kink in the dependence of the coefficient Γ_N on Δ leads to a singularity in the velocity of the Rayleigh wave of the type $|\Delta - \omega/\Omega|\ln|\Delta - \omega/\Omega|$, which is similar to the Kohn singularity in the phonon spectrum.[9]

To conclude this section, we shall clarify the problem of the invariance of the absorption integrated over the magnetic field in the high-frequency region. For this purpose, we integrate the expressions (2.24) and (2.25) with respect to Δ from 0 to 1. As a result we get, for N, $\Omega/\omega \gg 1$,

$$\int_0^1 d\Delta\Gamma = \mathcal{T}\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{\mathrm{Re}(B_\alpha B_\beta^*)}{\varkappa_\beta^2-\varkappa_\alpha^2}q_\alpha^2\left[\ln\left(\frac{\varkappa_\alpha v}{\omega}\right)^2-\ln(1+g_\alpha^{-1})\right]. \tag{2.31}$$

This formula differs from Γ_{cl} (the first term in the square brackets) by an additional component, which contains $\ln(1 + g_\alpha^{-1})$. If the characteristic energy yield is small in comparison with the quantum energy $\hbar\omega$, then this correction is also small. It can play a role only in the high-frequency region (2.29). In this case the absorption integrated over Δ is not invariant, inasmuch as the quantity Γ_{cl} depends logarithmically on the effective free path length $l = v/|\nu - i\omega|$ and consequently does not have a collisionless character.

3. ABSORPTION OSCILLATIONS AT FINITE TEMPERATURE

From the point of view of experiment, the most realistic case is that of finite temperatures, which satisfy the inequalities

$$\hbar(\omega + \nu) \ll T \ll \hbar\Omega. \tag{3.1}$$

The energy of thermal motion is much greater here than the energy of a quantum of the Rayleigh wave and the width of the energy levels of the electron, but is smaller than the distance between neighboring Landau levels. Evidently, the temperature is practically independent of the monotonic part of the absorption coefficient, since all the characteristic differences of the energies in the nonspecial terms of the sum (1.14) are much greater than T, by virtue of the right-hand inequality of (3.1).

The temperature can have a significant influence only on the value and shape of the special components with n = N and n = N + 1. As in the previous section, we shall consider separately the regions of low and high frequency.

1. Low frequencies ($\omega \ll \nu$). At absolute zero temperature, the resonance absorption line is described by the sum of the special components:

$$\Gamma_{res}(\sigma) = \Gamma_N(\Delta) + \Gamma_{N+1}(\Delta),\qquad \sigma = \sigma(\Delta) = \begin{cases}\Delta, & \text{if}\quad \Delta \ll 1\\ \Delta - 1, & \text{if}\quad 1-\Delta \ll 1\end{cases}. \tag{3.2}$$

In the limits of the line, $|\sigma| \ll 1$ and the quantity σ itself can be either positive or negative. For $\sigma > 0$ (the right wing of the line in the scale of the inverse magnetic field) $\Gamma_{res}(\sigma)$ is practically identical with $\Gamma_N(\sigma)$, and with $\Gamma_{N+1}(\sigma)$ for $\sigma < 0$. At finite temperature, the difference of the Fermi functions in (1.3) can be expanded in $\hbar\omega/T$ and we get for the contribution to the monotonic part of the coefficient Γ

$$\delta\Gamma = \Gamma - \Gamma_{mon} = \frac{\Omega'}{2}\int_{-N}^{\infty} d\sigma'\frac{\Gamma_{res}(\sigma')}{\mathrm{ch}^2[\Omega'(\sigma'-\sigma)]},\qquad \Omega' = \frac{\hbar\Omega}{2T} \tag{3.3}$$

The meaning of this formula is that the coefficient $\delta\Gamma$ at finite T is obtained by integration of Γ_{res}, found for T = 0, over the energy, with the weight $\partial f_0/\partial E$. It is evident that the lower limit in (3.3) can tend toward $-\infty$ and we rewrite this formula in the form

$$\delta\Gamma = \frac{\Omega'}{2}\int_0^\infty d\sigma'\left\{\frac{\Gamma_{res}(\sigma')}{\mathrm{ch}^2[\Omega'(\sigma'-\sigma)]}+\frac{\Gamma_{res}(-\sigma')}{\mathrm{ch}^2[\Omega'(\sigma'+\sigma)]}\right\}. \tag{3.4}$$

It is easy to understand from physical considerations that the effect of the temperature should be important near the maximum absorption, when

$$|\sigma| \lesssim T/\hbar\Omega. \tag{3.5}$$

On the wings of the line, where

$$|\sigma| \gg T/\hbar\Omega, \tag{3.6}$$

the absorption is a smooth function of σ and the temperature has slight influence on the value of $\delta\Gamma$. In the region (3.6), $\delta\Gamma$ is practically the same as $\Gamma_{res}(\sigma)$ at T = 0.

