Phase modulation of Mössbauer spectra and the generation of ultrasound in metals by a radio-frequency field

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Modulation of Mössbauer spectra by a radio-frequency field in nonferromagnetic metals is observed experimentally. The effect may be ascribed to generation of a sound wave by eddy currents induced in the metal by a radio-frequency field. Such a mechanism of modulation of Mössbauer spectra may also play an important role in ferromagnetic metals.

1. INTRODUCTION

Much work has been done on the excitation of ultrasound in metals and semimetals by a radiofrequency (rf) field. This phenomenon was observed experimentally and described in[1-4]. Theoretical investigations of the same effect were reported in[5-13].

The excitation of ultrasound is observed, as far as the authors are aware, only if the mean free path of electrons is long (pure samples and low temperatures) or if a strong external magnetic field H is applied so that the equation of motion of the lattice is dominated by the ponderomotive force $(\mathbf{j}\times\mathbf{H})/c$ ($\mathbf{j}$ is the density of the current induced by the rf field in a metal). Theoretical models of this effect have been developed for these conditions. If the mean free path is long, a deformation mechanism, first discussed by Silin,[14] plays an important role in the interaction between nonequilibrium electrons and the lattice. In this case, the topological features of the structure of the Fermi surface of the metal are important. On the other hand, in the case of high temperatures and short mean free paths, the Fermi surface topology is unimportant and the conversion of the energy of the rf field into ultrasonic energy is a very weak effect which could hardly be measured by standard acoustic and electromagnetic (surface impedance) methods.

In this situation ultrasound can be detected by the Mössbauer spectroscopy which is unique in respect of its sensitivity. The ultrasound generated by an rf field in an absorber causes the nuclei in the absorber to vibrate and this gives rise to a phase modulation of the Mössbauer lines. If the rf field frequency ω is greater than the width of the Mössbauer line, the Mössbauer spectrum exhibits satellites (side bands) at frequencies which are multiples of ω. The amplitude of the forced vibrations of the absorber atoms can be determined by measuring the intensities of these satellites. The mechanism of the generation of ultrasound by an rf field depends on the ratio of the skin depth δ and the mean free path of electrons l to the wavelength of ultrasound λ. If $(l/\lambda)(\delta/\lambda)^3 \gg 1$, the diffuse scattering of electrons by the surface (boundary) of a metal becomes important but if $l\delta/\lambda^2 \ll 1$ the spatial dispersion of the electrical conductivity or the nonlocal relationship between the current density and the electric field is important; finally, both these mechanisms are equally important if $l\delta/\lambda^2 \sim 1$. The case $l\delta/\lambda^2 \ll 1$ is easily attainable in ferromagnets with high values of the permeability μ. The inequality $(\delta/\lambda)^3 l/\lambda \gg 1$ may apply to a paramagnetic metal with a low electrical conductivity σ if the frequency is sufficiently high. In the $(\delta/\lambda)^3 l/\lambda \gg 1$ case, a study of the generation of ultrasound by an rf field may yield information on the surface scattering of carriers and on the structure of the surface layer of a metal.

Experimental investigations of the phase modulation of the Mössbauer spectra of metals have been carried out so far only for ferromagnetic materials.[17-24] It is suggested in[23] that the main mechanism of the generation of ultrasound in ferromagnets is the magnetoelastic coupling, i.e., the rf field disturbs the equilibrium magnetization of a sample and because of magnetostriction this excites elastic vibrations of the lattice. However, the discovery of the phase modulation of the Mössbauer spectra in paramagnetic metals, described in the present paper, suggests the possibility of a nonmagnetostriction mechanism of the excitation of satellites in ferromagnetic metals.

In Sec. 2 we shall give the theoretical formulas illustrating the mechanism of the generation of ultrasound by an rf field for the simplest model of the surface scattering. In Sec. 3 we shall present the experimental results and we shall discuss them in Sec. 4.

