Self-interaction of incoherent light beams

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A theory is developed for the self-interaction of incoherent light beams in a cubic medium. It is shown that, when the width of the space-time spectrum of the radiation is broad enough, it is possible to set up closed nonlinear equations describing the transformation of the field correlation function along the path of a ray. The variation in the frequency spectrum and in the spatial radiaion statistics is determined. The radius of the envelope and the correlation length of the spatially incoherent light beam are determined and the conditions under which self-focusing (or self-defocusing) effects are possible for continuous and pulsed radiation are discussed.

1. INTRODUCTION

The theory of self-interaction of incoherent optical radiation is a relatively new subject. In the first papers devoted to this question,[1-3] the analysis of the self-interaction of irregular waves in cubic homogeneous and randomly inhomogeneous media was given in terms of perturbation theory. This method was successfully used to investigate the initial stage of transformation of spatial statistics in a plane wave modulated by low-intensity noise distributed in a medium or localized on the boundary.[3] The solution of the analogous problem for a beam restricted in the transverse direction with a superimposed weakly modulated signal was given by Lyakhov.[4]

Another approach to the self-interaction of a spatially incoherent monochromatic beam of radiation in a cubic medium involves averaging of the nonlinear equations at an early stage of the solution.[5-8] This approach can be used to determine the character of the variation in the mean intensity of the incoherent beam and its radius. For example, the results of Vlasov et al.[7] show that self-focusing and the associated reduction in the radius of a multimode beam with increasing path length will occur only at a full radiation power greater than some threshold value which, in turn, is proportional to the square of the beam divergence at entry into the nonlinear medium. Vorob'ev[5,6] and Petrishchev[8] have found the law of transformation of the effective radius of a single-mode incident beam in a randomly homogeneous nonlinear medium. It is important to note that the transformation of the spatial statistics of incoherent radiation was practically ignored in [5-8]. This is connected either with the restrictive nature of the approximations employed (in [5,6,8] the analysis is confined to small-intensity fluctuations and is not concerned with beams which are highly inhomogeneous in the transverse direction) or with difficulties associated with taking into account the corresponding effects within the framework of the method of moments[7] (in the latter case, the transformation of spatial statistics in general requires the analysis of a chain of coupled equations for the higher-order moments).

In the context of the self-interaction of incoherent radiation, we must remember the similar problem of parametric and Raman transformation of radiation with broadband time and space spectra. Its solution is usually based on an analysis of the interaction and amplification of test waves in a given incoherent optical pump.[9-14] Recently, D'yakov[15] used the Dyson equation to generalize this approach to the case of the self-consistent problem. He considered stimulated Raman scattering under the conditions of saturation in the case of a plane nonmonochromatic pump with allowance for the reaction of the latter on the scattered wave.

The solution of the self-consistent problem is also of interest in connection with other nonlinear processes excited by radiation with spatially and temporally incoherent spectra. We have in mind, in particular, the self-focusing and self-defocusing of incoherent beams and the associated transformation of space and time statistics of a high-intensity light wave transmitted by a nonlinear medium. The analysis of these effects is also interesting because increasing attention is being paid[15-18] to the experimental investigation of nonlinear transformation of space and time statistics.

In this paper, we investigate the self-focusing and self-defocusing of incoherent beams of light propagating in an isotropic (in zero field) cubic medium with inertial nonlinearity. It is shown that when the total radiation power is fixed and the width of the space or time spectrum is large, it is possible to set up a closed equation for the field correlation function in the nonlinear medium. A separate analysis is given of the transformation of the time spectrum and the nonlinear transformation of spatial statistics of light beams (including nonstationary self-interaction of short incoherent pulses).

2. EQUATION FOR THE CORRELATION FUNCTION

Consider the transformation of an incoherent beam of light incident in the $z=0$ plane on a nonlinear layer. We shall assume that the width of the time spectrum of the radiation and the shift of the maximum of the spectrum which is possible in the course of the nonlinear interaction are small in comparison with the carrier frequency ω_0, and that the width of the angular spectrum is such that the quasioptical description of the corresponding fields can be used.

The complex amplitude of the electric field $\mathscr{E}$ in the light beam, written in terms of the coordinates z, $\mathbf{r}_\perp$, $\eta=t-z/v$ ($v=c/\sqrt{\epsilon_0}$ is the velocity of light in the medium, ϵ_0 is the unperturbed permittivity), will be described by the parabolic equation

$$\left(\frac{\partial}{\partial z}+\frac{i}{2k_0}\Delta_{r\perp 1}\right)\mathscr{E}(z,\mathbf{r}_{\perp 1},\eta_1)=\frac{ik_0}{2\varepsilon_0}\varepsilon^{NL}\mathscr{E}(z,\mathbf{r}_{\perp 1},\eta_1),\quad k_0=\omega_0/v. \quad (1)$$

The nonlinear correction to the permittivity, ϵ^{NL}, in the inertial medium with cubic nonlinearity will be assumed to satisfy the equation $(\tau\partial/\partial\eta+1)\epsilon^{NL} = \epsilon_2|\mathscr{E}|^2$ the solution of which is

$$\varepsilon^{NL}=\frac{\varepsilon_2}{\tau}\int_0 d\eta'\exp\left(-\frac{\eta-\eta'}{\tau}\right)|\mathscr{E}(\eta')|^2.$$

