Temporal effects in resonant nuclear interactions of γ quanta with crystals

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The time dependence of the intensities of the resonant γ radiation scattered by or transmitted through an ideal crystal is investigated. The case when the source is located in the crystal itself is also investigated. The influence of the suppression of inelastic channels and of the interference of the nuclear and electron scattering, and also of the photoeffect and conversion, is analyzed. The problem of the decay of collective excited state in an ideal crystal is analyzed on the basis of the dynamic theory of the scattering of γ quanta.

1. INTRODUCTION

In experiments of the Mossbauer type, the presence of long-lived excited states of nuclei makes it possible, besides the usual measurement of the spectrum of the transmitted or scattered γ radiation, also to study their time evolution. For the case of passage through a resonant absorber, the time dependence of the radiation was considered by Lynch et al.[1]. The first results of measurements of the spectrum with time delay are given in the same paper, and also in a paper by Wu et al.[2]. Thieberger et al.[3] investigated theoretically and experimentally the time dependence of radiation scattered by a polycrystal. In all these papers, the time was reckoned from the instant when an excited isomeric state was produced in the source. This instant was fixed by the γ quantum of the preceding decay.

This paper is devoted to the time dependence of the spectrum of radiation passing through a thick perfect crystal under diffraction conditions, when the source is located either outside or inside the crystal. The analysis is based on the dynamic theory of interaction of resonant radiation with a crystal, developed by Kagan and Afanas'ev (see[4,5]). (The dynamic theory for the case when the source is inside the crystal was developed in a paper by Aleksandrov and Kagan[6].) Particular interest attaches here to a determination of the influence of the suppression effect (SE) of the inelastic channels of a nuclear reaction [4,5] on the character of the time evolution, and also to the role of interference between the resonance and Rayleigh scatterings and between the photoeffect and conversion (see[7,8]).

It should be noted that the question of the time dependence of the intensity of radiation emerging from a crystal was considered earlier by Pham Zuy Hien[9]. Unfortunately, the results of his paper, which deals with the most interesting case corresponding to satisfaction of the Bragg condition, are in error (see below).

Among the problems involving time, particular interest attaches to an investigation of the character of the evolution of the radiation emerging from a crystal under conditions when the experiment is performed with chopping of the incident beam. This problem arises in connection with the possibility of directly proving, within the framework of such an experiment, the existence of a nuclear exciton, which is a collective excited nuclear state smeared out over the crystal (see the papers of Kagan and Afanas'ev[10,11,5]). It is clear that radiation that emerges with a time delay from a crystal after complete interruption of an incident beam is connected only with the decay of the resultant excited states.

This problem is considered here in detail and it is proved, in particular, that rapid chopping does not prevent experimental proof of the existence of a nuclear exciton.

2. INFLUENCE OF THE SUPPRESSION EFFECT

A. We assume that the source is thin enough so that the absorption in it can be neglected. In addition, we take for simplicity an unsplit line in the source and in the crystal. Then the amplitude of the radiation incident on the crystal is given by

$$A(\omega)=A/(\omega-\omega_0+i\Gamma/2). \tag{2.1}$$

The time dependence of the amplitude of the radiation field passing through the crystal can be represented in the form

$$a_s(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{E}_s(\omega)\,e^{-i\omega t}\,d\omega, \tag{2.2}$$

where $\mathbf{E}_S(\omega)$ is the Fourier component of the field as it emerges from the crystal.

We consider a crystal in the form of a flat plate and confine ourselves to the Laue case. If a plane wave with wave vector κ_0 close to satisfaction of the Bragg condition is incident on the crystal, then the field at the point $\mathbf{r}$ inside the crystal assumes the following value (see[4]):

$$\begin{aligned}E_s(\omega,\mathbf{r})=\frac{A(\omega)\,e^{i\varkappa_0 r}}{2(\varepsilon_{0s}^{(2)}-\varepsilon_{0s}^{(1)})}\Big\{&\mathbf{e}_{0s}\Big[(2\varepsilon_{0s}^{(2)}-g_{00}^{s})\exp\Big(\frac{i\varkappa_0\varepsilon_{0s}^{(1)}\mathbf{nr}}{2\gamma_0}\Big)\\&-(2\varepsilon_{0s}^{(1)}-g_{00}^{s})\exp\Big(\frac{i\varkappa_0\varepsilon_{0s}^{(2)}\mathbf{nr}}{2\gamma_0}\Big)\Big]-\mathbf{e}_{1s}e^{i\mathbf{Kr}}g_{10}^{s}\Big[\exp\Big(\frac{i\varkappa_0\varepsilon_0^{(1)}\mathbf{nr}}{2\gamma_0}\Big)\\&-\exp\Big(\frac{i\varkappa_0\varepsilon_{0s}^{(2)}\mathbf{nr}}{2\gamma_0}\Big)\Big]\Big\}.\end{aligned} \tag{2.3}$$

