Nonlinear acoustic effects in superconductors

Yu. M. Gal'perin, V. L. Gurevich, and V. I. Kozub

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
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Nonlinear effects occurring upon propagation of supersonic waves in a superconductor are considered. A kinetic equation for quasiparticle excitations in the field of a strong sound wave is derived by a technique similar to the Keldysh technique. It is shown that for long-wave sound (the wavelength of which is much greater than the electron mean free path) the nonlinearity parameter is the ratio of the characteristic electron-wave interaction energy to the Fermi energy. In the case of short-wave sound, a momentum nonlinearity occurs and is similar to that known for a normal metal and is due to capture of part of the excitations by the sound wave field. Allowance for this mechanism may be imporatant, in particular, in studies of the gap anisotropy by acoustic methods. A specific nonlinear effect is considered which is manifest in the appearance of a phase gradient of the order parameter near a sample in which intense sound is propagating. The possibility of observing the effect is appraised.

INTRODUCTION

The attenuation of ultrasound in superconductors has been studied in a large number of researches. It has been shown that one can obtain a series of interesting data on the energy spectrum of superconductors by studying the sound absorption; in particular, information can be obtained on the anisotropy of the energy gap.[1,2] However, in all the researches known to us, the sound absorption was studied at small amplitude, i.e., the calculations were made within the framework of small-amplitude sound theory. At the same time, the experimentalists at the present date are able to introduce sound of rather high intensity (of the order of 1 W/cm^2) into a crystal. As is well known,[3] appreciable nonlinear effects in the sound absorption can take place in normal metals at such intensities. One of the purposes of the present research was to study these effects in superconductors. In addition, we have also studied the acousto-electric effect, which varies greatly in different superconductors. It consists of the following. A traveling wave interacts with the normal excitations of the superconductor and transfers momentum to them, thus creating a current of normal excitations in the crystal. This current should be compensated by a current of the condensate of Cooper pairs, since no volume current can exist in a bulky superconductor. At the same time, the superconducting current is connected with the gradient of the order-parameter phase. Thus, a phase difference is developed in the order parameter on the boundaries of the superconductor along which the sound travels. This phase difference depends on the sound intensity and can be measured, for example, by using the sample along which the sound travels as the arm of a superconducting interferometer. Thus we can determine directly the sound intensity introduced into the crystal.

This effect exists independently of the relation between the sound wavelength $2\pi/q$ and the electron mean free path l. However, the damping mechanism is itself different for $ql \gg 1$ and for $ql \ll 1$. In particular, at $ql \gg 1$, even at low sound intensities, a specific high-frequency nonlinearity should appear, analogous to that known for the normal metal.[3] The physical reason for this nonlinearity is as follows. At $ql \gg 1$, only a small group of excitations interact with the sound wave and, consequently, determines the magnitude of the absorption. This is the group for which the projection of the velocity in the direction of sound propagation is close to the sound velocity w. The distribution function of this small group can undergo a strong change under the influence of the sound wave, and this causes the dependence of the absorption coefficient on the amplitude. At the same time, the distribution function of all the other excitations, which have other sound velocities, does not undergo appreciable change.

In Sec. 1 we obtain the kinetic equation for the electrons in the field of a sufficiently strong sound wave with account taken of the motion of the superconducting condensate; in Sec. 2 we calculate the nonlinear absorption coefficient at $ql \gg 1$, and in Sec. 3 we calculate the current of excitations and analyze the conditions for its measurement in a superconducting interferometer.

1. KINETIC EQUATIONS IN A SUPERCONDUCTOR

We begin with the derivation of the kinetic equation for the excitations in a superconductor. The kinetic equation was derived in a number of researches.[4-8] Nevertheless, we consider it is useful to rederive this equation for the following reasons. The principal reason is that the specifics of our problem require us to consider the perturbation of the distribution of quasiparticles of the sound wave under conditions when the condensate moves. Therefore, we have to introduce the kinetic equation with allowance for the motion of the condensate. Moreover, it is necessary for us to consider systematically the nonlinear character of the perturbation produced by the sound wave.

To derive the equation, we use the diagram technique of Keldysh.[7] It will be methodologically more convenient for us to consider the case of a motionless condensate, and then take into account the changes which must be made in the theory to account for the motion.

We shall describe the interaction of the electrons with the deformation field of the sound wave by means of a deformation potential,[8] assuming the interaction energy to be equal to[1)]

$$\Phi_{\mathbf{p}}(\mathbf{r}) = \Lambda_{ik}(\mathbf{p})\, u_{ik}(\mathbf{r}), \tag{1}$$

where u_{ik} is the deformation tensor in the sound wave and Λ_{ik} the tensor of the deformation potential, the components of which are of the order of several electron volts. This expression is valid in the so-called co-moving system of coordinates, i.e., in a system connected with the crystal lattice that is deformed during the

propagation of the sound wave. All the subsequent calculations will be made in this system. The Stewart-Tolman effect will be neglected because of its relative smallness.

To generalize the Keldysh technique to the case of a superconductor, besides the matrix of the ordinary Green's functions[7]

$$G^{\alpha\beta}=\begin{pmatrix} G_c & G_+ \\ G_- & \tilde{G}_c \end{pmatrix} = \begin{pmatrix} -i\langle T\psi_\alpha(x)\psi_\beta^+(x')\rangle & i\langle\psi_\beta^+(x')\psi_\alpha(x)\rangle \\ -i\langle\psi_\alpha(x)\psi_\beta^+(x')\rangle & -i\langle\tilde{T}\psi_\alpha(x)\psi_\beta^+(x')\rangle \end{pmatrix} \tag{2}$$

we introduce the matrix of the anomalous Green's functions

$$\hat{F}^{\alpha\beta}=\begin{pmatrix} \langle T\psi_\alpha(x)\psi_\beta(x')\rangle & -\langle\psi_\beta(x')\psi_\alpha(x)\rangle \\ \langle\psi_\alpha(x)\psi_\beta(x')\rangle & \langle\tilde{T}\psi_\alpha(x)\psi_\beta(x')\rangle \end{pmatrix}. \tag{3}$$

Here α and β are the spin indices, ψ^+ and ψ are the creation and annihilation operators of the electrons, $x=(r, t)$, $T(\tilde{T})$ is the symbol of ordering (or of anti-ordering) in time, the angle brackets denote the averaging defined in [7]. In similar fashion, we define the matrix ${}^+\hat{F}\alpha\beta$, but with the replacement in (3) of all operators ψ by ψ^+.

