Drift-cone instability in the presence of a cold-ion source

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It is shown that collisions lead to considerable heating and accumulation of cold ions in an adiabatic trap, so that the presence of a relatively weak source of such ions impedes the buildup of drift-cone instability. In some cases the ions produced by charge exchange in the residual gas are sufficient for stabilization. Heating of cold ions does not result in any appreciable decrease of the energy lifetime.

INTRODUCTION

One of the characterisitc properties of a plasma confined in an adiabatic trap is the non-Maxwellian ion-distribution function which is associated with the presence of a "loss cone." For the most widely spread out velocity distribution, the so-called collisional-equilibrium distribution, the difference from a Maxwellian function is expressed mainly in a deficient number of ions having low transverse energy. Because of this unbalance in the ion-velocity distribution, the plasma becomes unstable even when the transverse gradients of the plasma density are relatively small.[1,2] But, as has been shown in[3], by adding to a hot plasma a small amount of plasma containing comparatively cold ions one can elevate considerably the critical gradient for this so-called drift-cone instability.

When the effect of adding cold plasma was considered in[3], an arbitrary cold-ion distribution was assumed. However, in the case of plasma confinement during a time that is of the order of the collision time the distribution function of cold ions (like that of the hot ions) cannot be arbitrary but must depend on the collisions, which induce heating of the cold ions and, consequently, longer confinement of the latter. The heating of the cold ions leads, in turn, to a change of the energy balance. Also, the drift-cone mode, having small phase velocity, is sensitive to the form of the cold-ion distribution.

All the foregoing considerations indicate the need for a more detailed study of the stabilizing action of cold ions on the drift-cone instability, taking into account the effect of collisions on the distribution of these ions and on their lifetime.

1. THE COLD-ION DISTRIBUTION FUNCTION

Let us consider a stationary model of an adiabatic trap where, in addition to the hot ions, a certain amount of cold ions is generated per unit time. Possible sources of the cold ions are charge exchange and ionization, as well as some form of external injection. For simplicity we shall assume that the initial distribution of the generated cold ions is Maxwellian. Without specifying the source (which it is not essential to do in the present case), we shall characterize it by the injection time

$$\tau^* = n_0 (dn^*/dt)^{-1} \tag{1}$$

(where n_0 is the hot-ion density and dn^*/dt is the number of cold ions generated per unit time per unit volume) and by the temperature T^* of the generated cold ions. If charge exchange is the source, then τ^* is the charge-exchange time.

In connection with the formulation of the cold-ion distribution function collisions among the latter are unimportant in the present case. Collisions with electrons also play no decisive role; therefore, for simplicity, we shall neglect them and shall make suitable corrections in the final expressions. Collisions with hot ions thus play the principal role and the collision term in the kinetic equation for the cold-ion distribution becomes linear. We shall also assume fulfillment of the condition

$$t_{tr} < \tau_{st} T / T_i, \tag{2}$$

where t_{tr} is the cold-ion transit time along the length of the trap, τ_{st} is the collision time of the hot ions, and $T_i = m_i \langle v \rangle / 3$ is the effective temperature of the hot ions. The condition (2) corresponds to neglect of the energy change of the generated cold ions during a single transit; when this condition is fulfilled we may assume zero distribution of cold ions inside the loss cone. Then, considering that $T^* \ll T_i$ and employing the "square well" model[4] (where it is postulated that all the plasma parameters and the magnetic field are uniform along the length of the trap and change abruptly in a mirror), and also disregarding ambipolar effects, we obtain the following equation for the cold-ion distribution function:

$$D \Delta_v f^* = -\frac{n_0}{\tau^*} \left(\frac{m_i}{2\pi T^*} \right)^{3/2} \exp\left\{ -\frac{m_i v^2}{2T^*} \right\},$$

$$D = aT / m_i \tau_{st}, \quad \tau_{st} = 3\sqrt{2\pi m_i}\, T_i^{3/2} / 8\pi e^4 \lambda n_0, \tag{3}$$

where the coefficient a depends on the hot-ion distribution and is close to unity, m_i and e are the ion mass and charge (all ions are of the same species), λ is the Coulomb logarithm, and Δ_v is the Laplacian in velocity space. This equation must satisfy the boundary conditions that f^* vanishes on loss-cone surfaces and at infinity.

