Propagation of surface acoustic waves in metals in a magnetic field

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(Submitted May 2, 1973)
Zh. Eksp. Teor. Fiz. **65**, 1571–1582 (October 1973)

A calculation is reported of the electron coefficient of the absorption of Rayleigh waves in metals subjected to magnetic fields. Resonance magnetoacoustic effects in the absorption of surface waves traveling across a weak magnetic field are considered. A study is made of the absorption of Rayleigh sound in a strong magnetic field oriented in an arbitrary manner with respect to the surface. The anisotropy and anomalies of the absorption are discussed as a function of the relative orientations of the wave vector of sound and the direction of the magnetic field.

The dispersion properties of the Rayleigh (surface) acoustic oscillations in pure metals at low temperatures are governed primarily by their interaction with the conduction electrons. A distinguishing feature of the Rayleigh sound is the strong inhomogeneity of the elastic deformations in the surface wave. In the case of a crystal which is acoustically isotropic, the tensor of elastic strains induced by a Rayleigh wave is the sum of the fields of the longitudinal ($\alpha = l$) and transverse (α = t) acoustic modes:[1]

$$u_{ik}(\mathbf{r},t)=\sum_{\alpha}u_{ik}{}^{\alpha}(0)\exp[i(\mathbf{kr}-\omega t)-\varkappa_{\alpha}x]. \quad (1)$$

It is assumed that a Rayleigh wave is traveling along the plane boundary of an elastic half-space $x > 0$; ω is the frequency and $\mathbf{k} = k\{0, k_y, k_z\}$ is the wave vector in the boundary plane. The rate of exponential attenuation of the amplitude of a mode α with the depth in a crystal is given by

$$\varkappa_{\alpha}=(k^2-\omega^2/s_{\alpha}{}^2)^{1/2},$$

where s_α is the phase velocity of the investigated mode.

The interaction between the conduction electrons and sound alters the dispersion law of electrons in the deformed lattice by the amount

$$\delta\varepsilon'(\mathbf{p}',\mathbf{r}',t)=\Lambda_{ik}(\mathbf{p}')u_{ik}(\mathbf{r}',t)+\frac{e}{c}\mathbf{A}'(\mathbf{r}',t)\mathbf{v}(\mathbf{p}'). \quad (2)$$

The change in the electron spectrum is expressed in a system of coordinates moving at a velocity $\dot{\mathbf{u}}$ together with the vibrating lattice. This situation is indicated by a prime attached to each of the quantities involved. The notation used in Eq. (2) is as follows: Λ_{ik} is the deformation potential tensor, which is a function of the electron momentum $\mathbf{p}$; $\mathbf{v}$ is the velocity; $-e$ is the electron charge; the dot denotes the partial derivative with respect to time. The vector potential $\mathbf{A}'$ of the alternating magnetic field, which accompanies the acoustic wave, is found by solving the Maxwell and constitutive equations. The potential $\mathbf{A}'$ is a functional of the nonequilibrium part of the electron distribution function $\chi(\mathbf{p}, \mathbf{r}, t)\delta(\epsilon - \epsilon_F)$. In the field of an acoustic wave the nonequilibrium electrons are those whose energy ϵ lies close to the Fermi level ϵ_F.

The function χ should be found by solving the transport equation

$$(\partial/\partial t+\mathbf{v}\nabla+\Omega\partial/\partial\varphi+\nu)\chi=\dot{\delta\varepsilon}'. \quad (3)$$

where φ is the azimuthal angle of rotation of the electrons in a plane perpendicular to a static homogeneous magnetic field $\mathbf{H}$; $\Omega = eH/mc$ is the cyclotron frequency; m is the effective mass; ν is the frequency of collisions between electrons and scattering centers in the bulk of the metal. The boundary conditions of Eq. (3) at the surface x = 0 will be considered later.

The general expression for the absorption coefficient Γ of Rayleigh waves is given in[2] and can be represented in the form

$$\Gamma=\frac{1}{4W}\operatorname{Re}\int_0^\infty dx\,\langle\dot{\delta\varepsilon}'^{*}\chi\rangle. \quad (4)$$

Here,

$$W=|u_x{}^l(0)|^2A_0\rho_L\omega^2k^{-1}$$

is the energy density in a Rayleigh surface wave averaged over one vibration period and normalized to a unit area of the surface; $\mathbf{u}^\alpha(0)$ is the amplitude of the acoustic field of the mode α at the boundary; A_0 depends on the ratio of the velocities of sound;[2] ρ_L is the density of the crystal; the asterisk denotes the complex conjugation and the angular brackets represent the integration over the Fermi surface.

The coefficient Γ is obtained in[2] in the absence of a magnetic field and for a magnetic field $\mathbf{H}$ perpendicular to the surface of the metal. We shall consider the absorption of Rayleigh waves in a weak magnetic field parallel to the surface of a metal when the cyclotron radius $R = cp_F/eH$ exceeds the acoustic wavelength. We shall also obtain an expression for Γ in the opposite case of strong magnetic fields ($kR \ll 1$) with the vector $\mathbf{H}$ oriented in an arbitrary manner with respect to the surface. A preliminary report of the present study has been made in[3].