Here

$$\begin{aligned}&\varepsilon_{0s}^{(1,2)}={}^1/_4(g_{00}^{s}+g_{11}^{s}-\alpha)\pm{}^1/_4[(g_{00}^{s}+g_{11}^{s}-\alpha)^2+4(\alpha g_{00}^{s}-\Delta_s)]^{1/2},\\&\Delta_s=g_{00}^{s}g_{11}^{s}-g_{01}^{s}g_{10}^{s},\ \gamma_0=\cos\sphericalangle(\varkappa_0\mathbf{n}),\ \alpha=\mathbf{K}(\mathbf{K}+2\varkappa_0)/\varkappa_0^2,\end{aligned} \tag{2.4}$$

where α is the deviation from the Bragg angle, $\mathbf{n}$ is the inner normal to the surface of the crystal, $\mathbf{K}$ is the reciprocal-lattice vector, and $\mathbf{e}_{1,2S}$ are two mutually perpendicular polarization vectors. The quantities $g^{S}_{\alpha\beta}$, which are proportional to the corresponding amplitudes for the scattering by an individual atom, can be represented in the simultaneous presence of the resonant nuclear and potential electron scattering in the form

$$\begin{aligned}g_{\alpha\beta}^{s}=\frac{4\pi}{\varkappa_0^2\Omega_0}\sum_j\exp[i(\varkappa_\alpha-\varkappa_\beta)\rho_j]\Big\{&\exp\Big[\frac{-Z_j(\varkappa_\alpha)-Z_j(\varkappa_\beta)}{2}\Big]f_{nj}^{s}(\varkappa_\alpha,\varkappa_\beta)\\&+\exp\Big[-\frac{Z_j(\varkappa_\alpha-\varkappa_\beta)}{2}\Big]f_{ej}^{s}(\varkappa_\alpha,\varkappa_\beta)\Big\}.\end{aligned} \tag{2.5}$$

Here $Z_j(\kappa)$ is the Debye-Waller factor, f^{S}_{nj} and f_{ej} are

the coherent parts of the nuclear and electronic amplitudes for scattering by the j-th atom in the unit cell:

$$f_{nj}^{s}(\varkappa_\alpha, \varkappa_\beta) = -\frac{1}{2\varkappa_0}\frac{\Gamma_1}{\omega-\omega_0'+i\Gamma/2}\frac{2I+1}{2I_0+1}P_n^s(\alpha,\beta),$$
$$f_{ej}(\varkappa_\alpha, \varkappa_\beta) = -r_0 F_j(|\varkappa_\alpha-\varkappa_\beta|)P_e^s(\alpha,\beta), \quad (2.6)$$

F_j is the atomic factor of the j-th atom, r_0 is the classical radius of the electron, and $P_{n,e}^{S}$ are the polarization factors.

We shall henceforth deal for simplicity with the crystal in which the conditions

$$\exp(i\mathbf{K}\rho_j)=1, \quad Z_j(\varkappa_0)=Z_j(\varkappa_1) \quad (2.7)$$

are satisfied, with $g_{11}^{S} = g_{00}^{S}$ and $g_{01}^{S} = g_{10}^{S}$.

If the radiation is incident on a crystal exactly at the Bragg angle ($\alpha = 0$), then the field inside the crystal is described by a superposition of four waves. It was shown in[4] that for one polarization, in the case of an E1 transition ($\mathbf{e}_{0s}$ perpendicular to the scattering plane ($\varkappa_0\varkappa_1$), which corresponds to s = 1), and for the second polarization for the transition M1 (both polarization vectors in the scattering plane, s = 2), the amplitude for the production of an excited state for one pair of waves vanishes. As a result, this wave pair propagates in the crystal without absorption, whereas the other pair, to the contrary, is absorbed more strongly than in the usual case. Let us determine the time dependence of the radiation passing through the crystal.