Because of the axial symmetry, the solution of (3) can be expressed conveniently as a Fourier series in the angular eigenfunctions of the problem, which are solutions of the Legendre equation

$$\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial M_n(\mu)}{\partial \mu} + l_n(l_n+1) M_n(\mu) = 0.$$

$$M_n(\pm\mu_0) = 0, \quad \mu_0 = \sqrt{1 - 1/R}, \tag{4}$$

where μ is the cosine of the angle between the velocity vector and the magnetic field and R is the mirror ratio. The solution of (3) then becomes

$$f = \frac{n}{\pi^{3/2} D \tau^*} \sum_{n=0}^{\infty} \frac{(2T_i/m_i)^{l_{2n}/2} \mathscr{M}_{2n}^{(1)}}{(2l_{2n}+1)\mathscr{M}_{2n}^{(2)}} \left\{ \frac{\Gamma((l_{2n}+3)/2)}{2v^{l_{2n}+1}} + \left(\frac{m_i}{2T^*} \right)^{(l_{2n}+3)/2} \right. \tag{5}$$

$$\left. \times \int_v^{\infty} \left(\frac{v^{l_{2n}}}{v_1^{l_{2n}+1}} - \frac{v_1^{l_{2n}}}{v^{l_{2n}+1}} \right) v_1^2 \exp\left(-\frac{m_i v_1^2}{2T^*} \right) dv_1 \right\} M_{2n}(\mu),$$

$$\mathscr{M}_n^{(i)} \equiv \int_0^{\mu_0} M_n^i(\mu)\, d\mu.$$

The derived distribution function falls off as a power function for $v \to \infty$; the first term of the series decreases most slowly. For the most interesting mirror ratios, $1.5 < R < \infty$, the eigenvalue l_0 lies within the limits $2 > l_0 > 0$. The cold-ion density obtained by integrating (5) for f^* then diverges, because (3) and, therefore (5) also, is invalid for $v \gtrsim (T_i/m_i)^{1/2}$. But at these velocities the function f^*, like the distribution function of the majority of the hot ions, must decrease as

$$\exp\left\{-\left(1+\frac{l_0}{3}\right)\frac{m_i v^2}{2T_i}\right\}$$

so that a more accurate expression for f^* can be obtained if the right-hand side of (5) is multiplied by this exponential.

Knowledge of the distribution function makes it easy to obtain an expression for the density of cold ions:

$$n^* \approx n_0 \frac{\tau_{st}}{\tau^*}\left(\frac{T^*}{T_i}\right)^{l_0/2} \frac{\Gamma((3+l_0)/2)\Gamma((2-l_0)/2)}{\sqrt{\pi}(2l_0+1)a}\left(\frac{6}{3+l_0}\right)^{1-l_0/2}\frac{(\mathcal{M}_0^{(1)})^2}{\mathcal{M}_0^{(2)}},$$

$$n^* = \frac{3}{8\sqrt{5}} n_0 \frac{\tau_{st}}{\tau^*}\left(\frac{T^*}{T_i}\right)\ln\frac{3T_i}{5T^*}, \quad R=1,5, \tag{6}$$

and for their mean energy:

$$\varepsilon^* \approx \frac{3}{2} T_i \frac{2-l_0}{3+l_0}, \quad R>1.5,$$

$$\varepsilon^* \approx \frac{3}{2} T_i \frac{2}{5\ln(3T_i/5T^*)}, \quad R=1.5. \tag{7}$$

The expressions (6) and (7) show that for $R > 1.5$ considerable heating and an accumulation of cold ions will result from collisions. For $R < 1.5$ the collisional effect becomes less important, so that the density of cold ions is determined by the transit time and their energy is determined by the injection energy.

2. STABILIZATION CONDITIONS

We shall now consider the effect of the cold-ion source on the stability of the drift-cone mode. It will be assumed that the magnetic field is parallel to the z axis and that the plasma density depends only on the coordinate z. Fulfillment of the following condition will also be postulated:

$$(\omega/\omega_{pe})^2 k_\perp^2 L^2 \ll 1, \tag{8}$$

where ω is the oscillation frequency, $\omega_{pe}^2 = 4\pi n_0 e^2/m_e$ is the plasma electron frequency, $k_\perp^2 = k_x^2 + k_y^2$, and L is the characteristic length of the machine. The condition for fluting of the drift-cone mode is given by (8), which is usually fulfilled for traps of the given type. Then the dispersion equation becomes[1]

$$1 + \frac{\omega_{pe}^2}{\omega_{Be}^2} = \frac{\omega_{pi}^2}{\omega\omega_{Bi}}\frac{\varkappa k_y}{k_\perp^2} - \frac{8\pi^2 e^2}{k_\perp^2 m_i}\int_0^\infty v_\perp\, dv_\perp$$

$$\times \int_{-\infty}^{\infty} dv_\parallel \frac{\partial f_i}{v_\perp \partial v_\perp}\sum_{n=-\infty}^{\infty}\frac{n\omega_{Bi} J_n^2(k_\perp v_\perp/\omega_{Bi})}{\omega - n\omega_{Bi}};$$

$$\varkappa = \frac{1}{n_0}\frac{dn_0}{dx}, \quad \omega_{pi}^2 = \frac{4\pi e^2 n_0}{m_i}, \quad \omega_{Bi} = \frac{eB}{m_i c}. \tag{9}$$

The most unstable oscillations are those with $k_x = 0$; therefore we shall assume henceforth that $k = k_\perp = |k_y|$.

