The radiations emitted by an ultrarelativisitc particle in a gravitational field

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(Submitted May 17, 1973)
Zh. Eksp. Teor. Fiz. **65**, 2137–2140 (December 1973)

The electromagnetic and gravitational radiations emitted by an ultrarelativistic particle in an external gravitational field are considered. Estimates are obtained for the intensities of the radiations and their angular and spectral distributions for the cases of circular and infinite trajectories in a Schwarzschild field.

Several papers have appeared recently[1-6], discussing the properties of the radiation (hypothetical scalar radiation, electromagnetic radiation and gravitational radiation) emitted by an ultrarelativistic particle which moves in a gravitational field. The peculiarity of such problems consists in the fact that the external field affects not only the particle trajectories but also acts directly on its field, thus leading to a considerable modification of the character of the emitted radiation. In[1-4] the problem of radiation by a particle moving along a circular orbit in a Schwarzschild field was considered. Use has been made of a separation of variables in the wave equation for the radiation field, a method which is applicable only for very special external fields and trajectories. Such an approach is not physically lucid and is rather complicated. This seems to be the source of the contradictory assertions regarding the time-dependence of the signal received by a remote observer, and regarding the angular distribution of the intensity of radiation[1-3]. On the other hand, the equations obtained in these papers are not by themselves in contradiction with one another nor with the qualitative results which are derived below.

In the present paper we discuss qualitatively the character of the radiation and derive simple estimates for the total intensity and for the spectral and angular distribution of radiation emitted in an external gravitational field. We do not restrict our attention to the case of motion along a circle in a Schwarzschild field, case which is of purely methodological interest, in view of the instability of a relativistic circular orbit. We also consider the case of infinite motion. The latter problem has been considered by Peters[5,6] for the weak field case. However, the expressions obtained by him for the total energy loss are erroneous[1)].

We start by considering motions along a circular orbit in a Schwarzschild field. The orbit radius is determined by the relation

$$r_0\approx\frac{3}{2}r_g\left(1+\frac{1}{3\gamma^2}\right),\quad \gamma=(1-v^2)^{-1/2},$$

where r_g is the gravitational radius. The velocity v is measured in local time, the velocity of light c is set equal to 1.

The radiation of the particle is concentrated in an angular region $\vartheta\sim 1/\gamma$ near the direction of its velocity. This fact is independent of the nature of the radiation and is a direct consequence of relativistic kinematics.

We now determine the portion of trajectory on which a signal is formed which is received by a given observer at infinity. Making use of the well known expression for the trajectory of a light ray in the Schwarzschild potential (cf.[7]), one can show that if the ray is emitted under an angle ϑ to the particle trajectory, then going off from a radius r_0 to infinity, it will describe an azimuthal angle $\varphi\sim\ln\vartheta^{-1}$. The difference between the initial azimuthal angles of two rays emitted under the angles ϑ_1 and ϑ_2 and reaching a given observer is obviously equal to $\Delta\varphi\sim\ln(\vartheta_1/\vartheta_2)$. Since the main fraction of the radiation is emitted into angles $\sim 1/\gamma$ we have $\Delta\varphi\sim 1$. Thus the length of formation of the signal is $l^{(c)}\sim r_0$.[2)] A similar reasoning for the case of usual synchrotron radiation leads to the well-known relation $l^{(f)}\sim r_0\vartheta\sim r_0/\gamma$, i.e., the corresponding length turns out to be considerably smaller. It is just for this reason that the characteristics of radiation emitted in a gravitational field and in flat space turn out to be different.

We now go over to an estimate of the characteristic frequencies of the radiation. The reception time t is related to the emission time t′ by means of the relation $t = t' + R(t')$, where $R(t')$ is the distance from the point where the particle was situated at time t′ to the observation point, measured along the trajectory of the light ray. Therefore the characteristic duration of the signal received by the observer is

$$\Delta t=\frac{\partial t}{\partial t'}\Delta t'\sim(\vartheta^2+\gamma^{-2})l. \tag{1}$$

In other words, the signal has the shape of a short pulse of duration

$$\Delta t^{(c)}\sim\gamma^{-2}r_0,\quad \Delta t^{(f)}\sim\gamma^{-3}r_0. \tag{2}$$

The corresponding characteristic frequency band ω_c of the radiation is

$$\omega_c^{(c)}\sim\gamma^2 r_0^{-1},\quad \omega_c^{(f)}\sim\gamma^3 r_0^{-1}. \tag{3}$$

We note that owing to the proximity of the trajectories of an ultrarelativistic particle and of a light ray in a gravitational field, the radiation in this case is reminiscent of flat-space radiation for small deflection angles $\alpha\ll 1/\gamma$. In particular, the duration of the received signal turns out to be γ^2 times smaller than the characteristic transit time.

Radiation on frequencies $\omega\gg\omega_c$ is exponentially small. We also note that according to (1) radiation for $\omega\ll\omega_c$ occurs into a cone with an opening angle

$$\vartheta^{(c)}\sim(\omega r_0)^{-1/2},\quad \vartheta^{(f)}\sim(\omega r_0)^{-1/3}. \tag{4}$$

It should be stressed that all relations listed above are of a purely kinematical nature and are therefore valid for any kind of radiation.

We now go over to the question of the intensity of radiation, which is usually defined by the relation

$$dI\sim\omega^2u^2R^2d\Omega(\partial t/\partial t'). \tag{5}$$

Here u is the potential of the radiation field and $R^2d\Omega$ is the area element of a remote spherical surface. The last factor $\partial t/\partial t'$ indicates that, as usual, one estimates the average intensity, and not the intensity in the pulse.