Substituting (2.3) in (2.2), we obtain at unity incident-radiation intensity, in the case of polarization where the suppression effect is fully pronounced (i.e., $P_n^S = 1$) and $\omega_0 = \omega_0'$:

$$\mathbf{a}_s(t) = {}^1/_2\exp(i\varkappa_0\mathbf{r}-i\omega_0 t-\Gamma t/2)\{\mathbf{e}_{0s}[1+J_0(2\sqrt{2Bt})] + \mathbf{e}_{1s}e^{i\mathbf{K}\mathbf{r}}[-1+J_0(2\sqrt{2Bt})]\}, \quad (2.8)$$
$$B=\frac{\varkappa_0 l}{2\gamma_0}g_0\frac{\Gamma}{2}, \quad g_0=|g_{00}^{s}(\omega=\omega_0')|;$$

here l is the thickness of the crystal. The terms of (2.8) with the Bessel function in the square brackets correspond to two strongly-absorbing waves. In a thick crystal at Bt $\gg$ 1, these terms become negligibly small and the radiation emerges from the crystal without changing its time dependence at all. It is interesting that as a result of the suppression effect it is as though no capture of γ quanta with excitation of the nuclei occurs in the crystal and by the same token the temporal "dragging" of the radiation does not take place.

It follows from (2.8), in particular, that in the limit of a thin crystal (Bt $\ll$ 1) the intensity of the diffracted wave (the second term in (2.8)) takes the form

$$I_1(t) = e^{-\Gamma t}(Bt)^2. \quad (2.9)$$

The quadratic dependence on the crystal thickness is connected with the coherent character of the interaction of the γ quanta with the nuclei. A quadratic dependence on the time also arises in the case of independent scattering by individual nuclei in a thin layer, as was first obtained by Thieberger et al.[3]

For the second polarization, all four waves attenuate rapidly in the general case. The time dependence of the amplitude then takes the form

$$\mathbf{a}_s(t) = {}^1/_2\exp[i\varkappa_0\mathbf{r}-i(\omega_0-i\Gamma/2)t]\left\{\mathbf{e}_s[J_0(2\sqrt{2aBt})+J_0(2\sqrt{(1-a)2Bt})] + \mathbf{e}_{1s}e^{i\mathbf{K}\mathbf{r}}\frac{p}{1-2a}[-J_0(2\sqrt{2aBt})+J_0(2\sqrt{(1-a)2Bt})]\right\}. \quad (2.10)$$

We have introduced here the following notation for the polarization factors:

$$P_n^s(0,1)\equiv p=\cos\varphi_{01}, \quad \varphi_{01}=\sphericalangle(\varkappa_0,\varkappa_1),$$
$$a=\frac{1-p^2}{4}\left(1-\frac{1-p^2}{4}\right).$$

If p tends to unity, then the time dependence of the amplitude becomes close to (2.8). This corresponds to the appearance of a partial suppression effect.

Let us consider the case when the radiation is incident on the crystal at an angle greatly deviating from the Bragg angle ($\alpha \gg g_0$). The amplitude of the reflected wave (second term in (2.3)) is then negligibly small. The time dependence of the intensity of the radiation passing through the crystal is given by

$$I_0(t) = e^{-\Gamma t}J_0^2(2\sqrt{Bt}). \quad (2.11)$$

This result, which is true for both polarizations, was first obtained by Lynch, Holland, and Hamermesh[1].

We note that allowance for the vanishing of the elastic width Γ_1 when γ quanta move in an ideal crystal (see[10]) leads to a formula somewhat different from (2.11), but that in a sufficiently thick crystal (Bt $\gg$ 1) we again arrive at an expression analogous to (2.11), where the width Γ in the argument of the exponential should be taken to mean the total width of the line in the source, and the coefficient B contains Γ_2 instead of Γ.

Let us consider the question of the time dependence of the intensity for radiation emerging from a crystal, i.e., a source in the form of excited nuclei is situated inside a crystal containing resonant nuclei in the ground state. The question of the angular distribution of the γ radiation was investigated in[6]. In the most interesting case of a thick crystal, as was shown above, the radiation emerges from the crystal only along surfaces of cones with axis along the reciprocal-lattice vector and with apex angles $90^\circ - \theta_B$ (θ_B is the Bragg angle). The character of the angular distribution was fundamentally different depending on whether the radiating nucleus is at a site or in an interstice.

In[6] we used a reciprocity theorem that connects the radiation field produced inside the crystal by an external source with the radiation field outside the crystal produced in γ decay of nuclei situated in the crystal. This theorem was used in[6] for the Fourier components of the field, but it is easy to show that a similar relation also holds true for the time-dependent field amplitudes, and in particular for $\mathbf{a}_s(t)$.

