Acoustic perturbations in a medium by the motion of a light focus

V. N. Lugovoĭ and V. N. Strel'tsov

P. N. Lebedev Physics Institute
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The excitation and propagation of acoustic perturbations under the action of electrostriction forces and heating of the substance during uniform supersonic motion of light foci of a multifocus structure are investigated theoretically. Expressions for the density variation are obtained and the distributions of the density and of the sound energy in the medium are investigated on their basis. It is shown, in particular, that for the two indicated perturbation mechanisms in the medium the energy distributions in the produced sound cone are very different. Conditions for which variation of the density of matter in the focal region is close to quasistatic are established for the case of electrostriction. It is found that in both cases an anomalous variation of the density in the focal region occurs when the focus velocity is close to that of sound. The contribution of heating and striction to nonlinear polarization of the medium under the conditions considered is estimated.

Most papers on the propagation of intense light beams in material media consider the Kerr (or a near-Kerr) nonlinearity of the medium, (see the review[1]). Such processes as heating and electrostriction do not make an appreciable contribution to the nonlinearity of the medium at the usual parameters of the light beams, with corresponding pulse durations $\tau \lesssim 10^{-8}$ sec. However, upon formation of a multifocus structure of the light beam,[2] the relative contribution of these processes to the total nonlinearity can increase considerably, due to the nonlinear (and large) light absorption in the focal regions of the multifocus structure, and as a consequence of sufficiently small value of the diameters of these regions. In particular, according to[1], the heating of the medium in the focal regions can in some cases lead to the disappearance of the multifocus structure that was originally formed, and, by the same token, significantly affect the character of beam propagation in the medium. Electrostriction can also make a significant contribution to the nonlinearity in solid dielectrics for pulse lengths $\tau \lesssim 10^{-7}$ sec (see[3]). Both of these forms of nonlinearity are directly connected with the redistribution of the density of the medium by means of sound perturbations in the medium. The present paper is devoted to theoretical investigation of the density perturbations of the medium under the influence of heating and electrostriction in the focal regions of a multifocus light-beam structure. For pulse lengths $\tau \lesssim 10^{-7}$–10^{-8} sec in previously focused beams, typical values of the velocities of the focuses of the multifocus structure amount to $v_{ph} \sim 10^7$–10^8 cm-sec^{-1}. For parallel incident beams and pulse lengths $\tau \le 10^{-8}$ sec, the values of v_φ can be greater than or of the order of 10^9 cm-sec^{-1}.[4] Thus, perturbations in the density of the medium take place under the action of sources moving with supersonic speed.

We now consider the formation and propagation of a sound wave arising as a result of heating and strictive compression for uniform motion of the focus with supersonic speed in an unbounded medium, assuming that the field in the focal region is given in this case. The equation for the departure of the density ρ in the sound field from its equilibrium value ρ_0 in a light field has, in the hydrodynamic approximation, the form

$$\frac{\partial^2\rho}{\partial t^2} - \Delta\left[v^2\rho + \Gamma\frac{\partial\rho}{\partial t}\right] = -\frac{1}{8\pi}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \rho_0\Delta|\mathbf{E}|^2 + \left(\frac{\partial P}{\partial S}\right)_\rho \Delta S, \quad (1)$$

where v is the sound velocity in the medium, Γ the damping coefficient for sound, S the perturbation of the entropy of the medium, $\mathbf{E}$ the complex amplitude of oscillation of the field. We shall assume that the heating of the medium, and therefore the change in the entropy, take place due to the multiphoton absorption of arbitrary order n. Then, neglecting the thermal conductivity, we can write for the change in the entropy per unit mass[1)]

$$\rho_0 T_0 \frac{\partial S}{\partial t} = \eta_n \frac{|\mathbf{E}|^{2n}}{8\pi}. \quad (2)$$

Here T_0 is the equilibrium value of the temperature of the medium and $\eta_n|\mathbf{E}|^{2n}/8\pi$ is the energy dissipated in the medium per unit time for n-photon absorption.