The absorption of the Rayleigh sound in metals in a parallel magnetic field is discussed in a recent paper of Mints and Sorokin.[4] We shall discuss this paper in greater detail at the end of the next section; here, we shall just mention that the results obtained by us do not agree with the conclusions reached in[4] because the initial formula for the coefficient Γ used in that paper differs from our expression (4).

We shall conclude this introduction with the following comment. It is quite clear that the surface nature of the absorption of Rayleigh waves is manifested most clearly if the spatial dispersion is strong so that

$$kl=kv/\nu\gg1. \quad (5)$$

If this condition is not obeyed, an electron traveling its mean free path l is effectively subjected to a homogeneous acoustic field. Consequently, the absorption of the Rayleigh sound is then of the order of the volume (bulk) absorption: $\Gamma \sim \Gamma_V \sim \omega^2/\nu$. Therefore, we shall assume that the criterion (5) is satisfied.

ABSORPTION IN A WEAK MAGNETIC FIELD PARALLEL TO THE SURFACE OF A METAL

It follows from Eq. (2) that the rate of change of the energy of the conduction electrons because of their interaction with an acoustic wave is given by

$$\delta\dot{\varepsilon}'=\Lambda_{ik}\dot{u}_{ik}-e\mathbf{v}(\mathbf{E}+c^{-1}[\dot{\mathbf{u}}\times\mathbf{H}]), \qquad (6)$$

where **E** is the electric field in the laboratory system of coordinates. Thus, the absorption of acoustic waves is the sum of the deformation and the Joule losses. The Joule losses include an additive correction due to the contribution of the eddy fields **E** and of the induction emf $c^{-1}(\dot{\mathbf{u}}\times\mathbf{H})$ to the absorption. If the condition (5) is satisfied and the acoustic wavelength is short compared with the thickness of the electromagnetic skin layer $\delta = (c^2/4\pi\omega\sigma)^{1/2}$ at the acoustic frequency ω, the contribution of the electric fields to Γ is small. It is shown by Kaner[5] that the absorption due to the eddy fields is significant and comparable with the deformation losses in the range of frequencies for which $k\delta \ll 1$.

A direct comparison of the first and third terms on the right-hand side of Eq. (6) shows that the deformation correction is kR times as large as the contribution of the induction absorption mechanism.[1)] In other words, in a weak magnetic field the contribution of the electric fields to the absorption is small or of the same order as the contribution of the terms containing $\Lambda_{ik}u_{ik}$. On the other hand, the nature of the resonance effects discussed below is governed only by the kinematics of the conduction electrons in a magnetic field. The magnitude and explicit form of $\delta\epsilon'$ governs only the absolute value of the coefficient Γ. Therefore, we shall not calculate the corrections to the absorption resulting from the electric fields.

1. We shall consider the model of a metal with an isotropic and quadratic dispersion law of electrons. In a magnetic field **H** directed along the z axis (Fig. 1) all the electrons can be divided in a natural manner into two groups. The "volume" electrons (trajectories I and III in Fig. 1) do not collide with the surface of the metal. In the z = const plane their trajectories are circles of radius $R_\perp = R\sin\theta$ (θ is the polar angle in the momentum space) and the coordinates of the centers X of the orbits of these electrons are greater than the radii of the trajectories: $X > R_\perp$. The position of an electron on a trajectory is governed by the direction cosines of the angles: $n_x = -\sin\theta\sin\varphi$ and $n_y = \sin\theta\cos\varphi$ (we shall assume that φ lies in the interval $[-\pi, \pi]$). The second group of electrons is of the "surface" type. Their trajectories intersect the surface x = 0 and reach the surface at a glancing angle ψ. The coordinate X of the surface electrons is related to ψ by

$$X=-R_\perp\cos\psi \qquad (7)$$

and does not exceed $R_\perp$. The trajectories of these electrons are (z = const)

$$x=R\int_{\psi}^{\varphi}n_x\,d\varphi', \quad y=R\int n_y\,d\varphi. \qquad (8)$$

This division of electrons into the volume and surface groups is useful in the solution of the transport equation (3) by the method of characteristics.

2. In solving Eq. (3) for the volume electrons, rotating along circular trajectories in the same manner as in an infinite sample, we shall assume that the boundary condition for the function $\chi(\mathbf{p}, \mathbf{r}, t) = \chi(\mathbf{p}, x)$

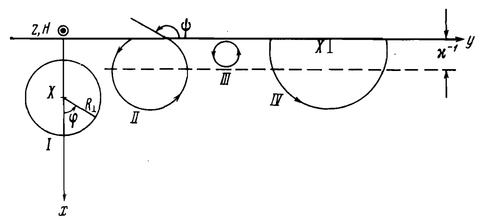

FIG. 1. Possible trajectories of electrons in a magnetic field parallel to the surface of a metal.