Let the radiating nucleus be situated in an interstice, and let its position be specified by the relative coordinate $\boldsymbol{\rho}$. We consider the time dependence of the radiation emerging in a direction satisfying the Bragg condition. Using the results of[6], we obtain directly

$$I(t) = \xi e^{-\Gamma t}|1-e^{i\mathbf{K}\rho}+J_0(2\sqrt{2Bt})(1+e^{i\mathbf{K}\rho})|^2. \quad (2.12)$$

Here ξ is a constant that does not depend on the time, and B contains the distance from the radiating nucleus to the surface of the crystal. It is seen from (2.12) that if the decaying nucleus is situated at a sufficiently large depth, then the emerging radiation has the same time dependence as in the case of total absence of resonant interaction between the γ quanta and the crystal nuclei. This is again a direct manifestation of the suppression effect.

It is interesting that if the radiating nucleus is situated at a site of the crystal lattice, then the time picture

becomes radically different. Indeed, using the same results, we obtain

$$I(t)=\xi e^{-\Gamma t}J_0^2(2\sqrt{2Bt}). \qquad (2.13)$$

In comparison with the preceding result, the time dependence is greatly altered. The reason is that by virtue of the suppression effect, the excitation amplitude vanishes for nuclei located at the lattice sites, and consequently the probability of emission of a coherent superposition of a wave pair producing an undamped state in the crystal also vanishes. Thus, the emerging radiation is connected only with the second pair of the strongly absorbing waves, and we thus arrive at (2.13).

If the exact Bragg condition is not satisfied and $\alpha \sim g_0$, the time dependence of the emerging radiation is strongly altered, and the intensity decreases with time much more weakly than in (2.13). The reason is that now the probability of generation of the first pair of waves in the decay is finite, and the damping is still weak (for more detail, see[6]). It is interesting that the time dependence of the intensity thus turns out in the general case to be very sensitive to a small change in the direction of the emission of the decay γ quanta from the crystal.

If the γ quanta emerging from the crystal are observed in an arbitrary direction ($\alpha \gg g_0$), the time dependence naturally remains exactly the same as in the case of (2.11) with the appropriate value of l.

As already noted in the Introduction, the question of the emission of excited nuclei from a crystal was considered in[9]. The expressions obtained in that paper for the intensity of the radiation from the crystal on the Bragg angle are naturally similar to (2.11), but the results corresponding to satisfaction of the Bragg angle do not coincide with (2.13) (the decaying nucleus was situated at a lattice site) due to incorrect accounting for the coherent picture in the crystal.

3. INFLUENCE OF INTERFERENCE ON THE CHARACTER OF THE TIME DEPENDENCE

We consider the interference between elastic resonant scattering by a nucleus and Rayleigh scattering by electrons. As seen from (2.5), the quantities $g^S_{\alpha\beta}$ can be represented in the case of interest to us in the form $g^S_{\alpha\beta} = g_N + g_e$. For simplicity we neglect absorption of the resonant γ quanta by the electrons, and by the same token regard g_e as a real quantity. When the radiation is incident on the crystal far from the Bragg angle, the field in the crystal is described by one wave, and in view of the fact that g_e does not depend on ω, we obtain for the time-dependent intensity a result similar to (2.11).

On the other hand, if the radiation with a polarization that is better from the point of view of the suppression effect is incident at the Bragg angle ($\alpha = 0$), then we obtain the following result for the field amplitude (we have introduced here the shift $\Delta\omega$ between the lines in the source and in the crystal):

$$\begin{aligned} a_s(t)=&\frac{1}{2}\exp\left[i\varkappa_0 r-i\left(\omega_0-\frac{i\Gamma}{2}\right)t\right]\cdot\\ &\times\left\{e_s\left[1+e^{2iB_e}\sum_{m=0}^{\infty}(i\Delta\omega)^m\left(\frac{t}{2B}\right)^{m/2}J_m(2\sqrt{2Bt})\right]\right.\\ &\left.+e_{1s}e^{ikr}\left[-1+e^{2iB_e}\sum_{m=0}^{\infty}(i\Delta\omega)^m\left(\frac{t}{2B}\right)^{m/2}J_m(2\sqrt{2Bt})\right]\right\}. \end{aligned} \qquad (3.1)$$

We have put here $B_e = \kappa_0 g_e l/2\gamma_0$. We see that the interference manifests itself both in the direct and in the diffracted wave. In a thick crystal, the two rapidly-absorbed waves drop out, and no interference takes place between the remaining pair of waves, i.e., electron scattering does not influence the time dependence. The clearest manifestation of the interference occurs in diffraction by a thin crystal. For the intensity of the diffracted wave we obtain from (3.1) at $Bt \ll 1$

$$I_1(t)=e^{-\Gamma t}\left(B_e^2+\frac{2B^2}{\Delta\omega^2}(1-\cos\Delta\omega t)+\frac{2B_eB}{\Delta\omega}(1-\cos\Delta\omega t)\right). \qquad (3.2)$$

The first and second terms of this formula correspond to potential scattering by electrons, which does not change the time dependence, and to nuclear resonant scattering. The third term is the manifestation of the interference of these two types of scattering.

