Interaction between an intense electromagnetic wave and a plasma

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A linear theory of parametric instabilities in an inhomogeneous plasma is considered. The linear amplification coefficients are calculated for small perturbations produced during backward parametric scattering and parametric absorption of an intense electromagnetic wave in a plasma. Nonlinear models are proposed which describe the backward parametric scattering process. In the case of "soft" turning-on of the instability, the quasilinear theory is sufficient as a rule to describe the relaxation of radiation penetrating the plasma. The relaxation length of the incident electromagnetic wave is estimated in the case of "hard" turning-on of the instability.

1. INTRODUCTION

Recently the possibility has been intensively studied of initiating thermonuclear fusion in a drop of deuterium-tritium mixture by means of high-power laser radiation specially profiled in time.[1] Absorption of this radiation by the plasma corona of the drop should lead to a rapid rise of temperature and pressure of the plasma, dispersion of the corona and, as a consequence, to compression of the internal part of the drop. Hydrodynamic calculations made with computers and other estimates show that the efficiency of the scheme discussed depends[2] on the absorption efficiency η of the electromagnetic wave by the shell of the drop as η^{-4}. Therefore the question of the mechanisms of interaction of the radiation with the plasma corona of the drop plays an extremely important role. The electromagnetic wave propagating into a region of increasing density reaches a reflection point ($\omega_0 \approx \omega_p(x)$) and turns back. At plasma corona temperatures of several keV and for reasonable assumptions as to the size of the dispersing shell (~ 1 mm) the absorption of the wave in this path, due to the imaginary part of the refractive index $\mathrm{Im}\{\epsilon(\omega)\}^{1/2} = \mathrm{Im}\{1 - \omega_p^2/\omega(\omega + i\nu_e)\}^{1/2}$ (ν_e is the electron collision frequency), is small. For oblique incidence there is an additional absorption due to transformation of part of the electromagnetic wave energy into longitudinal plasma oscillations in the vicinity of the point where $\epsilon(\omega, x) = 0$.[3] However, this absorption, generally speaking, is proportional to a small factor which describes the sub-barrier attenuation of the electromagnetic wave in the path from the reflection point to the transformation point.

The possibility of a strong interaction of the electromagnetic wave with the plasma is due to nonlinear effects. The main process of this type is three-wave decay. In this process the initial state in which a wave of sufficient amplitude with frequency ω_0 and wave vector $\mathbf{k}_0$ is excited (the pumping wave) is unstable with respect to small perturbations consisting of a pair of waves with frequencies and wave vectors $(\omega_1, \mathbf{k}_1)$ and $(\omega_2, \mathbf{k}_2)$ which satisfy the so-called decay conditions $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$, $\omega_0 = \omega_1 + \omega_2$, $|\omega_0| > |\omega_{1,2}|$. The growth rate γ of this instability is simply expressed in terms of the matrix element of the three-wave interaction $V_{\mathbf{k}_0,\mathbf{k}_1,\mathbf{k}_2}$—a quantity which has been well studied in turbulent plasma theory,[4]

$$\gamma = (\gamma_d^2 - \delta^2/4)^{1/2}, \quad \gamma_d = |V_{\mathbf{k}_0,\mathbf{k}_1,\mathbf{k}_2}|\,|C_0|,$$

$\delta = \omega_0 - \omega_1 - \omega_2$ is the detuning frequency, and C_0 is the pumping-wave amplitude normalized so that $|\omega_0|\,|C_0|^2$ is the energy of this wave.

The first example of a decay instability in a plasma was the decay plasmon → plasmon + sound.[5,6] The decay photon → plasmon + sound (t → l + s) occurs in the same way,[7,8] and the growth rates are precisely the same since it is not important whether the pumping electric field $\mathbf{E}_0 \cos(\omega_0 t - \mathbf{k}_0\mathbf{r})$ belongs to the plasmon or to the photon. The instability t → l + s in application to the interaction of laser radiation with the plasma corona indicates the transfer of laser energy to the energy of natural plasma oscillations. The latter are finally absorbed by plasma particles. (Discussion of the details of this absorption would take us beyond the scope of the present article).

Another type of instability of laser radiation is the decay t → l + l',[9] which occurs at a plasma density roughly four times smaller. All processes of this type have a parametric nature and can be interpreted as a parametric instability in a system of two coupled oscillators (the pair of waves arising with the instability). Coupling between these oscillators is accomplished as the result of the nonlinear properties of the medium (for example, nonlinearity of the dielectric properties) through the pumping wave. A pattern of instability zones similar to Mathieu zones for the decay instability is shown in Fig. 1.

As the pumping wave amplitude increases, it is eventually necessary to take into account the participation of a larger and larger number of oscillators (waves). However, this occurs only at very large amplitudes where $\Delta\omega$—the width of the main zone—becomes of the order of

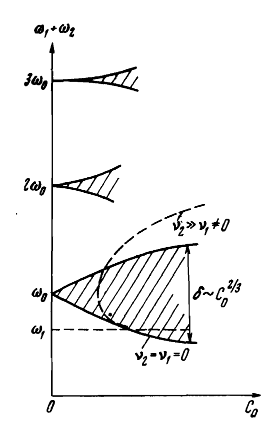

FIG. 1. Zones of parametric instability in the perturbation frequency-amplitude plane.

$\omega_{1,2}$. If the frequency of one of the waves, for example ω_2, is very small ($\omega_2 << \omega_{0,1}$), then for $\omega_1 >> \Delta\omega \sim \gamma > \omega_2$ it is still sufficient to take into account only two coupled oscillators. However, the wave (ω, k_2) is strongly modified. The growth rate of the instability in this case ($\gamma_d > \omega_2$) turns out to be $\gamma = (\sqrt{3}/2)(2\gamma_d^2\omega_2)^{1/3}$.[10,11] This limiting case sometimes is called the modified decay. The decay instability occurs, as has already been mentioned, for pumping-wave amplitudes above some critical amplitude (threshold). In a homogeneous plasma this threshold is $2\gamma_d^2 > \nu_1\nu_2$,[12] and depends on the damping decrements of the waves ν_1 and ν_2 (see Fig. 1).