$\times \exp[i(\mathbf{kr} - \omega t)]$ is its periodicity with respect to φ (period 2π). Consequently, the function χ_V of the volume electrons is exactly the same as for an infinite metal:

$$\chi_v=(e^{2\pi\bar{\gamma}}-1)^{-1}\int_{\varphi}^{\varphi+2\pi}\frac{d\varphi'}{\Omega}\Lambda_{ik}(\varphi')\,\dot{u}_{ik}\left(x+R\int_{\varphi}^{\varphi'}n_x\,d\varphi''\right)\exp\left(\int_{\varphi}^{\varphi'}\gamma\,d\varphi''\right),$$

$$\gamma=\frac{\nu+i(\mathbf{kv}-\omega)}{\Omega}, \quad \bar{\gamma}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\gamma\,d\varphi. \qquad (9)$$

In the case of the surface electrons which collide with the boundary, we shall use the diffuse boundary condition[2)]

$$\chi_s|_{\varphi=-\psi}=0, \qquad (10)$$

which corresponds to the absence of correlation between the distribution functions of the incident and reflected electrons. In other words, the condition (10) means that the electrons reflected from the surface of the metal are in equilibrium. The solution of Eq. (3), satisfying the boundary condition (10), can be written in the form:

$$\chi_s=\int_{-\psi}^{\varphi}\frac{d\varphi'}{\Omega}\Lambda_{ik}(\varphi')\,\dot{u}_{ik}\left(x+R\int_{\varphi}^{\varphi'}n_x\,d\varphi''\right)\exp\left(\int_{\varphi}^{\varphi'}\gamma\,d\varphi''\right). \qquad (11)$$

In the calculation of the coefficient Γ we must bear in mind that the electrons located in the bulk of a metal at depths $x > x_0$ do not reach the surface. The value x_0 can be found from the condition $X = R_\perp$ and is

$$x_0=R\int_{\pi}^{\varphi}n_x\,d\varphi'=R_\perp(1+\cos\varphi).$$

This means that in the integration with respect to x in Eq. (4) we can use the function χ_V from Eq. (9) for $x > x_0$ and the function χ_S of Eq. (11) for $0 < x < x_0$. Consequently, the coefficient Γ is the sum of the volume and surface absorptions: $\Gamma = \Gamma_V + \Gamma_S$.

It should be noted that the function $\chi(x)$ based on the characteristic $x = x_0(\varphi)$ (with $\psi = \pi$ and $X = R_\perp$) has a discontinuity. This is also true of an arbitrary law of reflection of electrons from the surface of the metal. Only in the case of the perfectly specular reflection is the electron distribution function continuous at $x = x_0$ and its first derivative loses the infinite discontinuity because $\partial\chi_S/\partial x$ at this point has a root singularity of the $(x_0 - x)^{-1/2}$ type.

3. We shall now calculate the coefficient Γ_V due to the volume electrons. We shall use the formula (4) and the expression (9) for χ_V. The integration over x can be performed in an elementary manner and the functions

$$\exp\left[\int_{\varphi}^{\varphi'}d\varphi''(\gamma-\varkappa_\alpha Rn_x)\right]$$

occurring in the integrals over the azimuthal angles should be expanded as a double Fourier series in φ and φ'. In this way we find that

$$\Gamma_V=\mathscr{T}\frac{kR}{2}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}\sum_{n=-\infty}^{\infty}\left(\frac{k_y-\varkappa_\alpha}{k_y+\varkappa_\alpha}\frac{k_y-\varkappa_\beta}{k_y+\varkappa_\beta}\right)^{n/2}$$

$$\times\int_0^\pi d\theta\sin\theta\exp[-(\varkappa_\alpha+\varkappa_\beta)R_\perp]J_n((k_y^2-\varkappa_\alpha^2)^{1/2}R_\perp)J_n((k_y^2-\varkappa_\beta^2)^{1/2}R_\perp)$$

$$\times\frac{\nu}{\Omega}\left[\left(\frac{\nu}{\Omega}\right)^2+\left(n+k_zR\cos\theta-\frac{\omega}{\Omega}\right)^2\right]^{-1}, \tag{12}$$

where

$$\mathcal{F}=\frac{3}{8}\xi\frac{N\varepsilon_Fk}{\rho_Lv},\quad B_\alpha=\frac{\Lambda_{ik}u_{ik}^\alpha(0)}{ku_x^l(0)\varepsilon_F(\xi A_0)^{1/2}}.$$

The quantity F is of the order of the collisionless absorption coefficient of the volume sound; N is the electron density. In the derivation of Eq. (12) it is assumed, for the sake of simplicity, that the factors B_α are independent of the angles; $\zeta = (\Lambda/\epsilon_F)^2$ is the dimensionless electron-phonon interaction parameter; $\Lambda \sim \epsilon_F$ is the characteristic value of the deformation potential; $J_\xi(q)$ is a Bessel function with the index ξ.

