Theory of superconductivity in systems with an inverse distribution

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The time of interband recombination in a semiconductor can exceed the time of interband relaxation, so that it is possible to produce an inverse electron distribution in the conduction band with the aid of an external source. If the semiconductor gap E_g does not exceed a certain critical value, then a superconducting state is produced with repulsion between the electrons. For a single-band metal with repulsion between the electrons there exists, besides the trivial solution with zero superconducting gap, a stationary nontrivial solution with inverse distribution if the superconducting gap exceeds the limiting phonon energy. It is shown that an undamped current corresponding to ideal paramagnetism exists in such systems. The Meissner effect is therefore missing in such superconductors and the magnetic field penetrates into the sample, and executes spatial oscillations. Models of a semimetal and a two-band metal with overlapping bands and different masses are also considered.

1. INTRODUCTION

Under equilibrium conditions, the value of the superconducting gap Δ is determined by the following expression:[1]

$$1=-g\int_0^{\tilde{\omega}}\frac{1-2n_p(T)}{\varepsilon}d\xi,\qquad \varepsilon=(\xi^2+\Delta^2)^{1/2},\qquad (1)$$

where g is the coupling constant, $\tilde{\omega}$ the cutoff energy of interaction among electrons and $n_p(T)=[e^{\epsilon/T}+1]^{-1}$ the distribution function of the excitations. We note that Eq. (1) for the gap Δ remains valid even in the nonequilibrium case if we understand by n_p the distribution function of quasiparticles that satisfy the kinetic equation (see Eq. (26) below). Recently, the possibility of increasing the critical temperature T_c of the superconducting transition by the effect of external fields on the distribution function $n_p(T)$ has been studied. Éliashberg[2] has considered the effect of a uhf field with frequency $\omega<2\Delta(T)$ on T_c. In this case, its effect reduces to a redistribution of the already existent quasiparticles with respect to the energy, so that the population of states with small ϵ decreases and T_c increases.

Aronov and Gurevich[3,4] have suggested a method of decreasing the effective electron temperature T_e by means of resonance interband transitions under the action of an electromagnetic field and the extraction of excitations in tunnel structures; here T_c does not change.

If the frequency $\omega\gtrsim 2\Delta$ is not a resonance one for interband transitions, then the absorption of such radiation is connected with the breaking of superconducting pairs, i.e., with an increase in n_p.

As is seen from Eq. (1), increase of n_p in the case of attraction between the particles leads only to a decrease of T_c (which has been observed experimentally[5–7]). However, it has been shown [8] that the superconducting state is possible in exactly the opposite situation, when $1-2n_p$ and g simultaneously change sign, i.e., for an inverted distribution of quasiparticles $n_p>1/2$ and repulsive interaction between the particles.

In order to understand the nature of the formation of the bound state in repulsion, it is convenient to transform to the representation of quasiholes, for which the distribution will be noninverted, the potential energy of the interaction will be positive (i.e., repulsive), and the kinetic energy will be negative because of the negative sign of the mass m_h. Evidently, because of the different signs of the kinetic and potential energies, the formation of a bound state becomes possible.

The basic difficulty here is in the creation and maintenance of the inverted population. It has been shown[8] how one can avoid this difficulty in the semiconductor model, when the presence of a forbidden band makes the inverted distribution between bands possible. Methods of obtaining the inverted population in semiconductors (such as optical excitation, excitation by electron beam, excitation by injection of nonequilibrium electrons and by breakdown in an electric field) have been described by Basov.[9]

For the single-band metallic model, the possibility of the existence of superconductivity in the case of an inverted distribution and repulsion between particles has been shown previously.[10] Attention was drawn to the possibility of the creation of an inverted population inside the conduction band in a paper by one of the authors.[11] In this work, it was shown that if the gap that arises as a result of the action of a resonance electromagnetic field[12] becomes greater than half the phonon frequency, then the single-phonon transition across the gap becomes impossible. This leads to a blocking of the quasiparticles. Such a blocking is also possible because of a superconducting gap, so that a self-maintaining state is developed in the repulsion with a superconducting gap $2\Delta_0>\omega_{ph}$ and an inverted distribution of quasiparticles.[10]

The present work is devoted to a theoretical study of the superconducting properties of systems with an inverted distribution. The basic result is a demonstration of the existence in the presence of scattering from impurities of an undamped current, the sign of which is opposite the sign of the current in the equilibrium superconductor. At the same time, in place of ideal diamagnetism (the Meissner effect), the system possesses ideal paramagnetism. These differences can be understood if we take into account the possibility mentioned above of a transition to the quasihole representation $(m_h<0)$.

2. INVESTIGATION OF THE VERTEX PART IN THE SEMICONDUCTOR MODEL

We consider a semiconductor with the Hamiltonian

$$\mathcal{H}=\sum_{\alpha=1}^{2}\left\{\sum_{p}\varepsilon_{\alpha}(p)a_{\alpha p}^{+}a_{\alpha p}+\frac{g_0}{2}\sum_{p}a_{\alpha p}^{+}a_{\alpha-p}^{+}a_{\alpha-p}a_{\alpha p}\right\}+\frac{g_1}{2}\sum_{p}a_{1p}^{+}a_{1-p}^{+}a_{2-p}a_{2p}+\text{h.c.} \qquad (2)$$

in a nonequilibrium state with a population which is characterized by the chemical potentials μ_1 and μ_2 of electrons in the conduction band and in the valence band with the respective dispersion laws $\epsilon_1(\mathbf{p}) = p^2/2m + E_g$ and $\epsilon_2(\mathbf{p}) = -(\mathbf{p}-\mathbf{w})^2/2m - E_g$ (see Fig. 1a).