We consider the case in which the acoustic field can be regarded as classical:

$$qv_F \ll \Delta, \quad \hbar\omega \ll \Delta,$$

where v_F is the Fermi velocity of the electrons in the normal state, and Δ is the value of the energy gap. In this case it is advantageous to carry out a Fourier transformation with respect to the difference of the coordinates and the time:

$$\hat{G}^{\alpha\beta}(r, t, \varepsilon, p)=\int d\tau\, d^3R e^{i(\omega\tau-pR)/\hbar}\hat{G}^{\alpha\beta}(r+\tfrac{1}{2}R,\ t+\tfrac{1}{2}\tau;\ r-\tfrac{1}{2}R,\ t-\tfrac{1}{2}\tau). \tag{4}$$

Assuming the interaction to be independent of the spins,

$$\hat{G}^{\alpha\beta}=\delta_{\alpha\beta}\hat{G}, \quad {}^+\hat{F}^{\alpha\beta}=I_{\alpha\beta}{}^+\hat{F},$$
$$\hat{F}^{\alpha\beta}=-I_{\alpha\beta}\hat{F}, \quad I_{\alpha\beta}=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

it is easy to obtain the set of Dyson equations for $\hat{G}$ and ${}^+\hat{F}$. In our approximation, these equations are diagonal in the momentum difference p. The Dyson equations take the following form:

$$\begin{aligned} \hat{G}_0^{-1}(p)\hat{G}(p)&=\hat{I}+i\hat{\Sigma}\hat{G}+i\hat{\Delta}\,{}^+\hat{F}, \\ \hat{G}_0^{-1}(-p)\,{}^+\hat{F}(p)&=i\hat{\Sigma}^T(-p)\,{}^+F(p)+{}^+\hat{\Delta}(p)\hat{G}(p), \quad p=(\varepsilon, \mathbf{p}). \end{aligned} \tag{5}$$

Here

$$G_0^{-1}(p)=[\tfrac{1}{2}i\hbar\hat{B}-(E_0(\mathbf{p})-\varepsilon_F)+\hbar\omega-\Phi_p(\mathbf{r}, t)]\sigma_z,$$
$$\hat{B}=\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}-\frac{\partial\Phi}{\partial\mathbf{r}}\frac{\partial}{\partial\mathbf{p}}, \quad \sigma_z=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$$

σ_z is a matrix which acts on the lower (not spin) indices of the Green's functions, $E_0(\mathbf{p})$ is the electron dispersion law in a nondeformed normal metal, $\hat{\Sigma}$ is the self-energy matrix defined in accord with the rules given in [7], and Δ and ${}^+\Delta$ are the self-energy matrices which contain the anomalous functions F and ${}^+F$. Solution of the set (5) for the causal functions $G_c\equiv G_{11}$ and ${}^+F_c\equiv{}^+F_{11}$ in zeroth order in the small parameter $\hbar/\tau_p\Delta$ (where τ_p is the relaxation time of the electron momentum) can be represented in the form

$$G_c=u_p^2\left[\frac{n_p}{\varepsilon-\varepsilon_p-is}+\frac{1-n_p}{\varepsilon-\varepsilon_p+is}\right]+v_p^2\left[\frac{1-n_{-p}}{\varepsilon+\varepsilon_p-is}+\frac{n_{-p}}{\varepsilon+\varepsilon_p+is}\right], \tag{6}$$

$${}^+F_c=-iu_pv_p\left[\frac{n_p}{\varepsilon-\varepsilon_p-is}+\frac{1-n_p}{\varepsilon-\varepsilon_p+is}-\frac{1-n_{-p}}{\varepsilon+\varepsilon_p-is}-\frac{n_{-p}}{\varepsilon+\varepsilon_p+is}\right], \tag{7}$$

where

$$\varepsilon_p=(\xi_p^2+|\Delta_p|^2)^{1/2}, \quad \xi_p=E_0(\mathbf{p})+\Phi_p(\mathbf{r}, t)-\varepsilon_F,$$
$$u_p^2=\tfrac{1}{2}(1+\xi_p/\varepsilon_p), \quad v_p^2=\tfrac{1}{2}(1-\xi_p/\varepsilon_p);$$

n_p is some not yet known function which, as we shall see, has the meaning of an excitation distribution function. We want to emphasize especially that ξ_p is the total energy of the electron (including the deformation component $\Phi_p(\mathbf{r}, t)$ which depends on the coordinates), measured from the Fermi level.

In the derivation of Eqs. (6) and (7), we used the following considerations. The order parameter of the system, equal to the 11 element of the anomalous self-energy matrix ${}^+\hat{\Delta}$, is determined from the self-consistency equation

$${}^+\Delta_c(\varepsilon, \mathbf{p})=mp_F\int\frac{d\xi_{p'}\,d\Omega\,d\omega'}{(2\pi)^3}D_c(\mathbf{p}'-\mathbf{p}, -\omega')\,{}^+F_c(\mathbf{p}', \omega'), \tag{8}$$

where D_c is the causal phonon Green's function. It is not difficult to verify that in the integral over ω', the important ω' are $\sim\omega_D$ (ω_D is the Debye energy). At the same time, in kinetic problems, the important values of ϵ are of the order of T or Δ, i.e., much less than ω_D. Therefore, the order parameter can be assumed to be independent of ϵ, i.e., we can set ${}^+\Delta_c(\epsilon, \mathbf{p})\equiv\Delta_p$. Using (6) and (7), and also the spectral representation for G_c,

$$G_c=\frac{1}{2\pi i}\int_{-\infty}^{\infty}dz'\left[\frac{G_+}{\varepsilon-\varepsilon'-is}-\frac{G_-}{\varepsilon-\varepsilon'+is}\right], \tag{9}$$

and a similar representation for ${}^+F_c$, we can obtain the following expressions for $G_\pm$ and ${}^+F_\pm$:

$$\begin{aligned} G_+&=2\pi i[n_pu_p^2\delta(\varepsilon-\varepsilon_p)+v_p^2(1-n_{-p})\delta(\varepsilon+\varepsilon_p)], \\ G_-&=-2\pi i[(1-n_p)u_p^2\delta(\varepsilon-\varepsilon_p)+v_p^2n_{-p}\delta(\varepsilon+\varepsilon_p)], \\ {}^+F_+&=2\pi u_pv_p[n_p\delta(\varepsilon-\varepsilon_p)+(n_{-p}-1)\delta(\varepsilon+\varepsilon_p)], \\ {}^+F_-&=2\pi u_pv_p[(n_p-1)\delta(\varepsilon-\varepsilon_p)+n_{-p}\delta(\varepsilon+\varepsilon_p)]. \end{aligned} \tag{10}$$

To determine the function n_p, it is necessary to obtain one more equation, which has the meaning of a kinetic equation. It is easiest to introduce it in the following fashion. According to (10),

$$n_p=\frac{1}{2\pi i}\int_0^\infty[G_+(p)-G_-(-p)]d\varepsilon.$$

We now write down the Dyson equation for G_+:

$$G_0^{-1}(p)G_+=i(\Sigma_cG_++\Sigma_+\tilde{G}_c+\Delta_c\,{}^+F_++\Delta_+\,{}^+\tilde{F}_c), \tag{11}$$

We combine it with the Hermitian-conjugate equation

$$-G_0^{-1*}(p)G_+=i(G_c\Sigma_++G_+\tilde{\Sigma}_c+F_c{}^+\Delta_++F_+{}^+\tilde{\Delta}_c). \tag{12}$$

It is taken into account here that

$$\tilde{G}^+=-\sigma_x\hat{G}\sigma_x, \quad {}^+\hat{F}^+=\sigma_x\hat{F}\sigma_x,$$
$$\hat{\Sigma}^+=\sigma_x\hat{\Sigma}\sigma_x, \quad \hat{\Delta}^+=-\sigma_x{}^+\hat{\Delta}\sigma_x, \quad \sigma_x=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{13}$$

With account of the Keldysh identity[7]

$$\Sigma_c+\tilde{\Sigma}_c+\Sigma_++\Sigma_-=0 \tag{14}$$

we have for the sum of the equations

$$\hbar\hat{B}(p)G_+=\Sigma_+G_--\Sigma_-G_++\Delta_c\,{}^+F_++{}^+\tilde{\Delta}_cF_++\Delta_+\,{}^+\tilde{F}_c+{}^+\Delta_+F_c. \tag{15}$$

Similarly, we have for G_-

$$-\hbar\hat{B}G_-=\Sigma_-G_+-\Sigma_+G_-+\tilde{\Delta}_c\,{}^+F_-+{}^+\Delta_cF_-+\Delta_-\,{}^+F_c+{}^+\Delta_-\tilde{F}_c. \tag{16}$$

