

Paramagnetic effect of the surface "mixed" state in type-I superconductors

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A two-dimensional "mixed" state is observed at the inner surface of a hollow cylindrical superconductor with a current. The response of the system to a weak longitudinal magnetic field is investigated. Calculation reveals the presence of a paramagnetic effect under these conditions.

1. A surface "mixed" state can arise upon destruction of type-I superconductivity by an electric current in a multiply connected sample. This surface "mixed" state, which was predicted by L. D. Landau in 1938^[1] for a hollow cylindrical superconductor, was subsequently discovered and investigated by I. L. Landau and Sharvin,^[2,3] and studied theoretically by Andreev and Tekel'^[4] and by Andreev and the author.^[5] The surface "mixed" state arises on the inner surface of a hollow cylinder in those cases in which the applied current is so strong that the radius of the region of the intermediate state^[6-8] becomes smaller than the radius of the inner surface of the cylinder. Here the purely normal state must be unstable, because of the fact that the magnetic field of the current is equal to zero inside the cylinder and is less than the critical magnetic field H_c near the inner surface of the cylinder. A purely superconducting layer is also impossible, since, as a consequence of the limited conductivity of the sample and the condition of continuity of the electric field at the phase boundary, an electric field should exist in the entire region, right up to the inner surface of the cylinder. In the theoretical work of Andreev and the author,^[5] the surface "mixed" state is described in terms of superconducting fluctuations,^[9,10] which arise close to the inner surface and are amplified or destroyed by the electric current. The authors calculated the value of the fluctuation current under the assumption that it is much smaller than the normal current in the "mixed" state of the surface layer. This theory is applicable to pure metals near J_{c2} and to alloys even in the case of currents that are small in comparison with J_{c2} , where J_{c2} is the second critical current, the concept of which was introduced by Andreev.^[11]

The aim of the present work is to study the response of the system to a magnetic field applied in the direction of the cylindrical axis for the case of alloys, where the applied current J is smaller than the second critical current. The calculations that are set down made clear the presence of a paramagnetic effect. A similar effect was discovered by I. L. Landau^[14] in his studies with pure metals.

2. In our calculations, we followed the method of calculation developed by Andreev and the author,^[5] but in the present case we took into account the presence of a longitudinal magnetic field H_a .

We consider a hollow cylindrical type-I superconductor with an inner radius r_1 and outer radius r_2 , along the axis of which an electric current J and a magnetic field H_a are applied. As a consequence of our assumption that the fluctuation current is small, the magnetic field perpendicular to the axis of the cylinder is determined principally by the field of the normal current

$$H_z(r) = \frac{2J}{cr} \frac{r^2 - r_1^2}{r_2^2 - r_1^2}.$$

The mixed state arises only near the inner surface ($r \approx r_1$), where the magnetic field can be expressed in the form

$$H_z(x) = \frac{4J}{c} \frac{x}{r_2^2 - r_1^2};$$

$x = r - r_1$ is the distance from the inner surface. The total vector potential of the magnetic field and the electric field is equal to

$$A_x = 0, \quad A_y = H_a x, \quad A_z = \frac{-2J}{c} \frac{x^2}{r_2^2 - r_1^2} - cEt. \quad (1)$$

A convenient method for estimation of the fluctuation current is the introduction of an external force in the time equation for the superconducting order parameter,^[12,13] which then takes the form

$$\frac{\partial \psi}{\partial t} - \nu \left\{ \psi - \xi^2 \left(\nabla - \frac{2ie}{c} \mathbf{A} \right)^2 \psi \right\} = f(\mathbf{r}, t), \quad (2)$$

where m is the mass of the electron, $\xi = \xi(T)$ is the coherence length, $\nu = 8(T_c - T)/\pi$, and $f(\mathbf{r}, t)$ is the external force, which satisfies the condition

$$\langle f(\mathbf{r}, t) f^*(\mathbf{r}', t') \rangle = 4mT\nu \xi^2 \delta(\mathbf{r} - \mathbf{r}') \delta(t - t'). \quad (3)$$