4. In calculating the absorption Γ_S due to the surface electrons it is convenient to modify Eq. (4) by going over from the integration with respect to x to the integration with respect to the glancing angle ψ. Using Eqs. (8) and (11), we can obtain the initial expression for Γ_S in the form

$$\Gamma_S=-\mathcal{F}\frac{(kR)^2}{4\pi}\operatorname{Re}\sum_{\alpha,\beta}B_\alpha B_\beta^*\int_0^\pi d\theta\sin\theta\int_0^\pi d\psi n_x(\psi)$$

$$\times\int_{-\psi}^{\psi}d\varphi\exp\left[-(\varkappa_\alpha+\varkappa_\beta)R\int_\psi^\varphi n_x\,d\varphi''\right]\int_{-\psi}^{\varphi}d\varphi'\exp\left[\int_\varphi^{\varphi'}d\varphi''(\gamma-\varkappa_\alpha Rn_x)\right]. \tag{13}$$

Here, the order of integration with respect to ψ (or x) and φ is interchanged. It is clear from the above formula that the contribution to Γ_S is made by all the surface electrons whose glancing angle ψ lies in the interval $(0, \pi)$ ($|X| \le R_\perp$). The integration over the azimuthal angle φ reduces to the integration over the trajectory from the moment of emergence of an electron from the surface $\varphi = -\psi$ to the moment $\varphi = \psi$ corresponding to its return to the surface.

The coefficient Γ_S can conveniently be represented as the sum of two terms:

$$\Gamma_S=\Gamma_{S1}+\Gamma_{S2}. \tag{13'}$$

Formally, such a division results from the integration by parts with respect to ψ in Eq. (13). The term outside the integral gives the component Γ_{S1}, which describes the absorption of a Rayleigh wave by the electrons with $\psi = \pi$ (the contribution of the point $\psi = 0$ is cancelled out because the length of the electron trajectory vanishes for $\psi \to 0$). Direct calculations show readily that the expression for Γ_{S1} can be written in the form:

$$\Gamma_{S1}=-\Gamma_V+\pi\mathcal{F}kR\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}\operatorname{Re}\int_0^\pi d\theta\sin\theta\frac{M(\varkappa_\alpha R_\perp)M(\varkappa_\beta R_\perp)}{\exp(2\pi\bar\gamma)-1}, \tag{14}$$

where the "matrix element" $M(\kappa_\alpha R_\perp)$ is represented by the integral

$$M(\varkappa_\alpha R_\perp)=\overline{\exp\{-\varkappa_\alpha R_\perp(1+\cos\varphi)+ik_yR_\perp\sin\varphi+\bar\gamma\varphi\}}$$

$$=\frac{1}{\pi}\int_0^\pi d\varphi\exp[-\varkappa_\alpha R_\perp(1+\cos\varphi)]\cos(k_yR_\perp\sin\varphi-i\bar\gamma\varphi). \tag{15}$$

Simple transformations of the integral term Γ_{S2} yield

$$\Gamma_{S2}=\mathcal{F}\frac{kR}{2\pi}\sum_{\alpha,\beta}k\frac{\operatorname{Re}(B_\alpha B_\beta^*)}{\varkappa_\alpha+\varkappa_\beta}\int_0^\pi d\theta\sin\theta\int_0^\pi d\psi\int_{-\psi}^{\psi}d\varphi\operatorname{Re}\exp\left[\int_\psi^\varphi d\varphi'(\gamma-\varkappa_\alpha Rn_x)\right]. \tag{16}$$

Thus, the total absorption Γ consists of two parts. The first part, $\Gamma_V + \Gamma_{S1}$ is identical with the second term in Eq. (14) and is due to the electrons which do not collide with the surface. The second part Γ_{S2} represents the absorption due to the electrons suffering diffuse reflection at the boundary (trajectories II and IV in Fig. 1).

5. We shall now obtain an asymptotic expression for the absorption coefficient of the Rayleigh sound Γ in the presence of an anomalous acoustic "skin" layer. This case corresponds to the inequality

$$kR\gg1. \tag{17}$$

We shall assume that a Rayleigh wave travels across a magnetic field **H** along the y axis. Then, the quantity $\bar\gamma = \xi = (\nu - i\omega)/\Omega$ is independent of the angle θ. The expression (15) for the matrix element M can be rewritten as follows (see[8]):

$$M(\varkappa_\alpha R_\perp)=\exp\left[-\varkappa_\alpha R_\perp+i\frac{\xi}{2}\operatorname{sgn}k_y\ln\frac{q_\alpha-1}{q_\alpha+1}\right]J_{i\xi\operatorname{sgn}k_y}(\varkappa_\alpha R_\perp\sqrt{q_\alpha^2-1})$$

$$+\frac{i}{\pi}\operatorname{sgn}k_y\operatorname{sh}\pi\xi\int_0^\infty dt\exp\left[-it\xi\operatorname{sgn}k_y-\varkappa_\alpha R_\perp\operatorname{sh}t\left(q_\alpha-\operatorname{th}\frac{t}{2}\right)\right], \tag{18}$$

where $q_\alpha = k/\kappa_\alpha > 1$ is the dimensionless wave number. The appearance of two terms in the above formula is due to the fact that, in the case of a strong spatial inhomogeneity corresponding to Eq. (17), the integral with respect to φ in Eq. (15) includes contributions from two characteristic regions. One is of width $(kR_\perp)^{-1/2}$ and lies in the vicinity of the point $\varphi = \pi/2$: this region corresponds to a stationary point of the cosine phase and represents the strong interaction of electrons with a Rayleigh wave. The presence of an exponentially small factor $\exp(-\kappa_\alpha R_\perp)$ in this term is due to the fact that such "strong-interaction points" ($\mathbf{k}\cdot\mathbf{V} = \omega$) lie at a depth $R_\perp$ from the surface where the inhomogeneous nature of the Rayleigh acoustic wave field is important. The second term in Eq. (18) corresponds to the interaction of sound with an electron in the range $\varphi \sim \pi$, where the acoustic field amplitude is of the order of unity. However, rapid oscillations of the cosine in the vicinity of this point give rise to a small factor of the order of $(kR_\perp)^{-1}$, which describes the relative time of interaction between an electron and sound in this region.