Replacing p in (16) by $-p$ and adding the equations, we obtain

$$\begin{aligned} &\hat{B}(p)G_+(p)-\hat{B}(-p)G_-(-p)=(\Sigma_+G_--\Sigma_-G_+)_p \\ &+(\Sigma_-G_+-\Sigma_+G_-)_{-p}+(\Delta_+\,{}^+F_--\Delta_-\,{}^+F_++{}^+\Delta_+F_--{}^+\Delta_-F_+)_p. \end{aligned} \tag{17}$$

Here the following properties of $\hat{F}$ and $\hat{\Delta}$ are taken into account:

$$\Delta_c + \bar{\Delta}_c + \Delta_+ + \Delta_- = 0, \quad {}^+\Delta_+ + {}^+\Delta_- + {}^+\Delta_c + {}^+\bar{\Delta}_c = 0,$$
$${}^+F_\pm(-p) = {}^+F_\mp(p),$$
$$F_\pm(-p) = F_\mp(p), \quad \Delta_\pm(-p) = \Delta_\mp(p), \quad {}^+\Delta_\pm(-p) = {}^+\Delta_\mp(p),$$

and follow from the Dyson equation and the definitions of these quantities.

Substituting (10) in (17), we obtain the following kinetic equation:

$$\frac{\partial n_p}{\partial t} + \mathbf{v}\frac{\xi_p}{\varepsilon_p}\frac{\partial n_p}{\partial \mathbf{r}} - \frac{\partial \Phi_p}{\partial \mathbf{r}}\frac{\xi_p}{\varepsilon_p}\frac{\partial n_p}{\partial \mathbf{p}} = \hat{I}_{coll}, \tag{18}$$

where $\mathbf{v} = \partial E_p/\partial \mathbf{p}$, and $\hat{I}_{coll}$ is the operator of collisions with phonons and impurities:

$$\hat{I}_{coll} = \frac{2\pi}{\hbar}\sum_Q |C_Q|^2 \{(1-n_p)\, n_{p-\hbar Q}\,(u_p u_{p-\hbar Q} - v_p v_{p-\hbar Q})^2 [(1+N_{-Q})\,\delta(\varepsilon_p - \varepsilon_{p-\hbar Q} + \hbar\omega_Q) + N_Q \delta(\varepsilon_p - \varepsilon_{p-\hbar Q} - \hbar\omega_Q)] - n_p(1-n_{p-\hbar Q})(u_p u_{p-\hbar Q} - v_p v_{p-\hbar Q})^2 [N_{-Q}\delta(\varepsilon_p - \varepsilon_{p-\hbar Q} + \hbar\omega_Q)] + (1+N_Q)\,\delta(\varepsilon_p - \varepsilon_{p-\hbar Q} - \hbar\omega_Q)] + (1-n_p)(1-n_{-p+\hbar Q})(u_p v_{p-\hbar Q} + u_{p-\hbar Q} v_p)^2 [(1+N_{-Q})\,\delta(\varepsilon_p + \varepsilon_{-p+\hbar Q} + \hbar\omega_Q) + N_Q\delta(\varepsilon_p + \varepsilon_{-p+\hbar Q} - \hbar\omega_Q)] - n_p n_{-p+\hbar Q}(u_p v_{p-\hbar Q} + u_{p-\hbar Q} v_p)^2 \times [N_{-Q}\delta(\varepsilon_p + \varepsilon_{-p+\hbar Q} + \hbar\omega_Q) + (1+N_Q)\,\delta(\varepsilon_p + \varepsilon_{-p+\hbar Q} - \hbar\omega_Q)] + \frac{2\pi}{\hbar} N_i \left[\frac{(2\pi\hbar)^2}{m}\right]^2 \int \frac{d^3Q}{(2\pi)^3} f_Q^2 (n_{p-\hbar Q} - n_p)(u_p u_{p-\hbar Q} - v_p v_{p-\hbar Q})^2 \delta(\varepsilon_p - \varepsilon_{p-\hbar Q}). \tag{19}$$

Here N_Q is the number of phonons with wave vector $\mathbf{Q}$, N_i is the concentration of impurity atoms, f_Q is the scattering amplitude of the normal electron by an individual impurity center, and $C_Q = Q_{p,p-\hbar Q}$ is the matrix element of the electron-phonon interaction. We note that in the case of a spatially homogeneous situation, a similar expression was obtained for the collision integral with phonons with the help of the Keldysh technique[10] for the case of a semiconductor placed in the strong electromagnetic-wave field that causes interband transitions. The collision operator for the case of a highly nonequilibrium isotropic distribution function of the excitations in the superconductor has been obtained by Éliashberg.[6]

The calculation that we carried out is valid in lowest order in the parameter $\hbar/\tau_p\Delta$. At low temperatures, in which we are interested, one usually needs to take into account the scattering from impurities, which also determines the momentum relaxation time τ_p, and scattering from phonons need be considered only in those problems in which the inelasticity of the scattering is important. To account for the next-order corrections in the parameter $\hbar/\tau_p\Delta$, the system (5) can be iterated with respect to this parameter. In first order in $\hbar/\tau_p\Delta$, a set of equations is used which is the nonlinear analog of the set (2.6) of the paper of Pokrovskiĭ and Savvinykh.[9] In that paper, the existence of a specific low-temperature attenuation was deduced on the basis of the solution of a set of kinetic equations. This attenuation was attributed with scattering of sound by impurities. In our view, this conclusion is a consequence of a numerical error in the expression for the collision term St_{gf} (see [9]). The error is analyzed by us in the Appendix I. We came to the conclusion that no such mechanism exists for sound damping in any case in that order in the parameter $\hbar/\tau\Delta$ which was considered in [9].

For the calculation of the acousto-electric current referred to in the Introduction, we must take into account the motion of the condensate. It is easiest to do this in the following fashion (cf., e.g., [11]). We make the following transformations in the Dyson equations (in the coordinate representation)

$$\hat{F}(x,x') = \tilde{\hat{F}}(x,x')\exp[ip_s(x+x')/\hbar],$$
$${}^+\hat{F}(x,x') = {}^+\tilde{\hat{F}}(x,x')\exp[-ip_s(x+x')/\hbar], \tag{20}$$
$$\hat{G}(x,x') = \tilde{\hat{G}}(x,x')\exp[ip_s(x-x')/\hbar].$$

For simplicity, we assume the electron spectrum $E_0(\mathbf{p})$ to be quadratic and isotropic.[2)] Then, for the condition $p_s v_F \ll \Delta$, we get the following kinetic equation in place of (18):

$$\frac{\partial \tilde{n}_p}{\partial t} + \mathbf{v}\frac{\xi}{\varepsilon} + \mathbf{v}_s\frac{\partial \tilde{n}_p}{\partial \mathbf{r}} - \frac{\partial \Phi_p}{\partial \mathbf{r}}\frac{\xi}{\varepsilon}\frac{\partial \tilde{n}_p}{\partial \mathbf{p}} + I_{coll} = 0, \tag{21}$$

and the energy $\tilde{\varepsilon}_p = \epsilon_p + \mathbf{p}_s\cdot\mathbf{v}$, where $\mathbf{v}_s = \mathbf{p}_s/m$, enters in $\tilde{I}_{coll}$ in the arguments of the δ functions. As a consequence of this, the collision integral I_{coll} vanishes for the function $n_0(\tilde{\epsilon}_p) = n_0(\tilde{\epsilon}_p + \mathbf{p}_s\cdot\mathbf{v})$, where $n_0 = [e^{\epsilon/\tau} + 1]^{-1}$ is the equilibrium distribution function.