For the problem considered the laser pulse powers practically always lie above the threshold calculated for a homogeneous plasma. Under these conditions a much more important role is played by the inhomogeneity of the plasma in the corona. In the simplest case of laminar outflow, this is the inhomogeneity in density $n = n(r)$ and the inhomogeneity in the velocity of gas-dynamical outflow $U = U(r)$. The temperature in the rarified shell can in most cases be assumed uniform as the result of the high electronic thermal conduction. The conditions for a decay resonance in this case can be satisfied only in a limited spatial region. Perturbation waves moving with their group velocities finally leave the instability zone. The question of whether the decay instability will succeed in appearing in a finite growth time depends on whether the initial fluctuations succeed in growing to a nonlinear level. For equilibrium thermal noise this means an amplification of approximately $e^{\mathcal{L}}$ times, where $\mathcal{L}$ is the so-called Coulomb logarithm: $\mathcal{L} = \ln(n\lambda_D^3)$.

Section 2 of the present article is devoted to establishment of the threshold of the decay instability of laser radiation as the result of plasma inhomogeneity. Specific cases of various decays selected in Sec. 3 show that as the laser power is increased, first the threshold is reached for $t \to l + s$, and then for $t \to l + l'$ instabilities. However with further increase of power of the incident radiation, parasitic decay instabilities[13] $t \to t' + s$ and $t \to t' + l$ appear which lead to development of parametric scattering of electromagnetic radiation and finally to an additional nonlinear reflection.[1)] This problem may turn out to be extraordinarily acute for laser initiation of a D − T drop. Therefore, in addition to calculation of the amplification coefficients for the processes of $t \to t' + s$ and $\to l' + l$ (and for the processes of induced scattering by ions and electrons co corresponding to them) in Sec. 3, we have attempted to discuss analytic models of the nonlinear stage of the parametric scattering process in Sec. 4. Nonlinear effects slow down the exponential growth of the instability but do not suppress the parametric scattering of the light. This problem requires further study, in particular, by the Monte Carlo method.

2. DECAY INSTABILITIES IN AN INHOMOGENEOUS PLASMA

Inhomogeneity of a plasma leads to destruction of the conditions for parametric resonance and consequently to limitation of the growth of parametrically coupled perturbations in the plasma. As a result the pair of perturbation waves grows only by a finite factor. If we consider as perturbations normal oscillations of an inhomogeneous plasma with frequencies satisfying the resonance condition and if we take into account that the resonance conditions for the wave vectors are satisfied only in a narrow zone of the spatial coordinates, we can construct a stationary picture of the amplification of oscillations in the system of two coupled oscillators.[16,17]

In the case in which the low-frequency partner in the decay pair is highly damped, the resonance region is smeared and the amplification coefficient turns out to be different.[18,19]

In addition to the method of normal modes, there is a simpler method, the approximate method of wave packets. More accurately, the frequency of one perturbation (the high-frequency perturbation) is assumed fixed and the second perturbation is represented in the form of a wide wave packet with various frequencies.[18] In the approximation of weak inhomogeneity, only the spectral component of the low-frequency packet with a wave number satisfying the resonance condition interacts with the high-frequency perturbation. The frequency of the perturbation component selected in this way turns out to be in resonance with the beat produced by the incident wave and the high-frequency perturbation only in a narrow zone of configuration space. Therefore, growth of the oscillations occurs, as in the first approach, up to some final amplitude. Depending on the type of problem, we must use the first or second appraoch. Generally speaking, the first approach is more rigorous. However, it is also more cumbersome and cannot always be used in practice.

By analogy with the problem of the linear instability of an inhomogeneous plasma, the first approach can be called the method of natural modes, and the second the quasimode approximation.[20]

The method of natural modes

The system of equations describing the interaction of coupled natural oscillations in a plasma inhomogeneous along the x axis has the form

$$\frac{\partial C_1}{\partial t} + v_1\frac{\partial C_1}{\partial x} = VC_0C_2^*\exp\left[i\int_0^x \varkappa(x)\,dx\right], \quad (1)$$

$$\frac{\partial C_2^*}{\partial t} + v_2\frac{\partial C_2^*}{\partial x} = V^*C_0^*C_1\exp\left[-i\int_0^x \varkappa(x)\,dx\right], \quad (2)$$

where V is the matrix element of the interaction operator, $v_i = \partial\omega_i/\partial k_{ix}$, $C_i(t, x; \omega_i(x, k))$ is the amplitude of oscillations with frequency ω_i, $\kappa(x) = k_{0x} - k_{1x} - k_{2x}$. The wave amplitudes are normalized in such a way that the quantity $|\omega_i|\,|C_i|^2$ is equal to the energy of the oscillations. Evaluation of the amplification coefficient for a parametric instability in an inhomogeneous plasma on the basis of Eqs. (1) and (2) was first done by Pilia.[16] A more detailed solution of these equations by the Laplace transform method was carried out by Rosenbluth.[17] This investigation made it possible to prove that the instability has a convective nature and the perturbations grow to a finite amplitude. To calculate the coefficient of (spatial) amplification of perturbations in Eqs. (1) and (2) it is possible to set $\partial/\partial t = 0$. As a result the intensity of the perturbations grows to a value[17]

$$|C_{1,2}|^2 \sim e^{[\nu]},$$

$$\nu = 2\int_{z_{t1}}^{z_{t2}} dz\left\{\frac{\gamma_d^2}{v_1(z)v_2(z)} - \frac{1}{4}[\varkappa(z)]^2\right\}^{1/2}. \quad (3)$$

Here the integration is carried out in the complex plane z between the turning points of the wave functions considered, $C_i(z)$, (i.e., between the zeroes of the radicand).

Selection of definite turning points must be accomplished in a way depending on the nature of the problem discussed. We will consider several specific examples.

1. Let the quasiclassical reflection points where the group velocities of the perturbations go to zero be far from the resonance zone over which the integration is carried out. In addition, for simplicity we will set $v_1 v_2 > 0$. Then the turning points of the functions $C_i(x)$ lie on the real axis x. Parametric amplification corresponds to subbarrier passage of the perturbations from $x \to -\infty$ to $x \to +\infty$, in which amplification of the oscillations occurs. Accordingly the integration is carried out over the segment joining the points x_{t_1} and x_{t_2} and gives the result[16,17]

$$\nu = 2\pi\gamma_d^2 / v_1 v_2 |\varkappa'|, \qquad \varkappa' = d\varkappa / dx. \tag{4}$$

As Rosenbluth has shown, for $v_1 v_2 < 0$ it is necessary in this expression to take the modulus of the product of the group velocities.

2. If the reflection point of one of the waves approaches the resonance zone and the detuning of the latter is determined by the change in the wave vector of that wave, then the amplification coefficient remains constant, since the quantity $v_1(dk_{ix}/dx)$ remains constant on approaching the reflection point and is equal to $(-\partial\omega_i/\partial x)$.

