Oscillation regimes in a rotating solid-state ring laser

E. L. Klochan, L. S. Kornienko, N. V. Kravtsov, E. G. Lariontsev, and A. N. Shelaev

Nuclear Physics Institute of the Moscow State University
(Submitted March 29, 1973)
Zh. Eksp. Teor. Fiz. **65**, 1344–1356 (October 1973)

Theoretical and experimental investigations are reported of the oscillation regimes in a solid-state ring laser. It is shown that such a regime may include the frequency locking of waves traveling in opposite directions, self-Q-switching, and unidirectional emission. It is demonstrated that the oscillation regimes depend strongly on the coupling between the waves running in opposite directions, on the resonator tuning, and on the rate of rotation of the laser.

INTRODUCTION

Crystal lasers are characterized by a homogeneously broadened gain profile and this gives rise to a considerable competition between the oscillation modes.

Investigations of the oscillation regimes in solid-state lasers with ring resonators are of considerable interest from the point of view of interaction between waves traveling along the ring in opposite directions. Achievement of unidirectional emission in a solid-state ring laser is important from the practical point of view. Such emission results in a considerable narrowing of the output spectrum and this makes it possible to reach high output powers in the single-mode case. Studies of the characteristics of rotating solid-state ring lasers should establish whether it would be possible to use such lasers in measurements of the angular velocities and angles of rotation.

At present, the properties of solid-state ring lasers are much less known than the properties of gas lasers. This applies particularly to the cw solid-state lasers. Theoretical investigations of the interaction between waves traveling in the opposite directions ("counter waves") in solid-state lasers have been reported in[1-5]. It is shown[1-3] that in the absence of coupling via backscattering the standing-wave regime in a ring laser is unstable. In the case of complex-conjugate coupling coefficients the standing-wave regime is also unstable, irrespective of the strength of the coupling. Studies have been made[4,5] of the stability of unidirectional emission and conditions under which such emission is impossible.

Experimental studies have been made[6,7] of the characteristics of solid-state ring lasers operating continuously. Unidirectional single-mode emission has been achieved[6] by establishing a small difference between the resonator Q factors for the counter waves. It has been found[7] that a strong coupling between counter waves due to reflection from the ends of a crystal in a ring laser gives rise to stable standing-wave conditions.

The present paper reports a theoretical and experimental investigation of the dynamics of oscillation in a solid-state ring laser. It is shown that several oscillation regimes may exist in such a laser. It is established that these regimes depend strongly on the coupling between counter waves, on the resonator tuning, and on the rate of rotation of the laser.

THEORY

Condition of stability of laser oscillations in the case of counter waves of constant amplitudes and different phases

The dynamics of oscillation in a ring laser can be described by the following system of equations for the complex amplitudes of the two counter waves $\tilde{E}_{1,2}$ and of the inverted population density N (it is assumed that a single mode is emitted):

$$\dot{\tilde{E}}_{1,2}=-\frac{\omega}{2Q}\tilde{E}_{1,2}+\frac{i}{2}\tilde{m}_{1,2}\tilde{E}_{2,1}\mp i\frac{\Omega}{2}\tilde{E}_{1,2}$$
$$+\frac{\sigma(1-i\varepsilon)}{2T}\left[\int_0^l Ndx\tilde{E}_{1,2}+\int_0^l Ne^{\pm i2kx}dx\tilde{E}_{2,1}\right], \qquad (1)$$
$$\dot{N}=W-N/T_1-aN|E|^2/T_1.$$

Here, ω/Q is the width of the resonator band; T is the transit time of light around the resonator; Ω is the splitting of the natural frequencies of the oscillator because of rotation; l is the length of the crystal; σ is the transition cross section; T_1 is the inversion relaxation time; W is the pumping rate; $a = \sigma c T_1/8\pi\hbar\omega$.

In Eq. (1) the coupling of the counter waves because of backscattering is assumed to be linear and it is introduced phenomenologically with the aid of complex feedback coefficients:

$$\tilde{m}_{1,2}=m_{1,2}\exp(\pm i\vartheta_{1,2}), \qquad (2)$$

where $m_{1,2}$ are the moduli and $\vartheta_{1,2}$ are the phases of the coupling coefficients. The electric field E in the ring laser is related to the complex amplitudes $\tilde{E}_{1,2} = E_{1,2}\exp(i\varphi_{1,2})$ as follows:

$$E=\frac{1}{2}\sum_{1,2}[\tilde{E}_{1,2}e^{i(\omega t\mp kx)}+\text{c.c.}]. \qquad (3)$$

Both waves are assumed to be polarized in the same way.

The parameter ϵ determines the detuning of the laser output frequency ω relative to the center of a homogeneously broadened luminescence line $\epsilon = (\omega - \omega_0)/\gamma_{ab}$, where γ_{ab} is the half-width of this line. In the case of solid-state lasers, the large width of the luminescence line usually leads to $\epsilon \ll 1$ and, therefore, we shall assume that $\epsilon = 0$.