The ion distribution function f is the sum of the collisional-equilibrium distribution function of the hot ions and the function (5) for the cold ions. It is convenient to integrate in (9) after transforming to spherical coordinates:

$$\int_0^\infty v_\perp dv_\perp \int_{-\infty}^{\infty} dv_\parallel \frac{\partial}{v_\perp \partial v_\perp}[f(v)M_0(\mu)]J_n^2\left(\frac{kv_\perp}{\omega_{Bi}}\right)$$

$$= 2\int_0^{\mu_0} d\mu \frac{M_0(\mu)}{1-\mu^2}\int_0^\infty J_n^2\left(\frac{kv}{\omega_{Bi}}\sqrt{1-\mu^2}\right)\frac{\partial}{\partial v}[vf(v)]dv. \tag{10}$$

The last term in (9) is a quite complicated function of k and ω, whose form depends on the magnitude of the mirror ratio. The analytic study of this equation is performed most simply for R = 3.277 with $l_0 = 1$ and $M_0(\omega)$ expressed in terms of elementary functions. The qualitative results obtained for R = 3.277 hold true within a broad range of the mirror ratios ($2 \lesssim R \lesssim 10$) that are most interesting from a practical point of view.

In view of the foregoing considerations, we shall consider (9) for R = 3.277. When f^* is substituted in this equation, we can confine ourselves to the first term in the curly brackets for the zeroth member of the series (5), because the contribution of the remaining terms will be smaller by the factor $(T^*/T_i)^{1/2}$. Then the complete expression for f_i becomes

$$f_i = \frac{0,38}{\pi} n_0 \frac{m_i^2}{T_i^2}\exp\left\{-\frac{2m_i v^2}{3T_i}\right\}M_0(\mu)\left(v + \frac{0,46}{v^2}\left(\frac{T_i}{m_i}\right)^{3/2}\frac{\tau_{st}}{\tau^*}\sqrt{\frac{T^*}{T_i}}\right);$$

$$M_0(\mu) = 1 - \frac{\mu}{2}\ln\frac{1-\mu}{1+\mu}. \tag{11}$$

Inserting (11) into (9), employing the transformation (10) and the asymptotic form of the Bessel functions for $k^2\rho^2 \gg 1$ (where $\rho^2 = T_i/m_i\omega_{Bi}^2$), we obtain an explicit form of the dispersion equation:

$$1 + \frac{\omega_{pe}^2}{\omega_{Be}^2} = \frac{\omega_{pi}^2}{\omega\omega_{Bi}}\frac{\varkappa}{k} - \frac{\omega_{pi}^2}{\omega_{Bi}^2}\left(\frac{0,47}{k^3\rho^3}\sum_{n=-\infty}^{\infty}\frac{\omega}{\omega - n\omega_{Bi}}\right.$$

$$\left. - \frac{0,17}{k\rho}\frac{\tau_{st}}{\tau^*}\sqrt{\frac{T^*}{T_i}}\sum_{n=-\infty}^{\infty}\frac{\omega_{Bi}}{\omega - n\omega_{Bi}}\frac{n}{n^2 - 1/4}\right). \tag{12}$$

Since, as shown in[1], we have the characteristic values $k\rho \sim (m_e/m_i)^{1/3}$, the condition $k^2\rho^2 \gg 1$ is fulfilled by a good margin.

The derived equation can be written in a more convenient form with the aid of the familiar formula

$$\sum_{n=-\infty}^{\infty}\frac{1}{x-n} = \pi \operatorname{ctg} \pi x, \tag{13}$$

and by converting it into a dimensionless form:

$$\frac{\pi}{\beta^2}x^2 \operatorname{ctg}\pi x - A\left(\pi \operatorname{ctg}\pi x - \frac{1}{x} + \frac{2x}{3(x^2-1)}\right) + \beta x = B; \tag{14}$$

$$x = \frac{\omega}{\omega_{Bi}}, \quad \beta = 1,29 k\rho\Omega^{1/3},$$

$$A = 0,22\frac{\tau_{st}}{\tau^*}\sqrt{\frac{T^*}{T_i}}\Omega^{-2/3}, \quad B = 1,29\varkappa\rho\Omega^{-2/3},$$

$$\Omega \equiv m_e/m_i + \omega_{Bi}^2/\omega_{pi}^2.$$

Stability of the oscillations corresponds to the absence of the complex roots of (14) in the upper half-plane of the variable x. The left side of (14) as a function of the real variable x possesses at least one maximum. If B is somewhat greater than this maximum value, Eq. (14) has two complex roots, one of which lies in the upper half-plane. Thus for the stability of all oscillations described by (14) it is necessary and sufficient that B shall be smaller than the smallest maximum. The magnitude of the maximum depends on β and A; since $\beta \sim k$, minimization of this value with respect to β yields the wavelength of the most unstable oscillations. It will be stated hereafter that we are investigating the stability of the n-th harmonic ($n > 0$) if the corresponding maximum lies in the interval $n-1 < x < n$.

