Effect of the medium on the bremsstrahlung of an electron in the optical frequency range

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The effect considered is the influence of absorption of virtual quanta owing to photoelectric effect and scattering from quasidiscrete levels of atoms of the medium on the bremsstrahlung spectrum of ultrarelativistic electrons. A classical method is used. Analytic expressions are derived for the bremsstrahlung spectrum in a dense medium over a wide range of atomic frequencies, as affected by the absorption virtual quanta. It is shown that in regions close to absorption lines and to the photoelectric absorption edge of the medium these processes lead to a decided change of the bremsstrahlung spectrum.

1. INTRODUCTION

The bremsstrahlung spectrum of high-energy electrons in a homogeneous medium depends on its density.[1-9] Deviations from the Bethe-Heitler spectrum, which holds in a rarefied medium, can be caused by multiple scattering of the electron in the medium, by polarization of the medium, and by the absorption of virtual quanta. The possibility of an effect of the absorption of virtual quanta was first pointed out by Landau and Pomeranchuk in their paper on the effect of the medium on bremsstrahlung.[1] The effect of the absorption of virtual quanta owing to production of electron-positron pairs has been considered in more detail in a number of papers.[4-7] It was shown [5-7] that at sufficiently high energies of the electron ($E \gtrsim 10^{14}$ eV) and radiation frequencies[1)] $\omega \gg 1$ the process of absorption of virtual quanta with electron-positron pair production in the medium leads to a considerable suppression of the bremsstrahlung over a wide range of frequencies (10^8 eV $\lesssim \omega \lesssim 10^{-20}$ E^2 eV).

The radiation of softer quanta ($\omega \lesssim 1$) can be affected, generally speaking, by absorption of virtual quanta of the field of the electron in other processes, in particular as a result of the Compton effect and the photoelectric effect. The Compton effect does not affect the true bremsstrahlung of the electron because of other effects of the medium.[2,8] The Compton scattering of virtual quanta does, however, lead to the appearance of additional radiation, which can be interpreted as emitted from the recoil electrons.[8] That an effect on the bremsstrahlung spectrum of high energy electrons owing to absorption of virtual quanta through the photoelectric effect is possible in principle was pointed out by Ter-Mikaelyan.[9]

In the present paper we consider the absorption of virtual quanta owing to such processes as the photoelectric effect and scattering by quasidiscrete levels of the atoms of the medium. It turns out that the photoelectric absorption of the virtual quanta by inner shells of the atoms leads to a decided change of the bremsstrahlung spectrum of the electrons in the frequency range close to the binding energies of inner-shell electrons.

These effects of the medium appear at relatively low frequencies of the radiation, $\omega \ll E$ (E is the energy of the electrons), and the characteristic lengths for the radiative processes affecting the fast electron (the coherence lengths) are of macroscopic scale. This allows us to use classical electrodynamics to solve the problem, and to take the effect of the medium into account in a phenomenological way by introducing the dielectric constant and the characteristic radiation length (cf. [2-7]). If in addition the coherence length of the radiation becomes comparable with the mean free path of the quanta, the way in which one separates the bremsstrahlung loss from the total energy loss becomes important in the analysis of the results.[7]

2. GENERAL EXPRESSION FOR THE ENERGY LOSS OF AN ELECTRON IN A MEDIUM

We use the results of our previous paper,[7] where the effect of the absorption of virtual quanta was considered in general form. The energy lost by an electron with energy $E \gg 1$ at frequency $\omega \ll E$ per unit path length in matter (the spectral density of the specific energy loss), averaged over all possible paths of the electron and including effects of polarization of the medium and of the absorption of virtual quanta, can be put in the form[7]

$$I(\omega)=\frac{e^2}{\pi l_s}\left\{\operatorname{Re}\frac{2^{5/2}}{i^{3/2}}\int_0^\infty\left[\frac{\psi_1(\lambda_s,\tau)}{\psi_2(\lambda_s,\tau)}-\frac{2^{1/2}}{i^{3/2}\tau}\right]\right.$$
$$\left.\times(s_1-{}^1/_8 i\Lambda_s)\exp[-2({}^1/_8\Lambda_s+is_1)\tau]\,d\tau+\Lambda_s\int_0^{s_\eta}\frac{s\,ds}{(s-s_1)^2+(\Lambda_s/8)^2}\right\}, \quad (1)$$

where $l_s=(q\omega)^{1/2}$ is the coherence length associated with the multiple scattering of the electron; $4q$ is the mean square of the angle in multiple scattering; $\lambda_s=l_s/L$, where L is the characteristic radiation length; $s_1=(l_0^{-1}+l_p^{-1})/8l_s^{-1}$, where $l_0=E^2/\omega$ is the coherence length of the radiation in vacuum and $l_p=-[4\pi\omega\chi'(\omega)]^{-1}$ is the coherence length associated with the polarization of the matter, $\chi'(\omega)$ being the real part of the electric susceptibility of the medium; $s_\eta=s_1-l_s/8\omega$; $\psi_1(\lambda_s,\tau)=H_0^{(1)}(\beta)H_1^{(2)}(\delta)-H_0^{(2)}(\beta)H_1^{(1)}(\delta)$; $\psi_2(\lambda_s,\tau)=H_0^{(2)}(\beta)H_0^{(1)}(\delta)-H_0^{(1)}(\beta)H_0^{(2)}(\delta)$; $\beta=2^{1/2}i^{3/2}\lambda_s^{-1}\exp(-\lambda_s\tau/2)$; $\delta=2^{1/2}i^{3/2}\lambda_s^{-1}$; $H_\nu^{(i)}$ is the Hankel function of order ν and index i; Λ_s is the ratio of the coherence length l_s to the mean free path of a quantum[2)] in the medium.

