Wave excitation by light and by a homogeneous alternating magnetic field

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Excitation and interaction of waves (magnetoacoustic, sound, spin, and electromagnetic) in a magnetic substance located in a homogeneous alternating magnetic field and irradiated by an intense light beam are investigated. The conditions for mixed excitation of magnetoacoustic waves by light and an alternating magnetic field and also for excitation of sound by magnetoacoustic waves and light are found. The effect of sound on parametric excitation of the spin waves is investigated. The theoretical results are compared with the experimental data.

If a magnet is placed in an external alternating field its physical characteristics, like those of any other medium, oscillate about their mean values, i.e., waves are excited. The excitation of different waves can be regarded as independent only in the limit of infinitesimally small amplitudes, when the linear approximation of the theory is valid and the excitation of the waves is determined only by their interaction with the external fields. At finite wave amplitudes, their interaction with one another, which is described by the nonlinear terms of the equations of motion, assumes an important role. The interaction of the waves with one another can compete with their interaction with the external fields and also can lead to excitation of waves that do not interact directly with the external fields. The study of wave excitation in magnets by specified external fields calls therefore for an analysis and for an account of the nonlinear wave-interaction processes.

Of particular interest in physics is wave excitation connected with instability of some stationary state of the motion, an instability that arises when the parameters of the external forces are altered. During the initial state of the instability, the amplitudes of a certain aggregate of waves increase exponentially. Subsequently, their growth is stopped by the nonlinear interactions. As a rule the amplitudes of the waves excited following development of an instability are large, so that these processes are easy to register, and an elucidation of the conditions under which the instability arises is of considerable interest. The state of motion with finite wave amplitudes is unstable because the waves that have become unstable interact both with the external fields and with waves of finite amplitudes, i.e., the instability is a nonlinear process. Even in those cases when the predominant role is played by the linear excitation mechanism, the influence of various nonlinear interactions can lead to qualitative changes of the excitation process.

In the present paper we investigate nonlinear excitation of waves (electromagnetic, spin, and acoustic) in a magnet exposed to light and to a uniform alternating magnetic field directed along a constant field (parallel magnetic pumping). From the equations of motion of the magnetic moment, of the elastic deformations, and Maxwell's equations for the electromagnetic field, we obtain equations for the amplitudes of the interacting waves by using the method of averaging over the rapid variables (Sec. 1).

At low amplitudes of the pump field, the only wave excited in the magnet is an electromagnetic wave having the frequency ν_0 of the external light source. When the pump-field amplitude is increased, this state of motion can become unstable, and it is the spin waves with frequencies ω_0 equal to half the pump-frequency which turn out to be unstable [1]. As a result of the interaction of the electromagnetic waves with the parametrically excited waves (PEW), electromagnetic waves with combined frequencies $\nu_0 \pm n\omega_0$ are excited and the effective damping of the PEW changes, and this affects their interaction with the pump field. Thus, a nonlinear mixed parametric excitation of spin waves takes place in an alternating magnetic field in the presence of an intense electromagnetic wave. This process is considered by us in Sec. 2 (see also [2,3]).

The states of motion, which constitute an aggregate of spin and electromagnetic waves with finite amplitudes, can become unstable against excitation of waves of diverse nature, primarily acoustic waves, since the constants for the interaction of sound with light and with spin waves are relatively large. This mechanism of sound excitation, and also of sound excitation by nonlinear mixing of electromagnetic waves, is considered in Sec. 3. The sound waves, as shown in Sec. 4, interact intensely with the spin waves, exerting an appreciable influence on the excitation thresholds and on the spin waves that are established in the case of above-threshold pumping. In the last section we discussed the results and compare the theory with the experimental results given in [4-8].

1. FUNDAMENTAL EQUATIONS

We introduce the thermodynamic potential [9]

$$\tilde{F}=F-\frac{1}{8\pi}[\mathbf{E}^{*}\mathbf{D}+\mathbf{H}^{*}\mathbf{B}+\text{c.c.}], \tag{1}$$

where F is the free-energy density, E and H are the electric and magnetic field intensities, and D and B are the electric and magnetic inductions. The equations of motion of the magnetic moment, of the elastic deformations, and Maxwell's equations can then be represented in the form

$$\begin{gathered}\dot{\mathbf{M}}=g[\mathbf{M}\times\mathbf{H}_{\text{eff}}],\quad \rho\ddot{u}_i=\partial\sigma_{ik}/\partial x_k,\\ \operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\quad \operatorname{rot}\mathbf{H}=\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t},\\ \operatorname{div}\mathbf{D}=0,\quad \operatorname{div}\mathbf{B}=0,\\ \mathbf{H}_{\text{eff}}=\partial\tilde{F}/\partial\mathbf{M}^{*},\quad \sigma_{ik}=\partial\tilde{F}/\partial u_{ik},\\ \mathbf{B}=4\pi\partial\tilde{F}/\partial\mathbf{H}^{*},\quad \mathbf{D}=4\pi\partial\tilde{F}/\partial\mathbf{E}^{*},\end{gathered} \tag{2}$$

Here **u** is the displacement vector, ρ is the density, and g is the gyromagnetic ratio.

We retain in the power-law expansion of free-energy density in terms of **E**, **H**, and **u**, besides the quadratic terms, also the terms with higher powers, since we are interested in phenomena due to wave interaction:

$$\begin{aligned} F={}&{}^1/_8\pi^{-1}(\varepsilon\delta_{ik}E_iE_k^*+\delta_{ik}H_iH_k^*)+\mathbf{M}^*(\mathbf{H}+\alpha\nabla\mathbf{M}+\mathbf{H}^{(m)}) \\ &+\mu u_{ik}^2+({}^1/_2k-{}^1/_3\mu)u_{ll}^2+{}^1/_8\varepsilon^2\pi^{-1}p_{ij,\,lk}u_{ij}E_lE_k^* \\ &+{}^1/_8 i\pi^{-1}\zeta e_{ikl}B_i{}^*E_kE_l+b_2M_i{}^*M_ku_{ik} \\ &+{}^1/_3Au_{ik}u_{il}u_{kl}+Bu_{ik}^2u_{ll}+{}^1/_3Cu_{ll}^3, \end{aligned} \tag{3}$$

where $p_{ij,\,lk}$ is the tensor of the elastooptical coefficients, and k and μ are the compression and shear moduli; the last three terms correspond to cubic anharmonicity in an elastically-isotropic medium with three elastic constants: A, B, and C[9]; ζ is the Faraday constant and b_2 is the coefficient of magnetoelastic coupling.