It follows from Eq. (1) that in the presence of coupling the threshold value of the inverted population density is

$$N_{\text{th}}=\frac{T}{l\sigma}\left[\frac{\omega}{Q}-(m_1m_2)^{1/2}\left|\sin\frac{\vartheta_1-\vartheta_2}{2}\right|\right]. \qquad (4)$$

Hence, we can see that in the case of equal phases of the coupling coefficients ($\vartheta_1 = \vartheta_2$) the threshold value of the inverted population density N_{th} is independent of the coupling. For fixed values of the moduli $m_{1,2}$ the coupling between the waves has the strongest influence on the threshold condition if the phase difference is $|\vartheta_1 - \vartheta_2| = \pi$.

The system (1) has steady-state solutions corresponding to the generation of counter waves of constant amplitudes and phase difference. A study of the stability of oscillation conditions is simplified considerably because, in the case of solid-state lasers we usually have

$$\omega\eta/Q \gg T_1^{-1} \tag{5}$$

($\eta = W/W_{th} - 1$ is the excess of the pumping rate above the threshold value).

The time dependences of the perturbations will be represented in the form $x(t) = x(0)e^{\lambda t}$. If the inequality (5) is satisfied, the perturbations can be divided into fast ($\lambda \propto \omega\eta/Q$) and slow [$\lambda \propto (\omega\eta/QT_1)^{1/2}$]. The slow perturbations give rise to damped transient processes. The standing-wave regime may be unstable under the influence of the fast perturbations. In the presence of such perturbations, the inversion does not change and remains equal to its steady-state value:

$$N = N_{st} = \frac{N_{th}(1+\eta)}{1 + aE_{10}^2 + aE_{20}^2 + 2aE_{10}E_{20}\cos(\Phi_0 - 2kx)}, \tag{6}$$

where E_{10}, E_{20}, and φ_0 are the steady-state values of the wave amplitudes and of the phase difference.

The characteristic equation for the fast perturbations can be written in the form:

$$(\lambda - \delta)^2 + M_0^2 = 0, \tag{7}$$

$$\delta = \frac{\sigma}{T}\int_0^l N_{st}\,dx - \frac{\omega}{Q}, \quad M_0 = \frac{1}{2}\left[m_1\cos(\Phi_0 - \vartheta_1)\frac{E_{20}}{E_{10}} + m_2\cos(\Phi_0 - \vartheta_2)\frac{E_{10}}{E_{20}}\right]. \tag{8}$$

In deriving the characteristic equation (7), no assumptions are made on the relationship between the steady-state values of the amplitudes and phases of the counter waves. It follows from Eq. (7) that the fast perturbations are oscillatory. The oscillation frequency is governed by the coupling via the backscattering and is equal to $\omega_M = M_0$, whereas the increment (decrement) is

$$\delta = \frac{\sigma}{T}\int_0^l N_{st}\,dx - \frac{\omega}{Q}.$$

If we use the system (1), we obtain two expressions for δ:

$$\delta = \left[\frac{\sigma}{T}\tilde{N} - m_1\sin(\Phi_0 - \vartheta_1)\right]\frac{E_{20}}{E_{10}}, \quad \delta = \left[\frac{\sigma}{T}\tilde{N} + m_2\sin(\Phi_0 - \vartheta_2)\right]\frac{E_{10}}{E_{20}},$$
$$\tilde{N} = -\int_0^l N_{st}\cos(\Phi_0 - 2kx)\,dx > 0. \tag{9}$$

The following conclusions are arrived at readily from the above expressions. A steady-state two-wave regime may be stable ($\delta < 0$) only if the feedback coefficients are sufficiently large:

$$m_{1,2} > \sigma\tilde{N}/T. \tag{10}$$

In the case of equal phases of the coupling coefficients ($\vartheta_1 = \vartheta_2$), the two-wave regime is unstable for all values of the moduli $m_{1,2}$. At fixed values of the moduli $m_{1,2}$ the coupling between the waves has its strongest stabilizing influence on the two-wave regime if the phase difference is $\vartheta_1 - \vartheta_2 = \pm\pi$.

The difference between the phases of the coupling coefficients $x_1 - x_2 = 0$ corresponds to the scattering of waves by inhomogeneities of the permittivity ϵ. In the case of scattering by inhomogeneities of the conductivity σ, the difference is $|\vartheta_1 - \vartheta_2| = \pi$.

