Cyclotron resonance in thin conductors

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The high frequency properties of thin metallic plates located in a strong magnetic field parallel to their surface are investigated (the curvature of the electron trajectory r is much smaller than the mean free path l. It is shown that new cyclotron resonance frequencies appear in place of the "cutoff" resonant frequencies at $d < 2r$. These new frequencies correspond to the frequency of revolution of electrons in orbits with a diameter equal to the plate thickness d. The impedance has a logarithmic singularity at these frequencies. The magnetic field dependence of the impedance is investigated at various plate thicknesses. This yields a relation between the electron revolution period and the diameter of its orbit over the entire Fermi surface.

1. In a magnetic field parallel to the surface of a conductor, under conditions of the anomalous skin effect, the interaction of the conduction electrons with the electromagnetic field is a multiple of the frequency of revolution of the electron in the magnetic field Ω.[1] For conduction electrons with an anisotropic, nonquadratic dispersion law, Ω turns out to be different for different values of the projection of the momentum p_z on the direction of the magnetic field. Therefore, only small groups of carriers can take part in cyclotron resonance, namely, electrons with the extremal frequency of revolution Ω_e, electrons near a limiting point of the Fermi surface [1], and electrons the diameters of whose orbits are extremal.[2] Knowing the frequency of cyclotron resonance, we can determine the effective masses of the indicated conduction electrons. To obtain sharp resonance lines, it is necessary that the carriers be in the narrow skin layer many times during the time of free flight t_0, i.e., the radius of curvature r of the trajectory of the conduction electron in the magnetic field should be much smaller than the free path length l, and the thickness of the conductor placed in the resonator should exceed the diameter of the largest orbit of the electron. Upon decrease in the thickness of the sample d, the orbits of the "resonance" electrons cannot fit in the cross section of the conductor, and, due to scattering of the carriers by the boundaries of the sample (in the case of nonspecular reflection), a cutoff takes place of the resonance frequencies first observed by Khaikin.[3] The method of cutting off of the resonant frequencies allows us to determine not only the extremal frequency of revolution of the electron in orbit in a magnetic field, but also the diameter of this orbit.

The investigation of cyclotron resonance in thin conductors allows us, as will be shown below, to determine the connection between the frequency of revolution and the diameter of the orbit for any cross section of the Fermi surface. We imagine that none of the orbits of the electrons with extremal effective mass is contained any longer in the cross section of the conductor. Then the frequency of revolution of the carriers that do not collide with the sample boundaries is a monotonic function of p_z. In this case, electrons moving along the orbit with the least and the greatest diameters are "isolated," and, in addition to the electrons close to a limiting point of the Fermi surface at resonance, electrons will take part whose orbit diameter is equal to the thickness of the conductor, i.e.,

$$2r(p_z)\,|_{p_z=p_1}=d,\quad 2r(p_z)\equiv cD(p_z)/eH, \tag{1}$$

where e is the electron charge, c the velocity of light, H the magnetic field, and $D(p_z)$ the diameter in the direction of the p_x axis of the cross section of the Fermi surface cut by the plane $p_z = \text{const}$. The y axis is directed along the normal to the surface of the plate.

The impedance will have a singularity of the logarithmic type if the condition

$$\omega=n\Omega(p_1) \tag{2}$$

is satisfied, i.e., the cyclotron resonance at the new frequencies will have exactly the same character as the resonance due to electrons near the limiting point of the Fermi surface at $\omega = n\Omega_0$.

The resonance frequencies corresponding to the frequency $\Omega_1 \equiv \Omega(p_1)$ and the frequency of revolution of the electrons near the turning point of the Fermi surface Ω_0 will be separated if their difference $\delta\Omega$ exceeds the collision frequency of the electron, i.e.,

$$t_0\delta\Omega\approx t_0\frac{\partial\Omega}{\partial p_z}\bigg|_{p_z=p_1}(p_0-p_1)\gg 1.$$

The condition of solvability of the resonance frequencies can be represented in another way if we recognize that

$$\frac{\partial\Omega(p_z)}{\partial p_z}\bigg|_{p_z=p_1}\approx\frac{\Omega}{p_1}$$

and the difference between p_1 and the value of the momentum at the limiting point p_0 can be set equal to $p_1(d/r)^2$ in order of magnitude, where $2r = cD_{max}/eH$. Then the condition $t_0\delta\Omega \gg 1$ is equivalent to the condition $r \ll d^{2/3}l^{1/3}$ and the resonance at the frequencies $\omega = n\Omega_1$ will have the observed singularity if

$$d<2r\ll d^{2/3}l^{1/3}. \tag{3}$$

For the determination of the connection of $\Omega(p_1)$ and $D(p_1)$, it is necessary to obtain the set of curves $Z(H)$ experimentally at various values of the plate thickness d. It is expedient to plot the resonance curves by fixing the value of the diameter $D(p_1)$, i.e., at $Hd = \text{const}$. Then the resonance frequencies determine the frequency of revolution of the electron $\Omega(p_z)$ in the orbit whose diameter is equal to $D(p_z) = eHd/c$. Inasmuch as the quantity eHd/c can take on arbitrary values, we can in principle determine all possible frequencies of revolution of the electrons on the Fermi surface in a magnetic field and find their connection with the diameter of the orbit.