We now consider in more detail the behavior of both terms in the formula (3.4). For $\sigma > 0$, the first term describes the broadening of the absorption of the right wing Γ_{res} due to the temperature, and the second term describes the exponentially decaying (in $\Omega'\sigma$) contribution from the left wing of the absorption line at T = 0. Inasmuch as the quantity Γ_{res} on the left wing is small (see (2.17)), its contribution due to thermal broadening in the region $\sigma > 0$ is unimportant. In other words, at $\sigma > 0$ the second term in (3.4) can be neglected.

The shape of the left wing ($\sigma < 0$) is subjected to very strong temperature influence. In the region (3.5) the absorption is determined by the diffuse temperature contribution from the right wing $\Gamma_{res}(\sigma')$ (the first component in (3.4)). The second component in (3.4) is unimportant.

In the transition to the region (3.6), the value of the first component decreases exponentially and the absorption is determined by the second term of formula (3.4), in which the temperature can go to zero. Such is the qualitative picture of the effect of the temperature on the giant oscillations of low-frequency Rayleigh wave.

We now give the formula for the contribution $\delta\Gamma$ near the center of the line in the region (3.5):

$$\delta\Gamma = \Omega'\mathcal{T}\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{\operatorname{Re}(B_\alpha B_\beta^*)}{\varkappa_\beta^2-\varkappa_\alpha^2}q_\alpha^2\int_0^\infty dx\frac{\operatorname{ch}^{-2}(x-\sigma\Omega')}{x+b_\alpha^{-1}}, \tag{3.7}$$

where the parameter b_α has the form

$$b_\alpha = \varkappa_\alpha l(2T/\varepsilon_F)^{1/2}. \tag{3.8}$$

The integral (3.7) can be computed only in the limiting cases of large and small values of b_α. If $\ln b_\alpha \gg 0.1$, then the line shape is described by the equation

$$\delta\Gamma = \frac{\Omega'}{2}\mathcal{T}\operatorname{ch}^{-2}(\Omega'\sigma)\sum_{\alpha,\beta}\varkappa_\alpha\varkappa_\beta\frac{B_\alpha B_\beta^*}{\varkappa_\beta^2-\varkappa_\alpha^2}(q_\alpha^2\ln b_\alpha - q_\beta^2\ln b_\beta). \tag{3.9}$$

The maximum absorption takes place for $\sigma = 0$, and $\delta\Gamma_{max}$ is approximately $\Omega'/2$ times Γ_{cl}. Near the maximum in the region (3.5), the line is symmetric; asymmetry appears far from the peak ($|\sigma| > T/\hbar\Omega$).

In the opposite limiting case of small b_α, the coefficient $\delta\Gamma$ decreases, and the shape of the curve becomes asymmetric and is determined by the expression

$$\delta\Gamma = \frac{\Omega'}{2}b\mathcal{T}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}\int_0^\infty dx\operatorname{ch}^{-2}(x^2-\sigma\Omega'), \tag{3.10}$$

in which the quantity b differs from (3.8) by the replacement $\kappa_\alpha \to k$. The maximum of the absorption is located somewhat to the right of the point $\sigma = 0$ and the value of the integral for $\sigma = 0$ is about $(8/9)^{1/2}$. The absorption oscillations in this case are proportional to the oscillations of the static density of states at finite temperatures.

2. High frequencies ($\omega \gg \nu$). In this range of frequencies, the finite absorption on the left wing is due to collisions of electrons with scatterers and disappears as $\nu \to 0$. Therefore, the collisionless absorption at $T = 0$ is significant only if $\sigma > 0$. At finite temperatures, we obtain

$$\delta\Gamma = \frac{\Omega'}{2}\int_0^\infty d\Delta\frac{\Gamma_{res}(\Delta)}{\operatorname{ch}^2[\Omega'(\Delta-\sigma)]}, \tag{3.11}$$

where $\Gamma_{res}(\Delta)$ is determined by the formulas (2.25) and (2.23). In contrast with (3.4), the integral with $\Gamma_{res}(\sigma')$, which describes the small collision absorption on the left wing, is lacking here. The formula (3.11) means that collisionless resonance absorption at $T = 0$ should be averaged over the energy of the electron near the Fermi boundary, diffused by the thermal motion.