We transform to the dimensionless quantities

$$\begin{aligned} \tilde{\mathbf{r}} &= \frac{\mathbf{r}}{\xi}, & \tilde{t} &= \nu t, & \tilde{f} &= \frac{1}{2\nu} \left(\frac{\xi}{mT} \right)^{1/2} f, \\ \tilde{\psi} &= \frac{1}{2} \left(\frac{\xi}{mT} \right)^{1/2} \psi, & \epsilon &= \frac{2eE\xi}{\nu}, \\ \beta &= \frac{4eJ}{c^2} \frac{\xi^3}{r_2^2 - r_1^2}, & \gamma &= \frac{2e\xi^2}{c} H_a. \end{aligned} \quad (4)$$

Then Eq. (2) takes the form

$$\frac{\partial \tilde{\psi}}{\partial \tilde{t}} - \tilde{\psi} + \mathcal{L}\tilde{\psi} = \tilde{f}(\tilde{\mathbf{r}}, \tilde{t}), \quad (5)$$

where

$$\begin{aligned} \langle \tilde{f}(\tilde{\mathbf{r}}, \tilde{t}) \tilde{f}^*(\tilde{\mathbf{r}}', \tilde{t}') \rangle &= \delta(\tilde{\mathbf{r}} - \tilde{\mathbf{r}}') \delta(\tilde{t} - \tilde{t}'), \\ \mathcal{L} &= -\frac{\partial^2}{\partial \tilde{x}^2} - \left(\frac{\partial}{\partial \tilde{y}} - i\gamma \tilde{x} \right)^2 - \left(\frac{\partial}{\partial \tilde{z}} + i\epsilon \tilde{z} + i\beta \tilde{x}^2 \right)^2. \end{aligned} \quad (6)$$

In the discussion that follows, we shall use only dimensionless quantities and can therefore omit the tilde over the corresponding letters.

3. We seek a solution of Eq. (5) in the form

$$\psi = \sum_{k_y, k_z, n} a_{k_y k_z}^{(n)}(t) \psi_{k_y k_z}^{(n)}(\mathbf{r}, t), \quad (7)$$

where

$$\begin{aligned} \psi_{k_y k_z}^{(n)}(t) &= \frac{1}{L} e^{i(k_y y + k_z z + \epsilon t)} \varphi_{k_y k_z}^{(n)}(x) \\ (k &= k_x + \epsilon t) \end{aligned}$$

is the eigenfunction of the operator \mathcal{L} :

$$\mathcal{L} \psi_{k_y k_z}^{(n)}(t) = \lambda_{k_y k_z}^{(n)} \varphi_{k_y k_z}^{(n)}(t). \quad (8)$$

The function $\varphi^{(n)}$ satisfies the normalization condition

$$\int_0^{\infty} \varphi_{k_y k_z}^{(n)}(x) \varphi_{k_y k_z}^{(m)}(x) dx = \delta_{nm};$$

L is the normalization length.

Equation (5) is equivalent to the following condition for the function $\varphi^{(n)}$:

$$\frac{d^2 \varphi_{kyk}^{(n)}(x)}{dx^2} + (\lambda_{kyk} - (k_y - \gamma x)^2 - (k + \beta x^2)^2) \varphi_{kyk}^{(n)}(x) = 0. \quad (9)$$

In Sec. 4, we shall give the solution of this equation for the condition

$$\gamma \ll \beta. \quad (10)$$

In this case, the terms which are proportional to γ and γ^2 make up only a weak perturbation relative to the system without the longitudinal magnetic field, and we can replace the terms $2\gamma k_y x$ and $\gamma^2 x^2$ by the expressions $2\gamma k_y \bar{x}(k)$ and $\gamma^2 \bar{x}^2(k)$, where $\bar{x}(k)$ is the abscissa of the minimum of the potential $U(x) = (k + \beta x^2)^2$.