The appearance of a new frequency of cyclotron resonance instead of the "cut-off" frequency occurs when a part of the orbits of the electrons with Ω_e is still contained in the cross section of the conductor. However, cyclotron resonance at $\omega = n\Omega_e$ leads to the appearance in the impedance of a fractional-power-law singularity and not a logarithmic one, which complicates the separation of the resonance frequencies corresponding to Ω_1. For interpretation of the resonance curves, it is necessary to compare them with the experimental data obtained on bulk conductors. If the reflection of the electrons from the boundary of the sample is close to specular, then the analysis of the experimental resonance curves becomes still more complicated, since the resonance frequencies corresponding to Ω_e do not disappear with decrease in d but are shifted and depend on the thickness of the conductor.[4,5]

2. The complete set of equations for the determination of the surface impedance tensor that connects the electric field on the surface of a metal with the total current

$$E_\mu(0) = Z_{\mu\nu}\int_0^d J_\nu(y)\,dy, \tag{4}$$

consists of the Maxwell equations

$$\frac{\partial^2 E_\mu}{\partial y^2} = -\frac{4\pi i\omega}{c^2} J_\mu, \quad \mu = (x, y), \tag{5}$$

$$\rho' = 0 \tag{6}$$

and the Boltzmann equation, which allows us to determine the electron distribution function f(y, **p**) and the electric current density **J**, assuming the high-frequency field **E** to be given. The electromagnetic field is assumed to be monochromatic, and in Eqs. (4) and (5) and everywhere below, we mean by **E** and **J** the amplitudes of the field and the current. Using the condition of electric neutrality (6), we can find the electric field at any depth of the conductor and eliminate it from the expression for the electric current.

In the linear approximation in terms of a weak electric field, the current density is determined by the value of the nonequilibrium contribution to the distribution function on the Fermi surface:

$$\mathbf{J} = \langle \mathbf{v}\Psi\rangle, \quad \langle g\rangle \equiv \frac{2e^3H}{ch^3}\int_{\epsilon=\epsilon_0} g\,dt\,dp_z, \tag{7}$$

since the conduction electrons in the metal form a strongly degenerate Fermi gas. Here $e(\partial f_0/\partial\epsilon)\Psi = f(y, \mathbf{p}) - f_0(\epsilon)$, $f_0(\epsilon)$ is the equilibrium distribution function of the carriers, ϵ_0 is the Fermi energy, and h is Planck's constant.

The kinetic equation, linearized in terms of the weak alternating field **E**(y),

$$(-i\omega + 1/t_0)\Psi + v_y\frac{\partial\Psi}{\partial y} + \frac{\partial\Psi}{\partial t} = \mathbf{vE} \tag{8}$$

must be supplemented by the boundary conditions which take into account the character of the reflection of the electron from the boundary of the metal:

$$\Psi(0; t', p_z')_{v_y>0} = q_1(t, p_z)\Psi(0; t, p_z)|_{v_y<0} + \chi_1, \tag{9}$$

$$\Psi_z(d; t, p_z)|_{v_y<0} = q_2(t', p_z')\Psi(d; t', p_z')|_{v_y>0} + \chi_2, \tag{9'}$$

where q_1 and q_2 are the specularity parameters of scattering of electrons by the boundaries y = 0 and y = d of the sample (these generally depend also on the angle of incidence of the electron on the surface of the conductor), and t is the time of motion of the carriers in the magnetic field, i.e., the phase on the orbit ϵ = const, p_z = const. The specific form of the collision integral for the considered effects does not play a significant role and, for convenience in calculation, we have used the relaxation time approximation in Eq. (8).

The change in the chemical potential of the reflected electrons χ_1 and χ_2 can be expressed in terms of the distribution function of the incident electrons, making use of the condition of vanishing of the normal component of the current at the boundary of the conductor:

$$\chi_1 = \frac{\langle(1-q_1(t,p_z))v_y\Psi(0;t,p_z)(1-\theta(v_y))\rangle}{\langle v_y(1-\theta(v_y))\rangle}$$

$$\chi_2 = \frac{\langle(1-q_2(t',p_z'))v_y\Psi(d;t',p_z')\theta(v_y)\rangle}{\langle v_y\theta(v_y)\rangle};$$

$$\theta(x) = \begin{cases}1, & x>0\\ 0, & x<0\end{cases}.$$

Under conditions of the limiting anomalous skin effect, an important contribution to the current is made by electrons which have a turning point in the skin layer (the point at which the component of the velocity normal to the surface vanishes). Therefore, the electrons which collide with both surfaces of the plate are unimportant, and we shall not take such electrons into account in the calculation of the electric field

$$J_\mu(y) = \left\langle v_\mu(t)\frac{qI^{\lambda}_{\lambda-T(y,p_z)} + \chi\exp\{(\lambda(y,t)-t)(1/t_0-i\omega)\}}{1-q\exp\{T(y,p_z)(i\omega-1/t_0)\}}\right\rangle + \langle v_\mu(t)I^{t}_{\lambda(y,t)}\rangle;$$

$$I_A{}^B = \int_A^B dt'\exp\{(t'-t)(1/t_0-i\omega)\}\mathbf{v}(t')\mathbf{E}(y+y(t')-y(t)), \tag{10}$$

$$q = \{q_1, q_2\}, \quad \chi = \{\chi_1, \chi_2\},$$

where $T(y, p_z)$ is the period of motion of the electron, and $\lambda(y, t)$ is the instant of reflection of the electron from the boundary of the sample.

In what follows, it is very important that the electric field and the current are rapidly damped in the interior of the plate (away from the surface y = 0) and are significantly different from zero only in a layer of small thickness δ. This makes it possible to replace the upper limit of the integral in Eq. (4) by infinity and, extending **E**(y) and **J**(y) in even fashion along the negative semiaxis y < 0, we change over to the Fourier transform

$$j_\mu(k) = 2\int_0^\infty J_\mu(y)\cos ky\,dy, \quad \varepsilon_\mu(k) = 2\int_0^\infty E_\mu(y)\cos ky\,dy. \tag{11}$$

We note that the analysis that follows is applicable only in the fundamental approximation in δ/d. In this approximation, the Maxwell equation (5), after the transformation (11), takes the form

$$k^2\varepsilon_\mu(k) + 2\frac{\partial E_\mu(0)}{\partial y} = \frac{4\pi i\omega}{c^2}j_\mu(k). \tag{12}$$

Small terms proportional to $E_\mu(d)$ and $\partial E_\mu(d)/\partial y$ are omitted. Therefore, in the calculation of the impedance with the help of Eqs. (7), (11) and (12), we should limit ourselves only to the asymptotic expression when the ratio of the skin layer depth δ to the conductor thickness d tends to zero, and retention of the next terms of the expansion in the parameter δ/d is an exaggeration of the accuracy.