2. THEORY

If the mean free path of electrons is long, so that the deformation interaction between nonequilibrium electrons and the lattice is unimportant, the equation of motion of an isotropic continuous medium can be written in the form[5-7,15,16]

$$\ddot{\mathbf{u}} + \gamma\dot{\mathbf{u}} - s_l^2\nabla(\nabla\mathbf{u}) + s_t^2[\nabla[\nabla\mathbf{u}]] = \rho^{-1}\mathbf{F}(\mathbf{r},t), \qquad (1)$$

where $\mathbf{u}$ is the lattice displacement vector; γ is the total damping decrement of ultrasound; s_l and s_t are the velocities of longitudinal and transverse ultrasound; ρ is the density of the medium; $\mathbf{F}(\mathbf{r}, t)$ is the density of the force [see Appendix, Eq. (A.14)].

We shall consider a sample in the form of a planar film of thickness 2d bounded by the planes z = d and z = −d. We shall direct the y axis along the axis of an rf coil in which the sample is placed. Then, an alternating magnetic field h has only the component h_y and the electric field has only the component E_x. At the surfaces z = d and z = −d the electric field is of the eddy type and, therefore, has opposite directions:

$$E_x(d) = -E_x(-d) \equiv E(0). \qquad (2)$$

We shall assume that all the quantities depend on the coordinate z and are proportional to $\exp(-i\omega t)$. Subject to these assumptions, we find that Eq. (1) yields the following equation for the amplitude of a linearly polarized (along the x axis) transverse acoustic wave:

$$\frac{d^2u_x}{dz^2} + \left(\frac{\omega^2+i\omega\gamma}{s_t^2}\right)u_x = (\rho s_t^2)^{-1}F_x(z). \qquad (3)$$

Substituting here $F_x(z)$ from Eq. (A.14), we obtain the solution

$$u_x(z) = u_x^{\mathrm{df}}(z) + u_x^{\mathrm{an}}(z), \qquad (4)$$

where

$$u_x^{\mathrm{df}} = -\beta\frac{F_0}{\rho}\frac{l}{d}\sum_m \frac{(lk_m)^2\sin k_m d}{\omega^2-\omega_m^2+i\omega\gamma}\sin k_m z,$$

$$u_x^{an} = -\beta' \frac{F_0}{\rho} \frac{l}{d} \frac{l}{\delta} \sum_m \frac{\mathrm{Re}\{\psi(k_m) e^{ik_m z}\}}{\omega^2 - \omega_m^2 + i\omega\gamma};$$

here,

$$F_0 = |e| n_e E(0), \quad \lambda = 2\pi s_t / \omega, \quad k_m = \pi m / 2d \quad (m = 1, 2, 3 \ldots),$$

$$\omega_m = k_m s_t, \quad \delta = c / \sqrt{2\pi\mu\sigma\omega},$$

$$\psi(k_m) = \frac{(k_m\delta)^2 - 2}{(k_m\delta)^4 + 4} \{-i \sin k_m d\} \quad \text{при } d \gg \delta.$$

In order-of-magnitude estimates we can reduce the sum over m to just one term with $\omega_m^2 = \omega^2$. Then,

$$\frac{u^{df}}{u^{an}} \sim \frac{\delta}{\lambda} \frac{l}{\lambda} \frac{(\delta/\lambda)^4 + 4}{(\delta/\lambda)^2 + 2}. \qquad (5)$$

If $\delta/\lambda \ll 1$, we find that $u^{df}/u^{an} \sim l\delta/\lambda^2$, i.e., in this case, the surface effect in the generation of ultrasound by an rf field can be ignored and only the weak anomaly of the skin effect need be considered. If $\delta/\lambda \gg 1$ and $(l/\lambda)(\delta/\lambda)^3 \gg 1$, the generation of ultrasound is dominated by the diffuse scattering of electrons at the boundary of a metal. If $l\delta/\lambda^2 \sim 1$, we must allow for the diffuse scattering of electrons at the boundary and for the spatial dispersion of the electrical conductivity. The case $\delta/\lambda \ll 1$ is encountered in ferromagnets with a high permeability μ because $\delta \sim 1/\sqrt{\mu}$. In weak field the permeability μ is high. This is why the phase modulation of the Mossbauer spectra by an rf field is easiest to observe in ferromagnetic models ($u_x^{an} \sim \sqrt{\mu}$).

The following consequences flow from Eq. (4): the amplitude of ultrasound is proportional to the density of the currents induced in a sample by an rf field and inversely proportional to the mass of the lattice atoms. This result is in agreement with experiment.

