Surface and volume vortex pinning in type-II superconductors

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Experiments are carried out on the dependence of the pinning force F_p on magnetic field strength, temperature, and Ginzburg–Landau parameter κ in a system of superconducting Pb-In alloys in which pinning is mainly determined by surface pinning of the vortices. The main laws determining the behavior of F_p are elucidated. It is found that in the investigated alloys the pinning forces obey the law of corresponding states. It is shown that the expression for the pinning force can be represented as the product of three functions, each of which depends only on one of the essential variables, $t = T/T_c$, $h = H/H_{c2}(t)$, or κ. The results of the experiments are in good agreement with Schmidt's theory which consider vortex pinning at the plane boundary of a perfect type-II superconductor. The temperature dependence of F_p differs from that observed in alloys with volume pinning. It is shown that this difference is due to the fact that the depth of the layer in which surface pinning is effective is the same as the penetration depth and depends on the temperature.

Numerous experimental data on the vortex pinning forces in type-II superconductors (SC II), unfortunately, differ greatly. It seems that only Fietz and Webb[1] have carried out systematic investigations of the pinning forces in the superconducting alloy systems Nb–Ta and Nb–Ti, as a result of which they succeeded in establishing seemingly rather general empirical rules describing the temperature and field dependences of the pinning force F_p. In both systems, the pinning is of the volume type and is determined by the interaction of the vortices with the dislocations[1]. It can be assumed that similar regularities should be observed also for other SC II. It is of interest to ascertain the nature of these regularities in the case of pinning centers of a different type, particularly in the case of surface pinning.

To study surface spinning of vortices it is convenient to use alloys of the Pb-In system, where the volume pinning plays a secondary role[2]. We report here a detailed study of the pinning forces F_p as functions of the temperature T, the magnetic field H, and the alloy concentration. The interval of the investigated concentrations lies in the range 12--36 at.% of indium. The measurements were performed on single crystals and polycrystalline samples with different compositions, in the form of plates or cylinders. Unlike in[1], where F_p was determined from the magnetization curves, in the present study the pinning force was determined from measurements of the critical currents, using the relation $F_p = j_c H/c$ (F_p is the pinning force per unit volume of the vortex lattice, and j_c is the density of the critical current). As shown in[3], identical results are obtained when both procedures are applied to one and the same sample. In the present study, the critical current was determined either from the appearance of a measurable stress, or by extrapolating the linear sections of the current-voltage characteristics to zero. The temperature, field, and concentration dependences of F_p determined by these two methods were identical, but the scatter in the extrapolation method was much smaller. The data presented below are therefore those obtained by this method. Both measurements were performed on samples whose surfaces were chemically polished.

Figure 1 shows, for one of the samples of the Pb-In alloy, the dependence of the pinning force on the relative magnetic field $h = H/H_{c2}(t)$ at several temperatures. The corresponding families of curves for alloys

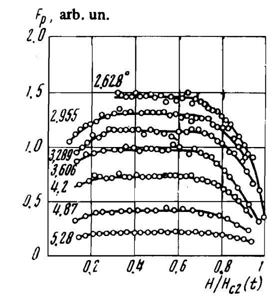

FIG. 1. Plot of the pinning force F_p on the relative magnetic field $h = H/H_{c2}(t)$ at different temperatures for a sample with 36 at. % In.

with other concentrations are perfectly similar to those shown in this figure. In relatively weak fields, with the exception of the immediate vicinity of the field H_{c1}, we have $F_p(h) = \text{const}$. In stronger fields, the values of F_p decrease with increasing h. It should be noted that the pinning force does not vanish at $H/H_{c2}(t) = 1$, this being undoubtedly due to the existence of surface superconductivity in fields exceeding H_{c2}. The pinning force increases with increasing temperature at any fixed value of h.

The plots of $F_{pmax}(t)$ are similar for all samples. Just as in[1], a correlation is observed between the temperature dependences of F_{pmax} and H_{c2}. An illustration of this correlation is Fig. 2, which shows the dependence of log F_p on $\log[H_{c2}(t)/H_{c2}(0)]$ for one of the samples. An analysis of this plot and those similar to it shows that F_p is proportional to $[H_{c2}(t)]^2$ for all the samples of the Pb-In alloy system. A somewhat different relation, namely $F_p \sim [H_{c2}(t)]^{5/2}$, was obtained in[1] for the systems Nb-Ti and Nb-Ta. The accuracy with which the exponent is determined in both cases is such that there is no doubt that the difference between the observed temperature dependences of F_p is real[1)]. Possible causes of this difference will be discussed below.