It follows from the kinetic equation for nonequilibrium electrons and holes in a semiconductor that if the dielectric gap $2E_g$ exceeds the maximum phonon frequency in the system:

$$2E_g > \omega_D, \qquad (3)$$

and the extrema of the bands are separated in momentum space by the vector $\mathbf{w}$, then the value of the recombination time τ_R of electrons excited by an external source reaches rather large values, of the order of $10^{-5}-10^{-6}$ sec. Inasmuch as the energy relaxation time of the carriers (electrons in the conduction band and holes in the valence band) is many orders of magnitude smaller than the τ_R found, then the inverted population mentioned above is entirely achievable; here μ_1 and μ_2 determine the quasi-Fermi levels of the electrons and holes, measured from the level $\mu = 0$ (the center of the forbidden band).

The investigation of the stability of such a nonequilibrium state relative to the superconductive pairing of electrons in the limits of each band reduces to the following set of equations for the vertex functions $\Gamma_{11}(q)$ and $\Gamma_{21}(q)$ (Fig. 2), which, for a small transferred 4-momentum $q = \{\omega, \mathbf{q}\}$, has the following form:

$$\Gamma_{11}(\omega_0)=[g_0+(g_1^2-g_0^2)\Pi_{22}]/\det, \quad \Gamma_{21}(\omega_0)=g_1/\det;$$
$$\det=1-g_0(\Pi_{11}+\Pi_{22})+(g_0^2-g_1^2)\Pi_{11}\Pi_{22}, \qquad (4)$$
$$\Pi_{\alpha\alpha}(\omega_0,\mathbf{q})=i(2\pi)^{-4}\int d\mathbf{p}\int d\omega\, G_\alpha(\mathbf{p},\omega)G_\alpha(\mathbf{q}-\mathbf{p},\omega_0-\omega).$$

In the limit $q = 0$, we easily obtain

$$\Pi_{11}=\int\frac{d\mathbf{p}}{(2\pi)^3}\begin{cases}[2\varepsilon_1(\mathbf{p})-\omega_0+i\delta]^{-1}, & \varepsilon_1(\mathbf{p})<\mu_1\\ -[2\varepsilon_1(\mathbf{p})-\omega_0-i\delta]^{-1}, & \varepsilon_1(\mathbf{p})>\mu_1\end{cases} \qquad (5)$$

while $\Pi_{22}(\omega_0, \mathbf{q})$ is obtained from $\Pi_{11}(\omega_0, \mathbf{q})$ by the transformation $1 \rightleftharpoons 2$. In the subsequent calculation of $\Pi_{\alpha\alpha}(\omega_0, \mathbf{q})$, we make the following assumption, which

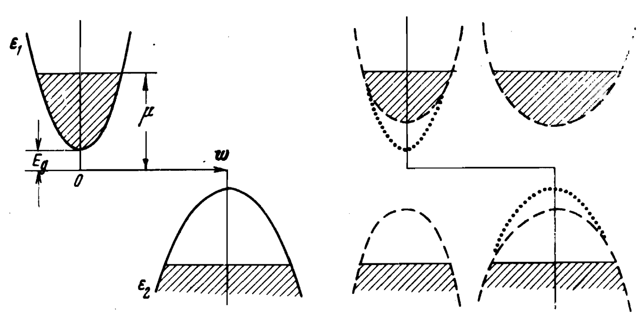

FIG. 1. Spectrum of excitations in a semiconductor with inverted distribution: normal (a) and superconducting (b) states of the system.

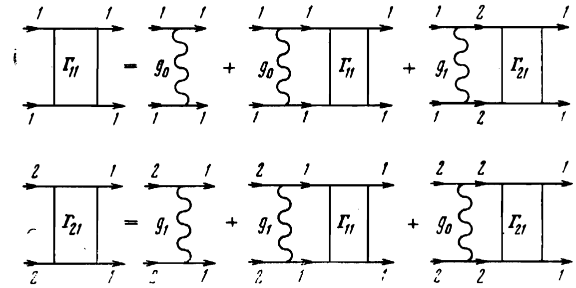

FIG. 2. Set of equations for the vertex functions Γ_{11} and Γ_{21}.

is valid only for quasi-two-dimensional structures:

$$\frac{d\mathbf{p}}{(2\pi)^3}=N(\varepsilon)d\varepsilon=d\varepsilon\begin{cases}N_0, & |\varepsilon|>E_g\\ 0, & |\varepsilon|<E_g\end{cases}. \qquad (6)$$

We note that such semiconductors are being intensively studied at the present time (see, for example, [13]).