A substantial increase in the amplification coefficient should be expected when the detuning of the resonance is due to the change of the wave vector of the other perturbation. In this case the change in group velocity of the first wave near the reflection point is determined from the equation

$$\omega_1^2(x, k_{1x}^2(x)) = \text{const}.$$

Placing the origin of coordinates at the reflection point, we obtain from this

$$v_1(x) = \mp 2 \frac{\partial\omega_1}{\partial x}\left(x \Big/ \frac{dk_{1x}^2}{dx}\right)^{1/2}. \tag{5}$$

In the expression for the detuning of the resonance of the wave vectors

$$\varkappa(x) = \varkappa(0) + (k_{0x} - k_{1x})' x \pm \left(\frac{dk_{1x}^2}{dx} x\right)^{1/2} \tag{6}$$

according to the condition we can neglect the last term. Substituting the expressions found into Eq. (3), we reduce it to the form

$$\nu = \nu_0 \int_{z_{t1}}^{z_{t2}} dz [z^{-1/2} - (\tilde{\varkappa} + z)^2]^{1/2},$$

$$\nu_0 = \left(2\gamma_d^2 \Big/ \left| v_2 \frac{\partial\omega_1}{\partial x}\right|\right)^{4/5} \left(\frac{dk_{1x}^2}{dx}\right)^{2/5} |\varkappa'|^{-3/5}, \tag{7}$$

$$\tilde{\varkappa} = \varkappa(0) / |\nu_0 \varkappa'|^{1/2}, \quad z = \varkappa' x / |\nu_0 \varkappa'|^{1/2}.$$

The existence of the reflection point introduces into the complex plane a cutoff going from z = 0 to the right along the real axis (see Fig. 2). Three turning points lie on one physical sheet. The solution for the first wave function $C_1(x)$ is the sum of two waves: a wave incident from the left and a wave reflected to the right after parametric amplification. The amplitudes of these waves are identical at the turning points z_{t_1} and z_{t_2}. Correspondingly the argument of the exponential in the amplification coefficient of the wave is calculated as the integral between these points with avoidance of the branch point (reflection point). The amplification $C_2(x)$ of the second wave traveling beyond the reflection point obviously is calculated as the integral between the points (z_{t_1}, z_{t_3}).

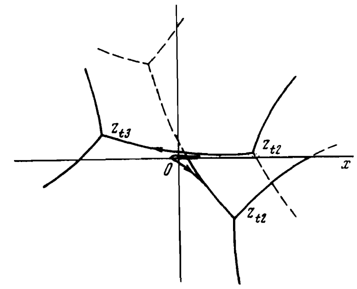

FIG. 2. Stokes lines for solution of the system of Eqs. (1) and (2) and the path of integration of Eq. (3). In the right-hand upper gradient read z_{t1}.

It should be noted that the condition of neglecting the last term in Eq. (6)

$$\nu_0 \gg (dk_{1x}^2/dx)^2 |\varkappa'|^{-3} \tag{8}$$

must be satisfied simultaneously with the condition of applicability of the quasiclassical approximation near the reflection point:

$$L > x \approx |\nu_0 / \varkappa'|^{1/2} > (dk_{1x}^2/dx)^{-1/2}. \tag{9}$$

Therefore Eq. (8) correctly describes the instability near threshold (i.e., is valid for ν_0 values of the order of the Coulomb logarithm), if the dimensionless quantity $(dk_{1x}^2/dx)^2 |\kappa'|^{-3}$ turns out to be less than or of the order of unity.

The quasimode approximation

In the approximation of quasimodes the argument of the exponential in the amplification coefficient is evaluated as the total integral of the instability growth rate over the time interval during which the more rapid wave packet intersects the instability zone:

$$\nu = 2\int \gamma(x) (\partial\omega_1/\partial k_{1x})^{-1} dx. \tag{10}$$

Here the integral is taken over the instability zone determined in the following way. In the case of decay into two perturbations the departure from the resonance is due to the change in the frequency of the second perturbation

$$\delta(x) = \omega_0 - \omega_1 - \omega_2(x, k_{0x}(x) - k_{1x}(x)). \tag{11}$$

The integral (10) takes the form:

$$\nu = 2\int dx \left(\frac{\partial\omega_1}{\partial k_{1x}}\right)^{-1} \left[\gamma_d^2 - \frac{1}{4}\delta^2(x)\right]^{1/2}. \tag{12}$$

Far from the turning point $\delta'(0) \approx -v_2 \kappa'(0)$, and we arrive at the result obtained by the method of normal modes. In the case in which the turning point of the first perturbation reaches the resonance zone, the frequency detuning is

$$\delta(x) = \delta(0) - v_2 (k_{0x} - k_{2x})' x \pm v_2 \left(\frac{dk_{1x}^2}{dx} x\right)^{1/2} + \frac{\partial^2\omega_2}{\partial k_{2x}^2} \frac{dk_{1x}^2}{dx} x. \tag{13}$$

If the third term is important here, we obtain a result not depending on the closeness of the turning point to the resonance zone (see Eq. (4)). If the second term is dominant, we obtain a result close to Eq. (7):[13]

$$\nu = \frac{8}{3}\{2\alpha^2 E(\alpha) - [E(\alpha) - (1-\alpha^2) K(\alpha)]\} \frac{\gamma_d^{3/2}}{|\partial\omega_1/\partial x| v_2^{1/2}} \left| \frac{dk_{1x}^2}{dx} \Big/ |\varkappa'(0)| \right|^{1/2} \tag{14}$$

where $K(\alpha)$ and $E(\alpha)$ are complete eliptical integrals, and

$$1 \geqslant \alpha^2 = {}^1/_2(1 - \delta(0)/2\gamma_d) \geqslant 0.$$

Thus, calculation of the amplification coefficient for a known dependence of the growth rate on the frequency detuning and, consequently, on the coordinate is not difficult in this approach. The numerical difference of the results of the two approaches (in the second case the amplification is somewhat smaller) turns out to be a quantity of the order of unity (more accurately $\sim(v_2/v_1)^{1/5}$).