These results have a simple physical meaning. In the presence of two counter waves the population inversion in the medium is depleted in a spatially inhomogeneous manner [see Eq. (6)]. The gains of the counter waves of amplitudes $E_{1,2}$ are given by the expression

$$\alpha_{1,2} = \sigma\left[\int_0^l N_{st}\,dx - \frac{E_{2,1}}{E_{1,2}}\tilde{N}\right]. \tag{11}$$

According to Eq. (11), the wave with a greater amplitude has a larger gain. In the absence of coupling of the waves via backscattering this leads to an instability of the standing-wave configuration and a complete suppression of the weaker wave. If allowance for the backscattering coupling is made, the effective gain of a wave during one pass through the resonator changes to

$$\alpha_{1,2}^{\text{eff}} = \sigma\left[\int_0^l N_{\text{ст}}\,dx - \frac{E_{2,1}}{E_{1,2}}\tilde{N}\right] \pm m_{1,2}T\sin(\Phi - \vartheta_{1,2})\frac{E_{2,1}}{E_{1,2}}.$$

If the inequality (10) is satisfied for the phase difference $\vartheta_1 - \vartheta_2 = \pm\pi$, it is found that the weaker wave has the greater gain. This leads to a stability of the standing-wave configuration in a ring laser.

Frequency locking of counter waves at near-threshold pumping levels

We shall now consider in detail the stability conditions of the standing-wave regime and the closely related regime of frequency locking between two traveling waves of approximately the same intensities. We can easily show that such regimes are possible only for equal or slightly differing moduli of the coupling coefficients. We shall assume that the moduli $m_{1,2}$ are similar in magnitude but that the phases $\vartheta_{1,2}$ are arbitrary. We shall first discuss the case when the damping level just exceeds the threshold value ($\eta = W/W_{th} - 1 \ll 1$).

The solution of Eq. (7) shows that two stable standing-wave regimes are possible and that they are distinguished by different values of the phase shift between the counter waves:

$$\Phi_{01} = \Theta + \arccos\frac{\Omega}{M}, \quad \Phi_{02} = \Theta - \arccos\frac{\Omega}{M},$$
$$M = \frac{1}{2}[m_1^2 + m_2^2 - 2m_1m_2\cos(\vartheta_1 - \vartheta_2)]^{1/2}, \tag{12}$$
$$\Theta = \operatorname{arctg}\frac{m_1\sin\vartheta_1 - m_2\sin\vartheta_2}{m_1\cos\vartheta_1 - m_2\cos\vartheta_2}.$$

The only stable regime ($\delta < 0$) is that for which the phase difference $\Phi = \Phi_{01}$. In this case, the sum of the dimensionless intensities $y = a(E_{10}^2 + E_{20}^2)$ and their difference $x = a(E_{10}^2 - E_{20}^2)$ are given by the expressions

$$y = \frac{2}{3}\left\{\eta + \frac{(M^2 - \Omega^2)^{1/2} - (m_1m_2)^{1/2}|\sin[(\vartheta_1 - \vartheta_2)/2]|}{\omega/Q - (m_1m_2)^{1/2}|\sin[(\vartheta_1 - \vartheta_2)/2]|}\right\}, \tag{13}$$

$$\frac{x}{y} = \frac{1}{2M^2\delta}\left[-\Omega m_1m_2\sin(\vartheta_1 - \vartheta_2) + \frac{m_2^2 - m_1^2}{2}(M^2 - \Omega^2)^{1/2}\right], \tag{14}$$

$$\delta = \frac{1}{3}\left[\frac{\omega}{Q}\eta - (1+\eta)(m_1m_2)^{1/2}\left|\sin\frac{\vartheta_1 - \vartheta_2}{2}\right| - 2(M^2 - \Omega^2)^{1/2}\right]. \tag{15}$$

The expressions (13) and (14) are valid if $x/y \ll 1$, $\eta \ll 1$, subject to the stability condition $\delta < 0$.

It follows from Eq. (14) that the difference between the wave intensities increases with increasing splitting of Ω between the resonator frequencies. If the laser is at rest ($\Omega = 0$), the intensities of the counter waves are equal ($x = 0$) for equal moduli of the coupling coefficients.

We shall analyze the stability condition in the case of frequency locking of the counter waves with equal moduli of the coupling coefficients $m_1 = m_2 = m$ and we shall assume that $M = m|\sin[(\vartheta_1 - \vartheta_2)/2]|$. If the laser is at rest ($\Omega = 0$), the standing-wave configuration is stable ($\delta < 0$) if

$$m\left|\sin\frac{\vartheta_1-\vartheta_2}{2}\right|>\frac{1}{3}\frac{\omega}{Q}\eta. \tag{16}$$

If the condition (16) is satisfied, the frequency-locking regime is stable if the difference between the natural frequencies of the resonator is $|\Omega| \leq \Omega_0$, where Ω_0 is the width of the frequency-locking band given by

$$\Omega_0=\left[m^2\sin^2\frac{\vartheta_1-\vartheta_2}{2}-\frac{1}{4}\left(\eta\frac{\omega}{Q}-m\left|\sin\frac{\vartheta_1-\vartheta_2}{2}\right|\right)^2\right]^{1/2}. \tag{17}$$

It is clear from Eq. (17) that for given values of the modulus m the width of the frequency-locking band has its maximum value when the difference between the phases of the coupling coefficients is $\vartheta_1 - \vartheta_2 = \pm\pi$. For equal phases ($\vartheta_1 = \vartheta_2$) the locking regime is unstable, irrespective of the strength of the coupling. The width of the locking band Ω_0 depends on the excess of the pumping level over the threshold value. In the case of a slight excess over the threshold ($\eta \ll 1$), we find that Ω_0 decreases with increasing η.