We further set $m_1 = m_2$, i.e., $\mu_1 = \mu_2 = \mu$; $N_{01} = N_{02} = N_0$. We then introduce the nondimensional constants $g_0' = N_0 g_0$ and $g_1' = N_0 g_1$ of intra- and interband interaction of the electrons (we shall omit the primes below). Then direct calculation of the values of $\Pi_{\alpha\alpha}(\omega_0)$ from Eq. (5) with the density of states (6) leads to the following equation for the poles of the vertex functions:

$$1-g_0\ln\frac{(2\mu)^2-\omega_0^2}{2\tilde{\omega}(4E_g^2-\omega_0^2)^{1/2}}+\frac{g_0^2-g_1^2}{4}\ln\frac{(2\mu-\omega_0)^2}{2\tilde{\omega}(2E_g-\omega_0)}\ln\frac{(2\mu+\omega_0)^2}{2\tilde{\omega}(2E_g+\omega_0)}=0. \qquad (7)$$

We first set $|g_0| = |g_1|$ in (7) for simplicity:

$$1-g_0\ln\{[(2\mu)^2-\omega_0^2]/2\tilde{\omega}(4E_g^2-\omega_0^2)^{1/2}\}=0. \qquad (7')$$

Depending on the sign of the logarithmic term, the vertex part will have a pole singularity for the case of attraction between electrons ($g_0 < 0$) or for the case of repulsion ($g_0 > 0$). It is seen that in the attraction case, the corresponding pole ω_0 turns out to be equal to $2\mu + i\Omega$ ($\Omega = 2\tilde{\omega}\exp(-1/|g_0|)$).[8] The appearance of the real part 2μ here is connected with the reference energy chosen and corresponds to the formation of an electron (hole) pair near the Fermi level μ.

We turn our attention to the case of repulsion between the electrons ($g_0 > 0$). Simple calculations give a purely imaginary pole:

$$\omega_0=i(\Omega^2-4E_g^2)^{1/2},$$

where

$$\Omega=\frac{2\mu^2}{\tilde{\omega}}\exp\left(-\frac{1}{g_0}\right). \qquad (8)$$

The fact that the real part of the pole is now equal to zero corresponds to pairing of the electrons not at the Fermi level μ but near the extrema of the bands of the semiconductor. It is also seen that instability takes place only for a sufficiently small width of the forbidden band $2E_g < \Omega$. In the general case $g_0 \neq g_1$, Eq. (7) is of second order relative to the logarithmic term and therefore has two roots:

$$\frac{1}{2}\ln\frac{(2\mu)^2+\Omega^2}{2\tilde{\omega}(4E_g^2+\Omega^2)^{1/2}}=\frac{g_0}{g_0^2-g_1^2}\pm\left[\frac{g_1^2}{(g_0^2-g_1^2)^2}-\beta^2\right]^{1/2},$$
$$\beta=\tfrac{1}{2}[\operatorname{arctg}(\Omega/2E_g)-\operatorname{arctg}(\Omega/2\tilde{\omega})]. \qquad (9)$$

Equation (9) determines the modulus of the purely imaginary pole of the vertex part in the case of an arbitrary relation between g_0 and g_1. From the two

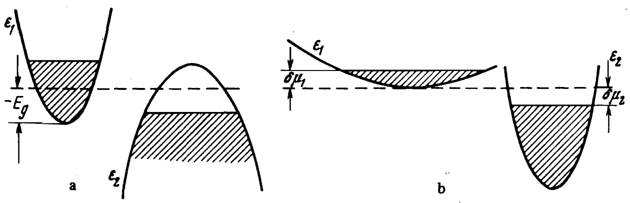

FIG. 3. Spectrum of excitations of a two-band systems with intersecting bands in the inverted state: a—semimetal, b—two-band metal. The broken lines indicate the position of the Fermi level in the equilibrium state.

solutions of Eq. (9), one must take that part which corresponds to the larger value of Ω. Taking into account that $E_g \ll \tilde{\omega}$, we note that the modulus of Ω is actually determined by the logarithmic term only, since β does not exceed $\pi/4$. Then the equation is easily solved:

$$\Omega=[\Omega_0^2-(2E_g)^2]^{1/2}, \qquad (10)$$

$$\Omega_0=\frac{2\mu^2}{\tilde{\omega}}\exp\left\{-2\left[\frac{g_0}{g_0^2-g_1^2}-\sqrt{\frac{g_1^2}{(g_0^2-g_1^2)^2}-\left(\frac{\pi}{4}\right)^2}\right]\right\}. \qquad (11)$$

Equations (10) and (11) show that superconductive pairing arises upon simultaneous satisfaction of two conditions:

$$\text{a) } 2E_g<\Omega_0, \quad \text{b) } g_1>\pi g_0^2/4 \qquad (12)$$

(see Fig. 2 of [8]).

Among systems that reveal instability in the inverted state relative to superconductive pairing for the case of repulsion between the electrons, substances with overlapping bands deserve mention. For example, the expression for the modulus of the imaginary pole for a semimetal is identical with that obtained for a semiconductor (10) and (11), except that we must now understand by E_g the amount of overlap of the electron and hole bands of the semimetal (Fig. 3a). Therefore the criterion for the production of superconductivity (12) in the case of a semimetal is that the band overlap not exceed some limiting value $(2|E_g|<\Omega)$.

When the Fermi level of the lower, partially filled band touches the bottom of a band (Fig. 3b) with significantly greater mass, then superconductive pairing also becomes possible in the case of repulsion, if an inverted population of the bands is achieved.