For calculation of the basic features of the considered process, we approximate the intensity distribution in the focal region in the following way:

$$|\mathbf{E}|^2 = \mathscr{E}_0^2 \exp\left(-4\frac{r_\perp^2}{d_{ph}^2}\right)\frac{1}{1+4(z+v_{ph}t)^2/l_{ph}^2}, \quad (3)$$

where $r_\perp = \sqrt{x^2+y^2}$. The expression that has been written down corresponds to the equilibrium (supersonic for $v_{ph} > v$) motion of the focus along the z axis.

Equations (1)–(3) form a closed system, which describes the process of sound propagation in an unbounded medium. For solution of the given equations, we make a Fourier transformation of the quantities in z and t that appear in them. We then get the following equation for the corresponding Fourier transform $\bar\rho$ of the departure of the density from equilibrium in the two remaining variables x, y:

$$(-v^2 + ik_t\Gamma)\Delta_{xy}\bar\rho + (k_z^2v^2 - k_t^2 - ik_tk_z^2\Gamma)\bar\rho = \Phi(x, y), \quad (4)$$

where

$$\Phi(x,y) = \Big\{Y_\rho^{(1)}\mathscr{E}_0^2(\Delta_{xy}-k_z^2)\exp\left(-4\frac{r_\perp^2}{d_{ph}^2}\right)\exp\left(-\frac{l_{ph}}{2}|k_z|\right) + \frac{i}{k_t}Y_{\rho n}^{(2)}\mathscr{E}_0^{2n}(\Delta_{xy}-k_z^2)\exp\left(-4\frac{nr_\perp^2}{d_{ph}^2}\right)$$
$$\times\frac{(-1)^{n-1}}{(n-1)!}\left[\frac{d^{n-1}}{dp^{n-1}}\frac{\exp(-pl_{ph}|k_z|/2)}{(1+p)^n}\right]_{p=1}\Big\}\pi^2 l_{ph}\delta(k_t - k_zv_{ph}); \quad (5)$$
$$Y_\rho^{(1)} = -\frac{1}{8\pi}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\rho_0, \quad Y_{\rho n}^{(2)} = \left(\frac{\partial P}{\partial S}\right)_\rho\frac{\eta_n}{8\pi\rho_0T_0}.$$

Equation (4) is an inhomogeneous equation of second order. The Green's function $G(x, y)$ of this equation depends on k_t, k_z as parameters. We can establish the fact that in the case of a vanishingly small damping $\Gamma \to 0$ (to which we shall restrict ourselves in what follows) G(x, y) is represented in the form

$$G(x,y)=\begin{cases}\dfrac{i}{4v^2}H_0^{(1)}\left(\left(\dfrac{k_t^2}{v^2}-k_z^2\right)^{1/2}|\mathbf{r}_\perp-\mathbf{r}_\perp'|\right), & k_z^2<\dfrac{k_t^2}{v^2},\ k_t>0\\ -\dfrac{i}{4v^2}H_0^{(2)}\left(\left(\dfrac{k_t^2}{v^2}-k_z^2\right)^{1/2}|\mathbf{r}_\perp-\mathbf{r}_\perp'|\right), & k_z^2<\dfrac{k_t^2}{v^2},\ k_t<0.\\ \dfrac{1}{2\pi v^2}K_0\left(\left(k_z^2-\dfrac{k_t^2}{v^2}\right)^{1/2}|\mathbf{r}_\perp-\mathbf{r}_\perp'|\right), & k_z^2>\dfrac{k_t^2}{v^2}\end{cases} \quad (6)$$

(here the roots are taken in the arithmetic sense.) The solution of (4) is obtained by convolution of $\Phi(x, y)$ with the Green's function (6):

$$\bar{\rho}=\iint\limits_{-\infty}^{\infty}G(x-x',y-y')\Phi(x',y')\,dx'\,dy'. \quad (7)$$

