Radiative corrections to bremsstrahlung at high energies

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Radiative corrections to the cross section for the single scattering of bremsstrahlung by colliding e^+e^- (e^-e^-) beams are obtained in an experiment in which the total energy carried away by the photons in the direction of one of the initial particles is recorded, while the radiation in the direction of the motion of the other particle is not recorded. The results can also be used to obtain the radiative corrections to bremsstrahlung produced by electrons scattered by nuclei.

The double-bremsstrahlung cross section in the collision of electrons (electrons and positrons) of high energy in a kinematic configuration in which the two photons are emitted in directions close to the direction of motion of the one of the initial particles was calculated recently[1]. The differential cross section with respect to the photon frequencies, obtained in[1], can be used in colliding-beam experiments in which the two registered photons travel in the same direction. In addition, this cross section is essential for the calculation of the radiative corrections to the single bremsstrahlung cross section

$$\frac{d\sigma^{(1)}}{d\Delta}=\alpha r_0^2\frac{\Delta}{1-\Delta}\left(2\gamma+\frac{8}{3}\right)[\ln(s^2\Delta\gamma^{-1})-1], \qquad (1)$$

$$\gamma=(1-\Delta)^2/\Delta,\quad \omega=(1-\Delta)E,\quad s=4E^2,\quad m_e=1,$$

in the presently employed colliding-beam experimental setup, in which one registers only the total energy $\omega_1 + \omega_2 = (1 - \Delta)E$ carried away by the photons in the direction of motion of one of the initial particles (E is the electron energy in the c.m.s.), and the radiation and the direction of motion of the other particle is not registered.

The purpose of the present paper is to calculate the radiative correction to (1) of order α^4 for the bremsstrahlung cross section, differentiated with respect to the total energy of the final photons, in the indicated formulation of the experiment. It is convenient to consider separately the case of emission of "high" energy $(1 - \Delta)E \sim E$, of the order of energy of the initial electron (with at least one of the photons remaining hard), and the case when the total energy of the photons is much less than the energy of the initial electron. A more rigorous relation between these cases will be considered later on. When the fraction $1 - \Delta$ of the initial-electron energy carried away by the photons is of the order of unity, the correction to the cross section (1) takes the form $\alpha^4(\ln s)h(\Delta)$. The main contribution to this correction, which comes from the region of small momentum transfers from the electron p_1 to the electron p_2, is calculated by the Weitzsäcker-Williams method. The corresponding correction to (1) is given in Sec. 1 (formula (7)). In the second section we consider the case when the energy fraction carried away by the photons is much less than unity, $1 - \Delta \ll 1$. The main contribution, $\sim\alpha^4/(1 - \Delta)$, comes in this case from the region of finite (~ 1) values of the momentum transfer and is calculated by the classical-current method[2]. The result of this calculation is given by formula (12).

The results are applicable for both e^-e^+ and e^-e^- collisions, inasmuch as in the former case the contribution of the annihilation diagrams is negligibly small ($\sim 1/s$), and in the latter case the contribution of the interference between the direct and exchange diagrams is also small, so that we need consider only direct diagrams and disregard identity. These results can also be used to determine the corrections to the bremsstrahlung of a fast electron of energy E on a nucleus, for which it is necessary to replace in formulas (7)–(12) $\alpha^2 r_0^2$ by $Z^2\alpha^2 r_0^2$ and s^2 by E^2 in the argument of the logarithm in (7).

1. In the case when the fraction of the energy carried away by the photons is on the order of unity, the correction to the main cross section (1) can be expressed in the form

$$\left(\frac{d\sigma^{(2)}}{d\Delta}\right)=\int\limits_{\omega_1+\omega_2=(1-\Delta)E} d\omega_1\frac{d^2\sigma}{d\omega_1\,d\omega_2}+\frac{d\sigma^{\gamma}_{\text{r. s.}}}{d\Delta}+\frac{d\sigma^{\gamma}_{\text{virt}}}{d\Delta}. \qquad (2)$$