The frequency dependence of the electric susceptibility $\chi'(\omega)$ can in general differ from the form we used earlier.[7] In deriving (1) we used the condition that the effective angles of radiation from the ultrarelativistic electron are small. The polarization of the medium does not lead to violation of this condition if the inequality $|\chi'(\omega)|\ll 1$ is satisfied (cf., e.g., [9]). Another difference is that in Eq. (14) of [7] we have formally replaced the quantity $\lambda_s^{(c)}=l_s n\sigma^{(p)}(\omega)$ by the quantity $\Lambda_s=l_s n\sigma^{(t)}(\omega)$, where n is the density of nuclei in the matter, $\sigma^{(p)}(\omega)$ is the cross section for production of an electron-positron pair by a quantum in the field of a nucleus, and $\sigma^{(t)}(\omega)$ is the total cross section for interaction of a quantum with an atom (specific cases are considered below).

The part of the total energy loss given by the first term in the curly brackets in Eq. (1) vanishes for $q \to 0$ and is identified with the true bremsstrahlung with the medium effects we have mentioned taken into account. The second term in the curly brackets does not depend on q and describes processes associated with the absorption and scattering of virtual quanta of the field of the uniformly moving particle. The superiority of this definition of the loss by bremsstrahlung in an absorbing medium as compared with other possible definitions has been discussed in detail earlier.[7]

3. THE BREMSSTRAHLUNG SPECTRUM IN THE REGION OF ATOMIC FREQUENCIES

Let us consider the bremsstrahlung of an electron in a range of frequencies ω in which the dielectric properties of the medium are determined by the interactions of photons with the bound electrons of the atoms of the medium. In this frequency range the electric susceptibility $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$ of the medium can be represented in the form

$$\chi(\omega) = \frac{n}{3}\sum_{\nu}\frac{2|\mathbf{d}_{\nu 1}|^2\omega_{\nu 1}}{\omega_{\nu 1}^2-\omega^2-i\omega\Gamma_\nu}, \tag{2}$$

where $\mathbf{d}$ is the operator of the dipole moment of an atom, $\omega_{\nu 1} = E_\nu - E_1$ is the energy difference of the excited and ground states of the atom, and Γ_ν is the width of the excited level. The summation is taken over all discrete excited states of the atom and over all states of the continuous and quasicontinuous spectrum (in this case the summation sign must be replaced by an integral over frequencies $\omega_{\nu 1}$ and the quantity Γ_ν is formally made to go to zero). The imaginary part $\chi''(\omega)$ of the electric susceptibility is connected with the value of the mean free path l_S/Λ_S of the quanta [cf. Eq. (1)] by the relation $\Lambda_S = 4\pi l_S \chi''(\omega)\omega$.

As was pointed out above, the expression (1) for the spectral density of specific energy loss is valid under the condition that the polarization of the medium is small: $|\chi'(\omega)| \ll 1$.

For condensed substances this condition is satisfied in the x-ray frequency range and in comparatively narrow frequency intervals corresponding to anomalous dispersion in the ultraviolet and optical regions.[3)] The effect of the change of the multiple scattering constant[7] does not appear in the indicated region of atomic frequencies even for high electron energies. The effects of polarization of the medium and of absorption of virtual quanta are more important: $\lambda_S \ll \max\{\Lambda_S, s_1\}$. As a result Eq. (1) is greatly simplified. Taking the last inequality into account, we can represent the frequency density of the specific loss in the form

$$I(\omega) = \frac{e^2}{\pi l_s}\left\{\mathrm{Re}\frac{2^{5/2}}{i^{1/2}}\int_0^\infty\left[-i\,\mathrm{cth}\frac{1+i}{2}\tau - \frac{2^{1/2}}{i^{1/2}\tau}\right]\right.$$
$$\left.\left(s_1-\frac{i\Lambda_s}{8}\right)\left(\exp\left[-2\left(\frac{\Lambda_s}{8}+is_1\right)\tau\right]d\tau + \Lambda_s\int_0^{s_\eta}\frac{s\,ds}{(s-s_1)^2+(\Lambda_s/8)^2}\right\}. \tag{3}$$