Here the equation for the modulus of the pole Ω in the vertex function has the solution

$$\Omega=\frac{2\delta\mu_1^2}{\tilde{\omega}}\exp\left(-\frac{1}{g_{\text{eff}}}\right), \quad g_{\text{eff}}=\frac{g_c N_1}{2+g_0 N_2\ln(\tilde{\omega}/\delta\mu_2)}, \qquad (13)$$

where the locations of the quasi-Fermi levels $\delta\mu_1$ and $\delta\mu_2$ and the densities of states N_1 and N_2 in each band are determined by the values of the corresponding effective masses:

$$m_1/m_2=N_1/N_2=\delta\mu_2/\delta\mu_1.$$

As in the case of a semiconductor and a semimetal, the amount of overlap of the edge of the upper "narrow" band with the Fermi level of the lower band is limited by a condition analogous to (12), i.e., it must not exceed the value of Ω calculated above for the case of touching.

3. EQUATIONS OF MOTION FOR THE GREEN'S FUNCTIONS

The investigation of the singularities of the Green's functions Γ_{11} and Γ_{12} in Sec. 2 shows that instability arises only in the particle-particle channel inside each band. In a state with an inverted population, the Coulomb interaction in the particle-hole channel corresponds to repulsion and the interband electron-hole pairing[14] does not take place. Therefore, for description of the stable state of the system it suffices to introduce into consideration, in addition to the ordinary Green's function

$$G_1(\mathbf{k},t)=-i\langle\hat{T}a_{1\mathbf{k}}(t)a_{1\mathbf{k}}^+(0)\rangle \qquad (14)$$

the anomalous function

$$F_1^+(\mathbf{k},t)\exp[i(E_{N+2}-E_N)t]=\langle N+2|\hat{T}a_{1,-\mathbf{k}}^+(t)a_{1\mathbf{k}}^+(0)|N\rangle, \qquad (15)$$

which characterizes only the intraband pairing of electrons (the Green's functions for the valence band are introduced similarly).

The form of the oscillating factor $\exp[i(E_{N+2}-E_N)t]$ is essentially determined by the position of the pole singularity in the vertex functions $\Gamma_{11}(\omega_0)$ and $\Gamma_{21}(\omega_0)$. In particular, for the usual Cooper singularity, $E_{N+2}-E_N=2\mu_0$, inasmuch as the instability is generated in the attraction case for states close to the Fermi surface.

In our case, the singularity is important for small ω_0 (measured from the center of the forbidden band). This circumstance will be specially taken into account in the derivation of the equations of motion for the functions G_1 and F_1^+ by setting $E_{N+2}-E_N=0$. In other respects, this derivation follows stationary techniques for $T=0$,[15] and the equations for the functions (14) and (15) take the following form after Fourier expansion:

$$(\omega-\varepsilon_1(\mathbf{p}))G_1(\omega,\mathbf{p})-i\Delta_1F_1^+(\omega,\mathbf{p})=1, \qquad (16)$$
$$(\omega+\varepsilon_1(\mathbf{p}))F_1^+(\omega,\mathbf{p})+i\Delta_1^+G_1(\omega,\mathbf{p})=0,$$
$$\Delta_1^+=g_0F_1^+(0)+g_1F_2^+(0). \qquad (17)$$

In a model with an inverted population, the spectrum of elementary excitations of the restructured phase is determined by the negative root $\omega^-=-(\epsilon_1^2+\Delta_1^2)^{1/2}$ of the determinant D of the set of equations (16) (see Fig. 1b). The least positive single-particle excitations in the system begin with energies determined by the position of the chemical potential μ. Therefore the solution of the system (16) has the following form:

$$G_1(\mathbf{p},\omega)=\frac{u_1^2}{\omega-\omega_1^+}+\frac{v_1^2}{\omega-\omega_1^-}, \quad F_1^+(\mathbf{p},\omega)=\frac{-i\Delta_1^+}{(\omega-\omega_1^+)(\omega-\omega_1^-)}; \qquad (18)$$

$$\omega_1^\pm=\pm\{(\varepsilon_1^2+\Delta_1^2)^{1/2}-i\delta\,\text{sign}\,[(\varepsilon_1^2+\Delta_1^2)^{1/2}-\mu_1]\},$$
$$u_1^2=\frac{1}{2}\left[1+\frac{\varepsilon_1(\mathbf{p})}{(\varepsilon_1^2+\Delta_1^2)^{1/2}}\right], \quad v_1^2=\frac{1}{2}\left[1-\frac{\varepsilon_1(\mathbf{p})}{(\varepsilon_1^2+\Delta_1^2)^{1/2}}\right]. \qquad (19)$$

The bypassing of the poles of the functions G_1 (18) used in (19) guarantees the correct population for the quasi-equilibrium state at $T=0$. A more general consideration with the use of kinetic equations is given in Sec. 4 for the metallic model.