We consider a uniaxial ferrodielectric and choose the Z axis parallel to the equilibrium-density magnetic-moment vector ($\mathbf{Z}\parallel\mathbf{M}_0$). We consider the case of parallel pumping, when the external field $H = H_0 + h\cos 2\omega_0 t$ is directed along M_0. Let light of frequency ν_0 be incident perpendicular to the plane boundary (the YZ plane) of a ferrodielectric that is linearly polarized with a polarization vector along the Y axis. As is well known[1], a longitudinal alternating magnetic field excites most intensely spin waves with wave vectors lying in the YX plane. At this orientation of the wave vectors, the elastic deformations connected with the spin waves are transverse, and the displacement vector is directed along the Z axis[10].

In addition to the high-frequency transverse sound, which plays an important role near the region of magnetoacoustic resonance (MAR), we consider also longitudinal acoustic oscillations, which can be excited in the magnet by light as well as by spin or magnetoacoustic waves.

From (2) and (3) follows a system of equations for the electromagnetic, magnetoacoustic, and acoustic oscillations:

$$\left(\Delta-\frac{\varepsilon^2}{c^2}\frac{\partial^2}{\partial t^2}\right)E_i=i\frac{4\pi}{c^2}\zeta e_{ikl}\frac{\partial^2}{\partial t^2}(E_km_l^*-E_l^*m_k)+\frac{\varepsilon^2}{c^2}p_{jl,ik}\frac{\partial^2}{\partial t^2}(u_{il}E_k), \tag{4}$$

$$d\mathbf{m}/dt=g[\mathbf{M}\times\mathbf{H}],\quad \tilde{\mathbf{H}}=\mathbf{H}+\mathbf{H}^{(m)}+\alpha\Delta\mathbf{m}+b_2(\mathbf{m}\nabla)\mathbf{u}-i\zeta\mathbf{E}^*\hat{e}\mathbf{E}, \tag{5}$$
$$\mathbf{m}=\mathbf{M}-\mathbf{M}_0,\quad \nabla(\mathbf{H}^{(m)}+\mathbf{m})=0,\quad \operatorname{rot}\mathbf{H}^{(m)}=0,$$

$$\begin{aligned} \left(\frac{\partial^2}{\partial t^2}-v_\perp^2\Delta\right)u_{\perp z}={}&\frac{b_2}{\rho M_0}(\nabla\mathbf{m})+\frac{\varepsilon^2}{4\pi\rho}p_{44}\frac{\partial}{\partial x_k}(E_zE_k) \\ &+\frac{1}{\rho}(Q_1+Q_2\delta_{kz})\frac{\partial}{\partial x_k}\left(\frac{\partial u_{\parallel l}}{\partial x_l}\frac{\partial u_{\perp z}}{\partial x_k}\right), \end{aligned} \tag{6}$$

$$\begin{aligned} \left(\frac{\partial^2}{\partial t^2}-v_\parallel^2\Delta\right)u_{\parallel i}={}&\frac{b_2}{\rho M_0^2}m_i(\nabla\mathbf{m})+\frac{\varepsilon^2}{4\pi\rho}p_{12}\frac{\partial}{\partial x_i}E^2 \\ &+\frac{1}{\rho}(Q_1+Q_2\delta_{iz})\frac{\partial}{\partial x_i}\left(\frac{\partial u_{\perp z}}{\partial x_l}\right)^2, \end{aligned}$$

$$v_\perp^2=\frac{\mu}{\rho},\quad v_\parallel^2=\frac{1}{\rho}\left(k+\frac{4\mu}{3}\right),\quad Q_1=B+k-\frac{2\mu}{3},\quad Q_2=\frac{A+2\mu}{4}, \tag{7}$$

where $p_{44} = p_{ij,\,kl}\delta_{ik}\delta_{jl}$, $p_{12} = p_{ij,\,kl}\delta_{ij}\delta_{kl}$ are the elastooptical coefficients of the interaction of the light with the transverse and longitudinal sound, $\hat{e}$ is a third-rank tensor that is antisymmetrical with respect to all pairs of indices, and δ_{ik} is the Kronecker symbol. Equations (4)—(7) should be supplemented by boundary conditions that require the continuity of E on the boundary S (we are considering only normal incidence of the light) and, in addition,

$$\sigma_{ik}|_S=0,\quad M|_S=0. \tag{8}$$

The investigated nonlinear phenomena become manifest at relatively low amplitudes of weakly-damped waves, when the main contribution to the free energy is made by the quadratic terms. Therefore the contribution of the nonlinear terms, and also of the dissipation and of the alternating magnetic field, is small in comparison with the contribution of the linear terms to (4)—(7), and we can use the well known method of asymptotic expansions. We seek the solutions of (4)—(7) in the form

$$\mathbf{E}=\sum_{\mathbf{k}}\mathbf{e}_\mathbf{k}a_\mathbf{k}A^{1/2}(\mathbf{k})e^{-i\nu_\mathbf{k}t+i\mathbf{k}\mathbf{x}},\quad A(\mathbf{k})=\frac{4\pi\nu_k}{\varepsilon},$$

$$\begin{aligned} \mathbf{m}&=\sum_\mathbf{q}\mu(\mathbf{q})b_\mathbf{q}B^{1/2}(\mathbf{q})e^{-i\omega_\mathbf{q}t+i\mathbf{q}\mathbf{x}},\quad B(\mathbf{q})=\frac{gM_0\Omega_1(\mathbf{q})(\omega_\mathbf{q}^2-v_\perp^2q^2)}{2\omega_q^2-v_\perp^2q^2-\omega_{sq}^2}, \\ \mathbf{u}_\perp&=\sum_\mathbf{q}\mu_3(\mathbf{q})b_\mathbf{q}B^{1/2}(\mathbf{q})e^{-i\omega t+i\mathbf{q}\mathbf{x}},\quad \Omega_1(\mathbf{q})=g\left[H_0+\alpha q^2+\frac{4\pi M_0}{q^2}q_x^2\right], \\ \mathbf{u}_\parallel&=\sum_\varkappa\mathbf{u}_\varkappa c_\varkappa C^{1/2}(\varkappa)e^{-i\Omega_\varkappa t+i\varkappa x},\quad C(\varkappa)=\frac{1}{2\Omega_\varkappa\rho}, \end{aligned} \tag{9}$$