In the further transformation of the expression (3) we must keep in mind that the coherence length l_p associated with the polarization of the medium, and along with it the parameter s_1, can take negative values in the atomic frequency range. Depending on the value of s_1, the integral over the variable τ can be transformed as follows by changing the path of integration:

$$\int_0^\infty f(\tau)\,d\tau = \begin{cases}\displaystyle\int_{C_1} f(\tau)\,d\tau, & s_1 \geqslant -\Lambda_s/8,\\[2ex] \displaystyle\int_{C_2} f(\tau)\,d\tau + 2\pi i\sum_k \mathrm{res}[f(\tau),\tau_k], & s_1 < -\Lambda_s/8,\end{cases}$$

where $\tau_k = (1+i)k\pi$, $k = 1, 2, \ldots$, are the singular points of the integrand $f(\tau)$ in the sector $0 < \arg\tau < 3\pi/4$, and C_1, C_2 are rays in the complex τ plane coming out from the point $\tau = 0$ in the respective directions $\arg\tau = 7\pi/4$, $\arg\tau = 3\pi/4$. The second term in the curly brackets can be integrated by elementary means. The result is

$$I(\omega) = \frac{e^2}{\pi l_s}\left\{8\,\mathrm{Im}\left(\frac{i\Lambda_s}{8}-s_1\right)\left[\ln\frac{\beta}{2}-\psi\left(\frac{\beta}{2}\right)-\frac{1}{\beta}\right]\right.$$
$$\left.+\mathrm{Re}\frac{8\pi\mu}{1+i}\frac{e^{-i\pi\mu}}{1-e^{-i\pi\mu}}\theta\left(-s_1-\frac{\Lambda_s}{8}\right)\right\}+\frac{e^2}{2\pi l_s}\Lambda_s\ln\frac{l_s^2}{(8\omega)^2[s_1^2+(\Lambda_s/8)^2]} \tag{4}$$
$$+\frac{8e^2 s_1}{\pi l_s}\left[\mathrm{arctg}\frac{8s_1}{\Lambda_s}-\frac{\pi}{2}\right],$$

where $\mu = 2(1+i)(s_1 - \Lambda_S/8)$, $\beta = \mu\,\mathrm{sgn}(s_1 + \Lambda_S/8)$, $\theta(x)$ is the Heaviside function, $\psi(x) = d(\ln\Gamma(x))/dx$ is the logarithmic derivative of the Γ function, and $\mathrm{sgn}(x)$ is the sign function.

The first term of the expression (4) gives the bremsstrahlung of the electron. The part of the specific energy loss associated with the second term, proportional to Λ_S, is due to excitation and ionization of atoms by the fast electron.[4)] Integrating this term over frequencies and making use of the fact that the main contribution to the ionization loss of a fast electron comes from a range of frequencies $\omega \gg \omega_{\nu 1}$, where $\chi'(\omega) = -Ne^2/\omega^2$ (N is the number density of electrons in the medium), we arrive at the well known expression for the ionization loss $I^{(i)}$ of an ultrarelativistic electron owing to distant collisions with the "density effect" [10] included:

$$I^{(i)} = 2\pi N e^4 \ln(1/4\pi N e^2).$$

Finally, the last term in Eq. (4) is different from zero even in the absence of absorption ($\Lambda_S \to 0$), and in that case represents the intensity of the Cherenkov radiation. In an absorbing medium ($\Lambda_S \gtrsim s_1$) the Cherenkov radiation, unlike the bremsstrahlung, in general cannot be separated out from the total energy loss given by Eq. (4).

In the region of relative transparency of the matter,[5)] where $|s_1| \gg \Lambda_S/8$, the intensity of the specific energy loss is given by an expression which does not take absorption of quanta into account ($\Lambda_S \to 0$):

$$I(\omega) = \frac{2e^2}{\pi l_s}[F(s_1) - 4\pi s_1\theta(-s_1)],$$

$$F(s_1) = 4s_1\,\mathrm{Im}\left[\psi(|\varkappa|)+\frac{1}{2|\varkappa|}-\frac{i\pi}{4}\right]+\mathrm{Re}\frac{8s_1 e^{-2\pi i\varkappa}}{1-e^{-2\pi i\varkappa}}\theta(-s_1), \tag{5}$$
$$\varkappa = (1+i)s_1, \qquad |\chi'(\omega)| \ll 1.$$

If the electron's speed $v \approx 1 - 1/2E^2$ is smaller than the phase velocity of light in the medium, the parameter s_1 is positive and the expression (5) is the same as Migdal's formula [3] for the bremsstrahlung intensity in a nonabsorbing medium. For negative values of s_1 the

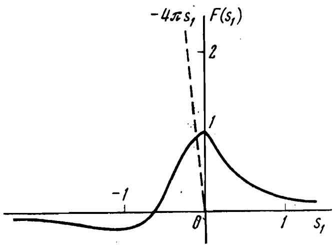

FIG. 1. The function $F(s_1)$ (solid curve) and the function $-4\pi s_1$ (dashed curve).