Separating out the components in (10) whose change with the magnetic field has a resonance character, we obtain the local connection of the Fourier components of the electric current and field:

$$j_\mu(k) = \sigma_{\mu\nu}(k)\varepsilon_\nu(k). \tag{13}$$

It must be noted that the exact account of the boundary conditions (9) and (9′) leads to an integral connection between $j_\mu(k)$ and $\epsilon_\nu(k)$. Azbel' and Kaner have shown[1,6] that in the solution of the problem of finding the shape of the cyclotron resonance lines in the case when the reflection of the electrons from the surface of the sample is not close to specular, account of the integral term in the formula for the electric field leads to the appearance only of an unimportant factor of the order of unity in the expression for the surface impedance. The exact solution of the corresponding integral equation, obtained by Hartmann and Luttinger,[7] and by Meĭerovich,[8] completely confirmed this result. Thus, in the analysis of the shape of the cyclotron resonance line, when the scattering by the surface of the plate $y = 0$ is almost specular, we can limit ourselves to the local connection between the Fourier components of the electric current and field, which simplifies the problem considerably.[1)]

The asymptotic expression for $\sigma_{\mu\nu}(k)$ for $kr \gg 1$, can be represented in the following fashion:

$$\sigma_{\mu\nu}(k)=\frac{2\pi e^3H}{ch^3k}\left\{\int\limits_{-p_0}^{+p_0}dp_z\theta(2r(p_z)-d)\frac{q_2(p_z)A_{\mu\nu}(p_z)}{1-q_2(p_z)\exp(i\omega^*T_\lambda(p_z))}\right.$$
$$\left.+\int\limits_{-p_0}^{+p_0}dp_z\theta(d-2r(p_z))\frac{A_{\mu\nu}(p_z)}{1-\exp(i\omega^*T(p_z))}\right\}. \tag{14}$$

Here $T(p_z) = 2\pi/\Omega(p_z)$, $T_\lambda(p_z)$ is the period of motion of the electron along the open orbit broken up by specular reflections from the surface of the plate:

$$A_{\mu\nu}(p_z)=\frac{v_\mu(p_z,0)v_\nu(p_z,0)}{|v_y'(p_z,0)|} \tag{15}$$

the prime indicates differentiation with respect to t and the start of the measurement of t is placed at the point of stationary phase, i.e., $v_y(p_z, 0) = 0$.

The second term in Eq. (14) takes into account the contribution to $\sigma_{\mu\nu}(k)$ of electrons which do not collide with the boundaries of the sample and which are returned many times to the skin layer by the magnetic field; the first term, which is sensitive to the character of the reflection of the carriers by the surface of the conductor, takes into consideration the contribution made to the electric conductivity by electrons which collide with the boundaries of the sample $y = d$ and which return many times to the skin layer at almost specular reflection.

It is easy to find the Fourier component of the electric field $\epsilon_\mu(k)$ from Eq. (12) for the case of a local coupling between $j_\mu(k)$ and $\epsilon_\mu(k)$ and then, making use of the definition (4), we obtain the following expression for the impedance:

$$Z_{\mu\nu}=-\frac{8i\omega}{c^2}\int\limits_0^\infty B_{\mu\nu}^{-1}(k)\,dk,$$
$$B_{\mu\nu}(k)=k^2\delta_{\mu\nu}-\frac{4\pi i\omega}{c^2}\sigma_{\mu\nu}(k). \tag{16}$$

Reducing the tensor $k\sigma_{\mu\nu}(k) = w_{\mu\nu} = \text{const}(k)$ to the principal axes and carrying out integration over k, we obtain an explicit expression for the diagonal components of the impedance:

$$Z_\mu=-\frac{16\pi i\omega}{3\sqrt{3}\,c^2}\left(-\frac{4\pi i\omega}{c^2}w_\mu\right)^{-1/3}, \tag{17}$$

where w_μ are the principal values of the tensor $w_{\mu\nu}$, the determination of which is clear from Eq. (14); the choice of the cube root in Eq. (17) is determined by the requirement that the real part of Z_μ be positive.

3. We shall now make clear the character of the resonance singularity of the impedance, which is due to electrons whose orbit diameter is equal to the thickness of the plate. Assuming the condition (3) to be satisfied and using the central symmetry of the Fermi surface, we transform the Fourier components of the tensor $\sigma_{\mu\nu}(k)$ to the following form:

$$\sigma_{\mu\nu}(k)=\frac{2\pi e^3H}{ch^3k}\left\{\int\limits_{p_1-\delta p}^{p_1}\frac{a_{\mu\nu}(p_z)q_2(p_z)\,dp_z}{1-q_2(p_z)\exp(i\omega^*T_\lambda(p_z))}\right.$$
$$\left.+\int\limits_{p_1}^{p_1+\widetilde{\delta p}}\frac{a_{\mu\nu}(p_z)\,dp_z}{1-\exp(i\omega^*T(p_z))}\right\},\quad a_{\mu\nu}(p_z)=A_{\mu\nu}(p_z)+A_{\mu\nu}(-p_z). \tag{18}$$