The calculation of (6) and the subsequent inverse Fourier transformation in z, t under the conditions

$$r_\perp\gg d_{\mathrm{ph}}, \quad (8)$$

$$l_{\mathrm{ph}}/d_{\mathrm{ph}}\gg(v_{\mathrm{ph}}^2/v^2-1)^{1/2}\ \text{(for striction)},$$

$$l_{\mathrm{ph}}/d_{\mathrm{ph}}\gg n^{-1}(v_{\mathrm{ph}}^2/v^2-1)^{1/2}\ \text{(for n-photon absorption)} \quad (9)$$

are given in the Appendix. The final expression for ρ (including thermal and striction parts) has the form

$$\rho=-\pi Y_\rho^{(1)}\mathscr{E}_0^2\frac{l_{\mathrm{ph}}d_{\mathrm{ph}}^2v_{\mathrm{ph}}^2}{4v^4}\operatorname{Im}\left\{\frac{\partial^2}{\partial l_{\mathrm{ph}}^2}[Q]_{p=1}\right\}+\frac{(-1)^n}{(n-1)!}\pi Y_{\rho n}^{(2)}\mathscr{E}_0^{2n}\cdot\frac{l_{\mathrm{ph}}d_{\mathrm{ph}}^2v_{\mathrm{ph}}}{8v^4n}\operatorname{Re}\left[\frac{d^{n-1}}{dp^{n-1}}\left\{\frac{1}{p(1+p)^n}\frac{\partial}{\partial l_{\mathrm{ph}}}Q\right\}\right]_{p=1}, \quad (10)$$

where

$$Q=\frac{1}{r_\perp(v_{\mathrm{ph}}^2/v^2-1)^{1/2}}[1+\Lambda^2]^{-1/2}\left[1-\frac{2i}{\pi}\ln(\Lambda+\sqrt{1+\Lambda^2})\right],$$

$$\Lambda=\frac{pl_{\mathrm{ph}}/2+i(z+v_{\mathrm{ph}}t)}{r_\perp(v_{\mathrm{ph}}^2/v^2-1)^{1/2}}. \quad (11)$$

Here the branch of the square root is fixed by the condition $\sqrt{1=1}$ and the cut of the plane of complex values of $1 + \Gamma^2$ is made along the negative real semiaxis.

We note immediately that, although the above equation formally describes the density perturbation throughout the space, it guarantees the correct value for ρ only in the region variables z, t, $r_\perp$ that satisfy the inequality

$$(z+v_{\mathrm{ph}}t)-r_\perp(v_{\mathrm{ph}}^2/v^2-1)^{1/2}\lesssim l_{\mathrm{ph}}, \quad (12)$$

inasmuch as the integrands in (20) oscillate rapidly outside this region over the interval of integration, and require more exact description. At the same time, the quantity ρ takes on maximum values (imr$\perp$) in the region of (12), because of the smoothness of the integrand functions. We also note that as $v_{\mathrm{ph}} \to v$, as is not difficult to see from (10), (11), $\rho \to \infty$ and, consequently, the initial, linearized equation (1) becomes inapplicable. Physically, such a divergence can mean, for example, the formation of a shock. Thus, the solution of (10), (11) is valid only for not too small a value of $v_{\mathrm{ph}}/v - 1$, which limitation we shall assume satisfied in the following.

We now consider the properties of the resultant solution. It follows from (10), (11) that the density perturbation differs from zero at all points in space. This is evidently associated with the fact that the electromagnetic field is itself distributed over the entire space. At the same time, according to (3), the major fraction of the electromagnetic energy is concentrated within the focal region, as a consequence of which there is a sharply expressed sound cone behind the moving focus in the perturbation distribution.