For the self-consistent determination of the ordering parameter Δ_1^+ according to Eq. (17), we need the value of the anomalous mean $F_2^+(0)$ for electrons of the valence band $(\alpha=2)$. The derivation of the corresponding equations is entirely analogous to the derivation of (16), and the functions G_2 and F_2^+ can be obtained formally from (18) after the replacement $1 \rightleftharpoons 2$, except that one must take into account the singularities of the population of the states in the lower band, analogous to (18) (see Fig. 1). We then find the set of equations of self-consistency for the ordering parameters Δ_1 and Δ_2

from the definition of the mean value at coinciding arguments:

$$\begin{aligned} \Delta_1^+ &= g_0 F_1^+(0) + g_1 F_2^+(0), \\ \Delta_2^+ &= g_0 F_2^+(0) + g_1 F_1^+(0), \end{aligned} \tag{20}$$

where

$$F_\alpha^+(0) = \Delta_\alpha^+ \int_{E_g}^{\tilde{\omega}} N(\varepsilon)\, d\varepsilon \frac{1}{2(\varepsilon_\alpha^2 + \Delta_\alpha^2)^{1/2}} \quad \operatorname{sign}[(\mu_\alpha^2 - \Delta_\alpha^2)^{1/2} - \varepsilon]. \tag{21}$$

For identical masses $m_1 = m_2$, the ordering parameter also turns out to be the same, $\Delta_1 = \Delta_2 = \Delta$, and obeys only one equation of self-consistency

$$\frac{2}{g_0 + g_1} = \int_{E_g}^{\tilde{\omega}} N(\varepsilon)\, d\varepsilon \frac{\operatorname{sign}[(\mu^2 - \Delta^2)^{1/2} - \varepsilon]}{(\varepsilon^2 + \Delta^2)^{1/2}}. \tag{22}$$

In the approximation of constant density of states $N(\epsilon) = N$, the equation (22) is solved for Δ (compare with (11)):

$$\Delta = \sqrt{\Delta_0(\Delta_0 - 2E_g)}, \quad \Delta_0 = \frac{2\mu^2}{\tilde{\omega}} \exp\left(-\frac{2}{N_0(g_0 + g_1)}\right).$$

In the three-dimensional case, when

$$N(\varepsilon) = (2m)^{3/2}(\varepsilon - E_g)^{1/2}/4\pi^2, \tag{23}$$

the conditions for the generation of superconductive pairing become more rigid, since the density of states $N(\epsilon)$ is small in the region where the superconducting gap Δ is formed.

However, in the case of repulsion between electrons considered here, the interaction constant g_0 can turn out to be sufficiently large as to compensate the smallness of the density of states near the extrema of the bands. This is also confirmed by the numerical solution of Eq. (22) in the three-dimensional case with the density of states (23). Figure 4 shows the dependence of the normalized superconducting gap Δ on the value of the semiconductor gap E_g for several values of the ratio $\tilde{\omega}/\mu$.

4. SELF-SUPPORTED SUPERCONDUCTING STATE IN THE "METALLIC" MODEL

At first glance, it may appear that the inverted distribution of electrons in the conduction band of a metal (or a doped semiconductor) is impossible even in the presence of a source I which transforms electrons from a state with energy below the Fermi surface of the metal μ_0 into a state above the Fermi surface, $\mu_0 = n_0^{2/3}(3\pi^2)^{2/3}/2m$. Then the equation for the gap (1) has only the trivial zero solution ($\Delta = 0$, $n_p < 1/2$) with repulsive interaction ($g > 0$).

However, as will be shown below, in addition to this solution, there exists a nontrivial solution (with $\Delta \neq 0$

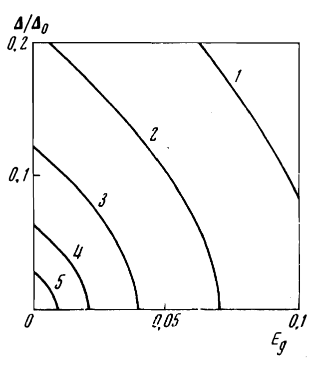

FIG. 4. Dependence of the superconducting gap Δ/Δ_0 on the value of the semiconducting gap E_g at $\tilde{\omega}/\mu$ = 1.00; 1.25; 1.50; 1.75 and 2.00 (curves 1-5, respectively). Δ_0 is the value of the gap for constant density of states.

and $n_p > 1/2$, which corresponds to an inverted distribution) of the joint system of equations for the gap (1) and the kinetic equation for the distribution functions of the quasiparticles n_p (25). Such a self-supporting state is realized under the condition (27). In the nonequilibrium case, the Green's functions of the electrons (14), (15) can be written in the form[16]

$$\begin{aligned} G(\mathbf{p}, \omega) &= \frac{\omega + \xi}{\omega^2 - \varepsilon_p^2 + i\delta} + 2\pi i n_p [u^2 \delta(\omega - \varepsilon_p) - v^2 \delta(\omega + \varepsilon_p)], \\ F^+(\mathbf{p}, \omega) &= \frac{i\Delta}{\omega^2 - \varepsilon_p^2 + i\delta} - \frac{\pi\Delta}{\varepsilon_p} n_p [\delta(\omega - \varepsilon_p) + \delta(\omega + \varepsilon_p)]. \end{aligned} \tag{24}$$

