# Excitation of acoustic oscillations by a low density beam of charged particles

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(Submitted June 20, 1973)
Zh. Eksp. Teor. Fiz. **65**, 2364–2368 (December 1973)

The size, shape, and time of creation of an overheated region near the track of a relativistic charged particle traversing a solid plate is estimated on the basis of the mechanism of ionization losses. The acoustic signal due to such an overheated region is calculated within the framework of thermoelastic theory. The possibility of acoustic recording of a relativistic multiply-charged ion is discussed.

1. It is well known[1-4] that in their passage through a solid beams of fast charged particles can excite various modes of acoustic oscillations in them. The mechanisms of excitation of the acoustic oscillations have been considered theoretically.[5-9] Macroscopic parameters have been introduced here for quantitative description of the stresses that are generated in the interaction zone of the beam with a solid. These include the bulk force and the thermoelastic stress. However, it is known that the transfer of energy and momentum from the beam to the solid takes place in microacts of interaction of the fast particles of the beam with the individual particles of the solid (electrons, nuclei), as a result of which the energy and the momentum transferred by the fast particles are first distributed in some small region of the material near the track of the passing particle. Calculation of the acoustic effect from such excited regions, which is important in the case of low-density beams (cosmic-ray showers), leads to results that differ from the results of calculation with macroparameters.[5-7]

2. We now determine the parameters which characterize the region of interaction of a single charged particle of the beam with the solid in the passage of the beam through a plate of thickness h. In what follows, we shall call such a region an "energy track." We limit ourselves to the case of a thin plate, when the energy lost by the particles in the plate is much smaller than its initial energy ($h \ll t_0$, where h is the thickness of the plate, $t_0$ the radiation thickness of the target substance). In this case, the value of the energy transferred by the particles to the substance is determined by the ionization losses of the charged particle in the substance, which are uniformly distributed along the track of the particle. The relaxation time $\tau_0$ of the transferred energy is determined by the mechanisms of interaction of the secondary electrons ($\delta$ electrons) with the substance. Taking into account that the characteristic time of electron-electron relaxation in a solid amounts to $\tau_1 \approx 10^{-13}$ sec, we get for the value of $\tau_0$

$$\tau_0 \approx (m_i/m_e)\tau_1 \approx 10^{-9}\ \text{sec}$$

($m_e$ is the mass of the electron, $m_i$ the mass of the nucleus of the substance). The radius $R_0$ of the energy track is determined by the diffusion of excited electrons in the substance in the time $\tau_0$:

$$R_0 = \sqrt{D\tau_0};$$

$D = l_\delta v_I/3$ is the diffusion coefficient of electrons in the substance, $l_\delta$ the free path length of an electron with velocity $v_I$ (here we have taken into account that the ionization losses are principally determined by secondary electrons with energies equal to the mean ionization potential I).

Estimation of $R_0$ gives results of the order of magnitude of $10^{-4}$ cm. It then follows that the initial radius of the track $R_0$ is a macroscopic quantity, such that one can introduce the temperature of the energy track. Here the excess of the temperature of the track over the temperature of the surrounding medium is equal to

$$\delta T_0 = \left(\frac{dE}{dx}\right)_{ion} \frac{1}{\pi R_0^2 c\rho},$$

where $(dE/dx)_{ion}$ is the ionization loss of the beam particle per unit path length in the substance, c is the specific heat, and $\rho$ the density of the target substance. In order of magnitude, $\delta T_0 \approx 10^{-4}$°K. The quantities $R_0$, $\tau_0$, and $\delta T_0$ characterize the initial stage of development of the energy track. At $t > \tau_0$, the track begins to spread; its radius R and the excess of temperature T, which depend on the time t, can be estimated by using the heat conduction equation. As a result, we obtain

$$R \approx R_0 + 2\sqrt{at}, \quad \delta T = \delta T_0 R_0^2/R^2, \tag{1}$$

where a is the coefficient of temperature conductivity of the material.

As is seen from (1), the radius of the energy track increases with time and, consequently, the temperature on it falls. Evidently, there will come a time $t_c$, corresponding to a track radius $R_c$, which the temperature excess $\delta T$ in the track becomes equal to the mean temperature fluctuation in it ($\langle \Delta T \rangle$):

$$\delta T = \langle \Delta T \rangle = T_0/\sqrt{c_0 N}; \tag{2}$$

here N is the number of particles in the energy track and $c_0$ the specific heat of the target substance, referred to a single atom.