In order to calculate the integral with respect to θ in Eq. (14), we shall need an asymptotic expression for M corresponding to large values of the argument:

$$M(\varkappa_\alpha R_\perp)=\sqrt{\frac{2}{\pi}}(q_\alpha^2-1)^{-1/4}(\varkappa_\alpha R_\perp)^{-1/2}\exp\left[-\varkappa_\alpha R_\perp+i\frac{\xi}{2}\operatorname{sgn}k_y\ln\frac{q_\alpha-1}{q_\alpha+1}\right]$$

$$\times\cos\left(\varkappa_\alpha R_\perp\sqrt{q_\alpha^2-1}-i\frac{\pi}{2}\xi\operatorname{sgn}k_y-\frac{\pi}{4}\right)+\frac{i}{\pi}\operatorname{sh}\pi\xi(k_yR_\perp+i\xi)^{-1}. \tag{18'}$$

The asymptotic expression (18′) confirms the qualitative considerations put forward above. Using now Eq. (18′) and the inequalities (5) and (17), we can easily represent $\Gamma_V + \Gamma_{S1}$ in the form:

$$\Gamma_V+\Gamma_{S1}=\frac{2\mathcal{F}}{kR}\sum_{\alpha,\beta}k^2\frac{B_\alpha B_\beta^*}{(\varkappa_\alpha+\varkappa_\beta)^2}\frac{(q_\alpha q_\beta)^{1/2}}{(q_\alpha^2-1)^{1/4}(q_\beta^2-1)^{1/4}}$$

$$\times\operatorname{Re}\left\{\exp\left[i\frac{\xi}{2}\operatorname{sgn}k_y\ln\left(\frac{q_\alpha-1}{q_\alpha+1}\frac{q_\beta-1}{q_\beta+1}\right)\right][\exp(2\pi\xi)-1]^{-1}\right\} \tag{19}$$

$$-\frac{\mathcal{F}}{2\pi kR}\operatorname{Re}\ln(2kl_-)(1-e^{-2\pi\xi})\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta},\quad l_-=\frac{v}{\nu-i\omega}.$$

This formula does not include terms which depend monotonically on the magnetic field and are proportional to $(kR)^{-1}$. The first term in Eq. (19) describes an acoustic cyclotron resonance in the absorption of the Rayleigh sound at the frequencies $\omega = n\Omega$, where n is an integer. This resonance should be observed at high frequencies, when $\omega \gg \nu$ and $\xi \approx -in$. The factor $(kR)^{-1}$ appears because there are only relatively few electrons in the vicinity of the limiting points of the type corresponding to trajectories III in Fig. 1. It should be noted that in the selected geometry the cyclotron resonance amplitude depends on the sign of k_y and is numerically greater if $k_y < 0$. This is due to the inhomogeneity of the acoustic wave field. The second term in Eq. (19) describes a logarithmic "antiresonance" and corresponds to the interaction of the acoustic wave with the electrons at the highest points in their trajectories ($\varphi = \pi$).

We shall now calculate the last term in the coefficient Γ, which is governed by Eq. (16). In a weak magnetic field (17) the electrons corresponding to the trajectories of type II and IV contribute to the absorption mainly at the beginning and end of their paths. Formally, this means that the contributions to the integral with respect to φ in Eq. (16) is made primarily in the vicinity of the lower and upper limits. Simple calculations yield the following expression for Γ_{S2}:

$$\Gamma_{s2}=\Gamma(0)+\frac{kR}{2\pi}\mathcal{T}\sum_{\alpha,\beta}k\frac{\mathrm{Re}(B_\alpha B_\beta^*)}{\varkappa_\alpha+\varkappa_\beta}\int_0^\pi d\theta\sin\theta$$
$$\times\mathrm{Re}\int_0^\pi d\psi\exp\left(-2\psi\xi-ik_yR\int_{-\psi}^{\psi}n_y\,d\varphi\right)[\varkappa_\alpha Rn_x(-\psi)-ik_yRn_y(-\psi)-\xi]^{-1}. \quad (20)$$

The first term $\Gamma(0)$ appears because of the summation of the contribution to the absorption of those electrons which approach the boundary at the angle $\varphi = \psi$ and it represents the absorption coefficient of a Rayleigh wave in the absence of a magnetic field:[2]

$$\Gamma(0)=\mathcal{T}\sum_{\alpha,\beta}k\frac{\mathrm{Re}(B_\alpha B_\beta^*)}{\varkappa_\alpha+\varkappa_\beta}\frac{q_\alpha}{(q_\alpha^2-1)^{1/2}}\mathrm{arctg}(q_\alpha^2-1)^{1/2}. \quad (21)$$

The second term in Eq. (20) is the correction due to the electrons which emerge from the surface at the angle $\varphi = -\psi$. The most effective among these electrons are those for which the average displacement

$$R\int_{-\psi}^{\psi}n_y\,d\varphi$$

in the direction of **k** has its extremal value. These electrons have trajectories of type IV and glancing angles close to $\pi/2$. The scatter of the angles ψ near this point is of the order of $(kR_\perp)^{-1/2}$ so that the centers of the orbits lie on the surface x = 0.