For convenience, we introduce the variables

$$\theta=\operatorname{arctg}\frac{r_\perp}{z+v_{\mathrm{ph}}t},\qquad r=[r_\perp^2+(z+v_{\mathrm{ph}}t)^2]^{1/2}$$

of a spherical set of coordinates with origin at the center of the moving focus and consider separately the perturbations due to thermal heating and to striction. As is seen from (10), (11), ρ, as a function of θ, has a sharp peak in both cases in the region[2] $\theta \approx \theta_{\mathrm{ac}}$, where $\theta_{\mathrm{ac}} \approx \operatorname{arc\,cot}(v_{\mathrm{ph}}^2/v^2 - 1)^{1/2}$, as was to be expected. The maximum value of ρ in this range of angles depends on r and is given by the following expressions:

$$\rho\approx-\frac{\pi}{n}Y_{\rho n}^{(2)}\mathscr{E}_0^{2n}\frac{l_{\mathrm{ph}}d_{\mathrm{ph}}^2v_{\mathrm{ph}}}{8v^4}\frac{1}{l_{\mathrm{ph}}^{1/2}(z+v_{\mathrm{ph}}t)^{1/2}} \quad (13)$$

for thermal heating, and

$$\rho\approx-\pi Y_\rho^{(1)}\mathscr{E}_0^2\frac{l_{\mathrm{ph}}d_{\mathrm{ph}}^2v_{\mathrm{ph}}^2}{4v^4}\frac{1}{l_{\mathrm{ph}}^{3/2}(z+v_{\mathrm{ph}}t)^{3/2}} \quad (13')$$

for striction. The characteristic radius of the decrease in the density $\Delta r_\perp$ in a cross section, perpendicular to the z axis, is constant along the generatrix of the cone and is equal in order of magnitude to[3]

$$\Delta r_\perp\sim\frac{l_{\mathrm{ph}}}{(v_{\mathrm{ph}}^2/v^2-1)^{1/2}}. \quad (14)$$

It also follows from (13), (13′) that the greatest energy density E_{ac} of the sound wave is reached in the region defined by the expression (12), i.e., near θ_{ac}, while in the case of thermal heating E_{ac} falls off with the distance $z + v_\varphi t$ from the focus as $1/(z + v_{\mathrm{ph}}t)$, and in the case of striction as $1/(z + v_{\mathrm{ph}}t)^3$. Thus, in the first case the energy of the sound perturbation in the medium grows linearly with increase in the distance $z + v_{\mathrm{ph}}t$ from the focus, while in the second the total energy is concentrated in the focal region.

The results given above describe the picture of the propagation of a sound wave at large distances (see (8)) from the region of the light focus. We now investigate the departure of the density inside the focal region, and for simplicity consider the perturbation on the axis of the focus ($r_\perp = 0$), where it evidently assumes its maximum values. Upon satisfaction of the condition (9) for $r_\perp = 0$, the Hankel function $H_0^{(1)}(x)$ in the integral (20) can be replaced by its expansion for $x \to 0$, with accuracy up to terms of order $x^4 \ln x$. Then we find for the departure of the density ρ_T due to heating, at arbitrary $z + v_\varphi t\ (r_\perp = 0)$,

$$\rho_T=-\frac{Y_{\rho n}^{(2)}}{n}\mathscr{E}_0^{2n}\frac{l_{\mathrm{ph}}}{8v_{\mathrm{ph}}v^2}\left[2\operatorname{arctg}\frac{2(z+v_{\mathrm{ph}}t)}{l_{\mathrm{ph}}}+\pi\right]+\frac{Y_{\rho n}^{(2)}}{n}\mathscr{E}_0^{2n}\cdot\times\frac{d_{\mathrm{ph}}^2l_{\mathrm{ph}}}{8v^2v_{\mathrm{ph}}}\operatorname{Im}\left\{\frac{1}{[l_{\mathrm{ph}}/2+i(z+v_{\mathrm{ph}}t)]^2}\left[C-2+2\ln\frac{4(l_{\mathrm{ph}}/2+i(z+v_{\mathrm{ph}}t))}{d_{\mathrm{ph}}(v_{\mathrm{ph}}^2/v^2-1)^{1/2}}\right]\right\}+\frac{Y_{\rho n}^{(2)}}{n}\mathscr{E}_0^{2n}\frac{d_{\mathrm{ph}}^2l_{\mathrm{ph}}}{4v^2v_{\mathrm{ph}}}\operatorname{Re}\frac{1}{[l_{\mathrm{ph}}/2+i(z+v_{\mathrm{ph}}t)]^2}. \quad (15)$$