The difference between the phases of the counter waves $\Phi_{01} = \Phi(\Omega)$ within the frequency-locking band depends on Ω. It follows from Eq. (12) that if we subtract the phase difference at the limit of the locking band and the phase difference at the center of this band, the result is $|\Phi_{01}(\Omega) - \Phi_{01}(0)| < \pi/2$. Bearing this point in mind and using the analogy with the case of strong coupling in gas lasers,[8] we may expect the frequency characteristic, i.e., the dependence $\dot{\Phi}(\Psi)$, of a solid-state laser to have a discontinuity at the limit of the locking band corresponding to the frequency difference $\dot{\Phi}(\Omega_0) \neq 0$.

We have so far considered the stability of the steady-wave regime under the influence of relatively fast perturbations. If the laser is at rest ($\Omega = 0$) and the moduli of the coupling coefficients are equal, we can carry out a more general analysis which is not limited to the fast perturbations. In this case, the roots of the characteristic equation are given by the following expressions:

$$\lambda_{1,2}=\delta\pm iM_0,\quad \lambda_3=-\frac{1}{T_1}\left(1-\frac{\sigma l}{3T}N_{\text{th}}\frac{\delta}{\delta^2+M_0^2}\right), \tag{18}$$

$$\lambda_{4,5}=-\frac{1}{2T_1}\pm i\left[\left(\frac{\omega}{Q}-m\left|\sin\frac{\vartheta_1-\vartheta_2}{2}\right|\right)\frac{\eta}{T_1}\right]^{1/2}, \tag{19}$$

$$\delta=\frac{\sigma l}{3T}N_{\text{th}}\,\eta-m\sin\left|\frac{\vartheta_1-\vartheta_2}{2}\right|. \tag{20}$$

The roots $\lambda_{4,5}$ describe a damped transient spiking process typical of solid-state lasers. All the roots have a negative real part (i.e., the standing-wave regime is stable) if

$$m|\sin{}^1/_2(\vartheta_1-\vartheta_2)|>{}^1/_3\omega\eta/Q. \tag{21}$$

The stability condition (21) is identical with the corresponding condition (16) in the case of fast perturbations.

Standing-wave regime at arbitrary pumping level

The standing-wave regime at an arbitrary pumping level will be considered for a ring laser at rest ($\Omega = 0$) on the assumption that the moduli of the coupling coefficients are equal. The steady-state value of the difference between the phases of the counter waves [see Eq. (12)] is now given by the expressions

$$\Phi_{01}=\Theta+\frac{\pi}{2},\qquad \Phi_{02}=\Theta-\frac{\pi}{2}. \tag{22}$$

As in the case of near-threshold conditions, the only stable regime is that characterized by the phase difference Φ_{01}. The stability condition of this regime in the presence of fast perturbations ($\delta < 0$) is of the form

$$m\left|\sin\frac{\vartheta_1-\vartheta_2}{2}\right|>\frac{\omega}{Q}\left[1-\frac{(1+8(1+\eta))^{1/2}-1}{2(1+\eta)}\right]. \tag{23}$$

If $\eta \ll 1$, the condition (23) reduces to (16). The right-hand side of the inequality (23) is a monotonically rising function of η and if $\eta \to \infty$, it tends to ω/Q. Since

$$m|\sin[(\vartheta_1-\vartheta_2)/2]|<\omega/Q,$$

the stability condition (23) is not obeyed if η exceeds a certain value. Thus, in a ring laser at rest a change in the excess of the pumping level over the threshold value may alter the oscillation conditions: for small values of the excess η, the standing-wave regime may be stable whereas, in the case of large values of η, beginning from a certain critical value η_0, this regime becomes unstable.