3. CALCULATION OF THE COEFFICIENTS OF PARAMETRIC AMPLIFICATION OF THE PERTURBATIONS

The theory of a weakly turbulent plasma[6,7,15] permits description of the interaction of high-frequency natural plasma oscillations (such as electromagnetic waves and longitudinal plasma oscillations) by means of the following expression for the nonlinear dielectric permittivity of the plasma:

$$\varepsilon_{NL}(\omega, \mathbf{k}) = \varepsilon(\omega, \mathbf{k}) + \int \frac{d^3\mathbf{k}'}{(2\pi)^3} K(\mathbf{k}, \mathbf{k}') \frac{|\mathbf{E}_{\mathbf{k}'}|^2}{4\pi}, \qquad (15)$$

where

$$K(\mathbf{k}, \mathbf{k}') = \frac{2\pi e^2}{m^2} \Pi \frac{(\mathbf{k} - \mathbf{k}')^2}{|\omega\omega'|^2} \frac{\varepsilon_e(\omega - \omega', \mathbf{k} - \mathbf{k}')}{\varepsilon(\omega - \omega', \mathbf{k} - \mathbf{k}')} [1 + \varepsilon_i(\omega - \omega', \mathbf{k} - \mathbf{k}')],$$

$\epsilon_j(\omega, \mathbf{k})$ is the contribution of particles of type j to the linear dielectric permittivity of the plasma; $|\mathbf{E}_{k'}|^2/4\pi$ is the spectral energy density of laser radiation with frequency ω' and wave vector $\mathbf{k}'$. The polarization factor Π is different, depending on whether the oscillation $(\omega, \mathbf{k})$ is longitudinal (l) or transverse (t):

$$\Pi_l = (\mathbf{k}\mathbf{e})^2/k^2, \quad \Pi_t = |e_\perp e_\perp' + e_\parallel e_\parallel' \cos\Theta|^2, \qquad (16)$$

where $e_\parallel$ and $e_\perp$ are the projections of the polarization vector $\mathbf{e}$ of the radiation $(\omega, \mathbf{k})$ on the scattering plane and the normal to it, and Θ is the scattering angle (i.e., the angle between the wave vectors $\mathbf{k}$ and $\mathbf{k}'$). Use of this extremely general and at the same time quite simple expression for the nonlinear dielectric permittivity permits the probabilities of the most interesting processes to be obtained without calculating each of them from first principles.

In the present paragraph we will limit ourselves to the case of plane-polarized monochromatic laser radiation

$$E = E_0 \cos(\omega_0 t - k_0 x), \qquad (17)$$

in which Eq. (15) reduces to the simpler form:

$$\varepsilon_{NL}(\omega, \mathbf{k}) = \varepsilon(\omega, \mathbf{k}) + K(\mathbf{k}, \mathbf{k}_0) E_0^2/8\pi \qquad (18)$$

We have broken down all of the instabilities described by Eq. (10) into two classes:

1) harmful instabilities which lead to reverse parametric reflection of the incident radiation by the plasma:

2) useful instabilities, as the result of development of which the energy of the incident radiation is transformed to energy of longitudinal oscillations (and finally to heat).

The results of our analysis of parametric instabilities in an inhomogeneous plasma are presented in the form of two tables in which we have shown the growth rates of the instabilities in the homogeneous-plasma approximation and the amplification coefficients expressed in terms of these growth rates and the scale of the inhomogeneity. Linear growth rates of an instability were calculated by means of Eq. (15). We will show how this is done for the case of one of the main processes—Mandel'shtam-Brillouin induced scattering; this process is also called the decay of a photon into a photon and a phonon $(t \to t' + s)$.

In the decay approximation when the instability growth rate is much less than the frequency of the acoustic perturbation, the kernel of the integral equation (18) contains a singularity of the pole type at the point where the beat frequency of the incident and scattered fields is equal to the frequency of the natural oscillations (in this case to the frequency of the nonisothermal ion sound). Expanding the quantities $\omega_1^2\epsilon(\omega_1, \mathbf{k}_1) - \mathbf{k}_1^2 c^2$ and $\epsilon(\omega_1 - \omega_0, \mathbf{k}_1 - \mathbf{k}_0)$ near the natural oscillation frequencies, we rewrite Eq. (18) in the form of a quadratic equation for $[\omega_1 - \omega_t(\mathbf{k}_1)]$:

$$\omega_1^{-2} \frac{\partial \omega_1^2 \varepsilon}{\partial \omega_1} (\omega_1 - \omega_t(\mathbf{k}_1)) = \frac{\omega_p^4}{2\omega_0^2\omega_1^2} \frac{E_0^2}{8\pi n T_e} \frac{\Pi_t}{(\mathbf{k}_1 - \mathbf{k}_0)^2 \lambda_D^2} \left[\frac{\partial \varepsilon(\omega_1 - \omega_0, \mathbf{k}_1 - \mathbf{k}_0)}{\partial(\omega_1 - \omega_0)} (\omega_1 - \omega_0 - \omega_s(\mathbf{k}_1 - \mathbf{k}_0)) \right]^{-1}. \qquad (19)$$

Solution of this equation leads to the well known expression for the decay instability growth rate:

$$\gamma_d^2 = \frac{\omega_p^4 \omega_s}{8\omega_0^2\omega_1} \frac{E_0^2}{8\pi n T_e} \Pi_t, \quad \delta = \omega_0 - \omega_t(\mathbf{k}_1) - \omega_s(\mathbf{k}_0 - \mathbf{k}_1). \qquad (20)$$

The same expression describes the decay of a photon into a plasmon and a phonon (the process $t \to l + s$) if we make this substitution[2)] $\Pi_t \to \Pi_l$. The decay of a photon to a photon + a plasmon is discussed in a similar manner.

In the other limiting case $\gamma \gg \omega_s$ we are dealing with modified decay.[10,11] In this approximation the kernel of Eq. (18) is inversely proportional to the square of the beat frequency:

$$K(\mathbf{k}_1, \mathbf{k}_0) \sim \frac{1 + \varepsilon_i}{\varepsilon} \approx \frac{\omega_s^2(\mathbf{k}_1 - \mathbf{k}_0)}{(\omega_1 - \omega_0)^2}.$$

As a result instead of the quadratic equation (19) we obtain a cubic equation whose solution can be conveniently expressed in terms of the growth rate of the ordinary decay instability:

$$\gamma_{md} = {}^1/_2\sqrt{3}(2\gamma_d^2\omega_s)^{1/3}. \qquad (21)$$

In an isothermal plasma, ion-sound oscillations are strongly damped and the parametric instability has the nature of induced scattering $(t \to t' + i)$ or conversion $(t \to l + i)$ in scattering by ions with a growth rate[3)] [21]

$$\gamma = -\frac{\omega_p}{4} \frac{E_0^2}{8\pi n T_e} \Pi \operatorname{Im}[(\mathbf{k}_1 - \mathbf{k}_0)^2 \lambda_D^2 \varepsilon(\omega_0 - \omega_1, \mathbf{k}_0 - \mathbf{k}_1)]^{-1}. \qquad (22)$$

In a plasma with a Maxwellian velocity distribution it is possible to express this growth rate in terms of the well known plasma dispersion function

$$\gamma = -\frac{\omega_p}{4} \frac{E_0^2}{8\pi n T_e} \Pi \operatorname{Im}\left\{ 1 + \frac{T_i}{T_e}[1 + i\pi^{1/2} z_i W(z_i)] \right\}^{-1}, \qquad (23)$$

where

$$W(z) = \frac{i}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-t^2} dt}{z - t + i0}, \qquad z_i = \frac{\omega_0 - \omega_1}{\sqrt{2}|\mathbf{k}_0 - \mathbf{k}_1| v_{Ti}}.$$

In the tables we have shown the maximum value of this growth rate. The decay instability amplification coefficients given in the tables were evaluated by means of the normal mode approach. This approach, as noted above, gives somewhat larger values of ν than follow from the more simple quasimode approximation used in ref. 13.