The current density in the system can be obtained by letting the current density operator act on G_+. As a result we have

$$\mathbf{j} = \sum_{p,\sigma}(\mathbf{v}+\mathbf{v}_s)\int \frac{d\varepsilon}{2\pi i}\check{G}_+(p). \tag{22}$$

2. THE SOUND ABSORPTION COEFFICIENT

As is shown below in Appendix II, and also by Pokrovskiĭ and Savvinykh[9], the sound absorption coefficient Γ_s of a superconductor can be expressed in terms of the nonequilibrium distribution function of the quasiparticles:

$$\Gamma_s = (\rho\omega^2 u^2)^{-1}\sum_{p,\sigma}\left\langle \frac{\xi_p}{\varepsilon_p} n_p \frac{\partial \Phi_p}{\partial x}\right\rangle_\eta, \tag{23}$$

where $\langle\,\rangle_\eta$ denotes averaging over the wave coordinate $\eta = qx - \omega t$, ρ is the density of the crystal, and $\mathbf{u}$ the lattice displacement vector.

It is easy to see from the equations of elasticity theory with account of the deformation interaction, from the condition of electric neutrality, and also from the kinetic equation, that the higher harmonics of the effective potential are of order Φ/ϵ_F in comparison with the fundamental. Inasmuch as this quantity is practically always small, we shall neglect terms of this order. We can thus assume that

$$\Phi_p(\mathbf{r},t) = \Phi_{p0}\cos\eta,$$
$$\Gamma_s = q\sum_{p,\sigma}\frac{(\Lambda_{ik}(\mathbf{p})v_i e_k)^2}{\rho w^2(\mathbf{v})}\frac{\xi_p}{\varepsilon_p}\int_{-\pi}^{\pi}(-\sin\eta)\, n_p(\eta)\frac{d\eta}{\Phi_{p0}}. \tag{24}$$

Here $\boldsymbol{\nu} = \mathbf{q}/q$, and $\mathbf{e}$ is the sound polarization vector. Thus, for the calculation of the absorption one should solve the kinetic equation (18). We shall assume that the dominant scattering mechanism is scattering from impurities. We divide the distribution function into two parts and separate the part $n_\epsilon(\epsilon_p)$. This part is averaged over the constant energy surface: $n_p(\mathbf{r},t) = n_\epsilon(\epsilon_p(\mathbf{r},t)) + n_p'(\mathbf{r},t)$. If we assume the function n_ϵ to be an equilibrium one, then, as can easily be established, no corrections to the gap are necessitated by the nonequilibrium character of the distribution function. Actually, for an isotropic spectrum, we have in place of (18)[3)]

$$\left(v_x\frac{\xi_p}{\varepsilon_p} - w\right)\frac{\partial n_p'}{\partial t} - \frac{\partial \Phi_p}{\partial x}\frac{\xi_p}{\varepsilon_p}\frac{\partial n_p'}{\partial p_x} + \frac{n_p'}{\tau_n}\frac{|\xi_p|}{\varepsilon_p} = w\frac{\partial \Phi_p}{\partial x}\frac{\xi_p}{\varepsilon_p}\frac{\partial n_0}{\partial \varepsilon_p}. \tag{25}$$

If we neglect the sound velocity w in comparison with

the electron velocity at the Fermi surface v_F, the solution of Eq. (25) will be odd in ξ_p. Therefore, the part n_p^1 of the distribution function does not make a contribution to the value of the energy gap, at least, with accuracy to within the small parameter $(w/v_F)^2$.

We now discuss the validity of the neglect of the difference between the function n_ϵ and the equilibrium function n_0. We have two types of corrections to n_0—corrections having the period of the sound wave and corrections which do not depend on time. We begin with the discussion of the former. They are calculated various ways, depending on the sound frequency. As an example, we consider the simplest case of low sound frequencies and assume that the acoustic oscillations are adiabatic. This means that the sound propagation in the conductor is accompanied by a lowering of the alternating contribution T' to the temperature T, which is proportional to $e^{i(\mathbf{q}\cdot\mathbf{r})-\omega t)}$ and is equal to (see [12])

$$\frac{T'}{T}=\frac{w^2\beta}{c_p}\operatorname{div}\mathbf{u}, \tag{26}$$

where β is the coefficient of thermal expansion of the conductor and ϵ_p is its specific heat (for constant elastic stresses). Then the alternative correction to the gap will take the form

$$\Delta'=T\frac{\partial\Delta}{\partial T}\frac{w^2\beta}{c_p}\operatorname{div}\mathbf{u}.$$

In the kinetic equation (25), this contribution appears in the combination

$$\frac{\partial\varepsilon}{\partial t}=\frac{\xi}{\varepsilon}\Lambda_{ik}u_{ik}+\frac{\Delta}{\varepsilon}T\frac{\partial\Delta}{\partial T}\frac{w^2\beta}{c_p}\operatorname{div}\mathbf{u}. \tag{27}$$

Direct estimates show that such a correction, which is due to the change in Δ, is negligible. For high frequencies, the estimate is made differently, similar to what we do below in the estimate for stationary corrections to the distribution function. However, even for such a situation, the effect turns out to be small in the most interesting cases. In the case of non-adiabatic sound oscillations, the alternating contribution to the temperature is even smaller than that determined by Eq. (26).

We now estimate the stationary corrections to the distribution function, which are connected with heating of normal excitations. For this purpose, we use the following consideration. The sound-wave energy absorbed per unit volume and per unit time is of the order of $\Gamma_s S$, where S is the sound intensity. This energy is redistributed among the normal excitations, the number of which is of the order of N_0T/ϵ_F for $T\sim T_c$ (N_0 is the electron concentration). The energy received by the excitations is relaxed by the acoustic phonons, and for each collision act, the energy of the individual excitation changes by an amount of the order of the energy itself, and the component of the momentum changes by an amount of the order of $p_F(T/\hbar\omega_D)^2$. Actually, it follows from the shape of the phonon distribution function that the important phonons are those having wave vectors $Q\sim T/\hbar w$, whence, with allowance for the energy conservation law for electron-phonon collisions, $\epsilon_{p+\hbar Q}-\epsilon_p-\hbar\omega_Q=0$, the estimate given above follows. Therefore, the energy relaxation time is connected with the momentum relaxation time on phonons τ_p^{ph} by the relation $\tau_\epsilon\sim\tau_p^{ph}(T)(T/\hbar\omega_D)^3$. Comparing the mean energy obtained by the excitations from the sound wave with the characteristic energies T and T_c-T (the second quantity is connected with the essential dependence of the energy gap on the temperature at $T_c-T\ll T_c$), we find that heating is unimportant if the following condition is satisfied:

$$S\ll\frac{N_0T_D}{\Gamma_s\varepsilon_F\tau_p^{ph}(T)T}\min(T_c-T,T). \tag{28}$$

A more rigorous quantitative calculation, based on the kinetic equation, confirms these estimates.

At helium temperatures, we can assume that $\tau_p^{ph}\sim10^{-6}-10^{-7}$ sec. With account of this, we find that at $(T_c-T)/T\sim1$ the heating is important at sound intensities of 10^4-10^5 W/cm^2 in a metal and 10^2 W/cm^2 in a superconducting semiconductor. Thus there exists a region of temperatures and sound intensities in which the heating of the excitations by the sound wave field is unimportant; however, as we shall see, other nonlinearity mechanisms are important. We note that at low temperatures $T\ll T_c$, and also for $T_c-T\ll T_c$, the heating can turn out to be significant; however, we shall not consider here the region of very low temperatures or temperatures lying in the immediate vicinity of T_c (bounded by the condition (28)). Thus, we shall assume that Δ does not change in the presence of a sound wave.