We introduce the quantity

$$\Lambda_{kyk}^{(n)} = \lambda_{kyk} - (k_y - \gamma \bar{x}(k))^2; \quad (11)$$

then Eq. (9) is written down in the form

$$\frac{d^2 \varphi_{kyk}^{(n)}}{dx^2} + (\Lambda_{kyk}^{(n)} - (k + \beta x^2)^2) \varphi_{kyk}^{(n)} = 0. \quad (12)$$

It is then seen that the functions $\varphi_{kyk}^{(n)}(x) \Lambda_{kyk}^{(n)}$ do not depend on k_y at all. As a consequence, we can represent them as

$$\varphi_{kyk}^{(n)}(x) = \varphi_k^{(n)}(x), \quad \Lambda_{kyk}^{(n)} = \Lambda^{(n)}(k).$$

We further assume that the electric energy $eE\xi$; is small in comparison with the relaxation frequency, i.e., that

$$\varepsilon \ll 1. \quad (13)$$

Substituting the expansion (7) in Eq. (5), we get, after simple transformations,

$$-\frac{\partial a_{kykz}}{\partial t} + (\lambda_{kykz} - 1) a_{kykz}^{(n)} = f_{kykz}^{(n)}(t), \quad (14)$$

where

$$f_{kykz}^{(n)}(t) = \frac{1}{L} \int d^3 r e^{-i\mathbf{k}\cdot\mathbf{r} - i\mathbf{k}_z z} \varphi_k^{(n)}(x) f(\mathbf{r}, t).$$

The new components of the external force satisfy the condition

$$\langle f_{kykz}^{(n)}(t) f_{kykz}^{(n)*}(t') \rangle = \delta_{kykz} \delta_{k_z k'_z} \delta_{n n'} \delta(t - t'). \quad (15)$$

In Eqs. (14), we have neglected the term with the time derivative of $\psi^{(n)}$: $a_{kykz}^{(n)} \partial \psi_{kykz}^{(n)} / \partial t$. It is small, inasmuch as the time dependence of the eigenfunction $\psi^{(n)}$ is given by the time dependence $k(t)$ for a given k_z . This dependence is very weak in the case $\varepsilon \ll 1$ and it can be neglected in comparison with the strong time dependence of the coefficients $a^{(n)}(t)$.

The solution of Eq. (14) has the form

$$a_{kykz}^{(n)}(t) = \exp(-p_{kykz}^{(n)}(t)) \int_{-\infty}^t \exp(p_{kykz}^{(n)}(t')) f_{kykz}^{(n)}(t') dt', \quad (16)$$

where

$$p_{kykz}^{(n)}(t) = - \int_0^t [1 - \lambda_{kykz}^{(n)}] dt', \quad k' = k_z + \varepsilon t'.$$

The dependence of the fluctuation current is determined by the well-known Ginzburg-Landau formula, which, in the usual units, has the form

$$\mathbf{j} = \frac{2e}{m} \text{Im} \left[\psi^* \left(\nabla - \frac{2ie}{c} \mathbf{A} \right) \psi \right]. \quad (17)$$

We then express the components of the current density in our dimensionless units:

$$\begin{aligned} \bar{j}_y &= \frac{8eT}{\xi^2} \text{Im} \left\langle \psi^* \left(\frac{\partial}{\partial y} - i\gamma x \right) \psi \right\rangle, \\ \bar{j}_z &= \frac{8eT}{\xi^2} \text{Im} \left\langle \psi^* \left(\frac{\partial}{\partial z} + i\epsilon t + i\beta x^2 \right) \psi \right\rangle. \end{aligned}$$

Substituting the expansion (7) and the solution (13) into the above, we find

$$\begin{aligned} \bar{j}_y &= \frac{8eT}{\xi^2} \sum_{n=0}^{\infty} \int \frac{dk_y dk_z}{(2\pi)^2} (k_y - \gamma x) \\ &\times |\varphi_k^{(n)}(x)|^2 \exp(-2p_{kykz}^{(n)}(t)) \int_{-\infty}^t \exp(2p_{kykz}^{(n)}(t')) dt', \end{aligned} \quad (18)$$

$$\begin{aligned} \bar{j}_z &= \frac{8eT}{\xi^2} \sum_{n=0}^{\infty} \int \frac{dk_y dk_z}{(2\pi)^2} (k + \beta x^2) \\ &\times |\varphi_k^{(n)}(x)|^2 \exp(-2p_{kykz}^{(n)}(t)) \int_{-\infty}^t \exp(2p_{kykz}^{(n)}(t')) dt'. \end{aligned}$$