The kinetic equations for n_p can be found with the help of the Keldysh technique[16] (see, for example, [17]) or that of Gor'kov and Éliashberg:[18]

$$\begin{aligned} \frac{\partial n_p}{\partial t} = 2\pi \sum_{\mathbf{p}', \mathbf{q}} |\Phi(\mathbf{p}, \mathbf{p}', \mathbf{q})|^2 \{ & [n_{p'}(1 - n_p) + N_q(n_{p'} - n_p)] \\ & \times (uu' - vv')\delta(\varepsilon - \varepsilon' + \omega_q) \\ & - [n_p(1 - n_{p'}) + N_q(n_p - n_{p'})](uu' - vv')^2 \delta(\varepsilon' - \varepsilon + \omega_q) \\ & - [n_p n_{p'} + N_q(n_p + n_{p'} - 1)](uv' + u'v)^2 \delta(\varepsilon + \varepsilon' - \omega_q) \}, \\ N_q = (e^{\omega_q/T} - 1)^{-1}, & \end{aligned} \tag{25}$$

where N_q is the distribution function of the phonons and $\Phi(p, p', q)$ is the matrix element of the electron-phonon interaction. We note that in this case the equations take into account only single-phonon processes of scattering and annihilation of quasiparticles (i.e., the transition of the particles through the gap Δ). The equation for the gap can be obtained if we substitute (24) in the self-consistent condition $\Delta = gF(0)$:

$$\Delta = -\frac{1}{g} \int \Delta \frac{1 - 2n_p}{\sqrt{\xi^2 + \Delta^2}} d\mathbf{p}. \tag{26}$$

Thus the nonequilibrium superconductor is described by the set of equations (25) and (26).

It follows from the form of (25) that if the condition

$$2\Delta > \omega_{ph}, \tag{27}$$

is satisfied, the annihilation term (the third term) in (25) vanishes. This condition is equivalent to the condition (3) for the "semiconductor" model. Consequently, transitions of quasiparticles through the gap are possible only due to multiphonon processes. Therefore the time τ_R is much greater than the time of energy relaxation of the quasiparticles above (below) the gap, and blocking of the quasiparticles takes place. With account of the effect of the source I, the distribution function of the quasiparticles is given by the expression

$$n_p = [\exp(\varepsilon - \mu)/T + 1]^{-1}, \tag{28}$$

where μ is the quasi-Fermi level of the quasiparticles, determined by the intensity of the source I and the time τ_R.

Substituting (28) in (26), we get at $T = 0$

$$\Delta_0 = 2\tilde{\omega}/(e^{1/g} - e^{-1/g}). \tag{29}$$

The obtained solution with the gap Δ_0 in the nonequilibrium state, is self-supporting in the presence of the source: the gap leads to an inverted distribution of the particles, which, in turn, leads to maintenance of the gap.

The problem of the transition of the system from the state with vanishing solution to the state of superconductivity with $\Delta \neq 0$, $n_p > 1/2$ is an important one. One of the methods was proposed previously.[10] It consisted of the creation on the Fermi level of a bare gap $\lambda > \omega_{ph}/2$ due to interband transitions under the action of a strong electromagnetic field. (Here $\lambda = d\epsilon_0$; d is the dipole moment of the transition, ϵ_0 is the amplitude of the field.)

5. CURRENT WITH ACCOUNT OF ELASTIC SCATTERING FROM IMPURITIES

Inasmuch as the superconducting gap is not formed on quasi-Fermi levels $\pm\mu$, then the minimal excitation energy of the quasiparticles is equal to zero, as also in the nonsuperconducting state, i.e., at $\Delta = 0$. Therefore, the question arises as to whether an undamped current exists in such a system in the presence of scattering. For the study of this problem it is necessary to find the current in an alternating field $E(t) = E_0 e^{i\omega t}$ with account of elastic scattering from impurities. We shall follow Mattis and Bardeen[19] in the method of taking the impurity scattering into account.

Then, with the help of the Keldysh technique[16] an expression is obtained for the current that is the same as the expression of [19] except that $n_p(\epsilon) = [e^{\epsilon/T} + 1]^{-1}$ is replaced by the nonequilibrium distribution functions of the quasiparticles n_p, which obey the kinetic equations (25):

$$\mathbf{j}(\mathbf{r}, t) = \frac{e^2 N(0) v_0}{2\pi^2} e^{i\omega t} \int \frac{\mathbf{R}[\mathbf{R}\mathbf{A}_\omega(r')] I(\omega, R, T) e^{-R/L}}{R^4} d\mathbf{r}',$$

$$I(\omega, R, T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ L(\omega, \xi, \xi') - \frac{n(\xi) - n(\xi')}{\xi' - \xi} \right\} \cos\alpha(\xi - \xi')\, d\xi\, d\xi',$$

$$L(\omega, \xi, \xi') = \frac{1}{4}\left(1 + \frac{\xi\xi' + \Delta^2}{\varepsilon\varepsilon'}\right)\left(\frac{n' - n}{\varepsilon - \varepsilon' - (\omega - is)} + \frac{n' - n}{\varepsilon - \varepsilon' + (\omega - is)}\right) \quad (30)^*$$

$$+ \frac{1}{4}\left(1 - \frac{\xi\xi' + \Delta^2}{\varepsilon\varepsilon'}\right)\left(\frac{1 - n - n'}{\varepsilon + \varepsilon' - (\omega - is)} + \frac{1 - n - n'}{\varepsilon + \varepsilon' + (\omega - is)}\right),$$

$$s \to +0, \quad \alpha = R/v_0\hbar, \quad R = |\mathbf{r} - \mathbf{r}'|.$$

We now find the expression for the current in the limiting case $(\omega/D) \to 0$ that is of interest to us, substituting the functions n_p of (28) in (30):

$$\mathbf{j}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t)\left[2\sigma_N + i\frac{\pi\Delta}{\omega}\sigma_N\right], \quad \mu \gg \Delta, \quad (31)$$

where $\sigma_N = 2e^2 n v_0 L/3m$ is the normal conductivity and L is the free path length.