Starting from (2), it is not difficult to obtain the following relations for $R_c$ and $t_c$:

$$R_c = \frac{\delta T_0}{T_0} R_0^2 \sqrt{\pi n h c_0}, \quad t_c \approx \frac{(R_c - R_0)^2}{4a},$$

where n is the density of atoms of the substance. An estimate of $R_c$ and $t_c$ at room temperature gives the following orders of magnitude:

$$R_c \approx 10^{-3}\ \text{cm}, \quad t_c \approx 10^{-6}\ \text{sec}.$$

At $t > t_c$, the excess of the track temperature over the temperature of the surrounding medium will be less than the mean temperature fluctuation and, consequently, it is impossible to separate the acoustic signal from the single track of thermal noise. It is therefore natural to assume $t_c$ to be the lifetime and $R_c$ the maximum radius of the energy track.

3. Let us calculate the acoustic signal from a single track. The ionization losses of a fast charged particle lead to the generation of the bulk thermoelastic force

$$F_T(\mathbf{r}, t) = -\Gamma \nabla P(\mathbf{r}, t) = -\alpha k \nabla T(\mathbf{r}, t), \tag{3}$$

  

in the solid. Here $\Gamma$ is the Grüneisen parameter of the target substance (we assume $\Gamma$ to be independent of direction in the target substance, as is the case in polycrystals and in single crystals of cubic structure), $P(r, t)$ is the density of the absorbed energy, $\alpha$ the coefficient of volume expansion, k the modulus of hydrostatic stress, and $T(r, t)$ the resultant temperature field.

We first solve the problem under the assumption that the track is an infinitely thin filament in which a continuous release of energy takes place with a constant intensity Sh (h is the length of the track). Taking into account the axial symmetry of the problem, we make use of the wave equation for longitudinal oscillations with the thermoelastic load on the right-hand side:[10]

$$\frac{\partial^2 u(r,t)}{\partial t^2} - c_l^2\left(\frac{\partial^2 u(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,t)}{\partial r} - \frac{u(r,t)}{r^2}\right) = \frac{F_T(r,t)}{\rho}; \quad (4)$$

here $u(r, t)$ is the amplitude of the generated diverging cylindrical sound wave, $c_l$ the longitudinal sound velocity in the target substance, and r the radius in the cylindrical system of coordinates.

The resulting temperature field $T(r, t)$ can be found as the solution of the heat conduction equation

$$\frac{\partial T(r,t)}{\partial t} = a\left(\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r,t)}{\partial r}\right) \quad (5)$$

with the boundary conditions

$$-2\pi\lambda \lim_{r\to 0}\left(r\frac{\partial T}{\partial r}\right) = S, \quad \lim_{r\to\infty} T(r,t) < \infty; \quad (6)$$

$\lambda = a\rho c$ is the coefficient of thermal conductivity of the target substance, and S the intensity of the source per unit length.

Solving Eqs. (4), (5) by means of Laplace transforms, we obtain the following expression for the amplitude of the acoustic signal:

$$u(r,t) = \frac{\Gamma S}{2\pi\rho c_l^2 r}\left\{\left(t^2 - \frac{r^2}{c_l^2}\right)^{1/2} - \int_0^t \exp\left(-\frac{r^2}{4au}\right) du - \int_{r/c_l}^t \exp\left[\frac{c_l^2}{a}(t-u)\right]\left(u^2 - \frac{r^2}{c_l^2}\right)^{-1/2} u\, du + \int_0^t \exp\left[-\frac{r^2}{4au} + \frac{c_l^2}{a}(t-u)\right] du\right\}. \quad (7)$$

Here it is assumed that the source is turned on at the time $t = 0$.

Assuming that the source acts only in the time interval $0 \le t \le \theta$, we finally obtain the expression

$$u_\theta(r, t) = u(r, t) - u(r, t-\theta). \quad (8)$$

for the amplitude of the acoustic oscillation.

4. Three characteristic times appear in the problem of the excitation of acoustic oscillations of the incident particle: the formation time of the track $\tau_0$, which is determined by the electron-ion relaxation time, $\tau_0 \approx 10^{-9}$ sec, the characteristic time for the diffusivity $\tau_a = a/c_l^2 \approx 10^{-12}$ sec, and the time of propagation of the signal from the source to the detector, $\tau_s = r/c_l$; for real experiments, $\tau_s \approx 10^{-4}$ sec. Thus, the inequalities

$$\tau_a \ll \tau_0 \ll \tau_s.$$

are satisfied. These inequalities allow us to simplify the general expression (8) materially. For this purpose, we consider first the idealized situation of a track which is formed after an infinitely short time, and then average the resultant amplitude of the sound signal over the time of formation of the track $\tau_0$.