One of the most interesting properties of the investigated alloys, which follow from the analysis of the experimental data, is the universal temperature dependence of the reduced pinning force $f_p = F_p(t, h)/F_p(0, h)$ ($F_p(0, h)$ is the value of F_p at $t = 0$, obtained by extrapolation) for all the investigated samples[2)]. The value of F_p obtained at any fixed value of h is independent of h. The temperature dependence of the relative pinning force is shown in Fig. 3.

It is known that a number of static and dynamic

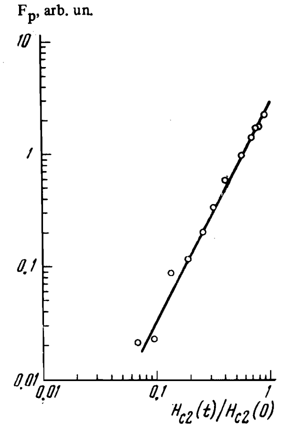

FIG. 2. Plot of the pinning force F_{pmax} against $H_{c2}(t)/H_{c2}(0)$ for a sample with 24 at. % In.

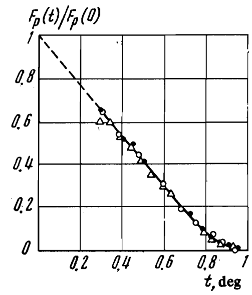

FIG. 3. Dependence of the relative pinning force f_p on the temperature for samples with different component concentrations: ○—12 at. %, ●—24 at. %, △—36 at. %. The values of $F_p(0)$ for all samples were determined by linear extrapolation of the plot of $F_p(t)$ to t = 0.

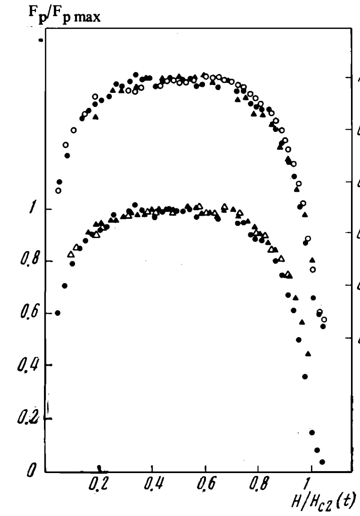

FIG. 4. Dependence of the normalized pinning force F_p/F_{pmax} on the relative magnetic field at different temperatures for different samples. Upper series of curves—sample with 36 at. % In; ○—1,988°K, ●—2,955°K, ▲—5.28°K. Lower series of curves—t ≈ 0.04: ●—36 at. %, △—24 at. %, ▲—at. % in.

characteristics of the mixed state of SC II are determined by the values of the relative parameters $t = T/T_c$ and $h = H/H_{c2}(t)$, so that at different absolute values of T, H, T_c, and H_c we observe in the first approximation corresponding states of different alloys. The investigations performed allow us to state that the law of corresponding states (or the similarity law) extends also to include the relative pinning force f_p. The field dependence of the normalized F_p/F_{pmax} does not vary with temperature (Fig. 4) and is universal for all samples whose surfaces are treated in like manner.

The foregoing properties allow us to draw general conclusions with respect to the field, temperature, and concentration dependences of F_p. It follows from the foregoing that the expression for the pinning force should constitute the product of three functions, each of which depends only on one of the essential variables:

$$F_p = F\varphi_1(h)\varphi_2(t), \qquad (1)$$

where F is a quantity with the dimension of force and depends on the alloy concentration, purity, and the degree of surface finish of the sample, while φ_1 and φ_2 are dimensionless functions that are universal for the given system of alloys. Thus, the quantity F is a characteristic of a given particular sample, and the functions φ_1 and φ_2 in (1) contain information concerning the given class of SC II.

On the basis of the foregoing experiments we can draw for the time being only a qualitative conclusion with respect to the dependence of F_p on the Ginzburg-Landau parameter $\varkappa$, namely, F decreases with increasing $\varkappa$. An analytic description of the function $\varphi_1(h)$ in the entire field interval where the mixed state is realized can not be obtained, whereas for $\varphi_2(t)$ it is possible to trace the function $[H_{c2}(t)/H_{c2}(0)]^2$. It is important to emphasize that the regularity reflected in relation (1) was observed also for alloys with volume pinning[1].