In the range of stability of the standing-wave configuration the intensities of the counter waves $aE_{01}^2 = aE_{02}^2 = aE_0^2$ depend nonlinearly on η:

$$aE_0^2={}^1/_2(1+\eta)-{}^1/_6[1+(1+8(1+\eta))^{1/2}]. \tag{24}$$

Conditions for self-Q-switching the case of complex-conjugate coupling coefficients

As mentioned earlier, the standing-wave regime in a laser at rest is unstable in the case of complex-conjugate coupling coefficients. If we study the stability of the unidirectional emission, we can show that such emission also becomes unstable if

$$m>\frac{1}{2}\left(\frac{\omega}{Q}\eta\frac{1}{T_1}\frac{1+\eta}{2+\eta}\right)^{1/2}. \tag{25}$$

Thus, if the inequality (25) is obeyed, both steady-state regimes (standing and traveling waves) become unstable. This should give rise to self-Q-switching of the intensities and a difference between the phases of the counter waves. It is interesting to note that instabilities of the standing and traveling waves are of oscillatory nature.

A numerical solution of the system (1) has been obtained earlier[3] for complex-conjugate coupling coefficients. It follows from the results reported above that self-Q-switching should arise for the coupling coefficients used in[3] because, in this case, the standing-wave and unidirectional emission regimes are unstable. The results of numerical calculations given in[3] are in agreement with this conclusion.

EXPERIMENT

We carried out an experimental study of the oscillation conditions in a ring laser utilizing neodymium-doped yttrium aluminum garnet (YAG : Nd^{3+}). This laser was operated continuously at $\lambda = 1.06\ \mu$. We studied the oscillation regimes as a function of the coupling of counter waves via backscattering and as a function of the rate of rotation (the laser was placed on a rotating platform).

Description of apparatus

Our investigation was carried out using three- and four-mirror ring lasers. The mirrors were of the dielectric type with a reflection coefficient r close to 99%. The three-mirror laser has a perimeter L = 117 cm and its area was S = 545 cm^2. It was formed by mirrors of R_1 = 5000 mm, R_2 = 2000 mm, and $R_3 = \infty$ radii of curvature. The corresponding parameters of the four-mirror laser were L = 172 cm, S = 1824 cm^2, R_1 = 2000 mm, R_3 = 5000 mm, and $R_2 = R_4 = \infty$. The active elements were garnet crystals of 5 mm diameter and l = 50 mm long. The experiments were carried out on crystals with untreated ends and with ends covered by an antireflection coating (the reflection coefficients of the crystal ends ranged from 8.5 to 0.4%).

The crystals were pumped with a gas-discharge krypton lamp of the DKRTV-3000 type. The crystal and the pump lamp were cooled by running water. The threshold pumping power was about 500 W.

A rotating platform enabled us to carry out measurements at rates of rotation down to 1 rev/sec. We recorded the intensities of the counter waves and of the beat signal between them. The output signal was photographed from the screen of an S1-17 oscillograph (transmission band ~ 10 MHz) and from the screen of an S4-8 panoramic spectrum analyzer.

Experimental results

The experimental results obtained indicated the existence of several oscillation regimes in the investigated ring laser. It was found that these regimes depended strongly on the coupling between the counter waves as a result of reflections from the ends of a crystal and they also depended on the detuning of the resonator and the rate of rotation of the laser. No significant difference was found between the oscillation regimes in the lasers with three- and four-mirror resonators.

We shall now describe the typical oscillation regimes observed in our solid-state ring laser.

1. Frequency-locking of counter waves. In the presence of a sufficiently strong coupling between the counter waves due to the reflection from the ends of a crystal (the reflection coefficient of each end was $r_e \geq 1.5\%$), we observed only the frequency locking of the counter waves. In this case, the amplitudes of these waves were equal and the fluctuations of the intensities due to external perturbations were always in phase. The frequency of these fluctuations increased with the pumping level and their amplitude rose when the external perturbations and the pumping were increased. It was not possible to break the frequency locking at the maximum rates of rotation which could be achieved in our study (ω_{rot} = 1 rev/sec). This indicated that the width of the frequency-locking region Ω_0 exceeded 1 MHz.

When the feedback was reduced ($r_e \sim 0.4\%$), we observed several oscillation regimes. In a laser at rest we observed two oscillation regimes, depending on the resonator tuning: these regimes were the frequency locking of the two counter waves and the self-Q-switching of the intensities of these waves. The resonator tuning could be varied by parallel displacement or rotation of the mirrors starting from 1° and by changing the position of the crystal inside the resonator (rotation by up to 1° and displacement by up to 5 mm at right-angles to the resonator axis).

The laser characteristics in the frequency-locking case were as follows. Inside the frequency-locking band the difference between the intensities of the counter waves was a function of the rate of rotation and it increased on approach to the limit of the frequency-locking region. When the laser was at rest the difference between the intensities of the counter waves depended on the resonator tuning. This tuning also governed the width of the frequency-locking region, and, at a fixed value of tuning, the locking could not be broken (the difference between the natural frequencies of the resonator in a rotating laser exceeded 1 MHz). It was found that the frequency locking of the counter waves could become unstable when the excess of the pumping level over the threshold reached a certain value that depended on the resonator tuning.

In these experiments the maximum value of the excess of the pumping over the threshold η was within the range $0 < \eta \leq 1$.