If the magnetic field H is close to the resonant value H_1, for which the relations (1) and (2) are satisfied, then, in the calculation of $\sigma_{\mu\nu}(k)$, one can use the saddle-point method. Here, as it is not difficult to see, a logarithmic singularity develops in the second component of the expression for $\sigma_{\mu\nu}(k)$ at $\omega = n\Omega_1$ and $t_0 \to \infty$. The first component in the formula (18) can have a singularity when $t_0 \to \infty$ only as $q_2 \to 1$. However, this singularity can arise only under purely random circumstances if $n\Omega_1$ for some number n is a multiple of the extremal frequency of revolution of the electron, $\Omega_\lambda \equiv 2\pi/T_\lambda$. Consequently, this singularity is not connected with the contribution to the first component of $\sigma_{\mu\nu}(k)$ in the immediate neighborhood of the upper limit of integration, and the first term in the expression for (18) gives a finite contribution to $\sigma_{\mu\nu}(k)$ at $\omega = n\Omega_1$ for arbitrary t_0. It is easy to establish this fact by representing the period of motion of the electrons $T_\lambda(p_z)$, for cross sections with a diameter close to the thickness of the plate, in the form of a function $T(p_z)$ and orbit diameter $2r(p_z)$ of these electrons in the bulk sample:

$$T_\lambda(p_z)=T(p_z)\left\{1-\frac{2}{\pi}\left[1-\frac{d}{2r(p_z)}\right]^{1/2}\right\}. \tag{19}$$

Expanding $T(p_z)$ and $r(p_z)$ in this expression in power series in $p_z - p_1$, we obtain

$$T_\lambda(p_z)\approx T(p_1)\left\{1-\frac{2}{\pi}\left|\frac{2}{d}\frac{\partial r(p_1)}{\partial p_z}\right|^{1/2}|p_z-p_1|^{1/2}\right\},$$

i.e., the period $T_\lambda(p_z)$ changes very rapidly near the point p_1, so that the derivative $\partial T_\lambda/\partial p_z$ is proportional to $|p_z - p_1|^{-1/2}$.

Thus the electrons which collide with the boundaries of the sample do not make a contribution to the considered resonance and the specularity parameter q_2 drops out of the final formulas. For the resonant part of $\sigma_{\mu\nu}(k)$, we have

$$\sigma_{\mu\nu}(k)=-\frac{ie^3H}{ch^3k}\frac{a_{\mu\nu}(p_1)}{\alpha n}\ln\left[\left(\frac{\alpha}{\beta}-1\right)\Delta+i\gamma\right],$$
$$\alpha=\frac{1}{T}\frac{\partial T}{\partial p_z}\bigg|_{p_z=p_1},\quad \beta=\frac{1}{D}\frac{\partial D}{\partial p_z}\bigg|_{p_z=p_1}, \tag{20}$$
$$\gamma=\frac{1}{2\pi n}\frac{T(p_1)}{t_0}\sim\frac{1}{n}\frac{d}{l},\quad \Delta=\frac{H-H_1}{H_1}.$$

If the symmetric tensor $a_{\mu\nu}$ is reduced to diagonal form and Eq. (17) is used, we get the following expression for the impedance:

$$R_\mu=\begin{cases}\dfrac{4^{5/3}\pi^{2/3}}{3\sqrt{3}}b_\mu\dfrac{h}{ec}\left(\dfrac{n\omega^2}{H}\right)^{1/3}\left(\dfrac{\alpha}{a_\mu(p_1)}\right)^{1/3}\mathscr{L}^{-4/3}, & \dfrac{a_\mu(p_1)}{\alpha}>0\\[2ex] \dfrac{4^{7/6}\pi^{2/3}}{3}\dfrac{h}{ec}\left(\dfrac{n\omega^2}{H}\right)^{1/3}\left|\dfrac{\alpha}{a_\mu(p_1)}\right|^{1/3}\mathscr{L}^{-1/3}, & \dfrac{a_\mu(p_1)}{\alpha}<0\end{cases}; \tag{21}$$

$$X_\mu = -\frac{\pi^{2/3}}{3\sqrt{3}} A \frac{h}{ec}\left(\frac{n\omega^2}{H}\right)^{1/3}\left|\frac{\alpha}{a_\mu(p_1)}\right|^{1/3}\mathscr{L}^{-1/3};$$

$$\mathscr{L}=\ln[(\alpha/\beta-1)^2\Delta^2+\gamma^2]^{-1/2},\quad 0<b_\mu\sim 1. \tag{22}$$

$$A=\begin{cases}4^{5/3}, & a_\mu(p_1)/\alpha>0\\ 4^{7/6}, & a_\mu(p_1)/\alpha<0\end{cases}.$$

The impedance is minimal at $\Delta = 0$; the phase relations (relation between the real part of the impedance R_μ and its imaginary part X_μ) at resonance, as is seen from the expressions (21) and (22), depend significantly on the sign[2)] of $a_\mu(p_1)/\alpha$.

If we construct resonance curves with fixed $D(p_1)$, then the shape of the curve $Z(d)$ close to resonance is also described by Eqs. (21) and (22), in which we must replace $\alpha/\beta - 1$ by unit, and take Δ to mean $(d-d_{res})/d_{res}$, by virtue of the condition $Hd = \text{const}$.

It is interesting to consider the case in which two resonant frequencies Ω_1 and Ω_2, are close, so that $\epsilon_1 = (\Omega_1 - \Omega_2)/\Omega_1$ is small in comparison with unity. Here the shape of the resonance curve is determined not only by the mean free path of the electrons but also by the parameter ϵ_1. In a thin plate, such a situation is realized if the cross section $p_z = p_1$ determined by Eqs. (1) and (2) is close to the cross section with the extremal effective mass for some resonance number n, or is close to the limiting point of the Fermi surface.