It is rather apparent that the thermal scatter leads first to the appearance of an exponentially damped "tail" at the left wing of the absorption line for $|\sigma| > T/\hbar\Omega$; second, it leads to a smoothing out of the breaks on the absorption curves (see Figs. 1, 2) and third, it leads to a change in shape and decrease in magnitude of the absorption at the maximum. In the region of not too high temperatures, where the parameter

$$a_\alpha = 8T/\hbar\omega(1+g_\alpha) \ll 1, \tag{3.12}$$

the temperature has practically no effect on the value of the absorption, cited in Fig. 1.[1)] This is connected with the fact that the coefficient $\Gamma_{res}(\sigma)$ depends smoothly on $\Delta = \omega/\Omega$; the absorption on the left of the maximum changes slowly with magnetic field in the characteristic range $|\sigma| \sim T/\hbar\Omega$. In order to obtain an explicit analytic expression for the contribution $\delta\Gamma$ in this case, it is necessary to replace the function G from (2.23) in the formula (2.25) for Γ_{res} by the following asymptotic form

$$G\left(\frac{\Omega}{\omega}\Delta;\, g_\alpha, g_\beta\right) = \Delta\frac{4\Omega}{\omega g_\alpha g_\beta}.$$

After this, the integral in (3.11) is computed exactly and $\delta\Gamma$ is represented in the form

$$\delta\Gamma = \frac{\Omega}{\omega}\frac{8T}{\hbar\omega}\mathcal{T}\ln\left[1+\exp\left(\frac{\hbar\Omega\sigma}{T}\right)\right]\sum_{\alpha,\beta}q_\alpha q_\beta\frac{B_\alpha B_\beta^*}{g_\alpha g_\beta}. \tag{3.13}$$

This formula describes the absorption at the left of the maximum and is valid up to the point where $\delta\Gamma$ is less than Γ_{max} from (2.28). Near the maximum and to the right of it, the absorption is given by the formula (2.25) with the function G from (2.23).

At higher temperatures, when $a_\alpha \gg 1$, it is convenient to use the general formula (3.11) for $\delta\Gamma$ in the unsymmetrized form:

$$\delta\Gamma = \frac{\Omega'}{2}\mathcal{T}\sum_{\alpha,\nu}\varkappa_\alpha\varkappa_\beta\frac{\operatorname{Re}(B_\alpha B_\beta^*)}{\varkappa_\beta^2-\varkappa_\alpha^2}q_\alpha^2\int_0^\infty dx\frac{\operatorname{ch}^{-2}(x-\sigma\Omega')}{x+a_\alpha^{-1}}, \tag{3.14}$$

which is like the expression (3.7). Just as in the case of large b_α, when $a_\alpha \gg 1$, the integral in (3.14) is equal to

$$\frac{\ln a_\alpha}{\operatorname{ch}^2(\sigma\Omega')} + 2\int_0^\infty dx\ln x\frac{\operatorname{th}(x-\sigma\Omega')}{\operatorname{ch}^2(x-\sigma\Omega')}. \tag{3.15}$$

It is not difficult to see that the maximum absorption lies somewhat to the right of the point $\sigma = 0$, and in the logarithmic approximation ($\ln a_\alpha \gg 0.2$[2)]) for $\delta\Gamma$, we obtain the expression (3.9), in which b_α should be replaced by $\sqrt{a_\alpha}$. We note that the parameter $\sqrt{a_\alpha}$ is identical with the quantity b_α, in which the effective path length at high frequencies v/ω is substituted in place of $l = v/\nu$. It is seen from Eq. (3.14) that the higher maximum, which is represented in Fig. 2, decreases by a factor of about $5T/\hbar\omega\ln a_\alpha$ in this case.

4. GIANT QUANTUM OSCILLATIONS OF RAYLEIGH WAVES IN A MAGNETIC FIELD PARALLEL TO THE SURFACE

Up to now, we have considered the effect of quantum electron states on the absorption of Rayleigh sound waves in a magnetic field perpendicular to the boundary surface. If the vector **H** is parallel to the surface of the metal, then the amplitude and shape of the quantum oscillations differ materially. The differences are due to the fact that the electrons drift along the lines of force in a parallel field, i.e., they move in planes with the same amplitude as the inhomogeneous sound wave. In a strong magnetic field, when the penetration depth of the Rayleigh wave κ^{-1} is much greater than the cyclotron radius R, the electronic absorption has an essentially volume character, inasmuch as the relative number of electrons colliding with the surface is small along with the parameter $\kappa R \ll 1$. On the other hand, for electrons which do not collide with the surface of the metal, the law of the conservation of the longitudinal (relative to the vector **H**) component of the momentum, $p_{za} - p_{zb} = \hbar k$ is satisfied exactly (here the z axis is directed along **H**). For this reason, the quantum effects have an influence on the absorption of the Rayleigh wave in precisely the same way as in the case of volume sound waves.