It is seen from Eq. (31) that such a system can be described by the two-fluid model of superconductivity. Here the first term in (31) corresponds to the normal (non-superfluid) component and is due to scattering of the excitations near the quasi-Fermi levels $\pm\mu$. The conductivity of the superfluid component (the second term) becomes infinite as $\omega \to 0$, i.e., it is insensitive to the scattering. We note that the sign of the second term is opposite the sign of the corresponding term in the current of the equilibrium superconductor.

The physical meaning of the obtained result is easily understood with the help of Fig. 5. Figure 5b corresponds to the current state (i.e., pairing of electrons with a nonvanishing total momentum) without account of scattering. Account of scattering leads to symmetrization of the electron momentum distribution on the quasi-Fermi levels $\pm\mu$ (see Fig. 5c), while at $p \approx p_0$ the asymmetry remains due to the suppression of the scattering because of the presence of the gap.

We note that at $\Delta = 0$, we get the usual expression for the current from (31):

$$\mathbf{j} = \mathbf{E}\sigma_N.$$

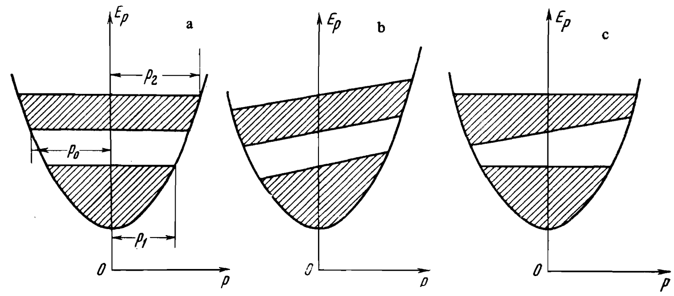

FIG. 5.

The corresponding calculation for the semiconductor model follows the scheme outlined above. In the approximation of constant density of states, the expression for the current has a form that is identical with (31), where by σ_N we understand the conductivity $\sigma_N = e^2 m\mu L/3\pi^2$ over the band with concentration $n = (2m\mu)^{3/2}/3\pi^2$ of electrons at a free path length $L = v_0\tau_{tr}$. In the three-dimensional case, when the density of states is proportional to $\sqrt{\epsilon}$, we must replace the factor Δ/ω in expression (31) for the current by $\Delta^{3/2}/(\mu\omega^{1/2})$, so that

$$\mathbf{j}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t)(2 + i\pi\beta\Delta^{3/2}/\mu\omega^{1/2})\sigma_N, \quad \beta \sim 1.$$

6. ANOMALOUS PARAMAGNETISM IN A NONEQUILIBRIUM SUPERCONDUCTOR

The paramagnetic portion of the Fourier component of the current under the action of a weak constant magnetic field in a nonequilibrium superconductor is given by the usual expression:

$$\mathbf{j}_\mathbf{q}^\mathbf{p} = -\frac{e^2}{m^2(2\pi)^4}\int d\omega\, d\mathbf{p}\,(\mathbf{p}\mathbf{A}_\mathbf{q})\mathbf{p}[G(\mathbf{p}, \omega)G(\mathbf{p}+\mathbf{q}, \omega) + F^+(\mathbf{p}, \omega)F(\mathbf{p}+\mathbf{q}, \omega)], \quad (32)$$

where the functions G and F are equal to (24) and the distribution function of the quasiparticles n_p obeys the kinetic equations (25).

Substituting (24) in (32) and integrating over ω, we get an expression for $\mathbf{j}_\mathbf{q}^\mathbf{p}$ that is formally identical with the well-known expression of [1] if in place of $[\exp(\epsilon/T) + 1]^{-1}$ we substitute the distribution function of quasiparticles that satisfy the kinetic equation (25):

$$\mathbf{j}_\mathbf{q}^\mathbf{p} = \frac{e^2}{m^2(2\pi)^3}\int d\mathbf{p}\,(2\mathbf{p}+\mathbf{q})\,\mathbf{p}\mathbf{A}_\mathbf{q} L(\varepsilon_\mathbf{p}, \varepsilon_{\mathbf{p}+\mathbf{q}}),$$

$$L(\varepsilon, \varepsilon') = \frac{1}{2}\frac{1 - n - n'}{\varepsilon + \varepsilon'}\left(1 - \frac{\xi\xi' + \Delta^2}{\varepsilon\varepsilon'}\right) + \frac{1}{2}\frac{n' - n}{\varepsilon - \varepsilon'}\left(1 + \frac{\xi\xi' + \Delta^2}{\varepsilon\varepsilon'}\right).$$

As $\mathbf{q} \to 0$, the first term in $L(\epsilon, \epsilon')$ vanishes because of the vanishing of the coherent factor, while the second term transforms into $\partial n/\partial\epsilon$. Then the total current, with account of the diamagnetic current, takes the form

$$\mathbf{j}_\mathbf{q}|_{\mathbf{q}\to 0} = -\frac{e^2 n_0}{m}\left(1 + \frac{2\mu_0}{p_0^3}\int_0^\infty p^4 \frac{\partial n}{\partial\varepsilon} d\mathbf{p}\right)\mathbf{A} \quad (33)$$

Substituting the expression for $n(\epsilon)$ in (33) at $T \ll \mu$, we finally obtain

$$j = \frac{e^2 n}{mc}\left(\frac{2\mu}{(\mu^2 - \Delta^2)^{1/2}} - 1\right)A \approx \frac{e^2 n}{mc}A, \quad \mu \gg \Delta, \quad (34)$$

i.e., the current is equal to the current in the equilibrium superconductor, but with opposite sign. This means that the considered system possesses anomalous paramagnetism, which leads to penetration of the magnetic field into the sample, and that the magnetic field undergoes oscillations with period $(4\pi n_0 e^2/m)^{1/2}$.