3. EXPERIMENT

The Mössbauer spectra were determined using a spectrometer operating under constant-velocity conditions. The radioactive source was Co^{57} embedded in palladium foil. The absorbers were foils made of iron, Fe-36% Ni alloy, and stainless steel of the 1Kh18N9T type in the form of squares of ~1.5 cm side. The foil was clamped between two mica plates and placed in a planar single-layer rf coil of rectangular cross section which was tuned with an air capacitor to the frequency of an rf oscillator. The absorber and rf circuit were screened by a metal casing which had beryllium windows for the admission of γ rays. The rf coil with the sample could be rotated so that the angle between the plane of the sample and the direction of propagation of γ rays was 45°. The intensity of the rf field was measured directly during the recording of the Mössbauer spectrum: for this purpose we used the amplitude of the high-frequency voltage induced in the rf coil with the sample. At high amplitudes of the rf field we employed air cooling of the sample.

The Fe-36% Ni foil was heated above the Curie temperature by the eddy currents induced by the rf field. The temperature of the sample was deduced from the thermal shift of the absorption line and in the case of ferromagnetic samples also from the hyperfine field at the Fe^{57} nuclei. The temperature dependences needed for calibration were determined first.

4. DISCUSSION OF EXPERIMENTAL RESULTS

The low-amplitude ultrasound generated by the rf field in a metal sample was manifested by the presence of satellites representing the phase modulation of the Mössbauer spectrum of the Fe^{57} nuclei in the sample (absorber). The mechanism of the conversion of the rf field energy into the energy of ultrasound was determined by performing experiments on ferromagnetic and nonferromagnetic metals. A comparison of the rf-field-modulated Mössbauer spectra of ferromagnetic and nonferromagnetic samples enabled us to estimate the importance of the magnetostrictive mechanism of ultrasound generation.

In the first of these experiments we studied the Mössbauer spectrum of an Fe-36% Ni foil, 4 μ thick. The Curie temperature of the Fe-36% Ni alloy was $T_C = 245°$ C. Figure 1c shows the Mössbauer spectrum of this sample recorded at $T > T_C$ ($T \sim 290°$ C) in an rf field of $\nu = 54$ MHz frequency and $h \sim 8$ Oe amplitude. The satellites can be seen clearly in Fig. 1. The amplitude of these satellites depended, in accordance with Eq. (4), on the intensity of the eddy currents induced by the rf field. When a sample was cut into strips about 1 mm wide and these strips were oriented with their long sides along the rf field, the intensity of the eddy currents fell strongly and this reduced the amplitude of the satellites right down to zero level. This is illustrated in the spectrum shown in Fig. 1e. When a sample was rotated through 90° about its normal to the surface, the long edges of the strips became perpendicular to the rf field. In this case, the Mössbauer spectrum was practically the same as in the uncut samples because such cuts had hardly any influence on the intensity of the eddy currents. A typical ferromagnetic Mössbauer spectrum with satellites was observed at temperatures $T < T_C$ (Fig. 1b).