The results obtained by investigating the stability of the first three harmonics are shown in the figure; the boundaries of their stable regions are shown with shading inside the edge of each respective unstable region. The region of stability with respect to all harmonics lies along the ordinate axis; its boundaries are determined by the stability of the first and second harmonics. The point C corresponds to the maximum (for R = 3.277) critical gradient, which is 4.1 times greater than the critical gradient in the absence of a cold plasma (A = 0) and 1.4 times smaller than the maximum critical gradient obtained in[3]; these results are associated with the different form of the cold-ion distribution function. The absolute value of the maximum critical gradient, and the corresponding parameters that characterize the cold-ion source, are

$$\varkappa\rho=1.78\Omega^{2/3},\quad \frac{\tau_{st}}{\tau^*}\sqrt{\frac{T^*}{T_i}}=3.67\Omega^{2/3}. \qquad (15)$$

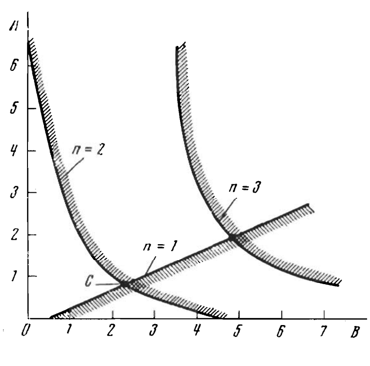
The density of cold ions, in accordance with (6), will now be

$$n^*=1.19\Omega^{2/3}n_0. \qquad (16)$$

3. DISCUSSION OF RESULTS

The considered conditions of the stabilizing action of cold ions on the drift-cone mode show that even a relatively low-power source of cold ions can appreciably enhance the critical density gradient that is required for the build-up of this mode. The principal limitation on the enhancement of the critical gradient as the cold-ion density increases is imposed by the second harmonic, which is associated with the so-called "double-humped" instability. However, the build-up of all harmonics above the first is influenced considerably by longitudinal nonuniformity of the magnetic field,[5] which can lead to a higher critical gradient. Increase of the mirror ratio is also accompanied by an increase of the critical gradient.

Calculations show that in experiments performed with the PR-6 machine[6] the ions resulting from charge exchange and ionization of the residual gas can have an appreciable stabilizing influence, because in this case we have $(\tau_{st}/\tau^*)(T^*/T_i)^{1/2} \sim 10^{-2}$. Ions produced by charge exchange and ionization can also comprise one of the causes that this instability is not observed in some other machines[7,8] where the critical gradient is reached without taking cold ions into account.



In the derivation of the cold-ion distribution function it was assumed that condition (2) is fulfilled. Such fulfillment may not occur in certain cases when the cold ions can acquire appreciable energy during a single transit. This energy should then naturally play the role of the initial cold-ion energy. Since the exact form of the initial cold-ion distribution is not important, all expressions will remain correct except for the substitution $T^* = T_i t_{tr}/\tau_{st}$.

From the consideration of how the cold-ion distribution function is formulated it is seen that the energy of these ions is considerably enhanced through collisions with hot ions. Since this energy is derived from the energy of the hot ions, it is of interest to determine how the cold ions affect the energy loss of the plasma. This loss is usually characterized by the energy lifetime of the ions, which in the present case is the ratio between the combined energy density of both the cold and hot ions, and the density of the energy flux carried away by the hot and cold ions that fall into the loss cone as a result of collisions. It is easily shown that this energy lifetime (for R = 3.277) is given by

$$\tau_\epsilon=\tau_{\epsilon0}\left(1-0{,}012\frac{\tau_{st}}{\tau^*}\sqrt{\frac{T^*}{T_i}}\right), \qquad (17)$$

where $\tau_{\epsilon0}$ is the energy lifetime without account of the cold ions. The small reduction of τ_ϵ is associated with the fact that as cold ions are heated their containment time is increased.

When collisions between cold ions and electrons are taken into account the collision time τ_{st} is effectively reduced, so that the corresponding corrections amount to the following substitution in the derived expressions:

$$\tau_{st}\to\tau_{st}\left(1+\frac{1}{a}\sqrt{\frac{m_eT_i}{m_iT_e}}\right)^{-1}. \qquad (18)$$

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