As can be seen from Table I, induced scattering both by plasmons and by phonons occurs mainly at an angle close to 90°. The amplification of the scattered wave is described by Eq. (7). In Table I we have given the argu-

TABLE I. Reverse parametric scattering

Process	Maximum growth rate	Maximum amplification coefficient	Remarks
$t \to t' + s$	$\gamma_d^2 = \frac{\omega_p^4 k_0 c_s}{4\sqrt{2}\omega_0^3}(1-\cos\theta)^{1/2}\frac{E_0^2}{8\pi n T_e}$	$\nu = \frac{4\pi\gamma_d^2\omega_0 L}{\omega_p^2 U}$ $\nu_0 = \left(\frac{4\gamma_d^2}{\omega_p k_0 U}\right)^{4/5}\left(\frac{U}{U'L}\right)^{3/5} k_0 L$	$\left(\frac{\mathscr{L}}{k_0 L}\right)^{1/2} > \frac{E_0^2}{8\pi n T_e} > \frac{\mathscr{L}}{k_0 L}$ $\left(\frac{mc^2}{T_e}\right)^{3/2}\left(\frac{\mathscr{L}}{k_0 L}\right)^{5/4} > \frac{E_0^2}{8\pi n T_e} > \left(\frac{\mathscr{L}}{k_0 L}\right)^{1/2}$
$t \to t' + l$	$\gamma_d^2 = \frac{\omega_p^3}{4\omega_0}(1-\cos\theta)\frac{E_0^2}{8\pi n m c^2}$	$\nu_0 = \left(\frac{8\gamma_d^2}{\omega_p^2}\right)^{4/5}\frac{k_0 L}{(6k_0^2\lambda_D^2)^{1/5}}$	$\left(\frac{mc^2}{T_e}\right)^{5/4} > \frac{E_0^2}{8\pi n T_e}\left(\frac{T_e}{mc^2}\right)^{3/4}\left(\frac{k_0 L}{\mathscr{L}}\right)^{5/4} > 1$
$t \to t' + i$	$\gamma \sim \frac{\omega_p^4}{\omega_0^3}\frac{E_0^2}{8\pi n T}$	$\nu = \frac{\gamma}{\omega_p}\left(\frac{c_s}{U'L}\right)^{1/2} k_0 L$	$T_i \approx T_e$ $\left\lvert\frac{\pi}{2}-\theta\right\rvert \lesssim \left(\frac{\omega_p}{\omega_0}\right)\left(\frac{c_s}{U}\right)^{1/2}$

ment ν of the exponential in the amplification coefficient, obtained after substitution of the specific dispersion characteristics of the oscillations excited.

1. For Mandel'shtam-Brillouin induced scattering we have

$$\frac{dk_{1x}^2}{dx} = \frac{\omega_p^2}{c^2 L}, \qquad L^{-1} = -\frac{d\ln n}{dx} > 0,$$

$$\varkappa'(0) \cong -k_2'(0) = -\frac{k_{2x}U'(0)}{v_s}, \qquad v_s = \frac{k_{2x}}{k_2}c_s + U(0),$$

where U(x) is the velocity of ultrasonic outflow of the plasma corona, $c_s = \sqrt{T_e/M}$ is the velocity of nonisothermal sound, and ω_p is the plasma frequency.

2. For Raman induced scattering we have

$$\frac{dk_{1x}^2}{dx} = \frac{\omega_p^2}{c^2 L}, \qquad \varkappa' \approx -k_2'(0) = -\frac{\omega_p}{2v_l L}, \qquad v_l = 3k_{2x}\lambda_D v_{Te}.$$

It is interesting to note that the detuning of the resonance of the wave vectors in the first of these processes is due to the inhomogeneous Doppler effect only for the condition $\nu_0 > (\omega_p/\omega_0)^4 k_0 L > \nu_0^{-3}$. Therefore Eq. (7) can be used only for high intensities of the incident radiation when the threshold of reverse parametric scattering is reached in the remote outer layers of the plasma corona. In the case of lower intensities the amplification of radiation scattered by 90° is limited by the detuning of the resonance with a rapid increase of the wave vector of the scattered wave leaving the reflection point. The amplification is determined by Eq. (4), which in a certain range of angles gives a constant value of amplification coefficient (see Table I).

The situation is more complicated in the case of induced conversion of transverse waves into longitudinal waves in ion sound ($t \to l + s$). Here, in view of the small size of the photon wave vector in comparison with the wave vectors of the plasmon and phonon, near the reflection point of the plasma waves the group velocity of sound oscillations is also small. In addition, the amplification coefficient turns out to be maximal if we formally set $k_{2x} = 0$. In this case in Eq. (2) the group velocity of sound waves is also zero and we must take into account the term with the second derivative with respect to the coordinate (see Eq. (26) for the case of modified decay).

Finally, the amplification coefficient of plasma waves excited near the point where $2\omega_p \approx \omega_0$ was taken by us from Refs. 17 and 18. The amplification of perturbations in the process of induced scattering by ions in an isothermal plasma is described by the simple equation (10).