The first term in (2) is the cross section, integrated with respect to the frequency of one of the photons at a fixed photon energy, of the double bremsstrahlung in one direction[1] under the condition that $\omega_{1,2} > \epsilon E$ and $\epsilon \ll 1$

$$\int\limits_{(\omega_1+\omega_2)/E=1-\Delta} d\omega_1\frac{d^2\sigma}{d\omega_1\,d\omega_2}=\frac{\alpha^2 r_0^2}{210\pi}(\ln s_1^2)$$
$$\times\frac{\Delta}{1-\Delta}\Big\{\eta^{-1}\int\limits_{\Delta}^{\Delta^{-1}}\frac{dt\ln t}{1-t}(32\gamma^3+206\gamma^2+570\gamma+1584)+\xi\eta^{-1}\ln\gamma(-32\gamma^3-416\gamma^2$$
$$-1310\gamma-1184)+\xi^2(-16\gamma^3-281\gamma^2-810\gamma-460-296\gamma^{-1}-192\gamma^{-2})$$
$$+\xi\eta(195\gamma^2-562\gamma+104-192\gamma^{-1})+\ln\gamma(195\gamma^2-32\gamma) \qquad (3)$$
$$+\frac{\pi^2}{6}(64\gamma^3+284\gamma^2+280\gamma)-64\gamma^2+78\gamma+456-192\gamma^{-1}-16\ln\frac{(1-\Delta)}{\varepsilon}$$
$$\times\left[4\gamma^3+50\gamma+46+\ln\gamma(2\gamma^3+28\gamma^2+35\gamma)+\xi\eta\left(2\gamma^3+24\gamma^2+\frac{87}{2}\gamma+23\right)\right]\Big\},$$

$$\xi=\ln\Delta,\quad \eta=(1+\Delta)/(1-\Delta),\quad s_1=s\Delta.$$

The second term in (2) takes into account the emission of a hard photon that carries away an energy $(1 - \Delta)E$, accompanied by emission of a soft photon with frequency not exceeding ϵE. The third term of (2) is the contribution of the interference of the Born amplitude of single bremsstrahlung (Fig. 1) and of the radiative correction to it. With logarithmic accuracy ($\sim\ln s$), it receives a contribution from only the diagram of Fig. 2b.

The last two terms of (2) were first calculated by Mork and Olsen[3] and the problem of the radiative correction to the bremsstrahlung in collisions between a fast electron and a nucleus. However, in view of the large number of misprints and inaccuracies in the results of[3], we had to recalculate anew the contribution made to the radiative correction by the emission of soft real and virtual quanta. As a result we have for the sum of the second and third terms of (2)

$$\frac{d\sigma^{\gamma}_{\text{r. s.}}}{d\Delta}+\frac{d\sigma^{\gamma}_{\text{virt}}}{d\Delta}=\frac{\alpha^2 r_0^2}{105\pi}(\ln s_1^2)\frac{\Delta}{1-\Delta}[-F_1(\Delta)+2F_2(\Delta)\ln\varepsilon], \qquad (4)$$

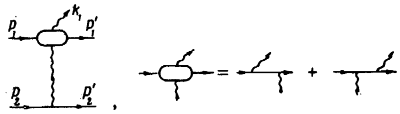

FIG. 1

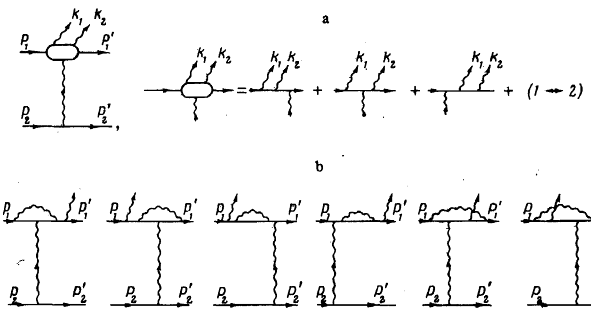

FIG. 2

where F_1 and F_2 are functions of Δ:

$$F_1=a_1+(a_2+a_3\xi)\xi+(a_4+a_5D)\eta+(a_6+a_7\xi+a_8\ln\gamma)\ln\gamma$$
$$+\xi\eta[a_9+a_{10}\xi+a_{11}\ln\gamma+a_{12}h(\xi/2)+4a_{10}h(\xi)],\tag{5}$$
$$F_2=a_2+a_7\ln\gamma+a_{10}\xi\eta.$$