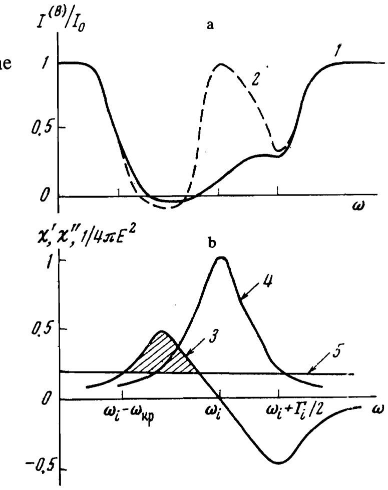

FIG. 2. Dependences of the bremsstrahlung intensity (diagram a) and the electric susceptibility (diagram b) on the frequency, near the absorption line: 1—bremsstrahlung spectrum with polarization of the medium and absorption of virtual quanta taken into account; 2—bremsstrahlung spectrum with polarization of the medium taken into account; 3—real part of susceptibility, χ'; 4—imaginary part of susceptibility, χ''; 5—the quantity $1/4\pi E^2$.

condition for Cerenkov radiation from the electron can be satisfied. The second term in (5) then is the same as the Frank-Tamm formula for the intensity of Cerenkov radiation in the limiting case $|\chi'(\omega)| \ll 1$, $E \gg 1$.

An example of a shape of the function $F(s_1)$ is shown in Fig. 1. For comparison the diagram also shows with a dashed line the plot of the function $-4\pi s_1$ associated with the Cerenkov radiation. In the range of frequencies where $F(s_1)$ is negative, bremsstrahlung must always appear along with the Cerenkov radiation at small angles $\theta \sim (|\chi'(\omega)|)^{1/2}$ with the direction of motion of the electron.[4,9] Here, strictly speaking, it is only the total radiation intensity that has meaning, and this is always positive. In this case it is incorrect (cf. [11] 6)) to define the bremsstrahlung as the first term in the expression (5) and the Cerenkov radiation as the second term, since the first term, proportional to $F(s_1)$, actually describes not only the bremsstrahlung but also the decrease of the Cerenkov radiation owing to multiple scattering in the medium. Negative values of $F(s_1)$ should rather be understood as decreases of the Cerenkov radiation, and not as "negative" bremsstrahlung.

The criterion characterizing the effect of absorption of virtual quanta on the bremsstrahlung is obtained, as usual, by comparing the coherence length of the radiation with the mean free path of the quanta:

$$\Lambda_s/8 \gtrsim \max\{1, s_1\}. \tag{6}$$

As was shown above, in the presence of the effect of absorption of virtual quanta the intensity of the bremsstrahlung is determined by the first terms of the expression (4). Further analysis shows (cf. Secs. 4 and 5) that in the range of atomic frequencies under consideration the effect of multiple scattering is less important in comparison with competing effects of polarization of the medium and absorption of virtual quanta. The quantity μ, which characterizes the relative influences of the various effects of the medium, in this case satisfies the inequality $|\mu| \gg 1$. As a result, up to terms $\sim|\mu|^{-4}$ the bremsstrahlung spectrum $I^{(B)}(\omega)$ can be represented in the form

$$I^{(B)}(\omega) = \frac{e^2}{\pi l_s} \operatorname{Im} \frac{\mu}{2(1+i)} \left[\frac{2}{15\mu^4} - \frac{1}{3\mu^2}\right] \tag{7}$$

Accordingly, to the accuracy indicated, for weak polarization of the medium, $|\chi'(\omega)| \ll 1$, the expression (7) determines the bremsstrahlung spectrum in the region of atomic frequencies. Let us examine in more detail two concrete cases in which it is necessary to take into account the effect of absorption of virtual quanta.

4. THE EFFECT OF SCATTERING OF VIRTUAL QUANTA BY QUASICLASSICAL LEVELS

We assume that in a certain range of frequencies the electric susceptibility $\chi(\omega)$ is determined by a single quasiisolated level ω_i, and that other levels do not contribute appreciably to $\chi(\omega)$ [the electric susceptibility $\chi(\omega)$ is determined by the i-th term in (2)]. According to Eq. (6) the absorption of virtual quanta must be taken into account in the range of frequencies ω that fall on the absorption line of the medium, $|\omega - \omega_{i1}| \lesssim \Gamma_i$, for electron energies $E \gtrsim \Gamma_i/4\pi n|d_{i1}|^2$. Thus, for x-ray frequencies $\omega_i \approx 10$ keV, when the level width Γ_i is governed by the radiation width ($\Gamma_i \sim \omega_{i1}^3|d_{i1}|^2$), the effect of absorption of virtual quanta in condensed media ($n \sim 10^{22}$ cm^{-3}) should be apparent at electron energies as low as $E \gtrsim 10^7$ eV.

A sample form of the bremsstrahlung spectrum, Eq. (7), near the absorption line ω_i is shown in Fig. 2a (solid curve). The value of the bremsstrahlung intensity I_0 at a given energy $E > \Gamma_i/4\pi n|d_{i1}|^2$, not including effects of dispersion on the bremsstrahlung process [$\chi(\omega) = 0$], is taken as unity.