where a_k, b_k, and c_κ are quantities that vary slowly over the periods ($2\pi/\nu$, $2\pi/\omega$, $2\pi/\Omega$) and over the wavelengths ($2\pi/k$, $2\pi/q$, $2\pi/\kappa$). The normalization of the amplitudes is chosen such that the equations for them have a canonical form. The polarization vectors and the dispersion laws are determined from the linear equations

$$\mu=(\mu_1,\mu_2),\quad \mu_1=1,\quad \mu_2=\frac{i\omega_s-\Omega(\mathbf{q})}{\Omega_1(\mathbf{q})},$$

$$\mu_3(\mathbf{q})=\frac{b_2}{\Omega_1(q)M_0\rho}\,\frac{i[\Omega_1(\mathbf{q})q_x-\Omega(\mathbf{q})q_y]-q_y\omega_s}{\omega_\mathbf{q}^2-v_\perp^2q^2}$$

$$\Omega(\mathbf{q})=4\pi gM_0\frac{q_xq_y}{q^2},\quad \Omega_2(\mathbf{q})=g\left[H_0+\alpha q^2+4\pi M_0\frac{q_y^2}{q^2}\right],$$

$$(\omega_\mathbf{q}^2-v_\perp^2q^2)(\omega^2-\omega_{s\mathbf{q}}^2)-\frac{gb_2^2}{\rho M_0\Omega_1(q)}[(\Omega_1(\mathbf{q})q_x-\Omega(\mathbf{q})q_y)^2-q_y^2\omega_\mathbf{q}^2]=0, \tag{10a}$$

$$\omega_{s\mathbf{q}}^2=\Omega_1(\mathbf{q})\Omega_2(\mathbf{q})-\Omega^2(\mathbf{q}), \tag{10b}$$

$$\Omega_\varkappa=v_\parallel\varkappa,\quad \mathbf{u}_\varkappa=\frac{\varkappa}{|\varkappa|},\quad \nu_\mathbf{k}=\frac{kc}{\sqrt{\varepsilon}},\quad \mathbf{e}_\mathbf{k}\mathbf{k}=0. \tag{10c}$$

Substituting (9) in (4)—(7) and neglecting the second derivatives of the amplitudes with respect to x and t, we obtain the following system:

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_\mathbf{k}\frac{\partial}{\partial\mathbf{x}}+\gamma_\mathbf{k}\right)a_\mathbf{k}=\langle\Sigma\{V_{12},a,b\}_{\mathbf{k}\nu}+\Sigma\{V_{13},a,c\}_{\mathbf{k}\nu}\rangle, \tag{11}$$

$$\begin{aligned} \left(\frac{\partial}{\partial t}+\mathbf{v}_\mathbf{q}\frac{\partial}{\partial\mathbf{x}}+\tilde{\Gamma}_\mathbf{q}\right)b_\mathbf{q}=\Big\langle&\Sigma\{V_{12},a,a\}_{\mathbf{q}\omega}+\Sigma\{V_{23},b,c\}_{\mathbf{q}\omega} \\ &+\sum_{\mathbf{q}'}h_{\mathbf{q}\mathbf{q}'}\exp[i(\omega_\mathbf{q}+\omega_{\mathbf{q}'}-2\omega_0)t+i(\mathbf{q}+\mathbf{q}')\mathbf{x}]\Big\rangle, \end{aligned} \tag{12}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_\varkappa\frac{\partial}{\partial\mathbf{x}}+\alpha_\varkappa\right)c_\varkappa=\left\langle\sum\{V_{13},a,a\}_{\varkappa\Omega}+\sum\{V_{23},b,b\}_{\varkappa\Omega}\right\rangle, \tag{13}$$

where $\mathbf{v}_k$, $\mathbf{v}_q$, and $\mathbf{v}_\kappa$ are the group velocities of the electromagnetic, magnetoacoustic, and acoustic waves,

$$h_{\mathbf{q}\mathbf{q}'}=hV_{\mathbf{q}\mathbf{q}'},\quad V_{\mathbf{q}\mathbf{q}'}=\frac{(\omega_\mathbf{q}^2-v_\perp^2q^2)\Omega_1(\mathbf{q})(\mu(\mathbf{q})\mu^*(\mathbf{q}'))}{4\omega_\mathbf{q}(2\omega_\mathbf{q}^2-v_\perp^2q^2-\omega_{s\mathbf{q}}^2)},$$

$$\Sigma\{V_{ik},a,b\}_{\omega_\mathbf{k}}=\sum_{\mathbf{k}'\mathbf{q}'}V_{ik}(\mathbf{k},\mathbf{k}',\mathbf{q}')a_{\mathbf{k}'}b_{\mathbf{q}'}\exp[i(\omega_\mathbf{k}-\omega_{\mathbf{k}'}-\omega_{\mathbf{q}'})t+i(\mathbf{k}'+\mathbf{q}'-\mathbf{k})\mathbf{x}],$$

$$V_{12}(\mathbf{k},\mathbf{k}',\mathbf{q})=\left[\frac{\varepsilon^2p_{44}}{4\pi}(\mathbf{q}\mathbf{e}_\mathbf{k})\mu_3+\zeta(\mu(q)\mathbf{e}_\mathbf{k})\right]e_{\mathbf{k}'z}[A(\mathbf{k})A(\mathbf{k}')B(\mathbf{q})]^{1/2},$$

$$V_{13}(\mathbf{k},\mathbf{k}',\varkappa)=\frac{p_{12}}{4\pi}\varkappa(\mathbf{e}_\mathbf{k}\mathbf{e}_{\mathbf{k}'})[A(\mathbf{k})A(\mathbf{k}')C(\varkappa)]^{1/2}, \tag{14}$$