We now turn to calculation of the absorption. At $ql\ll1$, the principal term on the left side of (25) is the third term. In this case, we have, for an isotropic electron spectrum,

$$n'=w\tau_n\frac{\partial\Phi}{\partial x}\operatorname{sign}\xi_p\frac{\partial n_0}{\partial\varepsilon_p}. \tag{29}$$

It is significant that the only nonlinearity parameter here is Φ_{p0}/ϵ_F, which is always small. The absorption coefficient in this case is completely described by linear theory and is obtained in [9,13].

In the opposite limiting case, $ql\gg1$, the term that contains the sound velocity on the left side of (25) is not important. In this case, the kinetic equation differs from the corresponding equation for the normal metal only in the form of the function $n^0(\epsilon)$ on the right side. As is known, the small group of electrons which belong to the "waist" of the Fermi surface $\omega=q\sigma\epsilon/\partial p$ makes the contribution to the absorption here.

In the linear case, the result is identical with the result obtained by Bardeen, Cooper and Schrieffer,[14] who regarded sound absorption as the process of the absorption of sound quanta by normal excitations. Here the absorption coefficient was directly connected with the value of the gap, averaged over the waist of the Fermi surface. If the sound intensity is so great that the condition

$$\omega_0\tau_n=q\tau_n(\Phi_{p0}/m)^{1/2} \tag{30}$$

is satisfied, the distribution function of the quasiparticles near the waist is strongly perturbed by the sound wave. In other words, a group of quasiparticles is trapped in the potential wave of the sound-wave field. The formation of such a group leads to a specific nonlinearity, which was studied previously for the case of normal conductors.[3]

The problem as applied to a superconductor is solved quantitatively in the following way. Inasmuch as the higher harmonics of the potential of the wave do not develop (as we have already noted) because of the smallness of the parameter Φ_{p0}/ϵ_F, the entire nonlinear part of the problem reduces to the solution of the kinetic equation (25), which is itself a linear partial differential

equation and, upon neglect of terms of the order w/v_F in comparison with those considered, takes the form

$$\tilde{v}_x\frac{\partial n^1}{\partial\eta}-\frac{d\tilde{\Phi}}{d\eta}\frac{\partial n^1}{\partial\tilde{v}_x}+\frac{\operatorname{sign}\xi}{\omega_0\tau_n}n^1=\Phi_{p0}\frac{w}{(\Phi_{p0}/m)^{1/2}}\frac{\partial n_0}{\partial\varepsilon}, \tag{31}$$

where $\tilde{v}_x=v_x/(\Phi_{p0}/m)^{1/2}$, $\omega_0=q(\Phi_{p0}/m)^{1/2}$, and $(\Phi_{p0}/m)^{1/2}$ are the characteristic frequency of oscillations in the well and the velocity of motion of the captured electrons, respectively; $\tilde{\Phi}_p=\Phi_p/\Phi_{p0}$. The solution of this equation can be obtained by the method of characteristics, by imposing on the distribution function of the untrapped quasiparticles the condition of periodicity in the wave coordinate, and by imposing on the distribution function of the trapped quasiparticles the condition of specular reflection from the walls of the potential wells. In the general case, the solution is obtained in [3] and has a rather complicated form. However, in the case of a strong nonlinearity ($\omega_0\tau_n\gg1$), it can be expanded in powers of $(\omega_0\tau_n)^{-1}$ and the contribution to the absorption is made by a term of the order $(\omega_0\tau_n)^{-1}$.

In this case, the distribution function of the quasiparticles is of the form (see [3])

$$n^1(\tilde{v}_x,\eta)=\frac{\operatorname{sign}\xi}{2\omega_0\tau_n}\int_{\eta}^{\eta+2\pi}U_0g(\eta')\left(\frac{\psi(\eta,\eta')}{\psi(0,2\pi)}-1\right)\psi(\eta,\eta')\,d\eta' \tag{32a}$$

for the untrapped quasiparticles and

$$n^1(\tilde{v}_x,\eta)=\frac{1}{2\omega_0\tau_n}\frac{\operatorname{sign}\xi}{\psi(\eta_1,\eta_2)}\left[\int_{\eta_1}^{\eta_2}U_0g\psi(\eta,\eta_1)\,d\eta'-2\int_{\eta}^{\eta_2}U_0g\psi(\eta,\eta')\psi(\eta,\eta_2)\,d\eta'-2\int_{\eta_1}^{\eta}U_0g\psi(\eta',\eta)\psi(\eta_1,\eta)\,d\eta'\right] \tag{32b}$$

for the trapped quasiparticles. Here $g(\eta)=(2(\mathcal{E}-\tilde{\Phi}(\eta))^{1/2}$, $\mathcal{E}=\frac{1}{2}\tilde{v}_x^2+\tilde{\Phi}(\eta)$ is the dimensionless energy of one-dimensional motion, U_0 is the right side of Eq. (31),

$$\psi(x,y)=\int_x^y g(\eta)\,d\eta.$$

Substituting (32a) and (32b) in the expression for the absorption coefficient (24), we can obtain the following result (cf. [3]):

$$\frac{\Gamma_s}{\Gamma_{s0}}=1.1\left\langle\frac{(\Lambda_{ik}e_ie_k)^2}{q\tau_n(\Phi_{p0}/m)^{1/2}}\frac{2}{e^{\Delta/T}+1}\right\rangle\left[\left\langle(\Lambda_{ik}e_ie_k)^2\frac{2}{\exp(\Delta/T)+1}\right\rangle\right]^{-1}, \tag{33}$$

where Γ_{S0} is the linear absorption coefficient (see, for example, [2]). $m^{-1}=\partial^2E_0(p)/\partial p_x^2$ (the derivative is taken at the point $\omega=q\partial\epsilon/\partial p$), the angle brackets denote averaging over the waist of the Fermi surface[4] $q\partial\epsilon/\partial p=\omega$. This ratio is equal to $1/q\tau_n(\Phi_{p0}/m)^{1/2}$ in order of magnitude.

In Eq. (33) there are integrals over the waist of the Fermi surface which, with account of the inequality $w\ll v_F$, is almost identical with the line $q\partial\epsilon/\partial p=0$. If $\Delta_p\gg T$, and the anisotropy of the gap is sufficiently large, then a contribution is made to the integrals only by the region in the vicinity of the points where Δ_p has a minimum. There are at least two such points (if the sound is propagated along an axis of high symmetry, their number can be greater). In this case, the ratio $\tau_n/\sqrt{m}$ can be determined at these points, for example, from an analysis of the nonlinear absorption coefficient for a known deformation potential.