As is not difficult to see, the term $\sim\ln[2l_{\mathrm{ph}}/d_{\mathrm{ph}}(v_{\mathrm{ph}}^2/v^2 - 1)^{1/2}]$, which corresponds to the logarithmic term of the expansion of the Hankel function, diverges as $v_{\mathrm{ph}} \to v$. Thus, this term is greatest for a velocity of the focus close to the sound velocity and determines the anomalous increase (in absolute value) of the density at the edges of the focal region; here the distribution of ρ_T is antisymmetric relative to the center of the focus. If the velocity of the focus is appreciably greater than the sound velocity, the first term in (15) is determinative. Accordingly, the value of the density departure ρ_T changes monotonically from zero to the limiting (as $z + v_{\mathrm{ph}}t \to +\infty$) value

$$\rho_T = -2\pi\frac{1}{n} Y^{(2)}_{\rho n}\mathscr{E}_0^{2n} l_{\rm ph}/8v_{\rm ph}v^2.$$

The last term in (15) determines only a small correction to the terms considered and, as follows from (13), (13′), a sound cone develops in the same order of smallness as this term.

In the case of striction, calculation of the integral (20) with use of a similar expansion of the Hankel function leads to the following expression for the density perturbation $\rho_{\rm str}$:

$$\rho_{\rm str} = \rho^{\rm stat}_{\rm str} + \rho^{\rm dyn}_{\rm str.s} + \rho^{\rm dyn}_{\rm str.a}. \qquad (16)$$

The first term

$$\rho^{\rm stat}_{\rm str} = \left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \rho_0 \frac{|\mathbf{E}|^2}{8\pi v^2}$$

determines the "static" (vanishing in the limit $d_{\rm ph}v_{\rm ph}/l_{\rm ph}v \to 0$) striction change in the density. Thanks to the symmetric distribution of the light intensity (with respect to $z + v_{\rm ph}t$), $\rho^{\rm stat}_{\rm str}$ also turns out to be symmetric in this variable in the focal region (relative to its center). The dynamic part of the perturbation, which is associated with the movement of the focus, is the sum of the two terms $\rho^{\rm dyn}_{\rm str.s}$ and $\rho^{\rm dyn}_{\rm str.a}$. The first term $\rho^{\rm dyn}_{\rm str.s}$ is symmetric in $z + v_{\rm ph}t$ relative to the center of the focus and has the form

$$\rho^{\rm dyn}_{\rm str.a} = Y^{(1)}_\rho \mathscr{E}_0^2 \frac{l_{\rm ph}d_{\rm ph}v_{\rm ph}}{32 v^4} \times 2\,{\rm Re}\left\{\frac{1}{[l_{\rm ph}/2 + i(z+v_{\rm ph}t)]^3}\left[3 - C - \ln\frac{4(l_{\rm ph}/2 + i(z+v_{\rm ph}t))}{d_{\rm ph}(v_{\rm ph}^2/v^2-1)^{1/2}}\right]\right\}. \qquad (17)$$