$$V_{23}(\mathbf{q},\mathbf{q}',\varkappa)=\left\{\frac{b_2}{M_0^2\varkappa}|\mu\varkappa|^2+\left(Q_1\varkappa+Q_2\frac{\varkappa_z^2}{\varkappa}\right)\right. \tag{14a}$$

$$\left.\times(\mathbf{q}\mathbf{q}')\mu_3(\mathbf{q})\mu_3^*(\mathbf{q}')\right\}[B(\mathbf{q})B(\mathbf{q}')C(\varkappa)]^{1/2}, \tag{14b}$$

$$\tilde{\Gamma}_\mathbf{q}=[\Gamma_\mathbf{q}(\omega_\mathbf{q}^2-v_\perp^2q^2)+\alpha_{\perp\varkappa}(\omega_\mathbf{q}^2-\omega_{s\mathbf{q}}^2)]/(2\omega_\mathbf{q}^2-v_\perp^2q^2-\omega_{s\mathbf{q}}^2). \tag{14c}$$

The angle brackets $\langle\ldots\rangle$ denote averaging that eliminates the rapidly varying terms in the right-hand sides of (11)–(13). We have introduced in (11)–(13) terms that take into account the wave dissipation: γ_k, Γ_q, α_κ, and $\alpha_{\perp\kappa}$ are the damping coefficients of the light, of the spin waves, and of the longitudinal and transverse sound. To obtain the boundary conditions for the amplitudes b_q and c_κ it is necessary to substitute (9) in (8). As to the amplitudes a_k, only the amplitude of light with frequency ν_0 is different from zero at the boundary x = 0:

$$a_{k\nu}|_{x=0}=a_0\delta_{\nu_0\nu_k}.$$

A solution of Eqs. (11)–(13) with boundary conditions (8) is

$$a_k=a_0e^{-\gamma x}\delta_{\nu_k\nu_0},\quad b_q=c_\kappa=0, \tag{15}$$

meaning the absence of oscillations of the magnetic moment and of elastic deformations in the magnet. The light and the alternating magnetic field, however, can excite both types of oscillation. This means that the solution (15) becomes unstable against variation of the amplitudes b_q and c_κ at certain values of a_0 and h. The onset of instability of the spin waves in the presence of light and of an alternating magnetic field is a mixed spin-wave excitation process. The light and the excited spin waves can excite, separately and jointly, acoustic waves which in turn can exert a noticeable influence on the excitation of the spin waves and on the absorption of the pump power in the sample. We now proceed to study these processes.

2. MIXED EXCITATION OF SPIN AND MAGNETOACOUSTIC WAVES

To find the condition for the onset of instability of a magnetoacoustic wave (ω, q) in an alternating magnetic field in the presence of an electromagnetic wave it suffices to consider the equations for the amplitudes (11)–(13), which are linearized with respect to the amplitude b_q in the vicinity of the solution (15). Simultaneously with the equations for b_q, we must consider all the equations whose right-hand sides are not equal to zero at $b_q \neq 0$ in the linear approximation. It is seen from (11)–(13) that excitation of a magnetoacoustic wave (ω, q) brings about also excitation of a magnetoacoustic wave $(2\omega_0-\omega, -q)$ and of an electromagnetic wave $(\nu_\pm, k_\pm)$, $\nu_\pm=\nu_0\pm\omega_0$, $k_\pm = k\pm q$).

Parametric resonance takes place when $2\omega_0-\omega=\omega=\omega_{-q}$. We consider waves for which this condition is satisfied. In order for the right-hand sides of the equations for the light-wave amplitudes to be different from zero, and consequently for the light to influence the excitation of the spin wave, it is necessary to satisfy the resonance condition

$$k_\pm=|k-q|=\nu_\pm/c\approx k. \tag{16}$$

This condition leads to a relation that determines the angle between the wave vectors k and $k_\pm$:

$$2\sin(\theta/2)=q/k. \tag{17}$$

The system (11) and (12), when linearized with respect to the amplitudes of the wave (ω, q) and of the waves that interact resonantly with it, takes the form

$$\left(\frac{\partial}{\partial t}+\gamma_k\right)a_+=V_{12}(k-q,q)a_0b_{-q},\quad a_+=a_{k,\nu+\omega_0},$$
$$\left(\frac{\partial}{\partial t}+\gamma_k\right)a_-=V_{12}^*(k-q,q)a_0b_q^*,\quad a_-=a_{k,\nu-\omega_0},$$
$$\left(\frac{\partial}{\partial t}+\tilde\Gamma_q\right)b_q=V_{12}^*(k-q,q)a_-^*a_0-\tilde h b_{-q}^*,\quad \tilde h=h_{q,-q},$$
$$\left(\frac{\partial}{\partial t}+\tilde\Gamma_q\right)b_{-q}=V_{12}(k-q,q)a_+a_0^*-\tilde h b_q^*. \tag{18}$$

We have neglected here the damping of the incident electromagnetic wave over the length of the sample, assuming its amplitude, as well as the amplitudes of the remaining waves, to be independent of x and to equal a_0. The right-hand sides of the remaining equations contain nonresonant terms and vanish after averaging.

By solving (18) it is easy to find that the exponential growth of the initiating amplitudes b_q, b_{-q}, and $a_\pm$ occurs if

$$\tilde h^2+\frac{V_{12}^4(k-q,q)}{\gamma_k^2}|a_0|^4>\tilde\Gamma_q^2. \tag{19}$$

This is the condition of mixed parametric excitation of spin waves. It is seen from (14a) that the threshold of the mixing excitation is minimal when the scattered-light polarization vector is directed along the Z axis, i.e., $e_\pm \parallel Z$. In this case

$$V_{12}^2(k-q,q)=\left|\frac{\varepsilon^2p_{44}q_y}{4\pi}\mu_3(q)-\zeta\right|^2\frac{gM_0\Omega_1(q)(\omega_q^2-v_\perp^2q^2)\pi}{\varepsilon(2\omega_q^2-v_\perp^2q^2-\omega_{sq}^2)}. \tag{20}$$