The quantities ξ, η, γ, and s_1 were defined earlier (see (1) and (3)),

$$D=-\int\limits_{-\Delta(1-\Delta)^{-1}}^{(1-\Delta)^{-1}}\ln(1-t)\frac{dt}{t},\quad h(x)=x^{-1}\int\limits_0^x t\,\mathrm{cth}\,t\,dt,$$
$$a_1=(16\gamma^3+581\gamma^2+735\gamma)\frac{\pi^2}{6}-32\gamma^2+\frac{59941}{105}\gamma+\frac{48916}{105},$$
$$a_2=-16\gamma^2-200\gamma-184,\quad a_3=-16\gamma^3-\frac{785}{4}\gamma^2-\frac{1763}{4}\gamma-424-396\gamma^{-1},$$
$$a_4=105(\gamma-6)\frac{\pi^2}{6},\quad a_5=-315\gamma^2-175\gamma+210,$$
$$a_6=\frac{9653}{105}\gamma^2+\frac{102655}{210}\gamma+28,\quad a_7=-8\gamma^3-112\gamma^2-140\gamma,\tag{6}$$
$$a_8=\frac{315}{4}\gamma^2+\frac{525}{4}\gamma,\quad a_9=\frac{11333}{105}\gamma^2+\frac{9443}{210}\gamma+\frac{6608}{105},$$
$$a_{10}=-8\gamma^3-96\gamma^2-174\gamma-92,\quad a_{11}=-8\gamma^3-\frac{21}{2}\gamma+119,$$
$$a_{12}=32\gamma^3+426\gamma^2+752\gamma-80.$$

The values given in[3] for a_8, a_9, and a_{11} are incorrect.

Substituting (4) and (3) in (2), we obtain for the correction to the distribution over the summary photon energy, in the case when the energy carried away by them is of the order of the initial-electron energy,

$$\left(\frac{d\sigma^{(2)}}{d\Delta}\right)_{\text{ж}}=\frac{\alpha^2r_0^2}{105\pi}(\ln s_1^2)\frac{\Delta}{1-\Delta}\Big\{\eta\int\limits_\Delta^{\Delta^{-1}}\frac{dt\ln t}{1-t}(16\gamma^3+333\gamma^2+276\gamma+290$$
$$-1104(\gamma+4)^{-1})+\eta\int\limits_\Delta^{\Delta^{-1}}\frac{dt\ln t}{1+t}(-16\gamma^3-192\gamma^2-348\gamma-184)$$
$$+\xi^2\left(8\gamma^3+\frac{223}{4}\gamma^2+\frac{143}{4}\gamma+194+248\gamma^{-1}-96\gamma^{-2}\right)$$
$$+\xi\eta\left(-16\gamma^3-\frac{795}{2}\gamma^2-330\gamma-382+1104(\gamma+4)^{-1}\right)\ln\gamma$$
$$+\ln^2\gamma\left(-8\gamma^3-\frac{763}{4}\gamma^2-\frac{1085}{4}\gamma\right)\tag{7}$$
$$+\xi\eta\left(-\frac{313}{30}\gamma^2-\frac{9779}{30}\gamma-\frac{164}{15}-96\gamma^{-1}\right)$$
$$+\ln\gamma\left(-\frac{313}{30}\gamma^2-\frac{4229}{6}\gamma-212\right)+\frac{\pi^2}{6}(16\gamma^3-439\gamma^2-595\gamma)$$
$$-\frac{7978}{15}\gamma-\frac{3568}{15}-96\gamma^{-1}+\frac{\pi^2}{6}\eta(945\gamma^2+420\gamma)\Big\}.$$