In Fig. 2b the curve of the electric susceptibility is plotted in arbitrary units. The quantity $1/4\pi E^2$ is also shown. The shaded area under the curve of $\chi'(\omega)$ corresponds to frequencies at which Cerenkov radiation is possible.

In the frequency range $|\omega_{i1} - \omega| \lesssim \omega_{cr}$, where ω_{cr} is fixed by the relation $\chi'(\omega_{cr}) = 1/4\pi E^2$, the polarization of the medium leads to a considerable change of the bremsstrahlung spectrum (curve 2 in Fig. 2a). The absorption of virtual quanta in the frequency range $|\omega_{i1} - \omega| \lesssim \Gamma_i$ diminishes the bremsstrahlung spectrum still more (curve 1 in Fig. 2a).

5. EFFECT OF ABSORPTION OF VIRTUAL QUANTA OWING TO PHOTOELECTRIC EFFECT

Let us now consider the emission of quanta with energy ω close to the binding energy of an electron of an inner shell of a sufficiently heavy atom. We shall carry through the analysis of the specific case of absorption of virtual quanta owing to ionization from the K shell. In this case there is a simple formula for the imaginary part of the electric susceptibility in terms of the K-shell photoelectric cross section $\sigma_1(\omega)$ (cf. e.g., [12]):

$$\chi''(\omega) = n\sigma_1(\omega)/4\pi\omega. \tag{8a}$$

The contribution to the real part $\chi'(\omega)$ of the susceptibility from the K electrons is mainly due to the continuous absorption. In this case $\chi'(\omega)$ has been

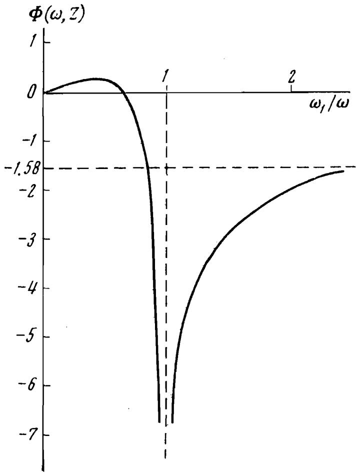

FIG. 3. Dependence on frequency of the function $\Phi(\omega, Z)$ which determines the susceptibility $\chi'(\omega)$, for aluminum, near the absorption line.

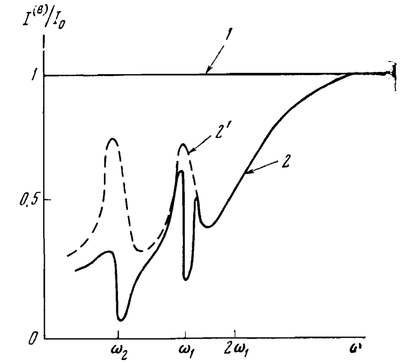

FIG. 4. Frequency dependence of the bremsstrahlung intensity near photoabsorption edges, with influence of absorption of virtual quanta included (curve 2) and not included (curve 2′). Curve 1 is the Bethe-Heitler spectrum.

calculated by Hönl [13] and can be put in the form

$$\chi'(\omega)=-\frac{Ne^2}{\omega^2}\left(1-\frac{\Phi(\omega,Z)}{Z}\right), \qquad (8b)$$

where Z is the number of electrons per atom of the medium. The characteristic shape of the function $\Phi(\omega, Z)$ is shown in Fig. 3 for $Z = 13$.

In the frequency region $\omega \sim \omega_1$ of interest to us (ω_1 is the binding energy of a K electron) and for energies $E \gg 1$ the effect of multiple scattering is unimportant ($\max\{s_1, \Lambda_S/8\} \gg 1$), and for the bremsstrahlung intensity we can use the expression (7), which takes into account only the polarization of the medium and the absorption of virtual quanta. Furthermore, for the description of the electric susceptibility we are to use Eqs. (8a), (8b).

In the region of radiation frequencies and electron energies given by the inequalities

$$[4\pi\omega\,|\chi'(\omega)|\,]^{-1} \ll E^2/\omega,\ 1/8\sqrt{q\omega}, \qquad (9)$$

the bremsstrahlung intensity, Eq. (7), takes the form

$$I^{(B)}(\omega)=-\frac{2e^2q}{3\pi^2}\frac{\chi'(\omega)}{|\chi(\omega)|^2}. \qquad (10)$$

For radiation quantum energies $\omega \gg \omega_1$ the real part of the electric susceptibility takes the form $\chi'(\omega) = -Ne^2/\omega^2$ ($\Phi(\omega, Z) \ll Z$). In this frequency range the absorption of virtual quanta is unimportant and we arrive at the expression of Ter-Mikaelyan[2] for the bremsstrahlung intensity in a nonabsorbing medium with allowance for the polarization of the medium.