We shall show that the structure of expression (1) for F_p and the temperature dependence of the pinning force can be obtained from a theory that considers the surface pinning of vortices. Shmidt[4] determined how the deformation of the current structure of the vortices near the surface affects vortex-lattice pinning on a flat boundary of an ideal SC II sample. In this case the pinning force acting in a unit volume is given by[4]

$$F_p = \frac{j_c H}{c} = -\frac{3^{1/4}\Phi_0^{1/2} M(H) H}{\sqrt{2}\,\delta d B^{1/4}}, \qquad (2)$$

where j_c is the average critical-current density, Φ_0 is the magnetic-flux quantum, M(H) is the magnetization of the sample, δ is the depth of penetration of the magnetic field, d is the thickness of the plate in the direction perpendicular to the magnetic field, and B is the magnetic induction.

A similar expression for F_p in a model close to that considered here was obtained in[5]. In the region of fields above $0.5H_{c2}(t)$, the magnetization curves for the investigated samples are linear, and there is no hysteresis. We can therefore use for M(H) the expression[6]

$$-M = (H_{c2}-H)/4\pi\beta_A(2\varkappa^2-1) \quad (\beta_A = 1.16),$$

which is valid for all temperatures if we use for $\varkappa$ the parameter $\varkappa_2(t)$ introduced by Maki[7]. The quantity B can be replaced with sufficient accuracy by H in the same region of fields. Expressing δ in terms of $\varkappa$ and the coherence length ξ and recognizing that $\xi(t) = [\Phi_0/2\pi H_{c2}(t)]^{1/2}$, we obtain from (2)

$$F_p = \frac{3^{1/4}(H_{c2}-H)[HH_{c2}(t)]^{1/2}}{4\sqrt{\pi}\,\beta_A(2\varkappa^2-1)\varkappa d}. \qquad (3)$$

If we change over to the relative magnetic field h, we can rewrite (3) in the form

$$F_p = \frac{3^{1/4}H_{c2}^2(0)}{4\sqrt{\pi}\,\beta_A d(2\varkappa^2-1)\varkappa}\{\sqrt{h}(1-h)\}\left\{\left[\frac{H_{c2}(t)}{H_{c2}(0)}\right]^2\right\}. \qquad (4)$$

This expression is valid for all temperatures in fields $H > 0.5H_{c2}(t)$, and agrees well with the presented experimental data. Expression (4) is a product of functions of three independent variables. There is qualitative agreement between the experimental and theoretical dependences of F_p on $\varkappa$ and h. The experimentally obtained temperature dependence of F_p coincides with (4) if the weak temperature dependence of $\varkappa$ is disregarded.

We note that numerical calculations of F_p in accordance with (4) yield for alloys of the Pb-In system results that are quite close to the experimental values. Thus, e.g., for one of the samples, the critical current calculated from (4) at h = 0.6 is 0.52 A, and the experimental value is 0.8 A. In those cases when the plate surfaces were mechanically ground or etched, the critical current density was always somewhat higher than for polished samples, but the main features of the behavior of F_p did not change even in these cases. One can therefore assume that the principal mechanism of surface pinning in alloys of the Pb-In type, which determines its temperature dependence, is the mechanism considered by Shmidt, and the pinning of the vortices by surface imperfections in fields below H_{c2} plays a secondary role. It is possible that the ratio of the experimental value of F_p to the theoretical one and the proximity of this ratio to unity is a measure of the perfection of the surface.

In the case of pinning by dislocations, i.e., by volume defects, Fietz and Webb[1] obtained the temperature dependence of F_2 in the form $F_p \sim [H_{c2}(t)]^{1/2}$. The reason for the difference between the temperature dependences of F_p in our experiments and in those of Feitz and Webb[1] can be found from simple considerations. In experiment one usually measures the critical current, i.e., one determines the total pinning force acting on a unit length of the entire vortex lattice in the sample. Assume that the average pinning force acting on a single pinned vortex (on a unit of its length) can be obtained for either the volume pinning or for the surface pinning by dividing the total force by the number of pinned vortices. If the number of defects is larger than the vortices, the number of pinned vortices coincide with the total number of vortices and is equal to $N = nS = hH_{c2}dw/\Phi_0$ (n is the density of the vortices and $S = dw$ is the sample cross-section area in which the vortices are pinned; here and below we consider for simplicity samples in the form of plates). In the model of Feitz and Webb[1], which considers pinning of vortices on dislocations and is based on taking into account vortex-lattice deformation due to interaction with the pinning centers, we obtain for the force f_1 acting on a unit length of one vortex the expression

$$f_1 = \frac{F_p wd}{N} = \frac{\alpha H_{c2}^{5/2}(t)\varphi(h)wd}{N} = \alpha'\varphi'(h)H_{c2}^{3/2}(t). \tag{5}$$