It then follows that, as in the case of thermal heating, an anomalous increase occurs in the density $\rho_{\rm str}$ at a velocity of the focus close to the sound velocity. However, in contrast to ρ_T, this increase is concentrated at the very center of the focal region. Here the term $\rho^{\rm dyn}_{\rm str}$ is large in comparison with the static perturbation $\rho^{\rm stat}_{\rm str}$. In the case of a significant excess of the velocity of the focus over the sound velocity, the principal contribution to $\rho_{\rm str}$ is made by the first term $\rho^{\rm stat}_{\rm str}$. The last term $\rho^{\rm dyn}_{\rm str.a}$ determines the antisymmetric part of the strictive perturbation of the density, and makes a small correction to the first two terms:

$$\rho^{\rm dyn}_{\rm str.a} = -\pi Y^{(1)}_\rho \mathscr{E}_0^2 \frac{l_{\rm ph}d_{\rm ph}^2 v_{\rm ph}^2}{16 v^4}{\rm Im}\frac{1}{[l_{\rm ph}/2 + i(z+v_{\rm ph}t)]^3}. \qquad (18)$$

As is seen, the sound cone (see (13)) has the same order of smallness as the term $\rho^{\rm dyn}_{\rm str.a}$.

We note that the anomalous change in the density ρ in the focal region for $v_{\rm ph} \to v$ that has been considered above takes place without restriction as to its absolute value within the framework of the approximations used for both forms of the perturbations. In this connection, we direct attention to the possibility of fractures in solids on passage of the velocity of the focus through the sound velocity. We also note that a similar anomalous increase can also develop in the self-consistent propagation of light in a medium with a strictive nonlinearity.

Expressions (15), (16) allow us to estimate the contribution of thermal heating and striction to the nonlinear polarization of the material in the focal regions, and thus establish the limits of applicability of the theory of propagation of light beams in a medium without account of the reciprocal effect of the given mechanisms on the field distribution of the beam. According to (15), for an appreciable excess of the velocity of the focus over the sound velocity, the maximum value of the increment to the dielectric permittivity of the material $\epsilon_{nl} = (\partial\epsilon/\partial\rho)_S\rho$ in the case of thermal heating can be conveniently written in the form

$$\varepsilon_{nl}^T \approx \left(\frac{\partial\varepsilon}{\partial\rho}\right)_S \left(\frac{\partial P}{\partial S}\right)_\rho \frac{1}{\rho_0 T_0}\frac{\varkappa c}{32 v_{\rm ph}v^2}\frac{1}{n_0 n}\mathscr{E}_0^2, \qquad (19)$$

where

$$\varkappa = \eta_n \mathscr{E}_0^{2n-2} l_{\rm ph} n_0/c$$

characterizes the relative fraction of the light power transformed into thermal power over the length of the focus, c/n_0 is the light velocity in the medium. Under the usual conditions, for single-photon absorption at a focus length $l_{\rm ph} \approx 10^{-1}$ cm, κ has a value of the order of 10^{-3} in liquids. Then, for ϵ^T_{nl} at the values $v \approx 3 \times 10^5$ cm/sec that are typical for liquids,

$$\left(\frac{\partial\varepsilon}{\partial\rho}\right)_S \rho_0 \approx 1, \quad \left(\frac{\partial P}{\partial S}\right)_\rho \frac{1}{\rho_0 T_0} \approx 2,$$

and values of $v_\phi \approx 10^7$ cm-sec^{-1}, we obtain $\epsilon^T_{nl} \approx 2 \times 10^{-11}\mathscr{E}_0^2$. Thus we see that for velocities $v_{\rm ph} \lesssim 10^7$ cm-sec^{-1}, the quantity ϵ^T_{nl} can be comparable with the Kerr nonlinearity ($\epsilon^{\rm Kerr}_{nl} \approx 10^{-11}$–$10^{-12}\mathscr{E}_0^2$). With increase in the intensity of the light beam, ϵ^T_{nl} can increase due to multiphoton processes and make a significant contribution to the total nonlinear polarization of the medium for higher values of the velocities of motion of the focus.