For smaller frequencies $\omega \sim \omega_1$ departures of the bremsstrahlung intensity (10) from the corresponding Ter-Mikaelyan expression can arise from two causes. First, in the range of frequencies where $\Phi(\omega, Z) \gtrsim Z$, we must use for $\chi'(\omega)$ the more exact expression (8b), which takes into account the binding of the K electrons in the atom. Second, it is in general necessary to allow for the absorption of virtual quanta owing to the photoelectric effect from the K shell. The region of frequencies where the latter influence appears is found from the condition

$$l_p/l_f=\chi''(\omega)/\chi'(\omega) \geqslant 1, \qquad (11)$$

where $l_f = [n\sigma_1(\omega)]^{-1}$ is the mean free path of the quanta. A detailed analysis of the inequalities (9) and (11) shows, for example, that for Al (Z = 13) the absorption of virtual quanta from the K shell has a decided effect on the bremsstrahlung spectrum in the frequency range $2.3\ \text{keV} \lesssim \omega \lesssim 3.5\ \text{keV}$ for electron energy $E \gtrsim 10^{10}$ eV.

For heavier elements the effect of absorption of virtual quanta by the K shell is less important. In substances with larger atomic numbers Z the effect of virtual-quantum absorption can appear from absorption by other shells (L, M, etc.), since the photoelectric cross section near the ionization threshold for a shell increases with the number n of the shell. The necessary analysis can be carried out in a similar way. For example, for matter with Z = 39 (Cu) the effect of absorption of virtual quanta by the L shell should appear in the frequency range $2.1\ \text{keV} \lesssim \omega \lesssim 4$ keV at electron energies $E \gtrsim 4\times 10^9$ eV.

The qualitative shape of the bremsstrahlung spectrum (7) in the frequency range including the atomic frequencies ω_1, ω_2 (ω_2 is the binding energy of an L electron) is shown in Fig. 4. According to the theory not including the influence of virtual-quantum absorption, the Bethe-Heitler spectrum for energies $E \gtrsim \omega/(4\pi Ne^2)^{1/2}$, in the frequency range $\omega_1 \ll \omega \lesssim E(4\pi Ne^2)^{1/2}$ (sic), goes over into the Ter-Mikaelyan spectrum (curve 2′).

At still lower frequencies $\omega \lesssim \omega_1$ the decrease of the polarization of the medium [cf. Eq. (8b) and Fig. 3] should lead to a relative increase of the bremsstrahlung intensity (curve 2′). Precisely in this region, however, for not too heavy elements, the absorption of virtual quanta sharply reduces the intensity of the radiation. Through the joint effect of these two factors the bremsstrahlung spectrum acquires a rather complicated shape (curve 2 of Fig. 4).

6. THE INFLUENCE OF COMPTON SCATTERING ON THE RADIATION PROCESS OF THE ELECTRON

If the quantum energy ω greatly exceeds the binding energy ω_1 of the K electrons of the atoms in the medium, the absorption of the virtual quanta will be mainly due to the Compton effect on electrons of the medium and the production of electron-positron pairs in the fields of the nuclei (for $\omega \geq 2$). In this frequency range we have the relations: $n\sigma^{(t)}(\omega) \simeq N\sigma^{(C)}(\omega) + n\sigma^{(p)}(\omega)$, $l_p = \omega/\omega_0^2$; $\omega_0^2 = 4\pi Ne^2$ is the plasma frequency, and $\sigma^{(C)}(\omega)$ is the cross section for Compton scattering of the quanta by a free electron. The Compton scattering of virtual quanta actually has no effect on the bremsstrahlung spectrum of a fast electron because of the stronger influence of other medium effects (polarization and absorption of quanta through pair production).[2,8]

Estimates show that in the frequency range $\omega \gtrsim \omega_1$

the mean free path of virtual quanta associated with Compton scattering, $l_C = [n\sigma^{(C)}(\omega)]^{-1}$, is much larger than $\min\{l_p, (n\sigma^{(p)})^{-1}\}$. In fact, the ratio l_p/l_C is of the order of $10^{-2}\omega\sigma^{(C)}(\omega)/\sigma^{(0)}$, where $\sigma^{(0)} = 8\pi e^4/3$ is the Thomson scattering cross section. Since $\sigma^{(C)}(\omega)$ is not larger than $\sigma^{(0)}$, for $\omega \gtrsim 1$ the ratio l_p/l_C is small ($\lesssim 10^{-2}$) and the absorption is unimportant ($\Lambda_s/8 \ll s_1$). For large $\omega \gg 1$ we have $\sigma^{(C)}(\omega) \sim \sigma^{(0)} \times \ln \omega/\omega$, and consequently in the frequency range $\omega \gtrsim 10^5$, where $l_C \gtrsim 10^{-1} l_p$, the Compton effect of virtual quanta could appreciably affect the bremsstrahlung process. But precisely in this range of frequencies the absorption of virtual quanta owing to production of electron-positron pairs is much more important, since the condition $l_C \gg [n\sigma^{(p)}]^{-1} \approx L$ is satisfied.