The interference of two inelastic processes, photoeffect and conversion, were first considered in[8] (see also[12]). If we have a nuclear transition of type E1 and the projection of the spin of the ground state of the nucleus is not altered by the interaction, then the final states of the two processes are indistinguishable and a unique interference should be observed in the total cross section; this interference was first observed in[7] and leads to violation of the symmetry of the line in transmission through the absorber. The total cross section is then given by (see[8])

$$\begin{aligned} \sigma_t=&\sigma_{ph}+\frac{\hbar\Gamma_1\Gamma}{2\varkappa_0^2}\frac{2I+1}{2I_0+1}e^{-Z(\varkappa_0)}\frac{1+2\beta_\gamma(\omega-\omega_0)/\Gamma}{(\omega-\omega_0)^2+\Gamma^2/4},\\ \beta_\gamma=&\frac{2\alpha}{\alpha+1}\left[\frac{2I+1}{3(2I_0+1)}\frac{\sigma_{ph}}{\sigma_0^c}\right]^{1/2}. \end{aligned} \qquad (3.3)$$

Here α is the conversion coefficient, σ_{ph} is the cross section of the photoeffect, and σ_0^c is the conversion cross section at resonance. Returning to expressions (2.5) and (2.6), we recall that all the formulas presented above contain only the elastic part of the scattering amplitude. Thus, to take into account the interference of the photoeffect and conversion it is necessary to have a more accurate expression for the imaginary part of the scattering amplitude.

As is well known, the total cross section is connected with the aid of the optical theorem with the imaginary part of the zero-angle elastic-scattering amplitude:

$$\operatorname{Im} f=\varkappa\sigma_t/4\pi. \qquad (3.4)$$

Substituting σ_t from (3.3), we obtain Im f. Then, from the known imaginary part we reconstruct the real part of the scattering amplitude with the aid of the dispersion relation. Adding them, we obtain

$$f(\omega)=-\frac{2I+1}{2I_0+1}e^{-Z(\varkappa_0)}\frac{1}{2\varkappa_0}(1-i\beta_\gamma)\frac{\Gamma_1}{\omega-\omega_0+i\Gamma/2}+f_e. \qquad (3.5)$$

Further, taking the polarization factors into account, we substitute this scattering amplitude in the expression for $g^S_{\alpha\beta}$ and obtain the time dependence in the usual manner.

Let a beam of γ quanta be incident on the crystal in an arbitrary direction; we then obtain for the intensity of the transmitted radiation

$$\begin{aligned} I_0(t)=&e^{-\Gamma t}|J_0(2\sqrt{Bt(1-i\beta\gamma)})|^2e^{-L},\\ L=&\sigma_{ph}l/\gamma_0v_0. \end{aligned} \qquad (3.6)$$

We have also taken the photoeffect into account in the imaginary part of the amplitude for scattering by electrons, and this has led to the appearance in (3.6) of a factor that attenuates exponentially with depth. For a

thick crystal (Bt $\gg$ 1), using the asymptotic form of the Bessel function, we obtain for $\beta_\gamma \ll 1$:

$$I_0(t)=\frac{e^{-\Gamma t-L}}{\pi\sqrt{Bt}}\left[\exp\sqrt{\frac{2\Gamma_2 tL}{3}}+2\cos\left(4\sqrt{Bt}-\frac{\pi}{2}\right)+\exp\left(-\sqrt{\frac{2\Gamma_2 tL}{3}}\right)\right] \tag{3.7}$$

Here Γ_2 is the inelastic width of the nuclear level. The appearance of two exponential terms in the square brackets of (3.7) is due entirely to interference between the photoeffect and the conversion. Such an appreciable influence of the interference enables us to investigate the time dependence of this phenomenon experimentally. It must be noted that the interference between the photoeffect and conversion becomes manifest in transmission experiments, precisely where the interference between the nuclear and Rayleigh scatterings does not come into play.