As h → 0 and for magnetoacoustic waves that are nonresonantly coupled with the magnetic field, Eq. (18) goes over into the condition of stimulated scattering of light by mannetoacoustic waves:

$$|a_0|^2>|a_0|_c^2=\gamma_k\Gamma_q/V_{12}^2.$$

As $a_0 \to 0$ and for magnetoacoustic waves that scatter the light nonresonantly, Eq. (19) goes over into the well known condition $\tilde h^2>h_c^2=\Gamma_q^2$ for parametric excitation of magnetoacoustic waves in parallel pumping[1]. Far from the MAR region, the magnetoacoustic waves go over into spin and acoustic waves, so that the conditions obtained above for the excitations of magnetoacoustic waves go over respectively into the conditions for mixed excitation of spin waves

$$h^2>\Gamma_q^2-\left|\frac{\zeta^2gM_0\Omega_1(q)\pi}{\varepsilon\gamma_k}\right|^2|a_0|^4$$

or acoustic waves

$$\left|\frac{\varepsilon^2p_{44}q_y}{4\pi}\mu_3(q)\right|^2\frac{gM_0\Omega_1(q)}{\varepsilon\gamma_k}|a_0|^2>\alpha_\kappa.$$

We see that far from the MAR region the alternating magnetic field makes no contribution to the excitation of the elastic oscillations, and the light, whose scattering by the spin waves is much weaker than by the magnetoacoustic waves, has little effect on the parametric excitation of the spin waves.

3. EXCITATION OF LONGITUDINAL SOUND

Longitudinal acoustic oscillations, as is seen from (7), can be excited by either electromagnetic or magnetoacoustic waves[1)]. We consider first the case of an "unbounded" magnet when the excited sound waves can be regarded as traveling waves. To this end the linear dimensions in at least one direction should greatly exceed the wavelength and the sound-relaxation length, or else the sample must be connected to a sound conductor. We can neglect in this case the boundary conditions, and the excitation of an acoustic wave $(\Omega_1\kappa)$ does not bring about excitation of a reflected wave $(\Omega, -\kappa)$.

Excitation of the acoustic wave (Ω, κ) in the presence of an electromagnetic wave (ν_0, k) with amplitude a_0 and of a magnetoacoustic wave (ω, q) with amplitude b_0

brings about excitation of scattered light ($\nu_0 \pm \Omega$, $k \pm \kappa$) and of magnetoacoustic waves ($\omega \pm \Omega$, $q \pm \kappa$). Noticeable excitation takes place if the following conditions are satisfied:

$$\nu_- = \nu_0 - \Omega = k_- c, \quad k_- = k - \varkappa \tag{21}$$

$$\omega_- = \omega - \Omega = \omega_s(q_-), \quad q_- = q - \varkappa. \tag{22}$$

If both conditions (21) and (22) are satisfied, mixed excitation of sound by the light and by the magnetoacoustic wave takes place. The condition for this excitation follow from Eqs. (11)—(13), which are linearized in terms of the amplitudes of the aforementioned waves in the vicinity of

$$a_k = a_0 \delta_{\nu_k \nu_0}, \quad b_q = b_0 \delta_{q q_0},$$

and take the following form:

$$\frac{V_{13}^2(k, \varkappa)}{\gamma_k} |a_0|^2 + \frac{V_{23}^2(q, \varkappa)}{\tilde{\Gamma}_q} |b_0|^2 \geqslant \alpha_\varkappa. \tag{23}$$

If only condition (21) is satisfied or if $b_0 \to 0$, then the sound is excited only by the light, and (23) yields the well known condition for stimulated scattering of light by longitudinal sound (see, e.g., [11,12]):

$$|a_0|^2 > |a_0|_c^2 = \alpha_\varkappa \gamma_k / V_{13}^2(k, \varkappa). \tag{24}$$

As $a_0 \to 0$, or when only condition (22) is satisfied, expression (23) goes over into the condition for stimulated scattering of magnetoacoustic waves by sound:

$$|b_0|^2 > |b_0|_c^2 = \alpha\, \Gamma_q / V_{23}^2(q, \varkappa). \tag{25}$$

This condition, in turn, goes over outside the MAR region into the condition for the excitation of sound by spin waves [13] or by transverse sound waves.

In bounded samples, where traveling sound cannot be excited, the foregoing analysis no longer holds. It can be shown that stimulated scattering of light by standing low-frequency ($\nu \gg \Omega$) acoustic waves is impossible. Standing sound contains besides the wave (Ω, κ) also the reflected wave (Ω, $-\kappa$), with $c_\kappa = c_{-\kappa}$. Assume that Ω and κ satisfy the condition (21), which ensures resonant scattering of the light (ν_0, k) with excitation of sound (Ω, κ) and of light ($\nu_0 - \Omega$, $k - \kappa$). Since $\Omega \ll \nu$, the condition $\nu_+ = \nu_0 + \Omega$, $k_+ = k_0 + \kappa$, corresponding to resonant scattering of light by the reflected sound wave (Ω, $-\kappa$) with excitation of light at the summary frequency (anti-Stokes component) is satsified with good accuracy simultaneously with (21). The interaction coefficients V_{13} (k_-, κ) and V_{13} (k_+, $-\kappa$) are equal to each other, accurate to terms $\sim \Omega/\nu$. Therefore, when light is scattered by standing sound, the Stokes component is equal to the anti-Stokes component, $a_{\nu - \Omega} = a_{\nu + \Omega}$, and consequently the total energy of the light remains unchanged. Therefore the light cannot excite standing sound in the approximation considered here.

$$|a_0|^2 \nu_0 + |a_+|^2 \nu_+ + |a_-|^2 \nu_- = |a_0|^2 \nu_0$$

We have assumed so far that the light is monochromatic. This is true if the spectral width $\Delta\nu$ of the incident light is so small that $\Delta\nu < \Omega_{min} = \nu_\| d/\pi$ (d is the thickness of the sample and determines the minimal sound frequency Ω_{min} in the sample). The line width of high-power lasers (ruby, neodymium) amount to $10^{-7} - 10^{-4}$ of the fundamental frequency, i.e., $\Delta\nu \sim 10^7 - 10^{10}$ sec^{-1}. As seen from (7), nonmonochromatic electromagnetic waves can excite in the magnet low-frequency standing sound oscillations in the frequency range $\Omega < \Delta\nu$. The excitation is the result of nonlinear mixing of the electromagnetic waves, and has no threshold.