It is important that the contribution to electromagnetic radiation comes only from the three-dimensionally transverse component of the vector potential, i.e., the component which is orthogonal to the direction n of the propagation vector

$$A_\perp \sim ev_\perp/R(1-\mathbf{nv}) \sim e\vartheta/R(\vartheta^2+\gamma^{-2}). \quad (6)$$

In the case of gravitational radiation only the doubly transverse part of the perturbation of the metric is important:

$$h_{\perp\perp} \sim \sqrt{k}\,\varepsilon v_\perp^2/R(1-\mathbf{nv}) \sim \sqrt{k}\,\varepsilon\vartheta^2/R(\vartheta^2+\gamma^{-2}), \quad (7)$$

where k is the Newtonian gravitational constant and ϵ is the energy of the radiating particle. The expressions (6), (7) correspond essentially to the Liénard-Wiechert potentials.

Substituting (6) and (7) into (5) and taking into account (4) we obtain the following expressions for the angular distribution for $\vartheta \gg 1/\gamma$:

$$dI_{em}^{(c)}/d\vartheta \sim e^2/r_0^2\vartheta^3, \quad dI_{em}^{(f)}/d\vartheta \sim e^2/r_0^2\vartheta^5, \quad (8)$$

$$dI_{gr}^{(c)}/d\vartheta \sim k\varepsilon^2/r_0^2\vartheta, \quad dI_{gr}^{(f)}/d\vartheta \sim k\varepsilon^2/r_0^2\vartheta^3. \quad (9)$$

It is easy to understand that in this case the angle ϑ coincides in order of magnitude with the polar angle measured from the orbit plane.

Taking into account the relations (4) which relate the characteristic values of the angles ϑ and frequencies ω, it can be seen easily that Eqs. (8) and (9) imply the following expressions for the spectral intensities for $\omega \ll \omega_c$:

$$dI_{em}^{(c)}/d\omega \sim e^2/r_0, \quad dI_{em}^{(f)}/d\omega \sim e^2(\omega r_0)^{1/3}/r_0, \quad (10)$$

$$dI_{gr}^{(c)}/d\omega \sim k\varepsilon^2/r_0(\omega r_0), \quad dI_{gr}^{(f)}/d\omega \sim k\varepsilon^2/r_0(\omega r_0)^{1/3}. \quad (11)$$

Taking into account the rapid decrease of intensity for $\omega > \omega_c$, Eq. (3), we find from (10), (11) the total intensities[3)]:

$$I_{em}^{(c)} \sim e^2\gamma^2/r_0^2, \quad I_{em}^{(f)} \sim e^2\gamma^4/r_0^2, \quad (12)$$

$$I_{gr}^{(c)} \sim k\varepsilon^2/r_0^2, \quad I_{gr}^{(f)} \sim k\varepsilon^2\gamma^2/r_0^2. \quad (13)$$

We note that from (11) it follows formally that $I_{gr}^{(c)} \sim (k\epsilon^2/r_0^2)\ln\gamma$. However, taking into account the logarithmic dependence on γ seems to be beyond the accuracy of our estimates. Moreover, it can be seen from the first equation (9) that although the intensity of the radiation is concentrated near the orbit plane, in this case an important contribution comes also from angles $\vartheta \gg 1/\gamma$. In the same manner frequencies $\omega \ll \omega_c$ are also important. However the contribution of harmonics to the magnitude of the field, rather than its intensity, decreases with ω slower than ω^{-1}. Therefore the relations (2) and (3) which refer to the form of the signal are valid also for gravitational radiation.

Let us now discuss the more realistic problem of radiation by an ultrarelativistic particle with impact parameter ρ, passing through a Schwarzschild field. The duration of the signal is in this case equal to $\Delta t \sim \gamma^{-2}\rho$, and the formulas for the total intensity can be obtained from Eqs. (12c) and (13c) by the substitution $1/r_0^2 \to r_g^2/\rho^4$. Indeed, if before the acceleration was $dv/dt' \sim r_0^{-1}$, we now have $dv/dt' \sim r_g/\rho^2$. The total energy losses as the particle flies by the field have the orders of magnitude

$$\Delta\varepsilon_{em}^{(c)} \sim e^2r_g^2\gamma^2/\rho^3, \quad (14)$$

$$\Delta\varepsilon_{gr}^{(c)} \sim k\varepsilon^2r_g^2/\rho^3. \quad (15)$$

The spectral and angular distributions are obtained easily in this case also.

The generalization of these estimates to arbitrary gravitational fields is obvious.

The authors are sincerely grateful to G. I. Budker for posing the question which triggered the present paper and to A. I. Vaĭnshteĭn for valuable discussions.

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1)According to Peters[7], the radiation losses in a Schwarzchild potential are γ times larger than in a Coulomb field. It is also clear that the influence of the field on the emission of radiation can only decrease the losses.

2)For comparison we list in parallel with the estimates for a given problem also the expressions for the radiation in flat space, indicating the former by the index *c* (curved) and the latter by the index *f* (flat).

3)We note that the expressions (9), (11), (13) for the gravitational radiation in flat space refer to the case of the relativistic rotator[8, 9]. For the case of motion of a charge in an electromagnetic field the result is different[9], which is related to resonant transition of the electromagnetic radiation into gravitational radiation[10].

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Translated by M. E. Mayer
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