In the striction case, the nonlinear increment is

$$\varepsilon^{\rm str}_{nl} \approx \left(\frac{\partial\varepsilon}{\partial\rho}\right)_S \left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \rho_0 \frac{|\mathbf{E}|^2}{8\pi v^2} \approx 10^{-13}|\mathbf{E}|^2$$

and thus usually $\epsilon^{\rm Kerr}_{nl} \gg \epsilon^{\rm str}_{nl}$ for liquids. In solids, where $\epsilon^{\rm Kerr}_{nl}$, is usually much smaller than in liquids, the striction increment $\epsilon^{\rm str}_{nl}$ can have a significant effect on the total nonlinear polarization of the material.

In conclusion, we make a few remarks concerning the propagation of light in media with a striction nonlinearity. As follows from (16)–(18), we can neglect the dynamic terms for $d_\varphi v_\varphi/l_\varphi v \ll 1$, and assume

$$\rho_{\rm str} = \rho^{\rm stat}_{\rm str} = \left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \rho_0 \frac{|\mathbf{E}|^2}{8\pi v^2}.$$

In this case, the form of the nonlinearity of the medium in the vicinity of the focus

$$\varepsilon_{nl} = \left(\frac{\partial\varepsilon}{\partial\rho}\right)_S \left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \rho_0 \frac{|\mathbf{E}|^2}{8\pi v^2}$$

is identical with the form of the usual Kerr nonlinearity. It was shown in[2,6] that the propagation of a light beam in a medium with a Kerr nonlinearity is accompanied by a significant change in its initial transverse distribution, with development of multifocus structure at supercritical power in the beam. This circumstance, however, was not taken into account in the work of Kerr,[3] where it was assumed that the light beam keeps its initial Gaussian form during propagation in the nonlinear medium considered.

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APPENDIX

The expression (7), which was written down with account of (5) and (6) and the subsequent Fourier transformation mentioned in the text, has the form

$$\rho=\frac{i}{16v^2}Y_\rho^{(1)}\mathscr{E}_0^2 l_{ph}\int_{-\infty}^{\infty}\exp\left\{-\frac{l_{ph}}{2}|k_z|-ik_z(z+v_{ph}t)\right\}$$
$$\times\int_0^\infty\int_0^{2\pi}H_0^{(1)}\left(k_z\left(\frac{v_{ph}^2}{v^2}-1\right)^{1/2}|\mathbf{r}_\perp-\mathbf{r}_\perp'|\right)\exp\left(-\frac{4r_\perp'^2}{d_{ph}^2}\right)$$
$$\times\left[\frac{16}{d_{ph}^2}\left(\frac{4r_\perp'^2}{d_{ph}^2}-1\right)-k_z^2\right]r_\perp'\,dr_\perp'\,d\varphi\,dk_z+\frac{(-1)^n}{(n-1)!}\frac{l_{ph}}{16v_{ph}v^2}Y_{\rho n}^{(2)}\mathscr{E}_0^{2n}\quad(20)$$
$$\times\int_{-\infty}^{\infty}\frac{\exp\{-ik_z(z+v_{ph}t)\}}{k_z}\frac{d^{n-1}}{dp^{n-1}}\left[\frac{\exp(-pl_{ph}|k_z|/2)}{(1+p)^n}\right]_{p=1}$$
$$\times\int_0^\infty\int_0^{2\pi}H_0^{(1)}\left(k_z\left(\frac{v_{ph}^2}{v^2}-1\right)^{1/2}|\mathbf{r}_\perp-\mathbf{r}_\perp'|\right)\exp\left(-4n\frac{r_\perp'^2}{d_{ph}^2}\right)$$
$$\times\left[\frac{16n}{d_{ph}^2}\left(\frac{4r_\perp'^2 n}{d_{ph}^2}-1\right)-k_z^2\right]r_\perp'\,dr_\perp'\,d\varphi\,dk_z.$$

Here the contour of integration over k_z lies on the real axis, and on transition through the point $k_z = 0$ passes into the upper half-plane of complex k_z. It is not difficult to see that the second integral in (20) is obtained from the first by replacing d_{ph}^2 by d_{ph}^2/n and l_{ph} by pl_{ph} with subsequent integration over $pl_{ph}/2$. We therefore consider the first term, which corresponds to strictive compression, ρ_{str}.