The second experiment was carried out on a sample

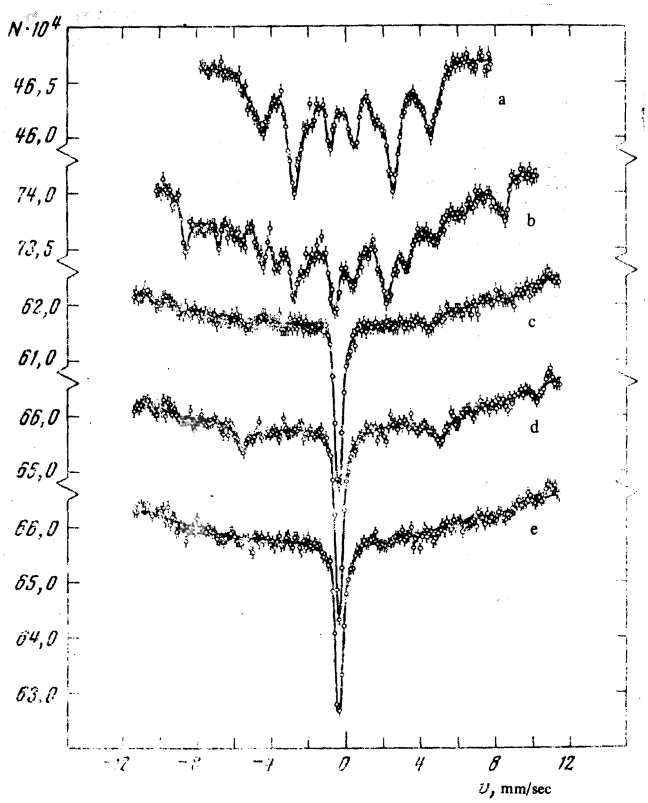

FIG. 1. Mössbauer spectra of Fe-36% Ni foils, 4 μ thick: a) h = 0, $T < T_C$: b) h ~ 15 Oe, $\nu = 70$ MHz, $T < T_C$: c) h ~ 8 Oe, $\nu = 54$ MHz, $T > T_C$: d) h ~ 8 Oe, $\nu = 63$ MHz, $T > T_C$, the plane of the foil inclined at 45° to the direction of propagation of the γ rays; e) h ~ 8 Oe, $\nu = 63$ MHz, $T > T_C$, sample cut into strips ~1 mm wide oriented along the rf field.

consisting of a powdered semiconducting paramagnetic compound β-$FeSi_2$. This powder was of 1 μ grain size and was placed on a thin mica plate. The Mössbauer spectrum of this sample showed no satellites. The sample was next coated with a film of copper ~1 μ thick. The particles of β-$FeSi_2$, embedded in a copper foil, were rigidly bound to this foil. The application of the rf field to this sample induced satellites, clearly visible in Fig. 2b.

In the third experiment, we observed satellites in the spectrum of a foil made of stainless steel of the 1Kh18N9T grade. This spectrum is shown in Fig. 2a.

In the fourth experiment the phase modulation of the spectrum of the Mössbauer nuclei Fe^{57} was observed in a 3-μ thick foil made of an alloy of gold containing 7 at.% Fe^{57}. The satellites were observed at frequencies of 60, 70, and 76 MHz. The rf field intensity was inhomogeneous along the sample and its maximum value exceeded 20 Oe. The Mössbauer spectrum was practically identical with that shown in Fig. 2a.

In these experiments we used nonferromagnetic samples and, therefore, the satellites could not be explained by the magnetostriction model put forward by Pfeiffer, Heiman, and Walker.[23] The mechanism responsible for the satellites observed in our experiments involved the eddy currents induced by the rf field. In accordance with Eq. (4), this mechanism should play an important role in ferromagnetic metals because of their high permeability. It is possible that, in general, the magnetostriction makes a small contribution to the modulation of the Mössbauer spectra by an rf field. This is supported by the results of an experiment carried out on iron foils ~2 μ thick. As in the experiments on Fe-36% Ni, the sample was cut into strips 1 mm wide. When the long edges of the strips were oriented along the rf field, the intensities of the satellites fell strongly but when the long edges were perpendicular to the rf field the satellite intensities were almost the same as in the case of uncut samples (due to the anisotropy of the eddy currents). Since the anisotropy of the eddy currents had the same effect on the intensities of the satellites exhibited by paramagnetic and ferromagnetic metals, we assumed that the lattice vibrations and, therefore, the satellites were excited mainly by the eddy currents and not by the magnetostriction.

We also studied the influence of the rf field amplitude on the Mössbauer spectrum. At high amplitudes of this field the intensity of the eddy currents increased and the satellites as well as the main lines became so broad that it was difficult to resolve the Mössbauer spectrum. This was evidently due to the excitation of the lattice vibrations of frequencies other than the rf field frequency, as a result of the interaction between elastic waves (nonlinear effects). In this case, the Mössbauer spectra of the uncut foils and of the cut strips 1 mm wide and oriented at right-angles to the rf field were practically identical: one of these spectra (for an uncut foil) is given in Fig. 3c (h ~ 6 Oe and ν = 54 MHz). However, when the sample was rotated through 90° about its normal to the surface, so that the long edges of the strips were aligned along the rf field, the intensity of the