In weak magnetic fields $l > r \gg d$, when the frequency Ω_1 is close to the frequency Ω_0, of revolution of electrons near the turning point, the expression for the high-frequency conductivity tensor takes the following form:[3)]

$$\sigma_z(k) = \frac{ie^3H}{2ch^3k}\frac{a_z(p_0)}{n\sqrt{\varkappa(p_0)}}\frac{1}{\mathscr{R}}\left\{\ln\frac{\tau-\mathscr{R}}{(d/r)^2+\tau-\mathscr{R}}+\ln\frac{(d/r)^2+\tau+\mathscr{R}}{\tau+\mathscr{R}}\right\} \tag{23}$$

$$\mathscr{R}=\sqrt{\tau^2+\Delta-i\gamma},$$

where

$$\varkappa(p_0)=\frac{1}{2T(p_0)}\frac{\partial^2T(p_0)}{\partial p_z^2},\qquad \tau=\frac{1}{T_0}\frac{\partial T(p_0)}{\partial p_z}\Big/2\sqrt{\varkappa(p_0)},$$

$$2\pi n(1-\Delta)=\omega T(p_0)$$

and, for definiteness, we assume $\kappa(p_0) > 0$, $\tau > 0$.

The effective mass of the conduction electrons $m^*(p_z)$ generally does not have an extremum at the limiting point of the Fermi surface and the resonance at frequencies that are multiples of Ω has a logarithmic character. However, at certain orientations of the magnetic field, vanishing of $\partial m^*/\partial p_z$ is possible in principle for $p_z = p_0$ and instead of a logarithmic singularity in the impedance, there is a fraction-power-law singularity, as is usual for resonance with electrons having an extremal effective mass. Therefore, in the case of expansion of the period of revolution of the electron in a magnetic field, $T(p_z)$, in a power series in $p_z - p_0$, the linear and quadratic terms are retained, i.e.,

$$T(p_z)\approx T(p_0)[1+2\tau\sqrt{\varkappa(p_0)}(p_z-p_0)+\varkappa(p_0)(p_z-p_0)^2]. \tag{24}$$

The shape of the resonance curves, as is seen from Fig. 1, depends in significant fashion on the parameter τ, which takes into account how slowly the effective mass of the charge carrier changes with p_z near the limiting point of the Fermi surface.[4)]

If the principal term in the expansion (24) over the entire interval (p_1, p_0) is the linear term, i.e.,

$$\tau \gg (d/r)^2, \tag{25}$$

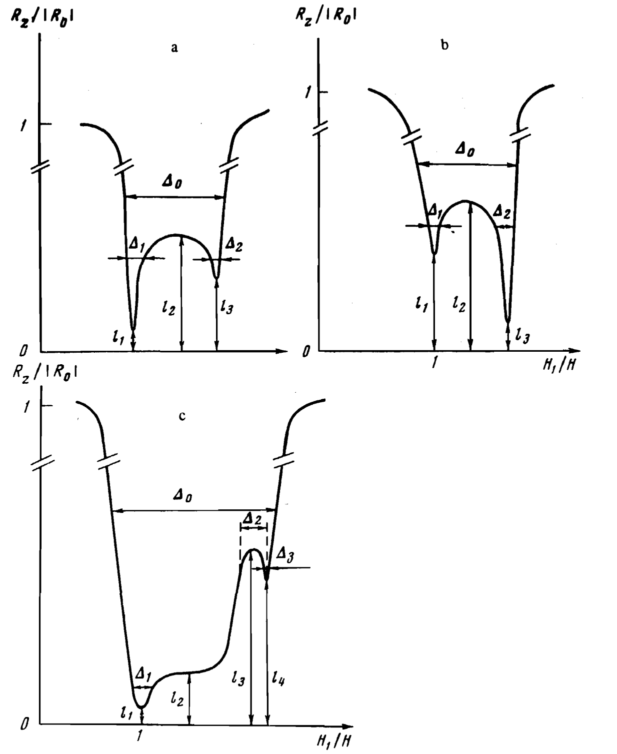

FIG. 1. Dependence of the real part of the impedance R_z, referred to $|R_0|$, on the magnetic field in the region of magnetic fields $l > r \gg d$. a) Total width of the resonance peak $\Delta_0 \approx \tau (d/r)^2$, in the middle of it the relative value of the impedance is of the order $l_2 \approx \tau^{1/3}$. The width of the deeper of the minima near the frequencies Ω_1 and Ω^0 is of the order $l_2 \approx \tau^{1/3}$. The width of the deeper of the minima near the frequencies Ω_1 and Ω_0 is of the order $\Delta_1 \approx \Delta_2 \approx \gamma$ and their depth is $l_1 \approx \tau^{1/3}$. $\ln^{-4/3}[\gamma^{-1}\tau(d/4)^2]$, $l_2 \approx \tau^{1/3}\ln^{-1/2}[\gamma^{-1}\tau(d/r)^2]$. b) The resonance curve has a more complicated structure in this case. The total width of the peak $\Delta_0 \approx (d/r)^4$, for $H_1/H \approx 1 + \Delta_0/2$; the ratio $Rz/|R_0|$ is of the order $l_2 \approx (d/r)^{2/3}$. In the region of order $\Delta_1 \approx \tau^2$ close to the frequency Ω_1, the ratio $R_z/|R_0| \approx l_{,1} < l_2$, where $l_1 \approx (d/r)^{8/3}$, $\ln^{-4/3}(1/\tau)$. The width of the peak Δ_2 near the frequency Ω_0 is also of the order of τ^2, and its height is $l_3 \approx \tau^{1/3}$. The additional narrow peak of width $\Delta_3 \approx \gamma$ has the height $l_4 \approx \tau^{1/3}$, $\ln^{-1/3}(\tau^2/\gamma)$. c) The symmetry of the fine structure of the peak is opposite to what is shown in Fig. 1a. The total width of the resonance peak $\Delta_0 \approx (d/r)^4$ at $H_1/H \approx 1 + \Delta_0/2$; the impedance takes on the value $l_2 \approx (d/r)^{8/3}$. The shape of the additional minimum near the frequency Ω_0 actually does not depend on the presence of a neighboring resonance frequency Ω_1, i.e., its width is $\Delta_2 \approx \gamma^{1/2}$, and its depth $l_3\ \gamma^{4/9}$. The minimum near the frequency Ω_1 is deeper than in the case when Ω_1 is not close to Ω_0, its depth is $l \approx (d/r)^{8/3}$, $\ln^{-4/3}$ X $[(d/r)^4/\gamma]$ and its width $\Delta_1 \approx \gamma$.

then the relative change in the frequency $(\Omega_1 - \Omega_0)/\Omega_0$ is of the order $\tau(d/r)^2$ and to be able to obtain the resonance frequencies Ω_1 and Ω_0, the condition

$$\tau(d/r)^2 \gg \gamma \tag{26}$$

must be satisfied. For $\tau \sim 1$, this expression is identical with the condition (3).