Although the Compton scattering of virtual quanta does not appreciably affect the competing process of bremsstrahlung of a fast electron, as noted above it does lead to the appearance of additional radiation. Owing to the scattering the virtual quanta are converted into real quanta of smaller frequency. This process can also be interpreted as bremsstrahlung of the recoil electrons.[8]

The intensity of the radiation of the recoil electrons can be obtained from the expression (1) for the specific energy loss of the electron at frequency ω. In fact, the second term in the curly brackets in (1) is proportional to the number of virtual quanta of the electron's field that are absorbed by the medium. The term associated with the absorption of quanta in the pair-production process has been considered earlier in [7] and must be excluded. The result is that the total intensity $W^{(r)}$ of the radiation of the recoil electrons, integrated over all frequencies, can be represented in the form

$$W^{(r)} = \frac{Ne^2}{\pi}\int_0^\infty \int_{\frac{\omega}{1+2\omega}}^\infty \sigma(\omega, \omega') \int_0^{s_\eta(\omega)} \frac{s\,ds\,d\omega'\,d\omega}{[s-s_1(\omega)]^2+[\lambda_s^{(c)}(\omega)/8]^2}, \tag{12}$$

where $\sigma(\omega, \omega')$ denotes the differential cross section for Compton scattering (cf., e.g., [12]).

Corresponding to this, we can write the intensity $I^{(r)}(\omega)$ of the radiation at frequency ω (the spectral density of the recoil-electron radiation) in the following way:

$$I^{(r)}(\omega) = \frac{Ne^2}{2\pi}\int_a^b \sigma(\omega_v, \omega) \int_0^{s_\eta(\omega_v)} \frac{s\,ds\,d\omega_v}{[s-s_1(\omega_v)]^2+[\lambda_s^{(c)}(\omega_v)/8]^2} \tag{13}$$

$a=0$, $b=\omega/(1-2\omega)$ for $\omega \leqslant 1/2$, $a=\omega$, $b=\infty$ for $\omega > 1/2$.

Let us first consider the frequency range $\omega \leq 1/2$, where there is no pair production ($\lambda_s^{(c)} = 0$). A calculation of $I^{(r)}(\omega)$ to logarithmic accuracy leads to the result obtained by Toptygin[8] by the method of equivalent photons

$$I^{(r)}(\omega) = {}^{32}/_{3}Ne^6(\omega^2+\omega-1)\ln(\omega^2E^{-2}+\omega_0^2). \tag{14a}$$

In the frequency range $\omega \gg 1/2$ the main contribution will come from the interaction of quanta with frequencies $\omega_v \sim E$. In this region the classical approach used in the present paper cannot be applied. For the same reason the result of Toptygin,[8] obtained by the method of equivalent photons, must be regarded as incorrect in the range $\omega > 1/2$.

The expression for the intensity of the radiation in the region $\omega \sim 1$ can be obtained to logarithmic accuracy by joining Eq. (13) with the result of Baĭer, Fadin, and Khoze,[14] where the radiation from the collision of a fast electron with a stationary electron was treated with a quantum mechanical method:

$$I^{(r)}(\omega) = \frac{8Ne^6}{3\omega}\left(1-\frac{1}{4\omega}+\frac{1}{16\omega^2}\right)\ln\frac{1}{(2E)^{-2}+\omega_0^2}. \tag{14b}$$

At energies $E \ll (2\omega_0)^{-1}$ the dependence of the intensity of radiation from recoil electrons on the medium disappears. Equation (14b) then agrees, to logarithmic accuracy, with the corresponding expression in [14], which was obtained without taking the effect of polarization into account. At larger energies, $E \gtrsim (2\omega_0)^{-1}$, as can be seen from Eq. (14b), the polarization of the medium has an important effect on the spectrum of the recoil-electron radiation.

The intensity of recoil-electron radiation in a medium, Eq. (14), differs from the corresponding vacuum expressions only in the argument of the logarithm. On the other hand, effects of the medium can lead to considerable suppression of the bremsstrahlung intensity from a fast electron (cf., e.g., [9]). Therefore, as Toptygin[8] has pointed out, in condensed media ($n = 3 \times 10^{22}$ cm^{-3}) for frequencies $\omega \lesssim 10^7$ eV and electron energies $E \gtrsim 10^5\omega$ the radiation is mainly due to recoil electrons.

Even in this case, however, it makes sense to consider these two types of radiation separately, since they can be distinguished experimentally. As was shown above, in the frequency range $\omega \lesssim 10^7$ eV the coherence length l_p of the bremsstrahlung is much smaller than the mean free path l_C of the quanta in the matter, associated with the Compton scattering. Therefore there are also spaces along the trajectory of the fast electron in which bremsstrahlung quanta have already been formed but on the other hand have not had time to be scattered by the electrons of the medium. Within such distances ($l_p \ll d \ll l_C$) the bremsstrahlung quanta travel mainly in a narrow cone around the direction of the electron's momentum, with an aperture angle $\theta \sim (l_0/l_p)^{1/2}/E$. On the other hand, the angular distribution of the quanta emitted by the recoil electrons at frequencies $\omega \ll E$ is almost isotropic.[14] Accordingly, these two types of radiation in a medium can be distinguished by their angular distributions.