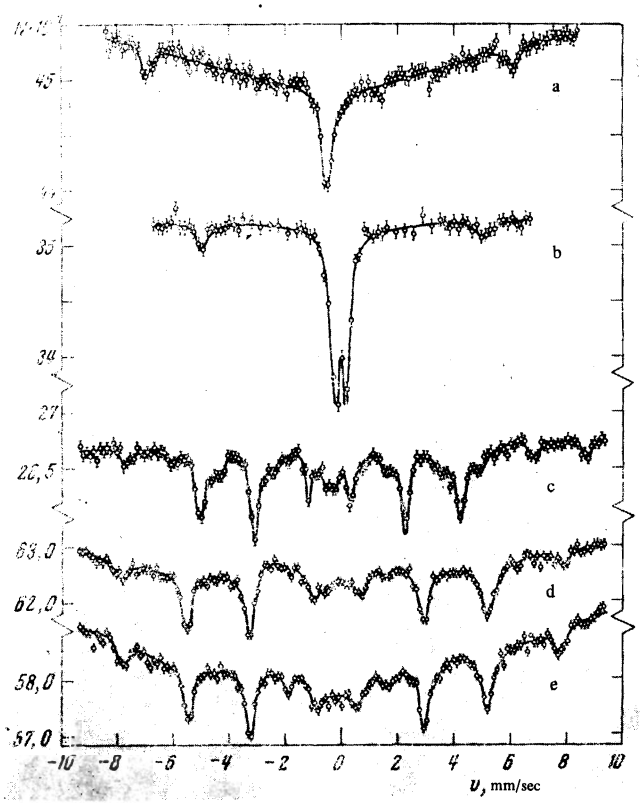

FIG. 2. Mössbauer spectra: a) stainless steel foil ~3 μ thick, h ~ 14 Oe, ν = 75 MHz; b) powder of β-$FeSi_2$ embedded in an evaporated copper film ~1 μ thick, h ~ 6 Oe, ν = 54 MHz; c) iron foil ~ 20 μ thick, h ~ 8 Oe, ν = 54 MHz; d) iron foil ~9 μ thick, h ~ 1 Oe, ν = 56 MHz; e) iron foil ~8 μ thick, h ~ 1 Oe, ν = 56 MHz (in this case a ferrite was inserted in the rf coil on one side of the sample in order to generate an inhomogeneous rf field).

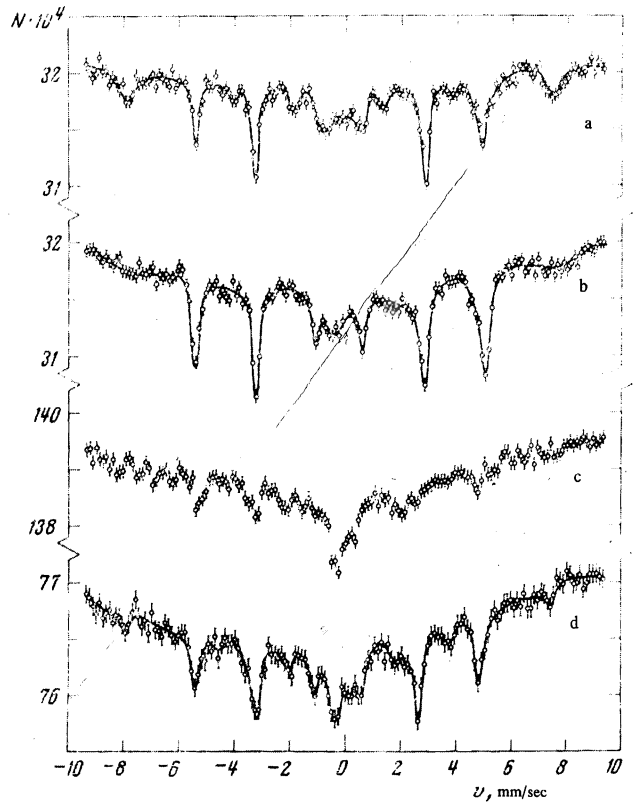

FIG. 3. Mössbauer spectra of iron foil ~2 μ thick; a) h ~ 1.5 Oe, ν = 54 MHz; b) same as a but foil wetted with glycerin; c) continuous foil, h ~ 6 Oe, ν = 54 MHz (the same spectrum is obtained for a sample cut into strips ~1 mm wide and oriented at right-angles to the rf field); d) foil cut into 1 mm wide strips and oriented along the rf field, h ~ 6 Oe, ν = 54 MHz.

eddy currents fell strongly and the satellites reappeared (Fig. 3d). It would seem to us that this was why the satellites were not observed in[23] for the perpendicular orientation of the long edges of the strips relative to the rf field, although the amplitude of the rf field was sufficiently high (12 Oe). Unfortunately, the Mössbauer spectra obtained in these experiments were not given in[23] and we were unable to compare these spectra with our results.