TABLE II. Parametric heating of plasma

Process	Growth rate	Amplification coefficient	Remarks
$t \to l + s$	$\gamma_d^2 = \frac{\omega_p(\mathbf{k})\,\omega_s(\mathbf{k})}{8}\frac{E_0^2}{8\pi n T_e}$	$\nu \sim \left(\frac{\gamma_d^2}{\omega_p\omega_s}\right)^{3/4}(k\lambda_D)^{1/2}kL$	$\lambda_D^{-1} \gg k \gg k_0$, $\omega_p \approx \omega_0$
$t \to l + l'$	$\gamma_d^2 = \frac{\omega_p^2}{4}\frac{k_0^2 E_0^2}{8\pi n m\omega_0^2}\sin^2 2\vartheta\cos^2\varphi$	$\nu = \frac{\pi k_0 L}{6}\frac{E_0^2}{8\pi n T_e}$	$\cos\vartheta = (\mathbf{k}\mathbf{E}_0)/kE_0$, $\cos\varphi = (\mathbf{k}\mathbf{k}_0)/kk_0$, $\omega_p \approx \omega_0/2$
$t \to l + i$	$\gamma \sim \omega_p\frac{E_0^2}{8\pi n T}$	$\nu \approx \frac{\gamma}{\omega_p}k_0 L$	$T_i \approx T_e$, $\theta \sim 1$

The departure from the resonance is due to the inhomogeneous Doppler effect, and the region of greatest amplification is located near the reflection point of scattered radiation. The value of the amplification coefficient is given by the following integral:

$$\nu = \nu_0\int_0^\infty \frac{dz}{\sqrt{z-\delta}}\,\mathrm{Im}\left[1 + \frac{T_i}{T_e}(1 + i\pi^{1/2}zW(z))\right]^{-1}$$

$$\nu_0 = \frac{2\omega_p^3}{\omega_0^3}k_0 L\left(\frac{c_s}{2U'L}\right)^{1/2}\frac{E_0^2}{8\pi n T_e}, \qquad \delta = \frac{\omega_0-\omega_1}{2k_0 v_{Ti}}. \qquad (24)$$

In the case of induced conversion by ions there is no distinctly bounded resonance zone and the convergence of the integral (10) is due to the decrease in the growth rate as the result of decrease in the factor $\Pi_l \sim (1 + k_x^2/k_\perp^2)^{-1}$ far from the reflection point:

$$\nu = \frac{E_0^2}{4\pi n T_e}k_\perp L\int_0^{\pi/2} d\varphi\,\mathrm{Im}\left\{1 + \frac{T_i}{T_e}[1 + i\pi^{1/2}z\sin\varphi W(z\sin\varphi)]\right\}^{-1} \qquad (25)$$

where $z = (\omega_0 - \omega_1)/\sqrt{2}k_\perp v_{Ti}$.

It was pointed out above that at high intensities the decay processes become more complex (are modified). In the homogeneous problem of the growth of perturbations with time the boundary of the transition to modified decay is the condition $\gamma_d \sim \omega_2$. In the case discussed by us of spatial amplification, the criterion of the transition to modified decay turns out to be different. Instead of the estimate $\mathrm{Im}\,\omega \sim \omega_2$ it is necessary to use $\mathrm{Im}\,k \sim k_{1,2}$. Taking into account that the imaginary part of k is determined by the amplification coefficient found by us, we obtain

$$\gamma_d^2 \sim v_1 v_2 k_{1,2}^2. \qquad (26)$$

For the process $t \to t' + s$ this can take place only for an energy density in the incident wave of the order $n_c T$ ($n_c \approx m\omega_0^2/4\pi e^2$). Thus, in the problem of the amplification coefficient for reverse parametric scattering the decay approximation turns out to be sufficient.

For the process $t \to l + s$ the boundary (26) is reached at a much lower intensity of radiation. Then in the case of modified decay in calculation of the amplification coefficient Eq. (2) must be replaced by the complete wave occasion for sound perturbations:

$$\left[\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial x^2}\right] C_2^* = -2i\omega_s V^* C_0^* C_1 \exp\left[-i\int_0^x \varkappa\, dx\right]. \tag{2'}$$

The system of two equations (1) and (2′) has not yet been studied in detail, and in particular the analog of criterion (3) has not been established for it. We are therefore forced to limit ourselves to a simple estimate based on the quasimode approximation.

In conclusion of this section we will make several remarks on how greatly the above results change in the case of laser radiation with a finite width of the spectral line and a finite angular spread of the laser beam.

Increasing the width of the spectral line δ leads to a decrease in the growth rate of the decay instability ($\gamma \sim \gamma_d^2/\delta$).[7] Amplification of perturbations in an inhomogeneous plasma in this case is easily calculated with the quasimode approach. In the case of propagation of perturbations along the direction of propagation of the incident wave, the decrease in growth rate is compensated by an increase of the resonance zone, so that the amplification coefficient remains as before. The amplification in a direction at 90° to the incident radiation is reduced in proportion to $\sim(\gamma_d/\delta)^{1/2}$.

The divergence of the laser beam is equivalent to appearance of an effective spread of frequencies as the result of variation of the x component of the wave vector ($\delta \simeq k_0\Theta^2 c$) and a variation of the resonance frequency of the oscillations excited ($\delta \simeq (\partial\omega/\partial k_\perp)k_0\Theta$).

4. NONLINEAR THEORY OF REVERSE PARAMETRIC SCATTERING

Only one ideal case is known in which the decay instability permits analytical solution even in the nonlinear stage of development. This case is the single-mode condition in a homogeneous medium where the pumping wave decays into a certain pair of waves.[23] Here the nonlinearity lies in the reverse reaction of the pumping wave on the growth of the perturbation. A state of nonlinear saturation is finally achieved (generally speaking, an oscillating state) in which a uniform distribution in the wave number ($|C_k|^2$) occurs, on the average, between the three oscillations (the Manley-Rowe relation). However, this state itself is unstable with respect to perturbations consisting of other possible pairs of waves. The pumping-wave energy is then dissipated into the energy of the entire ensemble of waves interacting with each other and with the plasma particles. These processes can be discussed in terms of the theory of weak turbulence if the decay instability is weak ($\gamma_d \ll \omega$). An example of the use of this approach is the discussion of nonlinear saturation of an instability of the induced conversion type $t \to l + i$.[24] Nonlinear saturation is achieved as the result of induced scattering of plasmons by ions ($\omega' - \omega'' = (\mathbf{k}' - \mathbf{k}'')\mathbf{v}$). This process carries the plasmons out of the spectral region where they are generated as the result of instability of the pumping wave. A state of saturation is achieved when creation of plasmons as the result of instability is balanced by their removal as the result of induced scattering.