In the limiting case when the photons carry away practically the entire energy of the initial electrons, $\Delta\to0$, we have from (7)

$$\frac{d\sigma^{(2)}}{d\Delta}\sim\frac{\alpha^2r_0^2}{2\pi}(\ln s_1^2)(\ln\Delta)^2,\quad\Delta\to0.\tag{7a}$$

If the electron loses practically no energy to photon emission, $\Delta\to1$, we have

$$\frac{d\sigma^{(2)}}{d\Delta}\sim\frac{\alpha^2r_0^2}{\pi}(\ln s^2)8\frac{\pi^2}{6},\quad\Delta\to1.\tag{7b}$$

Integration of (7) with respect to Δ yields for the correction to the total bremsstrahlung cross section

$$\int\limits_0^1\frac{d\sigma^{(2)}}{d\Delta}d\Delta=\frac{\alpha^2r_0^2}{105\pi}(\ln s^2)\left[1656\frac{\pi^2}{6}\ln2-688\zeta(3)-\frac{8129}{15}\frac{\pi^2}{6}+\frac{6719}{24}\right].\tag{7c}$$

2. If the energy fraction $1-\Delta$ carried away by the final photons, is much less than unity, it is necessary to take into account the contribution made to the radiative correction to (1) by the region of the finite (~1) values of the momentum transferred from the electron p_1 to the electron p_2. The contribution of this region, $\sim\alpha^4/(1-\Delta)$, becomes comparable with the contribution $\sim\alpha^4(\ln s^2)$ of the region of small momentum transfers (7b) at $1-\Delta\sim(\ln s^2)^{-1}$. The calculation is carried out by the classical-current method[2]. The cross section can be represented in the form

$$\left(\frac{d\sigma^{(2)}}{d\Delta}\right)=\int\limits_{(\omega_1+\omega_2)/E=1-\Delta}\frac{d^2\sigma^{\gamma\gamma}}{d\omega_1d\omega_2}d\omega_1+\left(\frac{d\sigma^\gamma}{d\Delta}\right)_{\text{virt}}+\left(\frac{d\sigma^\gamma}{d\Delta}\right)_{\text{vac}}.\tag{8}$$

The first term in the right-hand side of (8) corresponds to emission of two real soft quanta, whose summary energy is $(1-\Delta)E$ and is described by the Feynman diagrams shown in Fig. 3a:

$$\int\limits_{(\omega_1+\omega_2)/E=1-\Delta}d\omega_1\frac{d^2\sigma^{\gamma\gamma}}{d\omega_1d\omega_2}=-\frac{8\alpha^2r_0^2}{\pi(1-\Delta)}\int\limits_0^\infty\frac{dx}{x^3}\Phi(x^2)\frac{1}{8\pi}$$
$$\times\int\limits_0^{(1-\Delta)E}\frac{k^2dk\,do_\gamma}{(k^2+\lambda^2)^{1/2}}\left(\frac{p_1}{p_1k}-\frac{p_1'}{p_1'k}\right)^2$$
$$=\frac{8\alpha^2r_0^2}{\pi(1-\Delta)}\left\{\left(\frac{7}{8}\zeta(3)+\frac{5}{4}\right)\ln\frac{1-\Delta}{\lambda}+\int\limits_0^\infty\frac{dx}{x^3}\Phi(x^2)2\,\mathrm{cth}\,2\theta\int\limits_0^\theta u\,\mathrm{th}\,u\,du\right\};$$
$$\Phi(x^2)=\frac{2x^2+1}{x(x^2+1)^{1/2}}\ln[x+(x^2+1)^{1/2}]-1,\tag{9}$$
$$x^2=\frac{(\mathbf{p}_1-\mathbf{p}_1')^2}{4},\quad\mathrm{th}\,\theta=\frac{x}{(x^2+1)^{1/2}}.$$