It is important to note that the ratio of the nonlinear absorption coefficients in the superconductor and in the normal metal can be different from the ratio of the linear coefficients. Namely,

$$\frac{\Gamma_{s0}}{\Gamma_{n0}}=\left\langle(\Lambda_{ik}(p)v_ie_k)^2\frac{2}{e^{\Delta/T}+1}\right\rangle[\langle(\Lambda_{ik}(p)v_ie_k)^2\rangle]^{-1}, \tag{34}$$

while the following expression for the relation of the nonlinear absorption coefficients in the superconducting and normal states can be obtained from Eqs. (31), (32), and (24):

$$\frac{\Gamma_s}{\Gamma_n}=\left\langle\frac{(\Lambda_{ik}(p)v_ie_k)^{3/2}}{\tau_n}m^{1/2}\frac{2}{e^{\Delta/T}+1}\right\rangle\left[\left\langle\frac{(\Lambda_{ik}(p)v_ie_k)^{3/2}}{\tau_n}m^{1/2}\right\rangle\right]^{-1}. \tag{35}$$

3. THE ACOUSTO-ELECTRIC CURRENT

For the calculation of the constant current, it is natural to seek the solution of the kinetic equation (21) in the form

$$n_p=n_0(\varepsilon_p+pv_s)+n_p^1, \tag{36}$$

where n_p^1 is the nonequilibrium contribution associated with the sound. In the approximation linear in the small parameter pv_S/Δ, one can obtain from (22) the following expression for the current:

$$j=j_{ac}+j_c,$$

where

$$j_{ac}=\frac{e}{2}\sum_{p\sigma}v\langle n_p^1-n_{-p}^1\rangle_\eta$$

is the sound drag current (the brackets indicate averaging over the wave coordinate), and

$$j_c=e\sum_{p\sigma}\left\{v_s[u_p^2n_0+v_p^2(1-n_0)]+v(pv_s)\frac{\partial n_0}{\partial\varepsilon_p}\right\} \tag{37}$$

is the condensate current, which can be represented in the form

$$j_c=eN_Sv_S,$$

where

$$N_S/N_0=1-\int_0^\infty\operatorname{ch}^{-2}(x^2+(\Delta/2T)^2)^{1/2}\,dx,$$

N_S and N_0 are the number of "superconducting" electrons and the total number of electrons, respectively. The acousto-electric current can be calculated from Eq. (21), neglecting the motion of the condensate. We shall calculate the current without solving Eq. (21),[5] for which purpose we multiply it by the velocity of the electron v and sum over the constant energy surface. By definition,

$$\sum_{p\sigma}vI_{c\tau}\{n_p^1\}\delta(\varepsilon-\varepsilon_p)=\frac{1}{\tau_{tr}}\frac{|\xi|}{\varepsilon}\sum_{p\sigma}vn_p^1\delta(\varepsilon-\varepsilon_p), \tag{38}$$

and the electric current is expressed in terms of the remaining terms of the kinetic equation. The only term that does vanish at the accuracy of interest to us after averaging over the period of the wave is

$$-\sum_p v\left\langle\frac{\partial\Phi_p}{\partial r}\frac{\xi_p}{\varepsilon_p}\frac{\partial n^1}{\partial p}\right\rangle_\eta\delta(\varepsilon-\varepsilon_p). \tag{39}$$

As a result,

$$j_{ac}=e\int_\Delta^\infty d\varepsilon\,\tau_{tr}\frac{\varepsilon}{|\xi|}\sum_{p,\sigma}v\frac{\xi_p}{\varepsilon_p}\left\langle\frac{\partial\Phi}{\partial r}\frac{\partial n^1}{\partial p}\right\rangle_\eta\delta(\varepsilon-\varepsilon_p)=-e\sum_{p\sigma}\tau_{tr}(\varepsilon)m^{-1}\left\langle n^1\frac{\partial\Phi}{\partial r}\right\rangle_\eta\operatorname{sign}\xi. \tag{40}$$

Inasmuch as the important electrons are those close to the Fermi surface, one can take the transport relaxation time outside the summation sign, as a result of which we obtain a relation, similar to the Weinreich relation[16], between the acousto-electric current, the absorption co-

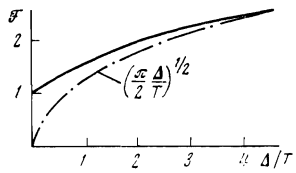

FIG. 1

efficient, and the sound intensity:

$$j_{ac}=\frac{\mu_n\Gamma_n S}{w}\frac{N_0-N_S}{N_0}=\frac{\mu_n\Gamma_s S}{w}\mathcal{F}\left(\frac{\Delta}{T}\right), \tag{41}$$

where μ_n is the electron mobility in the normal state,

$$\mathcal{F}\left(\frac{\Delta}{T}\right)=\int_\Delta^\infty \frac{\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\frac{\partial n_0}{\partial\varepsilon}d\varepsilon \Big/ \int_\Delta^\infty d\varepsilon\frac{\partial n_0}{\partial\varepsilon}$$
$$=\frac{1}{4}(e^{\Delta/T}+1)\int_0^\infty \frac{x+\Delta/T}{[x(x+2\Delta/T)]^{1/2}}\operatorname{ch}^{-2}\left(\frac{x}{2}+\frac{\Delta}{2T}\right). \tag{42}$$

A graph of the function $\mathcal{F}$ is shown in Fig. 1. It is seen that at $\Delta/T\gg1$ we have $\mathcal{F}(\Delta/T)\sim(\pi\Delta/2T)^{1/2}$ (see the dashed curve).

We have calculated the current of the normal excitations. Inasmuch as the total current in the volume of a bulk superconductor should be equal to zero, we have the following expression for p_S:

$$p_S=-\frac{\tau_{tr}\mathcal{F}(\Delta/T)}{wN_S}\Gamma_s S. \tag{43}$$

Integrating this equation over the length of the crystal, with account of the relation $\Gamma_S S=-\partial S/\partial x$, we obtain the following expression for the phase difference at the boundaries of the sample:

$$|\Delta\varphi|=\left|2\int_0^L\frac{p_S}{\hbar}dr\right|=\frac{2\tau_{tr}\mathcal{F}(\Delta/T)}{\hbar wN_S}(S_0-S_L). \tag{44}$$

In conclusion, we shall give some numerical estimates. In a pure superconductor, one can assume $\tau_{tr}\sim10^{-8}$ sec. Near the phase transition we have $N_S\sim N_0\cdot2(T_c-T)/T_c$). Setting $(S_{in}-S_{out})\sim1$ W/cm^2 and $N_0\sim10^{22}$ cm^{-3}, we have, in order of magnitude

$$\Delta\varphi\sim\frac{10^{-1}}{\delta}S\left[\frac{\mathrm{W}}{\mathrm{cm}^2}\right],\quad \delta=\frac{T_c-T}{T_c}.$$

If we assume that the phase difference can be measured with a superconducting interferometer accurate to 1% and that the temperature of the sample can be maintained near T_c with the same accuracy, then we can measure the sound intensity from 10^{-3} to one W/cm^2 by such a method.[6)] We note that the absolute value of the sound intensity that is propagated along the metal is measured here directly, something very difficult to do by other methods. In superconducting semiconductors, the electron concentration is considerably less (down to 10^{18} cm^{-3}); however, the scattering from impurities in them is considerably greater, and τ_{tr} is also much less. Inasmuch as the ratio τ_{tr}/N_0 enters into the expression for $\Delta\varphi$, estimates for superconducting semiconductors can turn out to be either more or less favorable, depending on the specific materials.