In the work considered[24] an extremely important additional simplifying assumption was that the scattered plasmons beyond the limits of the spectral region of the instability are absorbed as the result of ordinary collision damping. This assumption can be justified in the case of a weakly ionized plasma with a high frequency of collisions of electrons with neutral atoms (but not for our problem). In the general case a difficulty arises as the result of accumulation of scattered plasmons. As a rule, nonlinear processes lead to flow of energy along the spectrum to the region $\mathbf{k} \to 0$ with conservation of the number of quanta (and therefore with approximate conservation of total energy). In order to avoid the tendency to formation of a condensate of plasmons, it is necessary to find an effective outflow of their energy. In a sufficiently inhomogeneous plasma this outflow is achieved by the convection of plasmons into the region of falling density with subsequent Landau absorption and appearance of fast electrons.[18] In the case of a weakly inhomogeneous plasma the question of the mechanism of destruction of the plasmon condensate remains open. At the present time a promising idea is the modulation instability of a gas of plasmons whose energy density exceeds some critical value.[25] We can expect that with the density gradients arising as the result of this low-frequency instability, convection and Landau damping of plasmons will appear or plasmons will be absorbed as the result of direct collapse.[26]

Modulation instability of the plasmon gas must be taken into account also in the problem of reverse parametric scattering as a result of the decay instability $t \to t' + l$. However, as can be seen from the results of section 3, at least in the linear theory, the greatest amplification coefficient is achieved for the instability $t \to t' + s$. Below we discuss approximate nonlinear models of this process.

If the power of the laser beam is not too high (we give an upper-limit estimate below), and the profile of the plasma density drop at the edge of the corona is sufficiently smooth, the condition of soft inclusion of the decay instability[13] is achieved. The essence of this condition is that the electromagnetic wave propagating inside the corona smoothly enters a region with amplification coefficient $\sim e^{\mathcal{L}}$. The pairs of waves arising in this case (radiation scattered by 90° and sound) do not yet interact with each other but remove energy from the pumping wave. In this quasilinear model it is necessary to take into account only pairs of wave with the greatest amplification coefficient. The amplitude of the scattered radiation is given by the expression (see Eq. (7))

$$\frac{|E_{\mathbf{k}_1}|^2}{4\pi\omega_1} \approx \frac{T_e}{\omega_2} \exp\left\{\left[\frac{\omega_p^3}{\omega_0^3\sqrt{2}} \frac{E_0^2}{8\pi n T_e}\right]^{4/5} \frac{k_0 L c_s^{4/5}}{U^{1/5}(U'L)^{3/5}} \int_{z_{t1}}^{z_{t2}} [z^{-1/2} - (\tilde{\varkappa} + z)^2]^{1/2}\, dz\right\}, \tag{27}$$

where, in contrast to the linear theory, the amplitude of the pumping wave is not assumed to be given but is determined by a self-consistent method from the energy-conservation equation:

$$c\frac{d}{dx}\frac{E_0^2}{8\pi} = -\int \frac{d^3\mathbf{k}_1}{(2\pi)^3} v_1(x) \left\{\frac{\gamma_d^2}{v_1(x)v_2(x)} - \frac{1}{4}[\varkappa(x)]^2\right\}^{1/2} \frac{|\mathbf{E}_{\mathbf{k}_1}|^2}{4\pi}. \tag{28}$$

Rigorous integration over the phase space of the scattered waves would require additional study of the parametric amplification process in the case where the resonance condition is not accurately satisfied. However, the width of the resonance frequency zone, like the magnitude of the scattering angle interval, enters as a logarithm in the dependence of the incident radiation intensity on the coordinate.[13] Therefore in solution of Eq. (28) we can take into account only the largest pre-exponential factor $k_0^3/n(2\pi)^3 \gg 1$. As a result we obtain the simple result:

$$\nu\left(\frac{E_0^2(x)}{8\pi n T_e}, x\right) \approx \ln\left[n\left(\frac{2\pi}{k_0}\right)^3\right]. \quad (29)$$

It follows from this that the intensity of radiation passing into the corona falls in a length of the order of the characteristic length of variation of the density and escape velocity of the plasma. The condition $L \gg \gamma_d/\sqrt{v_1 v_2}$ guarantees, as can be seen from Eq. (28), that the intensity of scattered radiation per unit volume of phase space will be small in this case and therefore secondary scattering of this radiation can be neglected.

We can now estimate an upper limit of the amplitude of the incident electromagnetic wave below which the nonlinear effects discarded are unimportant. We can expect that the main nonlinearity occurs in the spectrum of the sound waves. The total energy supply pumped into sound oscillations W is determined by the balance between the arrival due to instability and the departure due to convection with the group velocity (in our model)

$$c\frac{E_0^2}{8\pi}\frac{\omega_s}{\omega_0} \approx c_s\frac{W}{L}, \quad (30)$$

i.e., $W \approx E_0^2/8\pi$. Nonlinear effects should become important if a substantial nonlinear distortion of the spectrum can occur during the time of development of the instability. In the wavelength region $\lambda \gg \lambda_D$ the sound dispersion law is linear. In the decay being discussed $t \to t' + s$ this inequality is always satisfied, since $k_0 \sim \omega_p/c \ll \lambda_D^{-1}$. Therefore the nonlinear distortion of the spectrum of sound oscillations is similar to the case of a gas-dynamically compressible gas. We will use an estimate of the time of nonlinear distortion (twisting) for an almost isotropic sound spectrum made in ref. 27[4]:

$$\tau_{NL}^{-1} \approx \omega_s \frac{k^3 W(k)}{n T_e}. \quad (31)$$

Here W(k) is the spectral energy density of the sound waves. The condition $\gamma_d \tau_{NL} > 1$ now leads to the desired upper limit for the amplitude of the laser radiation

$$E_0^2 / 8\pi n T_e < \gamma_d / \omega_s. \quad (32)$$

If the amplitude of the incident electromagnetic wave exceeds this critical value, i.e., the sound waves are not able to be carried away before they are nonlinearly distorted, another approximate model of nonlinear parametric scattering can be constructed. We will assume that a stationary center of the sound oscillations is established as the result of a balance between arrival due to instability and departure due to nonlinear twisting[5]

$$0 = V^* C_0^* C_1 e^{i\delta t} - [i\delta(x) + \tau_{NL}^{-1}] C_2^*, \quad (33)$$

where τ_{NL} is determined by the estimate (31). Scattering of the radiation is described as usual in the following way:

$$v_1(x)\frac{\partial C_1}{\partial x} = V C_0 C_2^* e^{-i\delta t}, \quad (34)$$

and the attenuation of the pumping wave as the result of scattering is given by the expression

$$c\frac{d}{dx}|C_0|^2 \approx -v_1(x)\frac{d}{dx}|C_1|^2. \quad (35)$$

Substitution of C_2 found from (33) into (34) and (35) leads to the equations

$$\frac{dI_1}{dx} = \frac{\gamma_d}{v_1(x)}\left\{-2\tilde{\delta} + \sum_{\sigma=\pm 1} (\tilde{\delta}^3 + I_1^2 + \sigma[(\tilde{\delta}^3 + I_1^2)^2 - \tilde{\delta}^6]^{1/2})^{1/3}\right\}$$
$$\tilde{\delta} = \delta^2 / 3\gamma_d^2, \quad I_1 = (\omega_2^2/\gamma_d)(\mathbf{k}_0 - \mathbf{k}_1)^3 |C_1(\mathbf{k}_1)|^2 / nT_e; \quad (36)$$

$$c\frac{d}{dx}\frac{E_0^2}{8\pi} \approx -\int \frac{d^3\mathbf{k}_1}{(2\pi)^3}\omega_1 v_1(x)\frac{d|C_1|^2}{dx}. \quad (37)$$