Using (11), we can express the difference $2p^{(n)}(t') - 2p^{(n)}(t)$ in the following fashion:

$$\begin{aligned} 2p_{kykz}^{(n)}(t') - 2p_{kykz}^{(n)}(t) &= 2 \int_t^{t'} (1 - \lambda_{kykz}^{(n)}) dt' = 2 \left\{ (t - t') (1 - k_y^2) + k_y \frac{2\gamma}{\varepsilon} \int_{k-t}^k \bar{x}(k') dk' \right. \\ &\quad \left. - \frac{1}{\varepsilon} \int_{k-t}^k (\Lambda^{(n)}(k') + \gamma^2 \bar{x}^2(k')) dk' \right\} \end{aligned}$$

Carrying out the substitution $\tau = \varepsilon(t - t')$ and performing integration over k_y , we obtain

$$\begin{aligned} \bar{j}_y &= \sqrt{\frac{2}{\pi}} \frac{eT}{\pi \varepsilon} \sum_{n=0}^{\infty} \int dk \int_0^{\infty} \frac{d\tau}{\sqrt{\tau}} \left(\frac{\gamma}{\tau} \int_{k-\tau}^k \bar{x}(k') dk' \right. \\ &\quad \left. - \gamma x \right) |\varphi_k^{(n)}(x)|^2 \exp \left\{ \frac{2}{\varepsilon} \left[\tau - \int_{k-\tau}^k (\Lambda^{(n)}(k') + \gamma^2 \bar{x}^2(k')) dk' \right. \right. \\ &\quad \left. \left. + \frac{\gamma^2}{\tau} \left(\int_{k-\tau}^k \bar{x}(k') dk' \right)^2 \right] \right\}, \end{aligned} \quad (19)$$

$$\begin{aligned} \bar{j}_z &= \sqrt{\frac{2}{\pi}} \frac{eT}{\pi \varepsilon} \sum_{n=0}^{\infty} \int dk \int_0^{\infty} \frac{d\tau}{\sqrt{\tau}} (k + \beta x^2) |\varphi_k^{(n)}(x)|^2 \exp \left\{ \frac{2}{\varepsilon} \left[\tau - \int_{k-\tau}^k (\Lambda^{(n)}(k') \right. \right. \\ &\quad \left. \left. + \gamma^2 \bar{x}^2(k')) dk' + \frac{\gamma^2}{\tau} \left(\int_{k-\tau}^k \bar{x}(k') dk' \right)^2 \right] \right\}. \end{aligned} \quad (20)$$

4. For calculation of the expressions (19) and (20), we must solve Eq. (12). The dimensionless parameter $\beta \sim J/J_{c2}$. In what follows, we shall assume the currents are small in comparison with the second critical current J_{c2} :

$$\beta \ll 1. \quad (21)$$

In this case, Eq. (12) can be solved and the function $\Lambda^{(n)}(k)$ can be determined for all the essential values of k . For $k \gg \beta^{1/3}$ Eq. (12) is the Schrodinger equation for an oscillator with mass $1/2$, energy $\Lambda^{(n)}(k) - k^2$, eigenfrequency $(8k\beta)^{1/2}$ and small anharmonicities of third and fourth order. Using the well-known formula for the energy levels of an anharmonic oscillator, we get

$$\Lambda^{(n)}(k) = k^2 + (32k\beta)^{1/2} (n + 1/2) + 3\beta k^{-1} (n^2 + 1/2 n + 1/4). \quad (22)$$

It must be taken into account that the function φ should satisfy the boundary condition of Ginzburg-Landau: $d\varphi/dx = 0$ at $x = 0$. Therefore, only the even solutions have meaning. In accord with this, we get formula (22) from the ordinary formula for the linear oscillator by the substitution $n \rightarrow 2n$.