2. Self-Q-switching of intensities of counter waves in a laser at rest. For certain values of the resonator tuning we found that self-Q-switching (a regime with time-dependent intensities and phase difference of the counter waves) appeared in a laser at rest.

Typical self-Q-switching oscillograms are shown in Fig. 1. This self-Q-switching of the intensities of the counter waves could be periodic (Fig. 1a) or more complex.

We noted particularly the self-Q-switching when the emission occurred alternately in either of the two directions of the ring laser (Fig. 1b). The frequency of self-Q-switching depended strongly on the resonator tuning and ranged from several kilohertz to MHz. The switching of the intensities of the counter waves was nearly in antiphase. The switching frequency could be varied

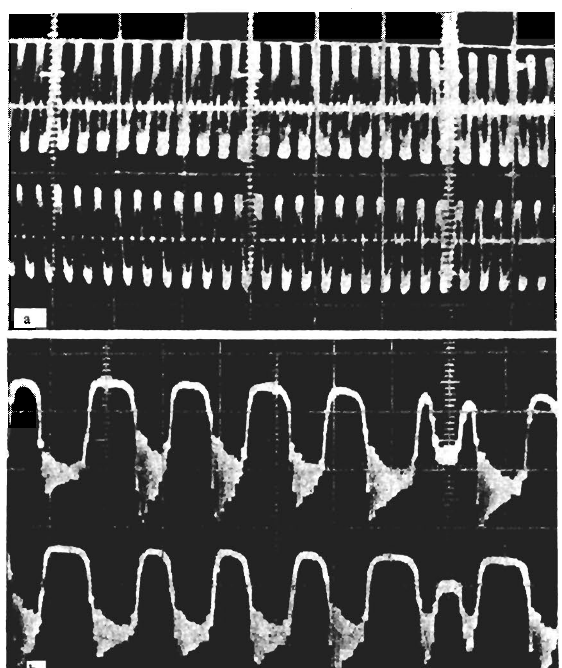

FIG. 1. Oscillograms in the case of self-Q-swtching (η = 0.2): a—scanning at 200 μsec/div.; b—scanning at 1 msec/div.

smoothly in a range of the order of 50 kHz by displacing the active element at right-angles to the resonator axis.

The depth of modulation and the average values of the intensities of the counter waves depended on the resonator tuning. For a fixed tuning the average values of the intensities and the amplitudes of the intensity fluctuations could be very different.

In the case of periodic self-Q-switching, i.e., in the absence of an additional low-frequency switching, the fluctuations of the intensity of the counter waves were usually in antiphase and the fluctuations themselves were nearly sinusoidal. Deviations from the sinusoids increased with decreasing switching frequency. It should be noted that the switching fluctuations (particularly the low-frequency envelope of these fluctuations) were unstable in time for a given tuning, i.e., the amplitude and frequency of the antiphase intensity fluctuations could vary (Fig. 2).

3. Influence of rotation on the oscillation regimes in a ring laser. As mentioned earlier, in the case of strong coupling between the waves as a result of reflection from the ends of a crystal, the rotation of a ring laser at rates up to 1 rev/sec did not alter significantly the intensities of the counter waves.

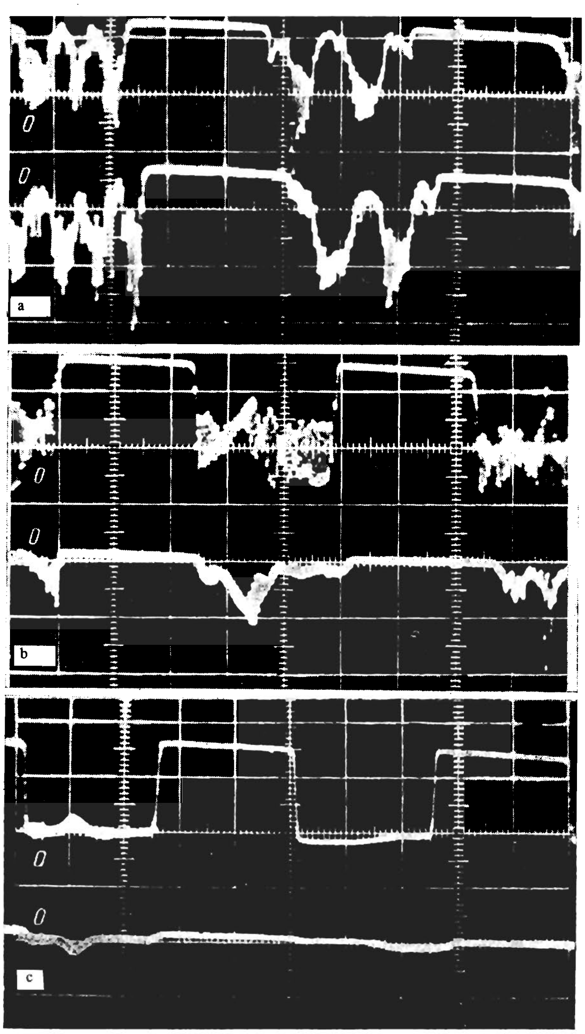

FIG. 2. Oscillograms of the intensities of counter waves in a ring laser at rest and in rotation (η = 0.2, scanning at 1 msec/div.): a—laser at rest ($\Delta\nu$ = 0); b—$\Delta\nu$ = 22 kHz; c—$\Delta\nu$ = 40 kHz.