Substituting the expression (23) for the conductivity in Eq. (17), we can easily calculate the impedance, which is determined by the following expression:

$$Z_z = \frac{16\pi^{2/3}}{3\sqrt{3}\cdot 2^{2/3}}\frac{h}{ec}\left(\frac{n\omega^2}{H}\right)^{1/3}\left(\frac{\sqrt{\varkappa(p_0)}}{a_z(p_0)}\right)^{1/3}\tau^{1/3}|\eta|^{-1/3}$$

$$\times\left(1+\frac{is}{3|\eta|}\right)\exp\left[i\left(-\frac{\pi}{2}+\frac{\pi}{6}(1-s)\right)\right], \tag{27}$$

$$\eta=\ln\left\{\frac{\Delta^2+\gamma^2}{[2\tau(d/r)^2-\Delta]^2+\gamma^2}\right\}^{1/2},\qquad s=\text{sign}\,\eta.$$

Analysis of this formula shows that the resonance peak has a fine structure: in the middle of the interval (Ω_1, Ω_0) the relative depth of the minimum of the impedance $Z_Z/|Z_0|$ is of the order of $\tau^{1/3}$ (here Z is the impedance in the absence of the magnetic field) and there are deep minima of the impedance in narrow regions with relative width of the order of γ, close to each of the frequencies Ω_1 and Ω_0. The shape of the resonance curve $R_Z(H)$ is shown schematically in Fig. 1a.

This curve can be realized only when the parameters τ and d/l satisfy the condition

$$\tau \gg (d/l)^{2/5}, \tag{28}$$

which becomes evident if we rewrite the inequalities (25) and (26) in the following equivalent form:

$$d/\sqrt{\tau} \equiv r_2 \ll r \ll r_1 \equiv (\tau l d^2)^{1/3}.$$

We now trace out how the shape of the curve varies with increase in the magnetic field, assuming the inequality (28) to be satisfied. For $r \sim r_1$, when the linear term in the expansion of the period (24) becomes of the order of the quadratic, the qualitative form of the curve $Z_Z(H)$ does not change. With further increase in the field, i.e., for $r \ll r_2$, the square-law term becomes fundamental in the expansion (24); this term also determines the total width of the resonance curve, which is of the order of $\epsilon \equiv (d/r)^4$.

Omitting the cumbersome term for the impedance, we show the schematic form of the dependence $R_Z(H)$ in Fig. 1b. In contrast with the preceding case, the width of the resonance peaks close to the frequencies Ω_1 and Ω_0 is determined by the parameter τ^2, and not τ. There is still one more narrow peak with width of the order of γ inside the resonance peak, close to the frequency Ω_0.

If the parameter τ is so small that the inequality (28) is reversed, i.e., if

$$\tau \ll (d/l)^{2/5}, \quad \text{(eq)}$$

then the linear term in the expansion of the period (24) does not play any significant role and the parameter falls out of the formula for the impedance everywhere:

$$Z_z = \frac{16\pi^{2/3}}{3\sqrt{3}} \frac{h}{ec} \left(\frac{\sqrt{\varkappa(p_0)}}{a_z(p_0)} \right)^{1/3} \left(\frac{n\omega^2}{H} \right)^{1/3} \delta^{1/6} (\xi^2 + \zeta^2)^{-1/6}$$
$$\times \exp\left\{ -\frac{i}{3} \left(\pi + \operatorname{Arctg} \frac{\xi\sqrt{\delta-\Delta} - \zeta\sqrt{\delta+\Delta}}{\xi\sqrt{\delta+\Delta} + \zeta\sqrt{\delta-\Delta} + \tilde{b}_z n\delta} \right) \right\};$$
$$0 < \tilde{b}_z \sim 1, \quad \delta = \sqrt{\Delta^2 + \gamma^2}, \quad \xi = \pi + 2 \operatorname{Arctg} \frac{\Xi}{1+\delta/\varepsilon}, \tag{29}$$
$$\zeta = \ln \left| \frac{1+\delta/\varepsilon - \Xi}{1+\delta/\varepsilon + \Xi} \right|.$$
$$\Xi = 2[(\delta+\Delta)/2\varepsilon]^{1/2}, \quad \operatorname{Arctg}(x/y) \equiv \arg(y+ix)$$

Figure 1c shows qualitatively the resultant dependence of R_Z on the magnetic field. As is seen from the figure, the resonant frequencies Ω_1 and Ω_0 are resolvable upon satisfaction of the condition $\epsilon \gg \gamma$, which is equivalent to the following:

$$d \ll r \ll (d^4 l)^{1/5}. \tag{30}$$

In the region of stronger magnetic fields, when $r \sim d$, the cross section $p_Z = p_1$ can be as close as desired to the cross section on which $\Omega(p_Z)$ has an extremum. For $0 < [2r(p_e) - d]/d \ll 1$, the electron orbits with diameters greater than $2r(p_1)$ do not fit within the thickness of the conductor. However, when ω is a multiple of Ω_e, resonance does take place, since the frequency is very close