APPENDIX I

We shall show how the kinetic equation is obtained in first order in $\hbar/\tau\Delta$. For this purpose, we carry out a Bogolyubov transformation on the electron operators:

$$a_{p\sigma}=u_p\alpha_{p\sigma}-v_p\sigma\alpha^+_{-p,-\sigma},\qquad a_{p\sigma}^+=u_p\alpha_{p\sigma}^+-v_p\sigma\alpha_{-p,-\sigma}. \tag{A.1}$$

It is then easy to obtain the following relation:

$$G_+=\{u_p^2\mathcal{G}_+(\mathbf{p})-v_p^2\mathcal{G}_-(-\mathbf{p})-iu_pv_p[{}^+\mathcal{F}_+(\mathbf{p})+\mathcal{F}_+(\mathbf{p})]\},$$
$$G_-=\{-u_p^2\mathcal{G}_-(\mathbf{p})+v_p^2\mathcal{G}_+(-\mathbf{p})-iu_pv_p[{}^+\mathcal{F}_+(\mathbf{p})+\mathcal{F}_+(\mathbf{p})]\}, \tag{A.2}$$
$${}^+F_+=u_p^2\mathcal{F}-v_p^2\mathcal{F}+uv[\mathcal{G}_+(\mathbf{p})-\mathcal{G}_-(-\mathbf{p})],$$

where

$$\hat{\mathcal{G}}_{p\sigma}(t_i,t_k')=-i\langle T_c\alpha_{p\sigma}(t_i)\alpha_{p\sigma}^+(t_k')\rangle,$$
$$\hat{\mathcal{F}}_p^{\alpha\beta}(t_i,t_k')=\langle T_c\alpha_{p\alpha}(t_i)\alpha_{-p\beta}(t_k')\rangle \tag{A.3}$$
$${}^+\hat{\mathcal{F}}_p^{\alpha\beta}(t_i,t_k')=\langle T_c\alpha_{p\alpha}^+(t_i)\alpha_{-p\beta}^+(t_k')\rangle.$$

The integral $\int d\epsilon\,\mathcal{G}_+/2\pi i$ has the meaning of the number of electrons. We therefore define:

$$\mathcal{G}_+=2\pi i n_p\delta(\varepsilon-\varepsilon_p),\ \mathcal{G}_-=-2\pi i(1-n_p)\delta(\varepsilon-\varepsilon_p). \tag{A.4}$$

The anomalous correlators $\mathcal{F}_\pm$ vanish in zeroth order in $\hbar/\tau\Delta$. In first order in this parameter, we determine these functions from the following considerations. In the derivation of the kinetic equation, we had to integrate over ϵ in the limits $0,+\infty$. Inasmuch as

$$\int_0^\infty d\varepsilon\,{}^+\mathcal{F}_+(\varepsilon)=\pi\,{}^+\mathcal{F}_+(\tau)|_{\tau=0}-\int_{-\infty}^\infty\frac{{}^+\mathcal{F}_+(\tau)}{i\tau}d\tau,$$

setting the correlator at the equal times to be ${}^+\mathcal{F}_+\equiv-\gamma^*$ (the minus sign is chosen so that the signs correspond to those of [9]) we define the function ${}^+\mathcal{F}_+$ by the expression

$${}^+\mathcal{F}_+(p)=-\pi\gamma_p\delta(\varepsilon-\varepsilon_p)+\mathrm{P}\frac{1}{i}\frac{\gamma_p}{\varepsilon-\varepsilon_p}, \tag{A.5}$$

where P denotes the principal value. We now repeat the calculations (11)–(16), and substitute the values of the Green's functions in the form (A.2) with account of (A.3) and (A.4). Integrating the difference $G_+(p)-G_-(-p)$ with respect to ϵ in the limits $(0,\infty)$, we get the following kinetic equation

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\xi_p}{\varepsilon_p}\frac{\partial}{\partial\mathbf{r}}-\frac{\partial\Phi}{\partial\mathbf{r}}\frac{\xi_p}{\varepsilon_p}\frac{\partial}{\partial\mathbf{p}}\right)n_p-\frac{\Delta}{\varepsilon}\left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}}-\frac{\partial\Phi}{\partial\mathbf{r}}\frac{\partial}{\partial\mathbf{p}}\right)\operatorname{Re}\gamma_p$$
$$+\frac{1}{\tau_n}\frac{|\xi|}{\varepsilon}(n_p-\langle n_p\rangle)+\frac{1}{\tau_n}\frac{\Delta}{\varepsilon}\operatorname{sign}\xi\operatorname{Re}\gamma_p=0. \tag{A.6}$$

A second equation connecting n_p and γ_p can be obtained by combining the Dyson equations for ${}^+F_+$ and ${}^+F_-$. Subtracting the equation for ${}^+F_-(-p)$ from the equation for ${}^+F_+(p)$, and using the symmetry properties of the functions, and also the spectral representation for ${}^+F_c$ in terms of ${}^+F_+$ and ${}^+F_-$, we can obtain

$$\left(i\hbar\frac{\partial}{\partial t}-2\xi\right){}^+F_+-i\operatorname{Re}{}^+\Delta_c[G_+(p)-G_-(-p)]$$
$$=\frac{i}{2}[{}^+\Delta_+(G_-(p)-G_+(-p))-{}^+\Delta_-(G_+(p)-G_-(-p))$$
$$+{}^+F_-(\Sigma_-(-p)-\Sigma_+(p))-{}^+F_+(\Sigma_+(-p)-\Sigma_-(p))]. \tag{A.7}$$

Substituting the expressions (A.2)–(A.4) in (A.7) and integrating in the range $(-\infty,\infty)$ with respect to ϵ, we get the following equation, after cumbersome but essentially straightforward transformations,

$$\left(i\hbar\frac{\partial}{\partial t}-2\varepsilon_p\right)\gamma_p+i\hbar\frac{\partial\gamma^0}{\partial t}\frac{\varepsilon}{\xi}=\frac{i\hbar}{\tau_n}\frac{\varepsilon}{|\xi|}\left(1+\frac{\Delta^2}{\varepsilon^2}\right)+\frac{i\hbar}{\tau_n}\operatorname{sign}\xi\frac{\Delta}{\varepsilon}n^+,$$
$$\gamma^0=-\frac{\Delta}{2\varepsilon_p}(1-2n^+),\qquad n^+=\frac{n_p+n_{-p}}{2}. \tag{A.8}$$

Here the condition $\hbar/\tau\Delta\ll1$ has been taken into account, and the only collision terms which are written out are those important for the calculation of the sound absorption. The collision terms are identical with the collision terms obtained in [9] in the form of Eq. (2.8). However, the second collision term in (A.6) differs from the first component of Eq. (5.10) of that paper by the coefficient $\frac{1}{2}$. Expressing the corrections to the excitation distribution function n_p in terms of the function γ_p with the help of Eq. (A.6), it is easy to establish the

fact that the corrections to the electron distribution function

$$f_p = \langle a_p^+ a_p \rangle = u_p^2 n_p + v_p^2 (1 - n_{-p}) + \frac{\Delta}{\varepsilon} \operatorname{Re} \gamma_p$$

are absent in the order $\hbar/\tau\Delta$ considered by us. Thus, if we are interested in quantities that are expressed in terms of the electron distribution function (for example, the sound absorption and the acousto-electric current), then we can use the kinetic equation (18) for the calculation of these quantities, at any rate accurate to first order in $\hbar/\tau\Delta$.

We note that in first order in $\hbar/\tau\Delta$ there are also corrections to the self-consistency equation, which are easily obtained by expressing ${}^+F_0$ in terms of ${}^+F_-$ and ${}^+F_+$ with the help of the spectral representation, and then substituting the expressions (A.2)–(A.4) for the latter functions. However, these corrections are not important for the calculation of the sound absorption, since they vanish as a consequence of the condition of electric neutrality in the sound wave.

APPENDIX II

We obtain an expression for the nonlinear sound absorption coefficient in a superconductor in terms of the electron Green's function.