For calculation of the integral over $r_\perp'$, φ, we use the addition theorem for cylindrical functions (see, for example,[7]):

$$H_0^{(1)}(M|\mathbf{r}_\perp-\mathbf{r}_\perp'|)=J_0(Mr_\perp')H_0^{(1)}(Mr_\perp)$$
$$+2\sum_{k=1}^{\infty}J_k(Mr_\perp')H_k^{(1)}(Mr_\perp)\cos k\varphi,\quad r_\perp>r_\perp'\quad(21)$$

(for $r_\perp < r_\perp'$, the arguments are exchanged). Inasmuch as the principal contribution to the integral over $r_\perp'$ in (20) is made by the region $r_\perp' \lesssim d_{ph}$, then, for $r \gg d_{ph}$, we can assume $r_\perp > r_\perp'$. Then, substituting the expansion (21) in (20) and integrating over $r_\perp'$, φ (here the terms with cos $k\varphi$ vanish), we obtain

$$\rho_{str}=i\pi Y_\rho^{(1)}\frac{d_{ph}^2 v_{ph}^2}{32v^4}l_{ph}\int_{-\infty}^{\infty}\left[H_0^{(1)}\left(r_\perp k_z\left(\frac{v_{ph}^2}{v^2}-1\right)^{1/2}\right)\right.$$
$$\left.\times\exp\left\{-\frac{l_{ph}}{2}|k_z|-ik_z(z+v_{ph}t)\right\}\exp\left\{-\frac{d_{ph}^2}{16}k_z^2\left(\frac{v_{ph}^2}{v^2}-1\right)\right\}\right]dk_z.\quad(22)$$

The last integral is easily computed for the condition (9), in which we can set

$$\exp\left\{-k_z^2 d_{ph}^2\left(\frac{v_{ph}^2}{v^2}-1\right)\Big/16\right\}=1.$$

in the integrand. As a result of the corresponding calculations, we find

$$\rho_{str}=-\pi Y_\rho^{(1)}\mathscr{E}_0^2\frac{l_{ph}d_{ph}^2 v_{ph}}{4v^4}\mathrm{Im}\left[\frac{\partial^2}{\partial l_{ph}^2}Q\right],\quad(23)$$

where

$$Q=\left[r_\perp^2\left(\frac{v_{ph}^2}{v^2}-1\right)+\left(\frac{l_{ph}}{2}+i(z+v_{ph}t)\right)^2\right]^{-1/2}$$
$$\times\left\{1-\frac{2i}{\pi}\ln\left[\frac{l_{ph}/2+i(z+v_{ph}t)}{r_\perp(v_{ph}^2/v^2-1)^{1/2}}+\left(1+\frac{(l_{ph}/2+i(z+v_{ph}t))^2}{r_\perp^2(v_{ph}^2/v^2-1)}\right)^{1/2}\right]\right\}.\quad(24)$$

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1)The validity of such an approximation will be seen below from the solution that is obtained.

2)We note that such a distribution of the density perturbations corresponds to a sound come, which is well known in hydrodynamics (see, for example, [5]), and to the distribution of the electromagnetic field in Cerenkov radiation in electrodynamics. The specifics of the case considered here lie in the fact that the source of the perturbations is distributed in space and its supersonic motion is generally not accompanied by the formation of a shock wave in this case.

3)It follows from (14) and (9) that the minimum value of $\Delta_{r\perp}$ is of the order of d_{ph}. For typical media and laser pulse lengths $\tau \lesssim 10^{-8}$ sec, the condition $v_{ph}d_{ph}^2/l_{ph}\chi \gg 1$ (χ is the coefficient of temperature conductivity) is satisfied with a reserve of several orders of magnitude, and consequently, the above neglect of thermal conductivity is valid.

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