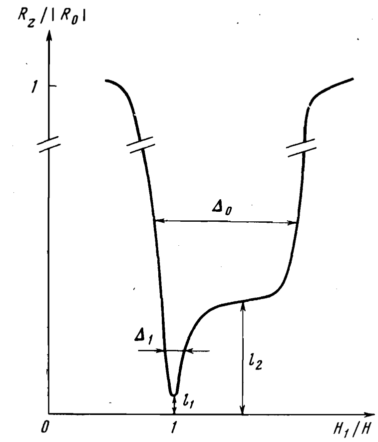

FIG. 2. The resonance peak of the real part of the impedance in the region of magnetic fields $0 < [2r(p_e) - d]/d \ll 1$. The total width of the peak is $\Delta_0 \approx (\Omega_1 - \Omega_e)/\Omega_e$ and its depth is $l_2 \approx \Delta_0^{2/3}$. The depth of the minimum of the impedance near the frequency Ω_1 is of the order of $l_1 \approx \Delta_0^{2/3} \ln^{-4/3} \Delta_0/\gamma$ and its depth is $\Delta_1 \approx \gamma$. The absence of an additional minimum close to the frequency Ω_e is connected with the fact the electrons with diameter of orbit $2r(p_e)$ are not contained in the thickness of the sample.

to the frequency of electrons moving along orbits with a diameter of the order of the thickness of the plate. The formula which describes the behavior of the impedance close to resonance under the condition Hd = const is identical with the formula (27) if we make the following substitutions in the latter:

$$Z_z \to Z_\mu, \quad a_z(p_0) \to a_\mu(p_e),$$
$$\varkappa(p_0) \to \varkappa(p_e),$$
$$\varepsilon \to |(\Omega_i - \Omega_e)/\Omega_e|, \quad \xi \to \xi - \pi,$$

Δ is determined by the relation $2\pi n(1-\Delta) = \omega T(p_e)$.

If $|\Omega_1 - \Omega_e/\Omega_e| \ll \gamma$, the form of the resonance curve is identical with the form of the curve at resonance at the extremal frequency in bulk samples, i.e., at $2r(p_e) < d$. The resonance curves corresponding to $|(\Omega_1 - \Omega_e)/\Omega_e| \gg \gamma$ are shown in Fig. 2.

4. In addition to the electrons that do not collide with the boundaries of the sample, those electrons can also take part in cyclotron resonance which are reflected in almost specular fashion from the surface y = d, the period of motion of which T_λ is extremal.[5] It is easy to establish this fact by analyzing the first component in the expression (14) for $\sigma_{\mu\nu}$, which is a resonant one if the condition

$$\omega T_\lambda(p_z^0) = 2\pi n, \quad \left.\frac{\partial T_\lambda(p_z)}{\partial p_z}\right|_{p_z = p_z^0} = 0 \tag{31}$$

is satisfied. It is also easy to calculate the resonant part of the impedance Z_μ by using the saddle-point method. The expression for Z_μ turns out to be the same as in the case of resonance in bulk samples with electrons having an extremal effective mass:[1]

$$Z_\mu = \frac{16\pi}{3\sqrt{3}} \frac{h}{ec} \left(\frac{\sqrt{\varkappa_\lambda}}{a_\mu(p_z^0)} \right)^{1/3} \delta^{1/6} \exp\left\{ -\frac{i}{3} \left(\pi + s \arctg \frac{\sqrt{\delta - s\Delta}}{\sqrt{\delta + s\Delta} + \tilde{b}_\mu n\delta} \right) \right\}; \tag{32}$$

$$0 < \tilde{b}_\mu \sim 1, \quad \varkappa_\lambda = \frac{1}{2} \frac{\partial^2 T_\lambda(p_z^0)}{\partial p_z^2} \Big/ T_\lambda(p_0),$$
$$\delta = \sqrt{\Delta^2 + \tilde{\gamma}^2}, \quad \omega T_\lambda(p_z^0) = 2\pi n(1-\Delta), \tag{33}$$
$$\tilde{\gamma} = \frac{1}{2\pi n} \left(\frac{T_\lambda(p_z^0)}{t_0} + (1 - q_2) \right).$$

However, the shape of the resonance curve in thin plates is determined not only by the mean free path of the electrons, but also by the specularity parameter q_2 and the thickness of the sample, inasmuch as $T_\lambda(p_z^0)$ depends on the relation between d and the radius of curvature of the orbit of the electron in the magnetic field $r(p_z^0)$.

In conductors with the number of conduction electrons of the order of one per atom, almost specular reflection of the charge carriers should be expected if the angle of incidence on the surface of the sample is small, i.e., where $r(p_z^0) \gg d$ for resonant electrons or

$$0<\frac{2r(p_z^0)-d}{d}\ll 1. \qquad \text{(eq)}$$

For $r(p_z^0) \gg d$, the period of motion of the carriers along an open orbit, broken by specular reflections from the surface of the plate $T_\lambda(p_z)$, is connected with the period of motion along this same cross section of the Fermi surface in the bulk sample $T(p_z)$ by the following relation:

$$T_\lambda(p_z)=\frac{1}{\pi}\left(\frac{2d}{r(p_z)}\right)^{1/2}T(p_z). \qquad (34)$$

The expression for the parameter $\bar{\gamma}$, which characterizes the form of the resonance curve, takes the form (after consideration of the relation (34)):

$$\bar{\gamma}=\frac{1}{2\pi n}\left[\frac{T(p_z^0)}{\pi t_0}\left(\frac{2d}{r(p_z^0)}\right)^{1/2}+(1-q_2)\right]\approx\frac{\sqrt{rd}}{l}+(1-q_2), \qquad (35)$$

For observation of cyclotron resonance, the satisfaction of the condition

$$[\sqrt{rd}/l+(1-q_2)]\ll 1. \qquad (36)$$

is necessary. In this case, cyclotron resonance already occurs in the weak field region, when the radius of curvature of the trajectory of the electron r exceeds not only the thickness of the sample, but also the free path length l; however, it is less than the quantity l^2/d.[5] The relations (31) and (34) determine the resonance values of the magnetic field.