Let us consider the case of incidence of the γ quanta at the Bragg angle. For the time dependence of the intensity of the radiation diffracted by a thin crystal we have

$$I_1(t)=e^{-\Gamma t}\left(B_e^2+\frac{2B^2}{\Delta\omega^2}(1+\beta_\gamma^2)(1-\cos\Delta\omega t)+\frac{2B_eB}{\Delta\omega}(1-\cos\Delta\omega t)+2B_eB\beta_\gamma\frac{\sin\Delta\omega t}{\Delta\omega}\right). \tag{3.8}$$

Comparing (3.8) with (3.2), we see that the interference of the photoeffect and the conversion has led to a change in the principal term of the nuclear scattering. Further, an additional term has appeared, which depends on both types of interference and has a time dependence that is essentially different from that of the other terms.

4. DECAY OF COLLECTIVE STATE

We consider the time-dependent problem with chopping. This problem is of importance when it comes to observing the decay of a collective state in a crystal[10]. Assume that the law governing the chopper operation is

$$\left[1+\exp\left(\frac{t-t_1}{\tau}\right)\right]^{-1}. \tag{4.1}$$

Using the time dependence of the radiation of a thin source and the law governing the chopping, we find the emission spectrum past the chopper with the aid of the Fourier transformation:

$$B_{t_0}(\omega)=\sqrt{\Gamma}\int_{t_0}^{\infty}\exp\left[\left(-i\omega_0-\frac{\Gamma}{2}\right)(t-t_0)+i\omega t\right]\left[1+\exp\left(\frac{t-t_1}{\tau}\right)\right]^{-1/2}dt. \tag{4.2}$$

Here t_0 is the instant when the excited state is produced in the source. Now, knowing the incident-radiation spectrum and the frequency dependence of the field in the crystal (2.3), we can obtain, using the inverse Fourier transformation, the time dependence of the amplitude of the radiation emerging from the crystal:

$$\mathbf{a}_{t_0}(t)=\int_{-\infty}^{\infty}d\omega\,B_{t_0}(\omega)\mathbf{E}_s(\omega)e^{-i\omega t}. \tag{4.3}$$

In this problem we do not fix the instant when the excited state is produced in the source, and therefore the intensity must be averaged over t_0, so that we obtain the intensity registered by the detector:

$$I(t)=i_0\int_{-\infty}^{t}|\mathbf{a}_{t_0}(t)|^2dt_0. \tag{4.4}$$

Here i_0 is the number of decays without recoil in the source in a given solid angle per unit time. The integration in (4.4) is over the entire measurement interval t_0.

The integral in (4.2) can be calculated (see[13]), and we have

$$B_{t_0}(\omega)=\frac{\tau\sqrt{\Gamma}}{2\pi i}\frac{1}{1/2-i\Omega\tau}\exp\left(-i\omega t_0+\frac{t_1-t_0}{2\tau}\right)\cdot\times F\left(\frac{1}{2},\frac{1}{2}-i\Omega\tau,\frac{3}{2}-i\Omega\tau;-\exp\left(\frac{t_1-t_0}{\tau}\right)\right), \tag{4.5}$$

where F is the hypergeometric function and $\Omega = \omega - \omega_0 + i\Gamma/2$.

Let the beam past the chopper be incident on a thin crystal at an angle greatly deviating from the Bragg angle. Then, substituting (4.5) and (2.3) in (4.3) we obtain for the decay amplitude

$$\mathbf{a}_{t_0}(t)=\mathbf{e}_s\exp[i\boldsymbol{\varkappa}_0\mathbf{r}+i(\omega_0-i\Gamma/2)(t_0-t)]\sqrt{\Gamma}\left\{i\left[1+\exp\left(\frac{t-t_0}{\tau}\right)\right]^{-1/2}-\frac{B}{i/2\tau-\Delta\omega}\exp\left(\frac{t_1-t}{2\tau}\right)F\left(\frac{1}{2},\frac{1}{2}+i\Delta\omega\tau,\frac{3}{2}+i\Delta\omega\tau;-\exp\left(\frac{t_1-t}{\tau}\right)\right)+\frac{B}{i/2\tau-\Delta\omega}\exp\left[i\Delta\omega(t-t_0)+\frac{t_1-t_0}{2\tau}\right]\times F\left(\frac{1}{2},\frac{1}{2}+i\Delta\omega\tau,\frac{3}{2}+i\Delta\omega\tau;-\exp\left(\frac{t_1-t_0}{\tau}\right)\right)\right\}. \tag{4.6}$$