The amplitude of elastic vibrations in a sample could be reduced considerably by wetting the sample with glycerin. In this case, some of the energy of the elastic vibrations generated by the eddy currents was lost because of the excitation of ultrasound in glycerin. The corresponding Mössbauer spectra are shown in Figs. 3a and 3b ($h \sim 1.5$ Oe, $\nu = 54$ MHz, iron foil 2 μ thick). Similar experiments were carried out by Albanese, Asti, and Rinaldi.[24]

The amplitude of the satellites also decreased with increasing thickness of the sample. For example, when the thickness of the iron foil was increased from 2 to 20 μ, it was necessary to raise the amplitude of the rf field from 1.5 to 8 Oe in order to observe the satellites (Fig. 2c). This was due to an increase in the dissipation of the ultrasonic energy, which was proportional to the volume of a sample.

Pfeiffer, Heiman, and Walker[23] studied the dependence of the satellite intensity on the angle between the direction of propagation of γ quanta and the plane of the sample. When this angle was reduced from 90 to 45°, the satellite intensity increased approximately by a factor of 1.5. The same effect was also observed for nonferromagnetic samples. We found this effect in samples of Fe-36% Ni at temperatures above the Curie point. Therefore, this effect could not be regarded as confirmation of the correctness of the magnetostriction model.

The effect just described can be explained easily by considering the mechanism of the excitation of ultrasound by eddy currents. In the case of a homogeneous isotropic metal sample subjected to a homogeneous rf field, the eddy currents excite a transverse elastic wave whose polarization vector lies in the plane of the sample [see Eqs. (3) and (4)]. This wave does not give rise to a component of the atomic vibrations along the normal to the plane of the sample which coincides with the direction of propagation of the γ rays. In this situation no satellites are observed. When the angle between the normal to the sample and the direction of propagation of the γ rays is increased, a component of the atomic vibrations along the direction of propagation of the γ rays appears and increases. This gives rise to satellites whose amplitude increases with this angle. Under real experimental conditions both the samples and rf fields are inhomogeneous and, therefore, the eddy currents excite atomic vibrations along the direction of propagation of the γ rays even when the normal to the surface of the sample coincides with this direction. This is due to the fact that, on a microscopic scale, this normal does not coincide with the direction of propagation of the γ rays. We found that the satellite amplitude increased considerably when the inhomogeneity of the rf field was made greater. This was done by introducing a small ferrite with sharp corners into the rf field coil (compare Figs. 2d and 2e).

The influence of a static magnetic field H on the satellite amplitudes was investigated only for ferromagnetic metals[23] for $\mathbf{H} \perp \mathbf{h}$ and $\mathbf{H} \parallel \mathbf{h}$. The results given in[23] can be explained within the framework of the mechanism of the generation of ultrasound by eddy currents put forward above. According to Eq. (4), the amplitude of ultrasound is proportional to the intensity of the eddy currents in a sample and this intensity is proportional to the permeability μ. In the case of ferromagnetic metals, the value of μ is a function of $\mathbf{H}$. Therefore, in weak magnetic fields, when the ponderomotive force $(\mathbf{j} \times \mathbf{H})/c$ can still be ignored, the influence of $\mathbf{H}$ reduces to a change in μ. If $\mathbf{H} \parallel \mathbf{h}$, the satellite amplitude decreases rapidly with increasing H even for H = 20 Oe. In our opinion, this is due to the fact if $\mathbf{H} \parallel \mathbf{h}$, the mean contribution to the permeability $\mu_{\parallel}$ (in the range of $\mathbf{H}$, $\mathbf{h}$ under consideration) is made by the displacements of the domain boundaries. It is clear from the hysteresis loop shown in Fig. 8 in[23] that the maximum values of $\mu_{\parallel}$ are reached in fields weaker than 15 Oe. Therefore, for $\mathbf{H} \parallel \mathbf{h}$ and $H > 15$ Oe the permeability falls strongly and this reduces the intensity of the eddy currents and the amplitude of the satellites in the Mössbauer spectra. If $\mathbf{H} \perp \mathbf{h}$, the permeability $\mu_{\perp}$ is smaller than for $\mathbf{H} \parallel \mathbf{h}$ and depends weakly on H up to $H \sim H_A$ (H_A is the anisotropy field); if $H > H_A$, we find that $\mu_{\perp} \propto H^{-1}$. This dependence of $\mu_{\perp}$ on H is not in conflict with the experimental results reported in[23].