When the ends of a crystal were covered with antireflection coatings ($r_e \leq 0.4\%$), the coupling between the counter waves became weaker and the oscillation regimes began to depend strongly on the rate of rotation of the laser.

In a laser at rest we observed the self-Q-switching of the counter waves, whereas an increase in the rotation rate weakened considerably the intensity of the wave traveling in the direction of rotation. At rates of rotation exceeding $\Omega_0 L\lambda/8\pi S$, the switching regime gave place to beats: the amplitude in each beam became modulated and a difference-frequency signal (beats) was observed as a result of interaction between the two waves. The frequency characteristics were similar to those reported for gas lasers under strong coupling conditions.[8]

If a laser at rest exhibited self-Q-switching, the rotation of the laser again strongly reduced the wave traveling in the direction of the rotation; it also increased the frequency and reduced the depth of the antiphase amplitude modulation of the intensities of the counter waves. Figure 2 shows oscillograms of the intensities of these waves in a laser at rest (Fig. 2a) and during a gradual increase of the rotation rate (Fig. 2b, $\Delta\nu$ = 22 kHz; Fig. 2c, $\Delta\nu$ = 40 kHz) in the case when the antiphase low-frequency modulation was very deep for the laser at rest. The radiation was also modulated by a vane placed outside the resonator in front of a photomultiplier detector.

The rate of rotation of the laser which produced a strong suppression (by a factor of over 50) of one of the counter waves was usually lowest in the case when the direction of emission was reversed in a laser at rest (Fig. 1b). The dependence of the average values of the intensities of the counter waves $I_\pm$ on the rate and direction of rotation of the laser was determined for this case (Fig. 3).

Figure 4 shows the dependence of the frequency of the self-Q-switching fluctuations ν_m on the rate of rotation ω_{rot}, plotted for several values of $\nu_m(\omega_{rot} = 0)$, which were 85, 470, and 860 kHz. The dependence of the frequency of these fluctuations on the rate of rotation (for a fixed resonator tuning) was described quite satisfactorily by the formula

$$\nu_m = [\nu_m^2(0) + \Delta\nu^2]^{1/2}, \quad \Delta\nu = 8\pi S\omega_{rot}/L\lambda. \tag{26}$$

DISCUSSION OF RESULTS

A solid-state ring laser is a complex spontaneously oscillating system characterized by several oscillation regimes. The investigations reported above show that the existence of a given regime depends on the coupling between counter waves (this coupling is due to reflections from the ends of a crystal) as well as on the resonator tuning, pumping level, and rate of rotation of the laser. It follows from our experiments that a solid-state ring laser is sensitive to the angular rates of rotation in the case of frequency locking of the counter waves (intensities of the counter waves and the phase difference between them depend on the rate of rotation) and outside the frequency-locking region (self-Q-switching and beats). Our theoretical discussion shows that the stability of the oscillation regimes in a ring laser at rest is governed by the feedback between the counter waves. In experiments carried out on antireflection-

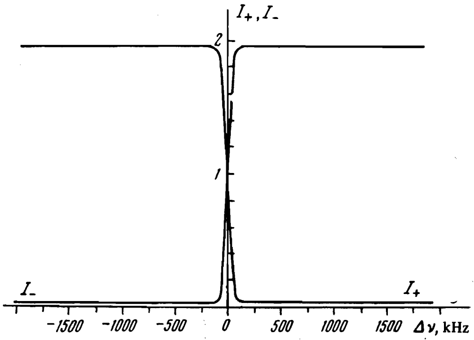

FIG. 3. Dependences of the average values of the intensities of the counter waves $I_\pm$ on the difference between the resonator frequencies $\Delta\nu$ as a result of rotation.

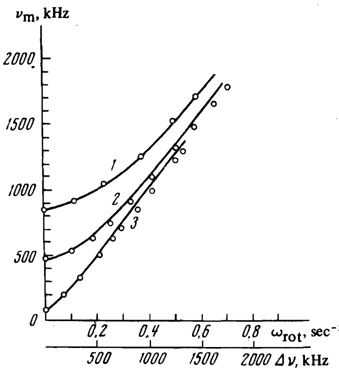

FIG. 4. Dependences of the frequency of antiphase amplitude modulation (switching) ν_m of counter waves on the difference between the resonator frequencies $\Delta\nu$ because of rotation. Curves 1, 2, and 3 correspond to $\nu_m(0) = 860$, 470, and 85 kHz, respectively.

coated crystals ($r_e < 0.4\%$) the oscillation regimes change with the resonator tuning. It is natural to assume that such tuning alters the coupling coefficients $\tilde{m}_{1,2}$.