As in the first model, the main contribution is from scattering into angles close to 90°, but inclusion of nonlinearity of the sound slows the rate of parametric scattering (beginning at certain amplitudes the exponential growth changes to power-law). In this way the nonlinear reflection zone shifts to the interior of the corona. If the profile of falloff of the plasma density at the edge of the corona is sufficiently steep so that the electromagnetic wave freely traverses the low-density region and immediately turns out to be in the zone of strong decay instability, the condition of hard inclusion should occur. For this case we have not been able to find solvable nonlinear analytic models. Therefore we limit ourselves below to certain semiquantitative estimates.

The least reflecting properties evidently should be possessed by a plasma corona with hot ions ($T_i \gtrsim T_e$) in which there is no sound. Instead of the decay $t \to t' + s$, here we must take into account induced scattering by ions $t \to t' + i$. An approximate nonlinear model of this process can be constructed as follows. After the exponential stage of growth of the instability from the thermal noise level, let there follow immediately a region where nonlinear interaction is important and where there is an effective attenuation of the incident wave (in a certain length Δx)

$$c\frac{I_0}{\Delta x} \sim \frac{\omega_p^4}{\omega_0^3}\frac{I_0}{nT}J_1 k v_{Ti}, \qquad I_0 = \frac{E_0^2}{8\pi} \quad (38)$$

where $J_1(\omega_1)$ is the spectral density (per unit frequency) of the scattered radiation. We assume that the pumping wave interacts directly with the scattered radiation in an interval of width $\sim kv_{Ti}$. Further evolution of $J_1(\omega_1)$ is determined, on the one hand, by relay pumping into the region of lower and lower frequencies (as the result of secondary induced scattering) and, on the other hand, by transport of radiation from the instability zone

$$c\frac{\omega_p}{\omega_1}\left(\frac{\Delta x}{L}\right)^{1/2}\frac{J_1}{\Delta x} \sim \frac{\omega_p^4}{\omega_1^3}\frac{k^2 v_{Ti}^2}{nT_e}J_1\frac{dJ_1}{d\omega_1}. \quad (39)$$

We will approximate the relay pumping of photons along the spectrum by a differential form by analogy with the problem of plasmon pumping,[24] and the angular spread of the scattered radiation about 90° is chosen with allowance for refraction (rotation of the wave vector) in the inhomogeneous plasma

$$\Delta\Theta \sim \frac{\omega_p}{\omega_0}\left(\frac{\Delta x}{L}\right)^{1/2}.$$

Then the scattered radiation spectrum falls linearly to zero between $\omega = \omega_0$ and $\omega = \omega_l$, and

$$J_1 \sim \frac{I_0}{kv_{Ti}}\left\{1 - (\omega_0 - \omega_1)\frac{\omega_0}{\omega_p k v_{Ti}}\left(\frac{I_0}{nTk_0L}\right)^{1/2}\right\}. \quad (40)$$

The constant of integration was found by considering the arrival-departure balance due to induced scattering near the upper limit of the scattered radiation spectrum ($\omega_0 > \omega_1 > \omega_0 - kv_{Ti}$). As a result the thickness of the nonlinear-scattering region turns out to be of the order

$$\Delta x \approx \frac{c}{\omega_0}\left(\frac{\omega_0}{\omega_p}\right)^4\frac{nT}{I_0}. \quad (41)$$

The fraction of the laser-wave energy absorbed by ions can be estimated as

$$\frac{\omega_0 - \omega_r}{\omega_0} \approx \frac{k v_{Ti} \omega_p}{\omega_0^2} \left(\frac{nT}{I_0} k_0 L \right)^{1/2}.$$

The applicability of this model can have an upper limit at high intensities of the incident radiation where the induced scattering is converted into modified decay. As still higher intensities $I_0/nT \gtrsim 1$ the interaction of the radiation with the corona must be considered quite differently, since the radiative forces become the principal ones in the gas dynamics of the corona.[13]

We will apply the equations obtained to the most frequently discussed case, the neodymium-glass laser. For $\omega_0 \approx 2 \times 10^{15}$ sec^{-1}, $\omega_0/\omega_p \sim 2$, $I_0/nT \sim 0.1$, the penetration depth at which effective reflection of the laser radiation occurs is of the order 10^{-2} cm $> \Delta x > 10^{-3}$ cm. This value is substantially less than the initial dimension of the D–T drop and, as can be expected, than the thickness of the plasma corona at the moment of critical compression. Consequently the processes discussed can play an important role in the physics of interaction of a high-power electromagnetic wave with plasma, and study of these processes takes on an important practical value. As a result of the complexity of analytical approaches to the problem, numerical experiments (Monte Carlo calculations) are extremely desirable. However, one-dimensional numerical models are hardly realistic,[28] since they do not include 90° scattering and the phase space of the unstable waves is too small.

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1)Among the processes of reverse parametric reflection of electromagnetic waves, the first process calculated was that of decay of Alfvén waves to a sonic wave and a backward going Alfvén wave [14,15].

2)We note that in the expression for the growth rate of the decay instability there enter only the electric field amplitude of the incident wave, its frequency, and the wave vector; the polarization drops out. Therefore after the substitution indicated Eq. (20) can be used both for the process $t \to \ell + s$ and for the process $\ell \to \ell' + s$.

3)Scattering by electrons [22] need be taken into account only in the remote region of the plasma corona where $k_0\lambda_D >$ (i.e., $\omega_p < \omega_0 (T_e/mc^2)^{1/2}$). However, for such a low density the amplification coefficient can be small. Therefore this case is not listed in Table I.

4)When scattering occurs mainly at 90°, the wave vectors of the sound waves lie on the surface of a cone formed in rotation of the set of three resonance wave vectors about the vector $\mathbf{k}_0$. In this case arguments similiar to those used in Ref. 27 lead to a somewhat different value:

$$\tau_{NL}^{-1} \approx \omega_s (k^3 W(k) / nT_e)^{5/6}.$$

5)The nonlinear model used by us is equivalent to the case of decay to a highly damped perturbation where the width of the wave packet in frequency turns out to be of the order of the damping decrement. It is already meaningless to describe the situation in the language of normal modes.

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