In the atomic frequency range $\omega \lesssim \omega_1$ the intensity of the radiation from the Compton recoil electrons can be represented in the form [cf. (14a)]

$$I^{(r)}(\omega) = \frac{8N_ce^6}{3}(1-\omega-\omega^2)\ln\frac{1}{[(l_0^{-1}+l_p^{-1})^2+(n\sigma^{(t)})^2]\omega^2} \tag{15}$$

where N_C is the number density of electrons in the medium with binding energies $E_{bd} \lesssim \omega$. In this frequency range the effect considered above, from absorption of virtual quanta, should show up relative to the radiation of the Compton electrons in the same ranges of frequency and of electron energy as in the case of the bremsstrahlung. The effect is not as large, however, since the expression (15) for the radiation spectrum from the Compton electrons contains characteristic parameters of the medium (coherence lengths) in the argument of the logarithm.

7. CONCLUSION

Accordingly, the bremsstrahlung spectrum of high-energy electrons in a condensed medium, in the region of atomic frequencies, depends strongly on the dispersion properties of the medium in this range of frequencies. The greatest influence on the bremsstrahlung spectrum comes from processes of absorption of virtual quanta owing to excitation and ionization of atoms (photoelectric effect and scattering by quasidiscrete levels). The main competing effect of the medium in the region of atomic frequencies is that of polarization, and the effect of multiple scattering is unimportant; at the same time, in the region of harder bremsstrahlung ($\omega \gg 1$) the competing effects are multiple scattering and polarization, and the effect of absorption appears along with that of the change of the multiple-scattering constant.[7] As has been shown above, owing to the competing effect of polarization on the bremsstrahlung spectrum the weaker effect of the Compton scattering of virtual quanta is not apparent.

A distinguishing feature of the bremsstrahlung spectrum in the presence of absorption of virtual quanta in the region of atomic frequencies is its nonmonotonic character. Because of the fact that the total energy loss of an electron in an absorbing medium can greatly exceed the loss owing to true bremsstrahlung, the question of correct separation of the bremsstrahlung loss from the total loss takes on great importance.

The bremsstrahlung spectrum (7) derived in the present paper corresponds to the loss of energy to bremsstrahlung in a single layer of matter which is sufficiently far from the boundary of the matter.[7)] In practice the distance to the boundary of the matter must be much larger than the distance in which a quantum of the radiation is formed (the coherence length). On the other hand, in frequency ranges where the absorption of virtual quanta is important, the radiation cannot be observed through the necessary distances along the trajectory of the electron. In this case the coherence length becomes comparable with the mean free path of the quanta, $\Lambda_s/8 \gtrsim s_1$, and at such distances a large fraction of the bremsstrahlung quanta is absorbed owing to excitation and ionization of atoms of the medium. Since such processes can also be caused directly by the fast electron, the problem arises of separating the bremsstrahlung loss from the total energy loss. According to our results,[7] this problem can be uniquely solved experimentally. To do so it is sufficient to separate out the asymptotic value I_a of the total energy loss at high electron energy $E \to \infty$ ($q \to 0$). By definition the remaining part $I - I_a$ of the total loss will be the bremsstrahlung energy loss in the absorbing medium.

In the range of atomic frequencies where Cerenkov radiation can appear, the bremsstrahlung cannot be considered separately from this radiation, since the polarization of the medium and the multiple scattering of the electron in the medium make these two types of radiation indistinguishable. In this case a separation of the total radiation of an ultrarelativistic electron into bremsstrahlung and Cerenkov radiation can be done only conventionally. On the other hand, the effects of the medium do not in principle make it impossible to distinguish between the additional radiation arising from Compton scattering of virtual quanta (radiation from recoil electrons) and the true bremsstrahlung. This result is directly due to the negligibly small effect of the Compton scattering of the virtual quanta on the true bremsstrahlung. Here, as in the case of a rarefied medium (vacuum) the criterion for separating the true bremsstrahlung and the radiation from recoil electrons is the decided difference between the angular distributions of these two types of radiation.

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1) The system of units with $\hbar = m = c - 1$ is used.

2) As was shown by Galitskiĭ and Yakimets, [5] the difference between the absorption cross sections for virtual and for real quanta is unimportant in the present case, and affects only the logarithmic accuracy of the second term in the curly brackets in Eq. (1), which is not connected with the true bremsstrahlung (see below).

3) In gases this condition is satisfied over a wide range of frequencies where $\chi''(\omega)$ is mainly due to the scattering of virtual quanta by the density fluctuations of the gas ("quasi-Rayleigh" scattering).

4) For gaseous media, in the optical frequency range the second term in Eq. (4) represents the intensity of the radiation that arises in the quasi-Rayleigh scattering of virtual quanta (cf. [9]).

5) The condition $|s_1| \gg \Lambda_s/8$ cannot be satisfied in a certain neighborhood of the point $s_1 = 0$, no matter how small the absorption is.

6) In the separation of the total loss into true bremsstrahlung and Cerenkov radiation, the paper of Yakimets [11] does not include (simply omits) the negative values of the bremsstrahlung intensity.

7) The problem of optical bremsstrahlung in an absorbing medium under different conditions, when the presence of boundaries of the matter is important, has been considered by Pafomov. [15]

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