Let the oscillation in the sound wave consist, with sufficient accuracy, of a single harmonic. We then define the damping coefficient as a quantity proportional to the imaginary part of the pole of the retarded phonon Green's function. It can be shown, in a manner similar to that used in [7], that $D_r(\omega, q)$ safisfies the Dyson equation

$$D_r^{-1} = D_{r0}^{-1} - i\Pi_r, \qquad (A.9)$$

where $\Pi_r = \Pi_c + \Pi_-$; Π_c and Π_- are the components of the polarization matrix

$$\hat{\Pi} = \begin{pmatrix} \Pi_c & \Pi_+ \\ \Pi_- & \tilde{\Pi}_c \end{pmatrix},$$

for the calculation of which the diagram technique of Keldysh is suitable.[7] Assuming the imaginary part of the pole to be much smaller than the real part, we obtain for the coefficient of sound power absorption

$$\Gamma = q \operatorname{Re} \Pi_r. \qquad (A.10)$$

As follows from Eq. (1), the vertex that describes the interaction of electrons with the sound wave can be represented in the form

$$\hat{A} = \hat{\gamma} \left(\frac{\Phi_{p0}}{\rho\omega^2 \mathbf{u}^2} \right) \delta_{p,\, p-\hbar q},$$

where

$$p = (\mathbf{p}, \varepsilon), \quad \Phi_{p0} = \Lambda_{ik} u_{ik}^0, \quad \gamma_{ij}{}^k = \delta_{ij} (\sigma_z)_{jk}.$$

We calculate the quantity

$$\int \frac{d^3p}{(2\pi\hbar)^3} \int \frac{d\varepsilon}{2\pi i} A_{11} G_c(p, p - \hbar q). \qquad (A.11)$$

The function $G_c(p, p-\hbar q)$ can be represented in the form of the sum of the graphs of Fig. 2. The numbers in parentheses denote the number of the corresponding matrix indices used in the Keldysh technique. It is seen that the quantity (A.11) is equal to

$$i(\Pi_{11} + \Pi_{12}) = i(\Pi_c + \Pi_+).$$

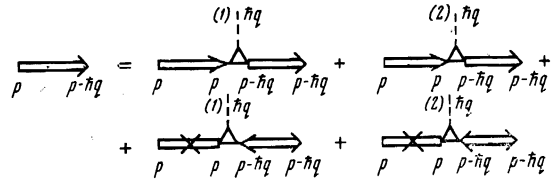

FIG. 2

Inasmuch as $\operatorname{Re}(\Pi_c + \Pi_+) = \frac{1}{2}(\Pi_+ - \Pi_-) = -\operatorname{Re}\Pi_r$, we have

$$\Gamma = \operatorname{Re} \int \frac{d^3p}{(2\pi\hbar)^3} \int \frac{d\varepsilon}{2\pi i} \frac{iq\Phi_{p0}}{\rho\omega^2 \mathbf{u}^2} G_c(p, p - \hbar q). \qquad (A.12)$$

It follows from the spectral representation for the causal Green's function that

$$\int \frac{d\varepsilon}{2\pi i} G_c(p, p - \hbar q) = \frac{1}{2} \int \frac{d\varepsilon}{2\pi i} [G_+(p, p - \hbar q) + G_-(p, p - \hbar q)]$$
$$= \int \frac{d\varepsilon}{2\pi i} G_+(p, p - \hbar q). \qquad (A.13)$$

In the classical case, we have

$$\Gamma = \frac{1}{\rho\omega^2 \mathbf{u}^2} \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{2\pi} \int d\eta \frac{\partial\Phi}{\partial\eta} \int \frac{d\varepsilon}{2\pi i} G_+(\eta), \qquad (A.14)$$

which is identical with Eq. (23) of the text.

[1] It is assumed that the contribution from longitudinal electric fields, which arise when sound propagates in a conductor, is included in the potential $\Phi_p(\mathbf{r})$ (these fields are determined from the condition of electric neutrality [6,9]). It can be verified that, neglecting small terms of the order w/v_F, which lie within the limits of accuracy of our calculation, the contribution from longitudinal fields to $\Phi_p(\mathbf{r})$ is "in phase" with the direct deformation contribution.

[2] The case of an arbitrary spectrum $E_0(\mathbf{p})$ can be considered in similar fashion.

[3] At $ql \gg 1$, the introduction of a relaxation time can be justified for any spectrum $E_0(\mathbf{p})$; [3] τ_n is the relaxation time in the normal state.

[4] In fact, in the calculation of Γ, the averaging is also carried out over the longitudinal momentum within the limit of width of the waist. This averaging, however, is not important if the anistropy of the gap is not anomalously large, so that the condition

$$\partial\Delta_p / \partial\vartheta \ll T v_F / w\Delta, \; ql, \; (\varepsilon_p / \Phi_{p0})^{1/2}$$

is satisfied, where ϑ is the angle between the direction of the vector $\mathbf{p}$ and the plane $p_x = 0$.

[5] We have chosen this method of calculation of the current because it allows us to introduce the transport relaxation time in a case in which the distribution function depends essentially on the angles. This method was first proposed by Kagan. [15]

[6] It is assumed, however, that the sound intensity here satisfies the condition (28).

[1] R. W. Morse, Progr. Cryog. 1, 220 (1959).
[2] C. L. Pokrovskiĭ, Zh. Eksp. Teor. Fiz. 40, 143, 898 (1961) [Soviet Phys.-JETP 13, 100, 628 (1961)].
[3] Yu. M. Gal'perin, V. D. Kagan and V. I. Kozub, Zh. Eksp. Teor. Fiz. 62, 1521 (1972) [Soviet Phys.-JETP 35, 798 (1972)].
[4] B. T. Geĭlikman, Zh. Eksp. Teor. Fiz. 34, 1042 (1958) [Soviet Phys.-JETP 7, 721 (1958)].
[5] B. T. Geĭlikman and V. Z. Kresin, Zh. Eksp. Teor. Fiz. 41, 1142 (1961) [Soviet Phys.-JETP 14, 816 (1962)].
[6] G. M. Éliashberg, Zh. Eksp. Teor. Fiz. 61, 1254 (1971) [Soviet Phys.-JETP 34, 668 (1972)].
[7] L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) [Soviet Phys.-JETP 20, 1018 (1965)].
[8] S. Titeica, Ann. der Phys. 22, 128 (1935); A. I. Akhiezer, Zh. Eksp. Teor. Fiz. 8, 1318 (1938).
[9] V. L. Pokrovskiĭ and S. K. Savvinykh, Zh. Eksp. Teor.

Fiz. **43**, 564 (1962) [Soviet Phys.-JETP **16**, 404 (1963)].
[10]V. F. Elesin, Zh. Eksp. Teor. Fiz. **59**, 602 (1970) [Soviet Phys.-JETP **32**, 328 (1971)].
[11]I. O. Kulik and I. K. Yanson, Effekt Dzhoefsona v sverkhprovodyashchikh tunnel'nykh strukturakh (Josephson Effect in Superconducting Tunnel Structures), Nauka, 1970, p. 129.
[12]L. D. Landau and E. M. Lifzhitz, Mekhanika sploshnykh sred (Fluid Mechanics) Gostekhizdat, 1954, p. 303. [Pergamon, 1958].
[13]V. Z. Kresin, Zh. Eksp.Teor. Fiz. **36**, 1957 (1959) [Soviet Phys.-JETP 9, 1392 (1959)].
[14]J. Bardeen, L. Cooper and J. Schrieffer, Phys. Rev. 108, 1175 (1957).
[15]V. D. Kagan, Fiz. Tverd. Tela **9**, 3293 (1967) [Soviet Phys.-Solid State **9**, 2752 (1968)].
[16]G. Weinreich, Phys. Rev. **197**, 317 (1957).

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