On the other hand, if $\Delta\omega = 0$, then the hypergeometric series that enter here can be summed (see[13]) and (4.6) goes over into

$$\mathbf{a}_{t_0}(t)=i\mathbf{e}_s\exp\left[i\boldsymbol{\varkappa}_0\mathbf{r}+i\left(\omega_0-\frac{i\Gamma}{2}\right)(t_0-t)\right]\sqrt{\Gamma}\cdot\left\{\left[1+\exp\left(\frac{t-t_1}{\tau}\right)\right]^{-1/2}+2B\tau\ln\left[\exp\left(\frac{t_1-t}{2\tau}\right)+\left[\exp\left(\frac{t_1-t}{\tau}\right)+1\right]^{1/2}\right]-2B\tau\ln\left[\exp\left(\frac{t_1-t_0}{2\tau}\right)+\left[\exp\left(\frac{t_1-t_0}{\tau}\right)+1\right]^{1/2}\right]\right\}. \tag{4.7}$$

The first term in these two formulas corresponds to radiation that does not interact with the crystal. The remaining two terms correspond to the fraction of γ quanta which have become absorbed and produced the excited state as a result of resonant interaction with the nuclei.

Let us consider the case of rapid chopping $\tau \ll 1/\Gamma$, and let $t - t_1 \gg \tau$. The condition $\tau \ll 1/\Gamma$ causes the time interval t_0 making the principal contribution to formulas (4.6) and (4.7) to satisfy the condition $t_1 - t_0 \gg \tau$. Thus, we have the following conditions:

$$\tau\ll 1/\Gamma,\quad t-t_1\gg\tau,\quad t_1-t_0\gg\tau. \tag{4.8}$$

When the first and second conditions from (4.8) are satisfied, it is possible to retain in (4.6) and (4.7) only the third term, whereas the first and second terms are small and are of the order of $\exp\{(t_1 - t)/2\}$.

From the form of the expression for the amplitude it follows that decay from the crystal past the chopper will occur only in direction of the vector $\boldsymbol{\varkappa}_0$, which characterizes the initial direction of the γ quanta. This important circumstance pertains, of course, only to the coherent part of the decay, i.e., to decay that does not preserve a trace of the position of the exciting nucleus. If, however, we have, for example, a change of the nuclear spin in the decay, then the intensity becomes distributed over the entire angle interval.

The incoherent part of the decay was not included in the expression for the radiation-field amplitude from the very outset because since the solution (2.3) corresponds only to the coherent part of the field. The presence of incoherent scattering is equivalent to effective absorption and is automatically taken into account in the elastic-scattering amplitude. The collective state should become manifest precisely in the coherent decay, and it

is important that the rapid chopping preserve fully the angular directivity of the radiation.

Let us find the intensity of the forward decay. We consider directly the case when $\Delta\omega \neq 0$ and the quantities $\Delta\omega(t-t_0)$ and $\Delta\omega(t-t_1)$ are arbitrary. In accordance with the conditions (4.8), we neglect the first and second terms in (4.6), and in the hypergeometric function of the third term we carry out an analytic continuation into the region $\exp\{(t_1-t_0)/\tau\} > 1$ and substitute the obtained expression in (4.4). The last of the conditions (4.8) makes it possible to carry out the integration with respect to t_0 only up to t_1, as the result of which we obtain for the decay intensity per unit time

$$I_0(t)=i_0\frac{2B^2}{\Delta\omega^2+\Gamma^2}e^{-\Gamma(t-t_1)}. \tag{4.9}$$

Let us analyze this formula. The number of excited states N* in the crystal at the instant of shutoff (at $\tau \ll 1/\Gamma$) is determined simply from the condition

$$i_0\tilde{\sigma}_t(\Delta\omega)\,l/a^3=N^*W_t^*. \tag{4.10}$$

Here W_t^* is the total decay probability of the excited nucleus, a is the interatomic distance, and $\tilde{\sigma}_t(\Delta\omega)$ is the total scattering cross section averaged over the frequency distribution of the incident γ quanta, i.e.,

$$\tilde{\sigma}_t(\Delta\omega)=\frac{2\pi}{\Gamma}\int_{-\infty}^{\infty}\sigma_t(\omega)\,\frac{(\Gamma^2/4)\,d\omega}{(\omega-\omega_0)^2+\Gamma^2/4}$$
$$=\frac{2\pi}{\varkappa_0^2}\frac{\Gamma_1\Gamma}{\Delta\omega^2+\Gamma^2}\frac{2I+1}{2I_0+1}e^{-Z(\varkappa_0)}.$$