In this case the solution of (7) is

$$u(x, t) = \sum_\varkappa \nabla c_\varkappa f_\varkappa(x) e^{-i\Omega_\varkappa t} + \text{c.c.}, \tag{26}$$

where

$$c_\varkappa = \frac{\varepsilon^2}{4\pi\rho} \int_{-\infty}^{\infty} \frac{E_{\Omega'}^2 d\Omega'}{\Omega_\varkappa^2 - \Omega'^2 - i\alpha_\varkappa},$$

$$E_{\Omega'}^2 = \frac{1}{2\pi v} \int_V \int_{-\infty}^{\infty} e^{i\Omega' t} f_\varkappa(x) E^2(x, t)\, dx dt,$$

$f_\kappa(x)$ are the eigenvectors of the self-adjoint operator

$$(\partial^2/\partial t^2 - v^2 \partial^2/\partial x^2) f = 0$$

with boundary conditions (8), and are normalized to unit volume. Since the damping coefficient of the sound decreases rapidly with increasing frequency, $\alpha \sim \Omega^2$, the light excites predominantly low-frequency acoustic modes, the wavelengths of which are comparable with the sample dimensions.

4. EFFECT OF SOUND ON PARAMETRIC EXCITATION OF MAGNETOACOUSTIC WAVES

Longitudinal sound excited by light may exert an appreciable effect on the parametric excitation of magnetoacoustic waves. The damping of PEW whose frequencies lie in the vicinity $\Delta\omega$ of the frequency ω_0 is noticeably compensated for by the external field. At low pump levels $h - h_c \leq h_c$ we have $\Delta\omega \sim \Gamma$ [1,11]. Scattering of PEW by sound leads to the appearance of combination waves. If the frequency Ω of the sound exceeds $\Delta\omega$, then these waves lie in the band corresponding to excitation by the external pump field, and attenuates rapidly, i.e., the scattering of PEW by sound increases the effective damping. At $\Omega < \Delta\omega$, the scattering process become manifest as a modulation (amplitude and phase) of the spin waves.

We shall first analyze in greater detail the effect of longitudinal sound excited by light on the PEW threshold, assuming $\Omega > \Delta\omega$. From (12) and (26) follow equations for the amplitudes of magnetoacoustic waves with frequencies ω_0 and $\omega_0 \pm \Omega$. By virtue of the condition $\Omega > \Delta\omega$, the interaction of the waves $\omega_0 \pm \Omega$ with the alternating magnetic field is nonresonant, and the term in (12) which describes this interaction vanishes after averaging. We then obtain, after simple calculations, the following condition for the excitation of waves with frequencies ω_0:

$$h = \tilde{\Gamma}_q + \frac{g^2 b_2^2}{M_0^2} \sum_{\Omega, l} |c_{\Omega l}|^2 \Phi_{\Omega l},$$

$$\Phi_{\Omega, l} = \frac{l_y}{|l|}(l\mu)\left\{\frac{(l\mu(q+l))^2}{\omega_{q+l}^2 - (\omega_q + \Omega)^2 - 2i\Gamma(\omega_q + \Omega)} + \frac{(l\mu(q-l))^2}{\omega_{q-l}^2 - (\omega_q - \Omega)^2 - 2i\Gamma(\omega_q - \Omega)}\right\}, \tag{27}$$

$$u = \sum_{\Omega, l} \frac{1}{|l|} c_{\Omega, l} e^{i\Omega t + ilx}.$$

The second term in (27) is the nonlinear increment to the spin-wave damping due to the scattering by sound and is proportional to the energy of the acoustic oscillations. Comparing (27) with (19) we see that the influence of the longitudinal sound leads to an increase of the effective damping of the PEW and to an increase of the threshold. It is seen from (27) that the increment to the damping plays an important role only in the case of resonant scattering of spin waves by sound, when

$$\omega_q \pm \Omega = \omega_{q\pm l}. \quad (28)$$

To each value of Ω there corresponds a complete set of wave vectors $\{l_\Omega\}$, so that the effect on the PEW will be most appreciable if the resonant scattering condition is satisfied for all $l \in \{l_\Omega\}$. For any l, the resonance conditions (28) $\omega_q - \omega_{q\pm l} = \pm\Omega = lv_\parallel$ yield at $\Omega \ll \omega_q$, $l \ll q$ the relation

$$(\omega_q - \omega_{q\pm l})/(q \pm l) = \partial\omega_q/\partial q \approx \Omega l/l^2 = v_\parallel, \text{ т. е. } v_g = v_\parallel.$$

Thus, in the region where the group velocity of the spin waves is equal to the velocity of the longitudinal sound, the nonlinear increment to the damping of the spin waves is resonantly large and is equal to

$$\Delta\Gamma_q = \frac{g^2 b_2^2}{M_0^2} \sum_{\Omega,l} |c_{\Omega l}|^2 \frac{2\omega_0^2 l_y^2 |\mathbf{l}\mu|^2}{|\mathbf{l}|\Gamma_q(\omega_q^2 - \Omega^2)}. \quad (29)$$

The interaction of magnetoacoustic waves with sound and with light influences also the amplitudes of the waves that are produced by above-threshold pumping. To find the steady-state amplitudes of the magnetoacoustic waves, we include in the equations for the amplitude b_q (12) the terms cubic in b_q, and also the change of the effective spin-wave damping coefficient as the result of their interaction with the light (19) and the sound (27):

$$\Gamma_q \to \tilde{\Gamma}_q = [(\Gamma_q + \Delta\Gamma_s)^2 + (\Delta\Gamma_{sw})^2]^{1/2} \equiv \Gamma_q + \Delta\Gamma_s + \Delta\tilde{\Gamma}_{sw}. \quad (30)$$