The phase modulation of the Mössbauer spectra may also be affected by other forces unrelated to the eddy currents. They include the magnetoelastic force (magnetostriction model) discussed in detail in[23], which may be of some importance in the case of ferromagnets. Moreover, ferromagnetic samples exhibiting a net magnetic moment $\mathbf{m}$ and subjected to an inhomogeneous rf field $h \propto \exp(-i\omega t)$ are acted upon by the force

$$\mathbf{F}_m = (\mathbf{m}\nabla)\mathbf{h}(\mathbf{r}, t).$$

This force causes forced vibrations of a sample as a whole at a frequency ω and this gives rise to satellites at frequencies which are multiples of ω. The same force appears also in ferrites.

The eddy currents also give rise to an additional force which can only make a contribution to the satellite intensity at frequencies which are multiples of 2ω. In fact, the eddy currents in a sample interact with the rf field which induces this current and give rise to a ponderomotive force whose density is*

$$\mathbf{f} = c^{-1}[\mathbf{jH}].$$

However, this effect is nonlinear in respect of $\mathbf{h}$ and evidently very small in the investigated range of values of $\mathbf{h}$ compared with the linear effect responsible for the generation of ultrasound by the rf field.

We shall conclude by pointing out that the experiments described above demonstrate the possibility of using the Mössbauer spectroscopy in investigations of the following phenomena: 1) diffuse scattering of electrons by the surface of a metal in the case when the mean free path of electrons is short; 2) very weak ultrasonic vibrations (of frequencies greater than the width of the Mössbauer line) generated by an rf field in a metal in general and particularly in polycrystalline samples at high temperatures, when the direct acoustic and electromagnetic (surface impedance) methods are inapplicable because of low susceptibility.

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APPENDIX

If the mean free path of electrons l is small compared with all the characteristic lengths in a given problem, we can develop a macroscopic theory of forces exerted on the lattice by the conduction electrons disturbed from the thermodynamic equilibrium state by an rf field applied to the metal in question. According to Kaganov and Fiks,[6] the density of the force acting on the lattice is, in this case,

$$F_x(z) = F_x^{(0)}(z) + F_x^{(s)}(z), \tag{A.1}$$

where $F_x^{(0)}(z)$ is the density of the force inside the metal,

$$F_x^{(0)}(z) = |e| n_e E_x(z) - \frac{2}{(2\pi\hbar)^3}\int dp\, p_x \left(\frac{\partial f}{\partial t}\right)_{st}, \tag{A.2}$$

$[(\partial f/\partial t)_{st}$ is the collision integral in the Boltzmann equation] and the density of the surface force due to the diffuse scattering of electrons by the boundary can be found from the condition of mechanical equilibrium in the metal as a whole (or from the law of conservation of momentum) and it is given (for example, at the boundary z = d) by

$$F_x^{(s)}(z) = \left\{\int_0^d dz\,(-F_x^{(0)}(z))\right\}\delta(z-d). \tag{A.3}$$

The electron distribution function $f(\mathbf{p}, \mathbf{r})$ is given by

$$v_z\frac{\partial f}{\partial z} + eE_x\frac{\partial f}{\partial p_x} = \left(\frac{\partial f}{\partial t}\right)_{st}. \tag{A.4}$$