The final expression for the coefficient Γ_{S2} can be represented in the form

$$\Gamma_{s2}=\Gamma(0)+\frac{\mathcal{T}}{4kR}\exp\left(-\pi\frac{\nu}{\Omega}\right)\sin\left(2kR-\pi\frac{\omega}{\Omega}\mathrm{sgn}\,k_y\right)\sum_{\alpha,\beta}B_\alpha B_\beta^* q_\alpha q_\beta. \quad (22)$$

The second term in this formula describes oscillations of the same type as in the Pippard geometric resonance. The quantity $\pi\,\mathrm{sgn}\,k_y\,\omega/\Omega$, which is in the argument of the sine, allows for the delay in the phase-matching condition. In the volume absorption case, when trajectories of type IV represent closed circles, this correction is compensated by the phase lead in the motion of an electron in the opposite direction.

The amplitude of the geometric oscillations differs in two characteristic ways from the analogous oscillations of the absorption of the volume acoustic vibrations. First of all, in a parallel magnetic field the geometric resonance oscillations are not modulated by the acoustic cyclotron resonance. This is due to the fact that the motion of electrons along trajectories of type IV is not periodic because of their diffuse reflection from the boundary. Secondly, the amplitude of the oscillations is $\sqrt{kR}$ times smaller than that of the volume oscillations. This is due to the fact that not all the extremal-section electrons interact effectively with the sound but only those whose orbit centers are located on the surface of the metal.

6. Mints and Sorokin[4] investigated the absorption of the Rayleigh waves in metals subjected to a magnetic field parallel to the surface and with **k** ⊥ **H**. The initial equation for the coefficient Γ used by Mints and Sorokin was

$$\Gamma^{[4]}=\frac{\nu}{4W}\int_0^\infty dx\langle|\chi|^2\rangle. \quad (23)$$

It is shown in[2] that for an infinite medium Eqs. (23) and (24) are identical, and for Rayleigh waves Eq. (23) gives an incorrect result. This can be shown if we use the transport equation (3), which easily yields the relationship

$$\Gamma^{[4]}=\Gamma+\Delta\Gamma,\qquad \Delta\Gamma=\frac{1}{8W}\langle v_x|\chi(0)|^2\rangle. \quad (24)$$

If the electrons are reflected diffusely, the surface correction $\Delta\Gamma$ is negative on the strength of Eq. (10) and is of the same order of magnitude as the other terms in Γ. For example, in the case of a strong spatial dispersion (17) and $\Omega \gg \nu$, we have

$$\Delta\Gamma=-\Gamma_{s2}+O(\nu/\Omega).$$

Therefore, in this limiting case the largest term Γ_{S2} in the expression obtained in[4] is compensated and the absorption has a deep minimum. This misunderstanding has led Mints and Sorokin[4] to conclude that the absorption of the Rayleigh sound is governed primarily by the volume electrons.

ABSORPTION COEFFICIENT IN A STRONG MAGNETIC FIELD

Let us assume that a magnetic field is inclined to the surface of a metal so that $H_x = H\sin\phi$, $H_y = 0$, $H_z = H\cos\phi$. The reflection of electrons from the metal-vacuum interface is of arbitrary nature and we can describe it by a specular reflection coefficient $\rho \le 1$, which is independent of the angle of incidence of an electron on the surface of a metal.

1. We shall consider the absorption of a Rayleigh acoustic wave in a strong magnetic field when the cyclotron radius R is much less than the acoustic wavelength:

$$kR\ll 1. \quad (25)$$

The condition (25) means that the cyclotron frequency Ω is the largest parameter of the problem. In other words, Γ in Eq. (4) should have a nonequilibrium correction in the form of the nonequilibrium part of the distribution function $\bar{\chi}$ averaged over the fast rotation of an electron in the magnetic field. The transport equation (3) for this quantity can be rewritten in the form

$$(\bar{\nu}-i\omega+ik_z v_H\cos\phi+v_H\sin\phi\,\partial/\partial x)\bar{\chi}(x)=\overline{\dot{\delta}\varepsilon'}(x). \quad (26)$$

We have taken into account the fact that $\bar{v}_y = 0$ and that $v_H \neq 0$ is the projection of the average velocity of the electron along the magnetic field.

It should be noted that whereas the expressions for the resonance effects described earlier include the total collision frequency corresponding only to the "departure" term in the collision integral, in a strong magnetic field the quantity ν should generally be the transport frequency. This is due to the fact that all the electrons on the Fermi surface contribute to the absorption in a strong magnetic field.