In the region of magnetic fields $[2r(p_z^0)-d]/d \ll 1$, when the diameter of the orbit of the resonance electrons is close to the thickness of the sample, we can use the expression (19) for $T_\lambda(p_z)$ and for $\bar{\gamma}$ we have

$$\bar{\gamma}=\frac{1}{2\pi n}\left[\frac{r}{l}+(1-q_2)\right], \qquad (37)$$

i.e., the condition for observation of cyclotron resonance is of the form

$$[r/l+(1-q_2)]\ll 1, \qquad (38)$$

and the resonance frequencies and resonant values of the magnetic field are determined by the relations (31) and (19). Here the values of p_z for which $T_\lambda(p_z)$ and $T(p_z)$ have an extremum are generally different, and if the orbit of the electron with extremal $T(p_z)$ no longer fits in the plate, then the given resonance frequency is cut off. Both $T_\lambda(p_z)$ and $T(p_z)$ reach their extremal values only on the central section $p_z=0$ of the Fermi surface. The shift of the resonant value of the magnetic field H_{res} relative to its value in the bulk sample H_{res}^∞ can be determined with the help of the formula (19):

$$H_{res}-H_{res}^\infty=-\frac{2}{\pi}H_{res}^\infty\left[1-\frac{d}{2r(0)}\right]^{1/2}. \qquad \text{(eq)}$$

Here the shape of the resonance curve at frequencies that depend on the thickness of the plate will be described by the expression (32) if its width is much less than the shift of the resonance frequency, i.e.,

$$[r(0)/l+(1-q_2)]\ll[(2r(0)-d)/d]^{1/2}.$$

We note that the phase relations are essentially different, depending on whether there is a maximum of a minimum in the period of the motion of the resonance electrons. In a thin plate at $0<[2r(0)-d]/d\ll 1$, the second term in the expression (19) for $T_\lambda(p_z)$ changes much more rapidly than the first and we have, with a sufficient degree of accuracy:

$$\varkappa_\lambda\approx-\frac{1}{\pi}\frac{\partial^2 r(0)/\partial p_z^2}{[d(2r(0)-d)]^{1/2}}$$

i.e., the form of the extremum $T_\lambda(0)$ is determined only by the character of the change in the diameter of the Fermi surface near the central cross section, while the relations between R_μ and X_μ in the bulk sample depend on the sign of $\partial^2 m^*(p_z)/\partial p_z^2$ on the central cross section of the Fermi surface.

Khaĭkin and Édel'man[4], observing the cyclotron resonance in a bismuth film, discovered that with decreasing the magnetic field, when the maximum diameter of the orbit of the electron exceeds the thickness of the film, the resonance frequencies were not cut off but were shifted, depending on the relation of r_{max} and d, i.e., the resonance occurred on electrons of the central cross section, which were reflected almost specularly by the surface of the film. The deviation of the parameter of specularity q_2 from unity leads, as we have shown, to a broadening of the resonance peaks of the impedance, which is also observed experimentally.

In a recently published paper by Volodin, Édel'man and Khaĭkin,[10] cyclotron resonance was observed in a thin plate of bismuth at nonextremal cross sections of the Fermi surface. As the authors observed, the amplitude of the resonance peaks of the impedance is approximately an order of magnitude smaller than in the case of resonance on electrons of the central cross section of the Fermi surface. This result is in complete agreement with ours. As is seen from Eq. (21) and (22), the cyclotron resonance on electrons of the central cross section, in which case the fractional-power-law singularity is observed in the dependence of the sample impedance on the magnetic field.

The experimental study of the shape of the resonance curve at "cutoff" of the resonance frequency, and also the study of cyclotron resonance at the frequencies mentioned above, is obviously a laborious problem. However, the obtaining of the non-extremal characteristics of the conduction electrons with the help of these investigations is very important for selecting the model of the electron electric spectrum and as a check on the assumptions lying at the basis of this model. This applies primarily to the transition metals, for which the assumed model of the Fermi surface is quite specific.[12,13]

[1)]The case of almost specular scattering of the electrons by the surface y = 0 is a special one. Here the principal contribution to the current is made by electrons which collide with the surface of the sample, as a result of which the resonant component has the form of a small addition to the impedance. The solution of this problem in the case of consideration of cyclotron resonance in bulk specimens under different limiting conditions is given in the work of Meierovich [8] and Zherebchevskii and Kaner. [9] Corresponding consideration of the dependence of Z(H) in a plate, which was studied in the present paper, can be carried out completely analogously (see Footnote 2).

[2)]If the scattering of the electrons by the surface of the plate y = 0 is nearly specular, then calculations similar to those given in [8,9] show that upon satisfaction of the inequality $\sqrt{\delta/r} \ll (1 - q_1) \ll 1 \ln \Omega t_0$, the resonant contribution ΔZ_{res} to the impedance is proportional to $\ln^{2/3} [(\alpha/\beta - 1)\Delta + i\gamma]$; if the scattering is still closer to specular, then $(1 - q_1) \ll \sqrt{\delta/r}$, then $\Delta Z_{res} \sim \ln [(\alpha/\beta - 1)\Delta + i\gamma]$. Thus the result in this case is not qualitatively changed – the resonance, just as for nonspecular reflection, has a logarithmic character. The length of this paper does not permit us to consider this problem in any further detail.

[3)]The velocity of an electron at a limiting point is parallel to the magnetic field and therefore only the components of the conductivity tensor $\sigma_z(k)$ and the impedance Z_z have a resonant character.

[4)]For brevity, we only give the curves $R_z(H)$. Making use of Eqs. (17) and (23), we can easily obtain similar relations for $X_z(H)$. The statement made in certain publications (see [1,11]) that the impedance has a fractional-power-law singularity at cyclotron resonance with the electrons close to the limiting point of the Fermi surface is the result of a misunderstanding, inasmuch as the vanishing of the Jacobian of the transformation to spherical coordinates p, θ, φ at $\varphi = 0$ was not taken into consideration.

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