Using Eq. (A.4), we can easily find that

$$F_x^{(0)}(z) = -\frac{\partial}{\partial z}\Pi_{xz}(z), \tag{A.5}$$

where

$$\Pi_{xz}(z) = \frac{2}{(2\pi\hbar)^3}\int dp\, p_x v_z f; \tag{A.6}$$

here, the function $\Pi_{xz}(z)$ vanishes unless the electrons are scattered diffusely by the boundaries of the metal. Ignoring the nonlocal relationship between Π_{xz} and the electric field, we find—in the linear approximation with respect to E_x—that

$$F_x^{(0)}(z) = -en_e\frac{\partial}{\partial z}(E_x(z)L(z)), \tag{A.7}$$

where L(z) has the dimensions of length and is of the order of l. Moreover, L(z) is finite and varies considerably in a distance of the order of l near the boundary of the metal, whereas the characteristic scale of the variation of $E_x(z)$ is given by

$$\delta \sim E_x|\partial E_x/\partial z|^{-1} \sim c(2\pi\mu\sigma\omega)^{-1/2},$$

where $\delta \gg l$. Under these conditions

$$F_x^{(0)}(z) = -en_e E_x(z)\,\partial L/\partial z. \tag{A.8}$$

On the basis of the above analysis, we find that Eqs. (A.8) and (A.3) yield

$$F_x^{(s)}(z) = en_e E_x(d) l\delta(z-d). \tag{A.9}$$

The appearance of this force and of $F_x^{(0)}(z)$ is due to the diffuse scattering of electrons by the boundaries of the metal. In fact, electrons acquire momentum from the electric field at distances of the order of l from the boundary and transfer this momentum (under the diffuse scattering conditions) on collision with the boundary. The force $F_x^{(0)}(z)$ appears because of the strong dependence of the current density on z at distances $\sim l$ from the boundary (due to the diffuse scattering). Because of this dependence, the first term in Eq. (A.2) is not compensated by the second term at distances $\sim l$ from the boundary but is compensated elsewhere.

Bearing in mind the foregoing analysis and the inequality $\lambda = 2\pi s_t/\omega \gg l$, we can replace $F_x^{(0)}(z)$ in Eq. (A.8) with the density of a force concentrated at some point z* at a distance $l^* = \beta l$ (β is a numerical factor smaller than or of the order of unity) from the boundary of the metal ($z^* = d - l^*$). Then, instead of Eq. (A.8), we can consider

$$F_x^{(0)}(z) = en_e E(d) l\delta(z - d + l'). \tag{A.10}$$

Adding Eqs. (A.9) and (A.10), we obtain the resultant force density near and on the boundary z = d. A similar expression can be obtained for the boundary z = −d. Consequently, allowance for the diffuse scattering of electrons by the boundaries of a planar film gives rise to the following formula:

$$F_x^{df}(z) = en_e E(0) l\{\delta(z-d) - \delta(z-d+l') - \delta(z+d) + \delta(z+d-l')\}, \tag{A.11}$$

in which we need retain only the higher terms of the expansion in respect of l^*.

In addition to $F_x^{df}(z)$, there is a force $F_x^{an}(z)$, which is due to the fact that Eq. (A.2) does not vanish if we allow for the nonlocal relationship between the current density and $E_x(z)$ or for the weak anomaly of the skin effect. In the linear approximation with respect to E_x, Eq. (A.2) gives $F_x^{an}(z)$ in the form

$$F_x^{an}(z) = |e| n_e\left\{E_x(z) - \int_0^d G(|z-z'|)E_x(z')\,dz'\right\}. \tag{A.12}$$

If $\delta \gg l$, we find that

$$F_x^{an}(z) \approx -|e| n_e \Lambda^2 \frac{\partial^2 E_x}{\partial z^2} + \ldots \tag{A.13}$$

The quantity Λ^2 in the above equation is equal to $\beta' l^2$, where β' is a numerical factor.

Adding $F_x^{an}(z)$ to Eq. (A.11), we find that the density of the combined forces acting on the lattice is

$$F_x(z) = F_x^{df}(z) + F_x^{an}(z). \tag{A.14}$$

The derivative $\partial^2 E_x/\partial z^2$ in Eq. (A.13) can be expressed by the following formula in the normal skin-effect approximation:

$$\frac{\partial^2 E_x}{\partial z^2} = E(0)p^2(e^{pd} - e^{-pd})^{-1}(e^{pz} - e^{-pz}), \tag{A.15}$$

$$p = (1+i)/\delta.$$

*$[jH] \equiv j \times H$.

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