Formally, the difference between the cases of normal and parallel magnetic fields is that in the expression (1.14), the exact δ function, which expresses the law of conservation of the z component of the electron momentum, appears in place of the d function. Therefore the situation in the parallel field does not differ at all from volume absorption, which was analyzed in detail in[4]. It was shown there that the quantum effect in absorption reduces to the multiplication of the classical coefficient Γ_{cl} by some universal function $\mathcal{F}$ which depends on the magnetic field, the frequency and the projection of the wave vector on the direction of **H**:

$$\Gamma = \Gamma_0 \mathcal{F}, \tag{4.1}$$

$$\mathcal{F} = \frac{\hbar\Omega\hbar k_z}{m} \cdot \times \sum_{n=0}^{\infty} \int_0^{\infty} dE \frac{f_0(E) - f_0(E+\hbar\omega)}{\hbar\omega} \int_{-\infty}^{\infty} dp_z D[E - \varepsilon_n(p_z)] D[E + \hbar\omega - \varepsilon_n(p_z + \hbar k_z)], \tag{4.2}$$

where the quantity Γ_0 is connected with the value of the classical absorption coefficient Γ_{cl} by the relation

$$\Gamma_0 = \Gamma_{cl} \frac{\pi}{\operatorname{arctg}(k_z l - \omega\tau) + \operatorname{arctg}(k_z l + \omega\tau)}, \quad \tau = \nu^{-1}. \tag{4.3}$$

For $|k_z| l \gg 1$ and $|k_z| l \gg \omega\tau$ the quantity Γ_0 is the same as Γ_{cl}. This very case ($\Gamma_0 = \Gamma_{cl}$) was considered in[4]. According to[10], the correct formula for Γ_0 in the strong magnetic field (1.13) is

$$\Gamma_0 = \frac{\pi}{2} \frac{k}{k_z} \mathcal{F} \sum_{\alpha,\beta} k \frac{B_\alpha B_\beta^*}{\varkappa_\alpha + \varkappa_\beta}. \tag{4.4}$$

The behavior of the function $\mathcal{F}$ has been studied in detail in[4] and we shall not consider it here. We only note that in the case of Rayleigh sound waves, there should be a strong anisotropy in the absorption and dispersion of the sound velocity in the range of small angles of inclination of the magnetic field relative to the surface of the metal.

[1)]The inequality (3.12) leads asymptotically to the condition (2.26), inasmuch as $8T \gg \hbar\omega$.

[2)]The integral in (3.15) is approximately equal to -0.1 for $\sigma = 0$.

[1] V. L. Gurevich, V. G. Skobov and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **40**, 786 (1961) [Soviet Phys.-JETP **13**, 552 (1961)].

[2] A. Ya. Blank and É. A. Kaner, Zh. Eksp. Teor. Fiz. **50**, 1013 (1966) [Soviet Phys.-JETP **23**, 672 (1966)].

[3] A. M. Grishin and É. A. Kaner, Zh. Eksp. Teor. Fiz. **63**, 2304 (1972) [Soviet Phys.-JETP **36**, 1217 (1973)].

[4] É. A. Kaner and V. G. Skobov, Zh. Eksp. Teor. Fiz. **53**, 375 (1967) [Soviet Phys.-JETP **26**, 251 (1968)].

[5] L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Theory of Elasticity) Nauka Press, 1965; Par. 24 [Addison-Wesley, 1971].

[6] V. G. Skobov, Zh. Eksp. Teor. Fiz. **40**, 1446 (1961) [Soviet Phys.-JETP **13**, 1014 (1961)].

[7] N. M. Makarov and I. M. Fuks, Zh. Eksp. Teor. Fiz. **60**, 806 (1971) [Soviet Phys.-JETP **33**, 436 (1971)].

[8] V. G. Skobov, Fiz. Tverd. Tela **6**, 1675 (1964) [Soviet Phys.-Solid State **6**, 1316 (1964)].

[9] W. Kohn, Phys. Rev. Lett. **3**, 393 (1959); A. B. Migdal, Zh. Eksp. Teor. Fiz. **34**, 1438 (1958) [Soviet Phys.-JETP **7**, 996 (1958)].

[10] A. M. Grishin and Yu. V. Tarasov, Solid State. Commun. **12**, No. 12 (1973).

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