In the derivation of this expression we took into account the fact that the defects on which the vortices are pinned are uniformly distributed over the entire volume of the sample, and that this volume, naturally, does not depend on the temperature. A similar temperature of the unit pinning force is obtained in a theory in which pinning by other types of defects, cylindrical or spherical cavities, is considered[8]. In particular, for cylindrical cavities parallel to the magnetic field we have

$$f_1 = \frac{1}{2}H_{cm}^2(t)\xi(t) = \frac{\Phi_0^{1/2}H_{c2}^{3/2}(t)}{4\varkappa^2\sqrt{2\pi}}, \tag{6}$$

where $H_{cm}(t)$ is the thermodynamic critical field.

In the case when the pinning is due to the interaction with the surface, we can state that the only pinned vortices are those situated at a depth $\sim\delta(t)$ (δ is the penetration depth of the magnetic field). Therefore the area of a unit height, in which the pinning of the vortices is significant, is

$$S = 2\delta(t)w. \tag{7}$$

In this case the number of pinned vortices depends on the temperature like $N = 2n\delta(t) \sim H_{c2}^{1/2}(t)$. If we use the obtained relation $F_p \sim H_{c2}^2(t)$, we obtain for the unit pinning force the expression

$$f_1 = \frac{F\varphi(h)H_{c2}^2(t)wd\Phi_0}{hH_{c2}(t)\cdot 2\delta(t)w} = F'\varphi'(h)H_{c2}^{3/2}(t). \tag{8}$$

It is interesting to note that in spite of the difference in the nature of the effect (pinning by dislocations, by cavities, by the plane boundary of the superconductor), the temperature dependence of the per-unit forces is the same in all three cases (see formulas (5), (6), and (8)). It is likely that this equality is caused by the fact that all the vortex-pinning mechanisms are based on the free-energy gain, which is proportional to the condensation energy in the volume of the normal core of the vortex $U \sim H_{cm}^2(t)\cdot 2\pi\xi^2(t)$[9], so that the characteristic pinning force is $f_1 \sim U/\xi(t) \sim H_{cm}^2(t)\xi(t) \sim H_{c2}^{3/2}(t)$. As seen from (7), the reason for the difference between the experimentally observed temperature dependence of the total pinning force is that in the case of surface pinning the volume of the region in which the pinning takes place varies with temperature. Thus, the temperature dependence of the total pinning force is determined by the temperature dependence of the elementary pinning force, which is common to different types of interactions, and by the number of pinned vortices. The form of the function $\varphi(h)$ and the coefficients of $\varphi(h)\varphi(t)$ then turn out to depend on the particular model.

Webb[10] gives data on the variation of the temperature dependence of F_p in Nb-Ti alloy with increasing annealing temperature. It turns out that annealing, which leads to an ordering of the dislocations and to the formation of regular dislocation walls, causes a decrease of the exponent of $H_{c2}(t)$ and to the appearance of a plateau in the plot of $F_p(h)$. Calculations (see[10,11]) have shown that this effect should be expected if some periodicity appears in the arrangement of the defect. In particular, if walls with high dislocation concentration are produced, or else walls consisting of cavities, then the transport current begins to flow along these walls[11]. Such walls can be regarded as internal surfaces, so that a temperature dependence close to $F_p \sim H_{c2}^2(t)$ is to be expected. On the other hand, if the defects form a regular two-dimensional lattice, the interaction of the vortex lattice with this system of defects is analogous to a single interaction between the vortex and the defect, and one should expect a temperature dependence close to $F_p \sim H_{c2}^{3/2}(t)$.

Thus, the data obtained in this paper enable us to establish general regularities in the behavior of the pinning forces for alloys with surface pinning of the vortices, and to determine the causes of the difference between these regularities and those observed in alloys with volume pinning of the vortices.

In conclusion, the author considers it his pleasant duty to thank I. M. Dmitrenko for interest in the work and V. V. Shmidt for a discussion of the results.

1)In the present paper, the measured value of the exponent is (2 ± 0.1). In [1] the exponent was determined with the same accuracy.

2)We note that the validity of this statement does not depend on the manner whereby F_p was extrapolated to zero temperature.

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