When these terms are taken into account, we obtain for the amplitudes of the magnetoacoustic waves the following equations (see [15]):

$$\left(\frac{\partial}{\partial t} + \tilde{\Gamma}_q\right) b_q = -\tilde{h} b_{-q}^* + 2i \sum_{q'} (T_{qq'} b_{q'} b_{q'}^*) b_q + \sum_{q'} (S_{qq'} b_{q'} b_{-q'}) b_{-q}^*,$$
$$\tilde{h} = hV_{q,-q} = h\frac{(\omega_q^2 - v_\perp^2 q^2)\Omega_1(q)\mu(q)\mu^*(-q)}{4\omega_q(2\omega_q^2 - v_\perp^2 q^2 - \omega_{sq}^2)} \quad (31)$$

where $T_{qq'} = T_{q,q',q,q'}$, $S_{q,q'} = T_{q,-q,q',-q'}$, and $T_{q,q',q'',q'''}$ are the coefficients of the four-wave interactions. Repeating the arguments and the calculations of [15], we obtain an expression for the amplitudes of the steady-state magnetoacoustic waves

$$|b_q|^2 = [\tilde{h}_c(\xi^2 - 1)^{1/2}]/|S_{q,-q}|, \quad \tilde{h}_c = \tilde{\Gamma}_q, \quad (32)$$

where $\xi^2 = h^2/h_c^2$ is the excess of the pump power over the threshold. We note that the obtained expression is valid if it is assumed that waves with constant amplitudes are established above the threshold. At noticeable excesses of the pump power over threshold, the steady-state oscillations can be more complicated (see [14,16]), but near the threshold expression (32) seems to correctly estimate the mean values of the amplitudes of the steady-state oscillations.

For the imaginary part of the magnetic susceptibility we obtain the expression

$$\chi'' = 2V_{q,-q}^2(\xi^2 - 1)^{1/2}/|S_{q,-q}|\xi^2, \quad (33)$$

where $v_{q,-q}$ is the coupling coefficient of the waves with the pump field h (31). Let us examine the corrections to χ'' necessitated by the interaction of the spin waves with the light and with the sound. To this end, we represent χ'' in the form of an expansion that is valid when $\xi > 1$:

$$\chi'' = \chi''|_{\Gamma=\Gamma_q}\left(1 + \frac{2(\xi^2 - 2)}{\xi^2 - 1}\frac{\Delta\Gamma_s + \Delta\tilde{\Gamma}_{sw}}{\Gamma_q}\right). \quad (34)$$

The second term in the parenthesis describes the sought corrections to χ''. It is seen from this expression that the correction is of alternating sign and vanishes at $\xi^2 = 2$. We note also that the nonlinear optical correction to χ'' differs in sign from the acoustic one.

5. DISCUSSION

It is easiest to determine in experiment the threshold power of the pump field $P_c \sim h_c^2$ and the imaginary part of the magnetic susceptibility χ''. The scattering of light by spin and magnetoacoustic waves can be assessed indirectly by studying the dependence of P_c and χ'' on the power of the incident light, or directly from the scattered light. A characteristic feature of the light scattered by spin and magnetoacoustic waves is its predominant polarization along the Z axis if the incident-light polarization vector is directed along the Y axis (see Sec. 2). This circumstance simplifies the experimental analysis of light scattered in a magnet.

Acoustic oscillations excited by electromagnetic and spin waves or magnetoacoustic waves can also be registered directly, or their presence can be assessed indirectly from measurement of P_c and χ''. In "pure form," mixed excitation of spin waves and magnetoacoustic waves by light and by an alternating magnetic field can be observed only when the no-threshold excitation of low-frequency sound by sound at $\Delta\nu < \Omega_{min}$ is eliminated. In this case the condition for mixed excitation of magnetoacoustic waves is given by expressions (19) and (27).

Let us estimate the light amplitude at which the influence of the light on the excitation of the spin and magnetoacoustic waves becomes noticeable, i.e., when both terms in (19) make comparable contributions. For yttrium iron garnet (YIG), which has a low spin-wave damping coefficient and is therefore frequently used in experiments on paramagnetic parametric excitation of spin waves, we have $M_0 = 10^3$ G, $\Gamma \sim 10^6\ \text{sec}^{-1}$, and $\omega_0 \sim 10^{10}\ \text{sec}^{-1}$ [17]. The Faraday rotation constant for light of 1μ wavelength (neodymium laser) is $\zeta \sim 10^{-6}\ \text{G}^{-1}$ [18], the Pockels constant is $P_{44} \approx 0.04$ [19], and $\gamma/\nu \sim 10^{-3}-10^{-4}$. Substituting these values in (19) we find that the contribution of the electromagnetic term $V_{12}^4|a_0|^4/\gamma^2$ in the case of mixed excitation of spin waves is appreciable at electric field amplitudes $E_0 \sim 10^5-10^6$ V/cm. Such electromagnetic-wave amplitudes, and even appreciably higher ones, are obtained at present by careful focusing of light from pulsed lasers. The transverse dimensions of the light beam are small in this case, and if they are smaller than the sample dimensions, then the strongest will be the influence of light on the excitation of the spin waves with wave vectors $q = -2k$, propagating along the light beam together with the back-scattered light. In this case the spin waves and the scattered light do not leave the excitation region and interact effectively with the incident light.

The interaction of the light with magnetoacoustic waves is more intense, and, as follows from (20), when the values given above are substituted, it becomes appreciable already at $E_0 \sim 10^3$ V/cm in the case of YIG. A qualitative picture of the function $\Delta P = P_c(0, q) - P_c(E^2, q)$ is shown in Fig. 1a. The first peak corresponds to the MAR region and the second to the back-scattering of light.

As follows from the results of Sec. 4, the low-frequency sound excited by light with large spectral width

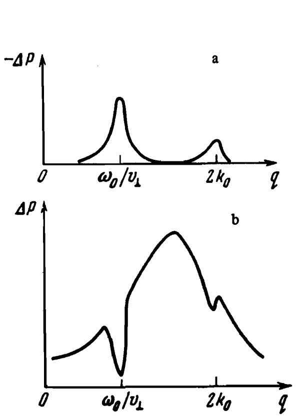

FIG. 1

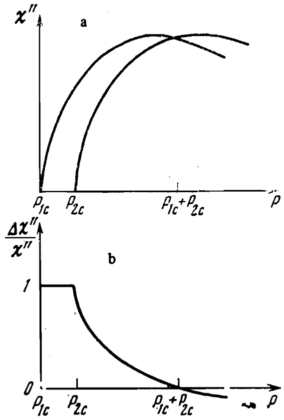

FIG. 2

$\Delta\nu > \Omega_{min}$ exerts the greatest influence on P_c in the region where $v_{\parallel} = v_q = \partial\omega_s/\partial q$. At $\Omega > \Delta\omega$ the scattering of the spin waves by sound leads to an increase of the effective damping of the PEW, and consequently to a raising of the threshold. A qualitative plot of the threshold-power increment due to the influence of the light and of the sound against the spin-wave vector q is shown in Fig. 1b. The maximum corresponds here to the value of q at which $v_{\parallel} = v_q$. The dips on the curve reflect the influence of the light in the MAR region also at $q = -2k$. We note that an appreciable raising of the PEW instability threshold when sound of frequency $\Omega > \Gamma$ is excited in the sample has been observed experimentally and described qualitatively in [14].