At $k < 0$, $|k| \gg 1/3$, the potential energy $(k + \beta x^2)^2$ has a sharp minimum for $x = \bar{x}(k) = (|k|/\beta)^{1/2}$. Introducing the new variable $x' = x - \bar{x}(k)$ we can rewrite Eq. (12) in the form of a Schrodinger equation for an

oscillator of mass $\frac{1}{2}$ energy $\Lambda^{(n)}(k)$, eigenfrequency $4(|k|\beta)^{1/2}$ and small anharmonicities of third and fourth order. Again using the formula for the energy levels of an anharmonic oscillator, we obtain

$$\Lambda^{(n)}(k) = 4(|k|\beta)^{1/2} \left(n + \frac{1}{2} \right) - \frac{\beta}{2|k|} (3n^2 + 3n + 1). \quad (23)$$

The argument of the exponential in the integrals (19) and (20) contains the large factor $2/\epsilon$. Therefore, the immediate neighborhood of the point of the maximum of the function

$$F_n(k, \tau) = \tau - \int_{k-\tau}^k \Lambda^n(k') dk' - \gamma^2 \left\{ \int_{k-\tau}^k \bar{x}(k') dk' - \frac{1}{\tau} \left(\int_{k-\tau}^k \bar{x}(k') dk' \right)^2 \right\}$$

makes the principal contribution to the integrals over k and τ . With account of the fact that $\bar{x}(k) = (|k|\beta)^{1/2}$ for $k < 0$, $\bar{x}(k) = 0$ for $k > 0$ and $|\tau_0| \approx |k_0 - \tau_0|$, the position of the maximum (k_0, τ_0) is determined by the condition

$$1 = \Lambda^{(n)}(k_0 - \tau_0) + \frac{1}{q} \frac{\gamma^2}{\beta} |k_0 - \tau_0| = \Lambda^{(n)}(k_0) + \frac{4}{q} \frac{\gamma^2}{\beta} |k_0 - \tau_0|.$$

Using Eqs. (22) and (23), we find

$$k_0 = 1 - (8\beta)^{1/2} (n + 1/4), \quad (24)$$

$$k_0 - \tau_0 = - \frac{1}{4\beta(2n+1)^2} + \frac{\gamma^2}{72\beta^2(2n+1)^4}.$$

The maximum value of F_n is then determined as

$$F_n(k_0, \tau_0) = \frac{1}{12\beta(2n+1)^2} + \frac{2}{3} - \frac{4}{3} (8\beta)^{1/2} \left(n + \frac{1}{4} \right) - \left(n^2 + \frac{3}{2}n + \frac{5}{12} \right) \beta \ln \beta - \frac{\gamma^2}{288\beta^2(2n+1)^4} + O(\beta) + O\left(\frac{\gamma^2}{\beta^2}\right). \quad (25)$$

We now expand the function F_n near the maximum in powers of $k - k_0$ and $\tau - \tau_0$ up to second order and integrate expressions (19) and (20) over k and τ . With account of the fact that the principal contribution to the integral is made by only the first term $n = 0$, we find the following expressions for the density of the fluctuation current

$$\bar{j}_v = \frac{2eT}{\xi^2} \sqrt{\frac{e}{2\pi}} \left(\frac{\gamma}{3\beta} - \gamma x \right) |\varphi_{k_0}^{(0)}(x)|^2 \exp \left\{ \frac{1}{6e\beta} + \frac{4}{3e} - \frac{8}{3e} \left(\frac{\beta}{2} \right)^{1/2} - \frac{5\beta}{6e} \ln \beta - \frac{\gamma^2}{144\beta^2 e} \right\}, \quad (26)$$

$$\bar{j}_i = \frac{2eT}{\xi^2} \sqrt{\frac{e}{2\pi}} (k_0 - \beta x^2) |\varphi_{k_0}^{(0)}(x)|^2 \exp \left\{ \frac{1}{6e\beta} + \frac{4}{3e} - \frac{8}{3e} \left(\frac{\beta}{2} \right)^{1/2} - \frac{5\beta}{6e} \ln \beta - \frac{\gamma^2}{144\beta^2 e} \right\}. \quad (27)$$