The boundary condition for Eq. (26) is the following relationship on the surface x = 0

$$\bar{\chi}^{\downarrow}(0)=\rho\bar{\chi}^{\uparrow}(0), \tag{27}$$

where the arrows ↓ and ↑ represent the electrons moving away from the boundary ($v_H > 0$) or toward the boundary. Equation (26) subject to the boundary condition (27) can be solved easily with the aid of the quadrature formulas and the solution obtained is

$$\bar{\chi}^{\uparrow}=\frac{1}{|v_H|\sin\phi}\int\limits_0^{\infty}dx'\exp\left[x'\frac{-\bar{\nu}+i(\omega-k_z v_H\cos\phi)}{|v_H|\sin\phi}\right]\overline{\delta\varepsilon'}(x+x',-|v_H|),$$

$$\bar{\chi}^{\downarrow}=\frac{1}{|v_H|\sin\phi}\int\limits_0^{x}dx'\exp\left[x'\frac{-\bar{\nu}+i(\omega-k_z v_H\cos\phi)}{|v_H|\sin\phi}\right]\overline{\delta\varepsilon'}(x-x',|v_H|)$$

$$+\frac{\rho}{|v_H|\sin\phi}\int\limits_x^{\infty}dx'\exp\left[x'\frac{-\bar{\nu}+i(\omega-k_z v_H\cos\phi)}{|v_H|\sin\phi}+2ik_z(x'-x)\operatorname{ctg}\phi\right]\overline{\delta\varepsilon'}(x'-x,-|v_H|). \tag{28}$$

Naturally, the dependence on ρ occurs only in the nonequilibrium correction to the distribution function of the electrons moving away from the surface.

2. Substituting the expression for $\bar{\chi}$ from Eq. (28) into Eq. (4) and integrating with respect to x′, x, and φ, we easily obtain the following formulas:

$$\Gamma^{\uparrow}=\frac{1}{2}\mathcal{T}\operatorname{Re}k\bar{l}_-\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}\int\limits_{-1}^{0}\frac{dn_H}{1+\mathcal{K}_\alpha\bar{l}_-|n_H|},$$

$$\Gamma^{\downarrow}=\frac{1}{2}\mathcal{T}\operatorname{Re}k\bar{l}_-\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}\int\limits_0^1 dn_H\left[\frac{1}{1+\mathcal{K}_\beta^*\bar{l}_-|n_H|}+\rho\frac{(\varkappa_\alpha+\varkappa_\beta)\bar{l}_-\sin\phi|n_H|}{(1+\mathcal{K}_\alpha\bar{l}_-|n_H|)(1+\mathcal{K}_\beta^*\bar{l}_-|n_H|)}\right]. \tag{29}$$

Here, as in the preceding section, we have considered only the deformation interaction of electrons with sound (see Footnote 1) and we have assumed that the parameters B_α are independent of the angle: $\bar{l}_- = \mathbf{v}/(\bar{\nu} - i\omega)$ is the effective mean free path of electrons in a strong magnetic field, $n_H = v_H/v$ is the unit vector of the magnetic field in the momentum space, and

$$\mathcal{K}_\alpha=\varkappa_\alpha\sin\phi-ik_z\cos\phi.$$

In Eq. (29) the arrows attached to Γ represent the contribution of the electrons moving toward the surface of the metal or away from it. It should be noted that the denominators of the integrands contain, instead of the usual frequency of collisions between electrons and scatterers $\bar{\nu}$, the effective quantity $\bar{\nu} + \kappa|v_H|\sin\phi$. The frequency $\kappa|v_H|\sin\phi$ is the reciprocal of the electron "lifetime" in an acoustic skin layer. Consequently, the mechanism of the selection of the effectively interacting electrons does not generally reduce to the conditions $\omega = k_z|v_H|\cos\phi$. This means that the integrals with respect to n_H include the contribution of all the electrons. Elementary calculations of the integrals yield the result

$$\Gamma=\frac{1}{2}\mathcal{T}\operatorname{Re}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}[Q_\alpha\ln(1+\mathcal{K}_\alpha\bar{l}_-)+Q_\beta^*\ln(1+\mathcal{K}_\beta^*\bar{l}_-)]$$
$$+\sin\phi\frac{\rho}{2}\mathcal{T}\operatorname{Re}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\mathcal{K}_\beta^*-\mathcal{K}_\alpha}[Q_\alpha\ln(1+\mathcal{K}_\alpha\bar{l}_-)-Q_\beta^*\ln(1+\mathcal{K}_\beta^*\bar{l}_-)], \tag{30}$$

where $Q_\alpha = k/H_\alpha$. It follows from Eq. (30) that in a strong magnetic field defined by Eq. (25) the deformation absorption Γ reaches saturation, i.e., it ceases to depend on the magnetic field.