The obtained nonlinear increments to the effective PEW damping lead to changes of the magnetic susceptibility χ''. The character of the changes can be easily established by comparing the plots of $\chi''(h^2, \Gamma)$ against h^2 at two values of the effective spin-wave damping, Γ_1 and $\Gamma_2 > \Gamma_1$ (Fig. 2a). Figure 2b shows a plot of

$$\Delta\chi''/\chi'' = [\chi''(h^2, \Gamma_1) - \chi''(h^2, \Gamma_2)]/\chi''(h^2, \Gamma_1),$$

from which it is seen that $\Delta\chi''/\chi''$ is a function that is equal to unity at $\Gamma_1^2 < h^2 < \Gamma_2^2$ and decreases at $h^2 > \Gamma_2^2$. At $h^2 = \Gamma_1^2 + \Gamma_2^2$ the function $\Delta\chi''/\chi''$ vanishes. The plot of $\Delta\chi''/\chi''$ agrees with the approximate analytic relation (34) in the region where the expansion (34) is valid.

The influence of low-frequency sound with $\Omega < \Delta\omega$, the scattering from which leads not to a growth of the PEW damping but to modulation of these waves, should become manifest as a low-frequency oscillation of χ''. This phenomenon was observed experimentally in a number of studies, see [14,16].

Parametrically excited waves in YIG samples under the influence of a strong laser beam from a pulsed neodymium laser were investigated experimentally in a number of works [4-8]. Even in the first of these studies [4] it was observed that the PEW instability threshold is decreased by the action of the powerful light pulse on the YIG sample, and that the absorbed power is increased. It was noted there also that the increment of χ'' during the time of action of the light decreases with increasing pump power and even reverses sign. An appreciable lowering of the threshold (in accordance with the estimate given above, see also [3]) could occur in the MAR region. The decrease of $\Delta\chi''$ with increasing pump agrees with the earlier results. The appearance of low-frequency oscillations of χ'', observed in [4] at large pump levels, can be interpreted as the result of mixed excitation of automodulation [14,16] or acoustic oscillations in the sample, with frequencies $\Omega < \Gamma$. However, the results obtained in [4] are not complete enough to be able to make a more detailed comparison with theory. The dependence of $\Delta\chi''/\chi''$ on the light power and on q at large pump powers, $P/P_c \sim 10-15$ dB, was investigated in [5]. The decrease of χ'' above the threshold during the time of the action of the light is explained in [5] as being due to the predominance of the anti-Stokes scattering of the light $\nu_+ = \nu_0 + \omega$ at large values of P/P_c. This interpretation seems to be unconvincing for the following reasons. A consistent allowance for the Stokes and anti-Stokes scattering shows (Sec. 2, see also [2,3]) that the interaction of the light with the spin waves leads to a lowering of the threshold, i.e., the Stokes scattering predominates. The predominance of the Stokes scattering is easily revealed in experiment, since the first to be excited should be precisely the pair of spin waves having the minimal threshold. On the other hand, predominance of anti-Stokes scattering should lead to a suppression of the pair of spin waves that scatters the light resonantly. This means that in this case there should be excited in the sample all kinds of other pairs of waves with wave vectors lying in a plane perpendicular to the z axis, and these waves hardly interact with the light. The experimental observation of a predominant anti-Stokes scattering of light by parametric excited waves would therefore not be a simple matter.

Important details of the process of wave excitation in YIG subjected to the action of a laser in the case of parallel pumping are clarified in the papers of Solomko and Maĭstrneko [6] and of Agartanov and co-workers [7,8]. In these works, the experimental conditions are similar to those in [4,5], but more complete information is given on the dependence of $\Delta\chi''/\chi''$ on the spin-wave vector. In addition, in [8] is given the dependence of $\Delta\chi''/\chi''$ on the pump-field power and on the laser power. In [6], as well as in [7,8], a rise in the PEW threshold and a decrease of χ'' were observed when the sample was exposed to light. The plots of $\Delta\chi''/\chi''$ against q obtained in [6,7] have a clearly pronounced maximum and dips that duplicate qualitatively the form of the $\Delta P(q)$ curve shown in Fig. 1b. Only one dip on the $\Delta\chi''/\chi''$ curve was observed in [7] and was found to lie in the MAR region. In [6], two dips were observed, one located according to the authors in the MAR region, and the other in the region of back scattering of the light. It is also suggested in [6] that the increase of the PEW threshold and the decrease of χ'' are due to low-frequency acoustic oscillations excited in the sample by the light. The dependence of $\Delta\chi''/\chi''$ on the pump power, measured in [8], duplicates the one shown in Fig. 2b.

We see that the experimental results can be explained within the framework of the theoretical concepts deduced in [4-8]. However, a more complete comparison of theory with experiment and a determination of the processes that have not been taken into account in our theory (such as temperature effects connected with wave absorption in the sample), but play an important role under various conditions, call for more complete experimental research. In particular, it is desirable to analyze the acoustic oscillations produced in the sample during the time of action of the light, and also the scattered light. Using thin samples and lasers with narrow lines, one can exclude the excitation of low-frequency sound by light and by the same token separate the influence of the light on the PEW from that of the light-excited sound.

[1)]Electromagnetic and magnetoacoustic waves can excite also transverse sound, but the constant for the interaction of transverse sound with electromagnetic waves is noticeably smaller than that of longitudinal sound, so that nonlinear processes in which transverse sound takes part are less strongly manifest. We consider here only excitation of longitudinal sound. Similar reasoning can be used also for transverse sound and does not lead to qualitatively new results.

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