The spatial dependence of the fluctuation current density is determined by the function $|\varphi_{k_0}^{(n)}(x)|^2$ with $k_0 \approx 1$ and $n = 0$; $\varphi_1^{(0)}(x)$ is the wave function of the oscillator, normalized in the interval $(0, \infty)$:

$$|\varphi_1^{(0)}(x)|^2 = 2(2\beta/\pi^3)^{1/2} \exp(-\sqrt{2}\beta x^2).$$

The quantity $d = (2\beta)^{-1/4}$ plays the role of the thickness of the intermediate-state layer. The second term in the parentheses of Eqs. (26) and (27) is small in comparison with the first, and therefore it can be neglected. It is significant that the second term in the parentheses of Eq. (26) corresponds to a diamagnetic current.

By integration of Eqs. (26) and (27) over dx , we obtain the total surface-current density:¹⁾

$$I_{sv} = \frac{2eT}{\xi} \sqrt{\frac{e}{2\pi}} \frac{\gamma}{3\beta} \exp \left\{ \frac{1}{6e\beta} + \frac{4}{3e} - \frac{8}{3e} \left(\frac{\beta}{2} \right)^{1/2} - \frac{5\beta}{6e} \ln \beta - \frac{\gamma^2}{144\beta^2 e} \right\}, \quad (28)$$

where I_{sv} is connected with I_{sz} by the simple relation

$$I_{sv} = \frac{\gamma}{3\beta} I_{sz}. \quad (29)$$

The additional magnetic field H_S produced at the mouth of the cylinder by the fluctuation current is determined to be

$$H_s = \frac{4\pi}{c} I_{sv} = 4\sqrt{2}\pi \gamma \frac{eT}{c\xi} \frac{\sqrt{e}}{\beta} \exp \left\{ \frac{1}{6e\beta} + \frac{4}{3e} - \frac{8}{3e} \left(\frac{\beta}{2} \right)^{1/2} - \frac{5\beta}{6e} \ln \beta - \frac{\gamma^2}{144\beta^2 e} \right\}. \quad (30)$$

5. We now attempt to determine the order of magnitude of the paramagnetic effect. Our analysis was based on the assumption that the fluctuation current I_{sz} is small in comparison with the normal current I_n , which flows through a layer of thickness d :

$$I_{sz} \ll I_n, \quad (31)$$

where

$$I_n = \frac{Jd}{r_2^2 - r_1^2} \sim \frac{cd\beta}{e\xi^2} \sim \kappa \beta^{1/2} c H_c, \quad (32)$$

κ is the parameter of the Ginzburg-Landau theory. In the work of Andreev and the author,^[5] it was shown that (31) is roughly equivalent to the condition

$$(a_0/\lambda_0)^2 \exp(l/\beta^2 \kappa^2 \xi_0) \ll 1,$$

where l is the mean free path length of the electron, λ_0 is the London penetration depth at $T = 0$, and a_0 is the interatomic distance.

This condition is satisfied in the case of type-I superconducting alloys ($\kappa \sim 1$, $l \ll \xi_0$). Introducing the parameter

$$\gamma = 2e\xi^2 H_a/c = H_s/H_c \sqrt{2} \kappa$$

and (32) in Eq. (31), we obtain

$$\frac{H_s}{H_a} \approx \frac{1}{\beta^{1/2}} \frac{I_{sz}}{I_n} \sim \left(\frac{a_0}{\lambda_0} \right)^2 \exp \left(\frac{l}{\beta^2 \kappa^2 \xi_0} \right).$$

In the limits of applicability of our theory, this corresponds to the experimental results of I. L. Landau (private communication), who found that $H_S \approx H_a$ for pure samples ($\kappa \ll 1$).

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¹⁾We note that the factor $1/2\sqrt{\beta}$ is omitted in Eq. (22) of [5].

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217