The second term in the right-hand side of (8) describes the contribution of the interference of the Born amplitude of Fig. 1 with the radiative correction to it, described by the diagrams 3b and 3c. Using the expression for the renormalized vertex function (see, e.g.[4] formula (36.4.13)), we express this term of (8) in the form

$$\left(\frac{d\sigma^\gamma}{d\Delta}\right)_{\text{virt}}=-\frac{8\alpha^2r_0^2}{\pi(1-\Delta)}\left\{\left(\frac{7}{8}\zeta(3)+\frac{5}{4}\right)\left(\ln\frac{1}{\lambda}-1\right)\right.$$
$$\left.+\int\limits_0^\infty\frac{dx}{x^3}\Phi(x^2)\left[\frac{\theta\,\mathrm{th}\,\theta}{2}+2\,\mathrm{cth}\,2\theta\int\limits_0^\theta u\,\mathrm{th}\,u\,du\right]\right\}.\tag{10}$$

The third term in (8) is the interference of the Born amplitude of Fig. 1 with the radiative correction to it

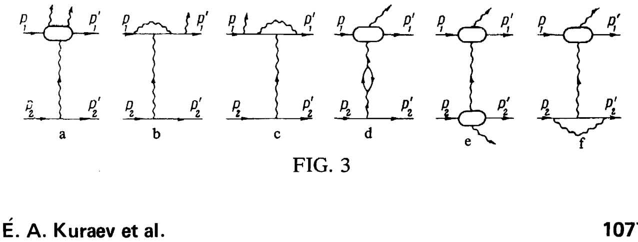

FIG. 3

that accounts for the polarization of the vacuum (Fig. 3d). Using the expression for the second-order polarization operator (see[4], formula (36.3.6)), we express it in the form

$$\left(\frac{d\sigma^{\gamma}}{d\Delta}\right)_{\text{vac}}=\frac{8\alpha^2 r_0^2}{\pi(1-\Delta)}\int_0^{\infty}\frac{dx}{x^3}\Phi(x^2)\left[\frac{1}{9}+\frac{1-2x^2}{3x^2}(1-\theta\,\text{cth}\,\theta)\right]=\frac{368}{81}\frac{\alpha^2 r_0^2}{\pi(1-\Delta)}. \quad (11)$$

The contribution of the diagram of Fig. 3e, which takes into account the emission of a soft photon with energy $\omega_1=(1-\Delta)E$ by the electron p_1 and of a photon of arbitrary hardness by the electron p_2 (in a direction opposite to that of p_1 in the c.m.s.), integrated over all the frequencies ω_2, is completely cancelled out by the contribution of the interference of the Born amplitude of Fig. 1 with the radiative correction to it, Fig. 3f, which takes into account the correction to the vertex function of the electron p_2.

This fact is general and can be proved rigorously by means of arguments analogous to those given by us earlier[5], where it was used to obtain the relations between the cross sections of different quantum-electrodynamic processes and corrections to the Lamb shift. The validity of our statement can be easily verified if it is noted that the radiative correction to the electron scattering in the Coulomb field, after subtracting the radiative correction necessitated by the polarization of the vacuum ([4], formula (39.3.3)), differs in sign from the integrated (with respect to the frequency ω_1 from ΔE to E) distribution over the photon frequencies and over the momentum transfer in the process of double bremsstrahlung in opposite directions, multiplied by $[2\alpha d\omega_2\Phi(x^2)/\pi\omega_2]^{-1}$ ([2] formula (52)).

Summing the contributions (9) and (10), and then carrying out an elementary integration and taking (11) into account, we obtain ultimately the expression for the correction to (1) (in the case when the photons carry away low energies:

$$\left(\frac{d\sigma^{(2)}}{d\Delta}\right)_{\text{M}}=\frac{8\alpha^2 r_0^2}{\pi(1-\Delta)}\left\{\left(\frac{7}{8}\xi(3)+\frac{5}{4}\right)\ln(1-\Delta)+\frac{7}{16}\xi(3)+\frac{127}{81}\right\}. \quad (12)$$

When the fields of the energy lost by the electron to radiation are of the order of unity, we should use formula (7) for the distribution with respect to the total energy carried away by the photons; if this fraction is small, $1-\Delta \ll (\ln s^2)^{-1}$, it is necessary to use formula (12), and in the intermediate case $1-\Delta\sim(\ln s^2)^{-1}$ the correct expression is given by the sum (7) + (12).

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