The physical meaning of this result is easily understood if we refer to Fig. 5a. A contribution to the paramagnetic field is made by electrons near $p = p_1$ $p = p_2$, while the contribution at $p = p_0$ vanishes due to the gap. Thus the paramagnetic current is doubled, so that the total current with account of the diamagnetic current is equal to the expression (34).

The corresponding expression in the case of the semiconductor model has the form

$$j_q|_{q\to 0} = \frac{e^2 n}{m} \frac{2\Delta^2}{\mu^2} A, \qquad (35)$$

where $n = (2m\mu)^{3/2}/3\pi^2$ is the number of nonequilibrium electrons in the conduction band of the semiconductor.

It should be noted that renormalization of the diamagnetic current due to superconductive pairing was taken into account in the calculation of (35).

We now consider the effect of temperature on the superconducting state. A change in the distribution function of the order of unity takes place in the layer kT near the quasi-Fermi levels $\pm\mu$ as the temperature is raised. Inasmuch as the change of $n_p(T)$ in the energy range near the superconducting gap turns out to be of the order of $e^{-\mu/T}$, it is clear that the transition temperature $T_c \sim \mu$.

We give estimates of the intensity I of the pump source at the frequency $\omega \gtrsim 2\mu$. The stationary state is established at $\kappa I = n/\tau_R$, $n = (2m\mu)^{3/2}/\hbar^3$, where κ is the absorption coefficient. We then obtain the following relation from the condition (27) of the existence of a self-maintaining solution

$$I > \frac{1}{\kappa\tau_R}\left[\frac{m^2\omega_{ph}\tilde{\omega}}{2}\exp\left(\frac{1}{g}\right)\right]^{3/4}. \qquad (36)$$

At $\tau_R \sim 10^{-8}$ sec, $\hbar\tilde{\omega} \sim 1$ eV, $m \sim (0.1-1)m_0$, $g = 0.5$, and $\kappa = 10^3-10^4$ cm^{-1}, we obtain a value of the order of 10^3-10^5 W/cm^2 for the intensity I. The corresponding critical temperature $T_c \sim \hbar(\kappa\tau_R I)^{2/3}/m$ turns out to be of the order of 10^3 °K. We note that our estimate of the intensity is rather illustrative, inasmuch as more optimal variants, the discussion of which lies beyond the limits of this paper, are possible.

For realization of superconductivity at $g > 0$ in the metallic model, as has been observed above, it is necessary to produce a dielectric gap λ over the whole Fermi surface, due to interband transitions, by means of a strong monochromatic field. For this purpose, the form of the surface of constant energy of the second zone at a distance $\hbar\omega$ from the Fermi level must be the same as that of the Fermi surface. Inasmuch as such an agreement is possible only near the extremal points, it is necessary to have the corresponding semiconductor, the Fermi population in which is established by doping with an impurity.

As was noted above, in semiconductors ($E_g > 0$) and semimetals ($E_g < 0$), superconductivity with an inverted population is possible if the condition $2|E_g| < \Delta_0$ is satisfied. Thus, one must investigate the narrow-band, layered semiconductors, and also semiconductor compounds of the type A_2B_6, the alloys BiSb, Bi_2Te_3, etc., and to obtain sufficiently long lifetime of the excitations, rather pure materials with noncoincident band extrema in momentum space are most suitable.

To establish the necessary conditions in two-band systems for the Fermi level of the lower, partially filled band (see Fig. 3b) to touch the bottom of an upper band with much larger mass, semiconductors with several noncoinciding minima (in momentum space) of the conduction band may be suitable—such as GaAs, which is used in Gunn diodes. The position of the Fermi level that is necessary for the superconducting state can be obtained in this case by appropriate doping with a donor impurity.

The condition $|E_g| < \Delta_0$ would appear at first glance to be impossible to meet, since the forbidden band should be larger than the Coulomb coupling energy because of the electron-hole Coulomb attraction in the stable state of the dielectric (exciton dielectric).[14] However, for an inverted population in the layer of ξ from 0 to μ, the Coulomb interaction will correspond to electron-hole repulsion. At $\mu \approx \Delta_0$, the condition $|E_g| < \Delta_0$ can be satisfied.

But if the pump intensity is below this threshold level, ($\mu < \Delta_0$) then the condition for superconductivity will nevertheless be satisfied because of the suppression of the effects of electron-hole pairing by scattering from the charged impurity.

Furthermore, at $\mu < \Delta_0$, the superconductivity condition can turn out to be satisfied at a temperature above some critical value if it is not satisfied at $T = 0$. The fact is that the critical temperature of electron-hole pairing is of the order of $E_g\{\ln[E_g/(\Delta_0 - E_g)]\}^{-1}$, while that of superconductivity is of the order of μ, as was shown above. Therefore, superconductivity is possible at $E_g\{\ln[E_g/\Delta_0 - E_g)]\}^{-1} < T < \mu$.

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*$[RA_\omega(r')] = R \times A_\omega(r')$.

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