For instantaneous formation of the track, we have from Eq. (8),

$$u_\delta(r,t) = \frac{\Gamma Q}{2\pi\rho a r}\left\{\int_0^t \exp\left[\frac{c_l^2}{a}(t-u) - \frac{r^2}{4au}\right] du - \int_{r/c_l}^t \exp\left[\frac{c_l^2}{a}(t-u)\right]\left(u^2 - \frac{r^2}{c_l^2}\right)^{-1/2} u\, du\right\}; \quad (9)$$

here Q is the energy released per unit length of the track. This expression is greatly simplified if we restrict ourselves to the most interesting case of not too large times between the instant of arrival of the signal and the instant of observation: $t - t_s \ll t_s$. As a result, we get

$$u_\delta(r,t) = \frac{u_m}{\sqrt{\pi}}\int_0^\infty \frac{dv}{e^v}\left[v + \frac{c_l^2}{a}(t - t_s)\right]^{-1/2}, \quad u_m = \frac{\Gamma Q}{2\rho(2\pi c_l^3 a r)^{1/2}}. \quad (10)$$

In the case of a real track, we must average expression (10) over the time of formation of the track $\tau_0$ and also over the cross section of the track. After averaging, we have

$$u(r,t') = \frac{4u_m a}{\pi^{3/2} R_0^2 c_l^2 \tau_0} \times \int_0^M [\xi(2R_0 - \xi)]^{1/2} d\xi \int_0^\infty e^{-v}\left\{\left[v + \frac{c_l^2}{a}\left(t' - \frac{\xi}{c_l}\right)\right]^{1/2} - \left[v + \frac{c_l^2}{a}\left(t' - \frac{\xi}{c_l} - \tau_0\right) \times \theta\left(t' - \frac{\xi}{c_l} - \tau_0\right)\right]^{1/2}\right\} dv, \quad (11)$$

where $M \equiv \min(c_l t', 2R_0)$,

$$\theta(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases};$$

$t'$ is the time measured from the instant of arrival of the acoustic disturbance at the point of observation; $\xi$ is the variable used to average over the cross section of the track.

The shapes of the sound signals are shown in Figs. 1, 2. Figure 1 corresponds to a real track, while Fig. 2 corresponds to an idealized, infinitesimally thin and instantaneously excited track. The dashed line corresponds to the level of thermal noise at room temperature ($T_0 = 300°K$). Evidently only those parts of the curves which are located above the dashed line have physical meaning.

5. Setting $\Gamma = 1$, $Q = (dE/dx)_{ion} = 5$ MeV/cm, $c_l = 2 \times 10^5$ cm/sec, $\rho = 3$ g/cm$^3$, $a = 1$ cm$^2$/sec, and $r = 1$ cm, we get the value $10^{-15}$ cm for the maximum acoustic displacement, to which a material stress of $10^{-3}$ dyn/cm corresponds.

Thus the acoustic stresses and displacements that are produced by a fast charged particle are very small.

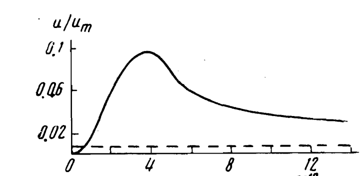


FIG. 1. Amplitide of the acoustic signal from a real track of a fast charged particle.

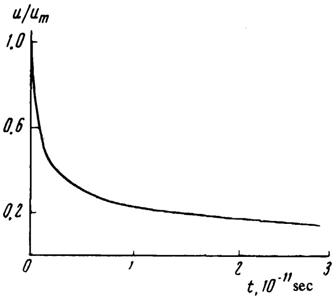


FIG. 2. Amplitude of the acoustic signal from an idealized, instantaneously excited track.

However, they grow strongly in the case of a fast, multiply charged ion. Inasmuch as the ionization losses of the charged particle are proportional to the square of charge, the amplitude of the acoustic signal from the fast ion increases by a factor of $Z^2$ ($Z$ is the ionic charge). Setting $Z = 82$, we obtain a value of $7 \times 10^{-12}$ cm for the maximum acoustic displacement. Acoustic signals with such amplitudes can in principle be recorded by existing detectors.

The acoustic signal from a beam of low intensity is not difficult to obtain by summing the acoustic signals from all the particles of the beam.

In conclusion, the authors express their gratitude to I. A. Akhiezer and V. D. Volovik for valuable discussions.


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Translated by R. T. Beyer
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