It is seen from this relation that the appearance of the squared width in the denominator of (4.9) is connected with the foregoing averaging over the frequency distribution in the source. Thus, the number of excited states is equal to

$$N^*=\frac{i_0 2\pi\Gamma\Gamma_1 l\,(2I+1)\,e^{-Z(\varkappa_0)}}{\varkappa_0^2a^3\,(\Delta\omega^2+\Gamma^2)\,(2I_0+1)\,W_t^*}. \tag{4.11}$$

Returning to (4.9), we represent the intensity at the instant $t-t_1$ in the form $I_0 = N^*W^*$. Here W* is the decay probability of the excited state in the crystal, and

$$W^*=W_0\pi l/(\varkappa_0 a)^2\gamma_0, \tag{4.12}$$

where W_0 is the probability of the elastic decay of the individual nucleus. It is interesting to note that this agrees fully with the result predicted by Afanas'ev and Kagan[10]. It has thus become possible to separate two processes—the formation of a collective excited state (see (4.11)) and its decay (4.12). The exponential dependence of the intensity in (4.9) on the time corresponds to a decrease in the number of excited states with increasing time.

When the external beam is incident at the Bragg angle, we can observe the decay of the collective state both forward and at the Bragg angle. Substituting (2.3) at $\alpha = 0$ in the thin-crystal limit in (4.3), we obtain for the forward-decay amplitude an expression similar to (4.6), but with the substitution $B \to 2B$. The amplitude of the decay at the Bragg angle is given by

$$a_{1,/_0}(t)=e_{1s}\frac{2B}{\Delta\omega-i/2\tau}\exp\left[i(\varkappa_0+\mathbf{K})\mathbf{r}+i\left(\omega_0-\frac{i\Gamma}{2}\right)(t_0-t)\right]\cdot$$
$$\times\sqrt{\Gamma}\left[\exp\left(\frac{t_1-t}{2\tau}\right)F\left(\frac{1}{2},\frac{1}{2}+i\Delta\omega\tau,\frac{3}{2}+i\Delta\omega\tau;-\exp\left(\frac{t_1-t}{\tau}\right)\right)\right. \tag{4.13}$$
$$\left.-\exp\left[i\Delta\omega(t-t_0)+\frac{t_1-t_0}{2\tau}\right]F\left(\frac{1}{2},\frac{1}{2}+i\Delta\omega\tau,\frac{3}{2}+i\Delta\omega\tau;-\exp\left(\frac{t_1-t_0}{\tau}\right)\right)\right]$$

We see therefore that the decay occurs in the direction of the vector $\boldsymbol{\kappa}_0 + \mathbf{K}$. Here, unlike the forward-decay amplitude, there is no term that corresponds to radiation that has not reacted with the crystal. This circumstance makes decay at the Bragg angle more convenient from the point of view of experiments aimed at observing the collective state. Using (4.8) and substituting (4.13) in (4.4), we obtain for the intensity of the decay at the Bragg angle

$$I_1(t)=\frac{i_0\cdot 8B^2}{\Delta\omega^2+\Gamma^2}e^{-\Gamma(t-t_1)}. \tag{4.14}$$

The intensities of the forward decay and of the decay at the Bragg angle are equal.

Let us examine the influence of the interference of nuclear and electronic interactions, and also that of the photoeffect and conversion, on the decay of a collective state. We substitute the elastic part of the scattering amplitude (3.7), determined with allowance for the interference between the photoeffect and the conversion, into the expression for $g^S_{\alpha\beta}$. Let an external beam be incident on the crystal at an angle remote from the Bragg angle. Calculations analogous to those used in the derivation of (4.9) yield for the forward-decay intensity

$$I_0(t)=i_0\frac{2B^2}{\Delta\omega^2+\Gamma^2}(1+\beta_\gamma{}^2)\,e^{-\Gamma(t-t_1)}. \tag{4.15}$$

Here, just as in the time dependence without chopping, the elastic scattering by the electrons exerted no effect on the intensity, and the interference between the photoeffect and the conversion was manifested in the form of a small increment that did not change the frequency dependence.

On the other hand, if the beam is incident on the crystal at the Bragg angle, then the intensity of the diffracted radiation will contain interference both between the electron Rayleigh and the nuclear scattering and between the photoeffect and conversion. The Rayleigh scattering is not connected with the formation of a long-lived excited state, so that the terms corresponding to this process turn out to be proportional to $(1+\exp(t-t_1)/\tau)^{-1}$, while the interference terms are proportional to the square root of this quantity. Consequently, in the limit of rapid chopping, i.e., when conditions (4.8) are satisfied, the Rayleigh scattering and its interference with the nuclear scattering can be neglected, and an expression similar to (4.15) can again be used for the intensity of the decay of a collective state at the Bragg angle.

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