Andronova and Bershteĭn [9] have assumed that the coupling between the counter waves is due to the backscattering by the resonator mirrors and they have shown that the resonator tuning (in particular, a change in the resonator perimeter) alters considerably the feedback $\tilde{m}_{1,2}$. It should be pointed out that the resonator tuning should be accompanied not only by a change in the coupling via the backscattering but also by a change in the amplitude and frequency non-reciprocities of the counter waves as a result of diffraction effects.[10] In view of this, the question arises as to what extent the changes in the oscillation regimes resulting from a change in the resonator tuning are governed by the diffraction effect (in particular, by the diffraction splitting of the natural frequencies of the resonator) and by changes in the coupling as a result of the scattering.

The experimentally observed change in the frequency of the switching fluctuations as a result of a displacement of the crystal at right-angles to the resonator axis may, in principle, be explained by either of these two phenomena. The diffraction splitting of the frequencies may give rise to beats in a laser at rest (if this splitting is greater than the frequency-locking band). However, the experimentally observed dependence of the frequency of the switching fluctuations on the rate of rotation (Fig. 4) excludes this possibility in our experiments. A quantitative comparison of the theory and experiment is complicated by the absence of direct experimental measurements of the coupling between the counter waves. Nevertheless, the results obtained can be used in drawing certain quantitative conclusions.

If the ends of a crystal are antireflection-coated ($r_e \leq 0.4\%$) the experimentally obtained investigations indicate a stable standing-wave oscillation regime. Using the condition of stability of this regime (23), we can estimate the coefficient of the coupling via the backscattering. If $\eta = 1$, the stability requires that

$$m \geqslant m|\sin(\vartheta_1-\vartheta_2)/2| > {}^1/_4\omega/Q.$$

Hence, it follows that $|m|$ is at least of the order of the width of the resonator band, i.e., $m \gtrsim 10^6$–10^7 sec^{-1}. The same estimate of $|m|$ is obtained from the measured values of the frequency-locking band. If we represent the coupling coefficient in the form $m = R^{1/2}/T$ (R is the ratio of the intensity of the backscattered wave to the intensity of the incident wave) and if we assume that $T = 0.3 \times 10^{-8}$ sec, we obtain $R \approx 10^{-5}$–10^{-3}.

An estimate of the strength of the coupling between the counter waves can also be obtained from the measured values of the frequency of switching in a laser at rest. In the case of coupling coefficients which are nearly complex-conjugate, the frequency of the switching fluctuations is $\nu_m(0) = m/2\pi$.[3] The maximum values of $\nu_m(0)$ found experimentally are of the order of 1 MHz and, in this case, $m \approx 2\pi \times 10^6$ sec^{-1} and $R \sim 10^{-4}$.

The authors are grateful to Leading Engineer N. I. Naumkin for his help in the experiments.

1. V. L. Zhelnov, A. P. Kazantsev, and V. S. Smirnov, Fiz. Tverd. Tela, 7, 2816 (1965) [Sov. Phys.-Solid State 7, 2276 (1966)].
2. V. N. Morozov, Fiz. Tverd. Tela, 8, 2256 (1966) [Sov. Phys.-Solid State 8, 1798 (1967)].
3. I. P. Efanova and E. G. Lariontsev, Zh. Eksp. Teor. Fiz. 55, 1532 (1968) [Sov. Phys.-JETP 28, 802 (1969)].
4. E. M. Belenov, Zh. Tekh. Fiz. 38, 871 (1968) [Sov. Phys.-Tekh. Phys. 13, 654 (1968)].
5. V. L. Zhelnov, V. S. Smirnov, and A. P. Fadeev, Opt. Spektrosk. 28, 744 (1970).
6. A. R. Clobes and M. J. Brienza, Appl. Phys. Lett. 21, 265 (1972).
7. L. S. Kornienko, N. V. Kravtsov, and A. N. Shelaev, Opt. Spektrosk. 35, 775 (1973).
8. P. S. Landa and E. G. Lariontsev, Radiotekh. Elektron. 15, 1214 (1970).
9. I. A. Andronova and I. L. Bershteĭn, Izv. Vyssh. Ucheb. Zaved., Radiofiz. 14, 698 (1971).
10. A. D. Valuev, S. A. Savranskiĭ, A. F. Savushkin, and B. A. Shokin, Opt. Spektrosk. 29, 410 (1970).

Translated by A. Tybulewicz
138