3. If we assume that $\phi = \pi/2$, we find that Eq. (30) easily yields the coefficient Γ in a magnetic field perpendicular to the surface of the metal.[2] The case when **H** is parallel to the surface of the metal ($\phi = 0$) is of special interest. In this geometry (**H** ∥ z) the expression (30) can be rewritten in the form

$$\Gamma=\eta(k_z)\mathcal{T}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta},$$
$$\eta(k_z)=\frac{k}{2k_z}[\operatorname{arctg}(k_z\bar{l}-\omega\tau)+\operatorname{arctg}(k_z\bar{l}+\omega\tau)],\quad \tau=\bar{\nu}^{-1},\quad \bar{l}=v/\bar{\nu}. \tag{31}$$

In the terms largest with respect to the parameter kR the absorption is independent of the nature of the reflection of electrons from the boundary. This is due to the fact that in a parallel field the relative number of electrons colliding with the surface of a metal is a small quantity of the order of kR. It also follows from Eq. (31) that the absorption coefficient Γ is strongly anisotropic in its dependence on the angle between the wave vector **k** and the magnetic field. A schematic dependence of the function η on k_z is plotted in Fig. 2.

In the longitudinal propagation case (**k** ∥ **H**) it follows from the condition (5) that $\eta = \eta_0 = \pi/2$ and the order of magnitude of Γ is the same as in the absence of a magnetic field $\Gamma(0)$. In the propagation across a magnetic field, when $k_z = 0$, the quantity η becomes

$$\eta_1=k\bar{l}/[1+(\omega\tau)^2],$$

so that the absorption exceeds $\Gamma(0)$ by a factor η_1. At the maxima of the curve corresponding to the $\omega\tau \gg 1$ case, the function η reaches the value

$$\eta_2=(kv/2\omega)\operatorname{arctg}2\omega\tau$$

at the points $|k_z| = \omega/v$. In this case, the order of magnitude of the absorption is given by $\Gamma \sim Fkv/\omega \sim \omega$, i.e., in one vibration period the amplitude of the wave decreases by the factor e. The strong anisotropy of the absorption is due to the following circumstances.

If a wave is traveling along a magnetic field, the acoustic energy is absorbed only by a small group of electrons near the central section and these electrons satisfy the phase relationship

$$\omega=\mathbf{k}\cdot\bar{\mathbf{v}}. \tag{32}$$

Consequently, the average value of the mean free time

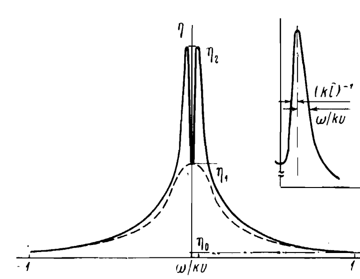

FIG. 2. Schematic form of the absorption curve in a strong magnetic magnetic field.H ∥ z. The continuous curve corresponds to the high-frequency limit $\omega\tau \gg 1$ and the dashed curve to the low-frequency limit $\omega\tau \gg 1$.

(the time between collisions) of all the electrons is small: $\tau_{eff}^{-1} \sim kv$. If we bear in mind that $\Gamma \sim \omega^2\tau_{eff}$, we obtain the correct order of magnitude of the absorption of the longitudinal Rayleigh sound from the formula (31) if we substitute $\eta = \eta_0$.

If $k_z = 0$ in a strong magnetic field, defined by Eq. (25), the relationship (32) is "satisfied" by all the electrons and the effective mean free time is

$$\tau_{eff}^{-1} \sim \nu - i\omega, \quad \mathrm{Re}\,\tau_{eff} \sim \frac{\tau}{1+(\omega\tau)^2}.$$

In this case, the absorption is due to the scattering of electrons.

In the high-frequency limit $\omega\tau \gg 1$ the absorption exhibits a sudden rise when the projection of the wave vector onto the direction of **H** becomes small: $|k_z| \sim \omega/v$. This is due to the fact that the "resonance" condition (32) selects that group of the effectively interacting electrons whose drift velocity along the magnetic field is $v_z = \omega/k_z$. In the range $|k_z| < k_{min} = \omega/v$ there are no electrons which can satisfy Eq. (32). The peak of the coefficient Γ corresponds to the edge of the collisionless electron attenuation curve and it is analogous to the tilt effect in the absorption of the volume acoustic waves, observed first by Reneker[9] in bismuth. It should be noted that the existence of a discontinuity in the dependence of Γ on k_z gives rise to a singularity in the Rayleigh sound velocity at $\nu \to 0$ and this singularity is of the type $\ln||k_z|/k - \omega/kv|$.

The authors are grateful to É. A. Kaner for discussing this investigation and for valuable comments.

1) It is shown in [6] that in a strong magnetic field when $kR \ll 1$ the induction effects can only be smaller than or of the same order as the deformation absorption. This is due to the fact that the induction part of the interaction [see Eq. (6)] is an odd function of **p**, whereas the deformation is an even function. In the presence of a strong magnetic field the process of averaging over the Fermi surface reduces the relative importance of the induction correction to $\delta\epsilon'$. A special case is the resonance coupling of a weakly damped electromagnetic wave with an acoustic wave in the case when the induction interaction predominates.

2) The interaction between electrons and Rayleigh waves in a very weak magnetic field parallel to the surface of the metal has been considered in [7] for the specular reflection of electrons.

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Translated by A. Tybulewicz
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