Let us substitute for ϵ^{NL} in (1) and multiply the corresponding equation by $\mathscr{E}(z, \mathbf{r}_{\perp 2}, \eta_2)$. We now add to this expression the equation for $\mathscr{E}^*(z, \mathbf{r}_{\perp 2}, \eta_2)$ multiplied by $\mathscr{E}(z, \mathbf{r}_{\perp 1}, \eta_1)$. After averaging the resulting relation, we obtain

$$\left(\frac{\partial}{\partial z}+\frac{i}{k_0}(\Delta_{r_{\perp 1}}-\Delta_{r_{\perp 2}})\right)\langle\mathscr{E}(z,\mathbf{r}_{\perp 1},\eta_1)\mathscr{E}^*(z,\mathbf{r}_{\perp 2},\eta_2)\rangle$$
$$=\frac{ik_0\varepsilon_2}{2\varepsilon_0\tau}\Big[\int_0^{\eta_1}d\eta'\exp\left\{-\frac{\eta_1-\eta'}{\tau}\right\}\langle\mathscr{E}(z,\mathbf{r}_{\perp 1},\eta')\mathscr{E}^*(z,\mathbf{r}_{\perp 1},\eta')$$
$$\times\mathscr{E}(z,\mathbf{r}_{\perp 1},\eta_1)\mathscr{E}^*(z,\mathbf{r}_{\perp 2},\eta_2)\rangle-\int_0^{\eta_2}d\eta'\exp\left\{-\frac{\eta_2-\eta'}{\tau}\right\} \tag{2}$$
$$\times\langle\mathscr{E}^*(z,\mathbf{r}_{\perp 2},\eta')\mathscr{E}(z,\mathbf{r}_{\perp 2},\eta')\mathscr{E}^*(z,\mathbf{r}_{\perp 2},\eta_2)\mathscr{E}(z,\mathbf{r}_{\perp 1},\eta_1)\rangle\Big].$$

We shall use the following approximation to obtain a closed equation for the second-order correlation function. We shall assume that the characteristic distance over which the nonlinear interaction between the field and the fluctuations $\delta\epsilon^{NL}=\epsilon^{NL}-\langle\epsilon^{NL}\rangle$ takes place is much greater than the size of the region of longitudinal field correlation $z_0=k_0\rho_0^2$ which is determined by the length of diffraction spreading of characteristic inhomogeneities in the cross section of the incoherent beam (the inhomogeneity scale ρ_0 is assumed to be much smaller than the radius r_0 of the beam envelope). The additional correlation between random fields introduced by the nonlinear interaction will then be small, and to obtain the closed equation for the second-order correlation function we can use, for example, the kinetic-equation approximation [19] (or the randomized phase approximation widely used in the theory of turbulent plasma). Assuming that the radiation distribution function at entry to the nonlinear medium is nearly normal, and $\langle\mathscr{E}(0, \mathbf{r}_\perp, \eta)\mathscr{E}(0, \mathbf{r}'_\perp, \eta')\rangle 0$, we can replace the fourth-order correlation function in (2) by the sum of the products of the second-order correlation functions.

The equation for the function

$$B_{ij}(\eta_1,\eta_2)=\langle\mathscr{E}(z,\mathbf{r}_{\perp i},\eta_1)\mathscr{E}^*(z,\mathbf{r}_{\perp j},\eta_2)\rangle\quad(i,j=1,2)$$

is

$$\left(\frac{\partial}{\partial z}+\frac{i}{k_0}\nabla_r\nabla_\rho\right)B_{12}(\eta_1,\eta_2)=\frac{ik_0\varepsilon_2}{2\varepsilon_0\tau}\left[\int_0^{\eta_1}d\eta'\exp\left(-\frac{\eta_1-\eta'}{\tau}\right)\right.$$
$$\times(B_{11}(\eta',\eta')B_{12}(\eta_1,\eta_2)+B_{11}(\eta_1,\eta')B_{12}(\eta',\eta_2)-\int_0^{\eta_2}d\eta'\exp\left(-\frac{\eta_2-\eta'}{\tau}\right)$$
$$\times(B_{22}(\eta',\eta')B_{12}(\eta_1,\eta_2)+B_{22}(\eta',\eta_2)B_{12}(\eta_1,\eta'))], \tag{3}$$
$$\mathbf{r}={}^1/_2(\mathbf{r}_{\perp 1}+\mathbf{r}_{\perp 2}),\qquad\boldsymbol{\rho}=\mathbf{r}_{\perp 1}-\mathbf{r}_{\perp 2}.$$

We must now determine the conditions under which the nonlinear interaction between the field and the fluctuations $\delta\epsilon^{NL}$ will not substantially alter the field correlation. We must therefore estimate the field phase change $\delta\varphi^{NL}$ over the longitudinal correlation length z_0 connected with the effect of $\delta\epsilon^{NL}$. Suppose that the characteristic times of amplitude and phase modulation are roughly the same and equal to τ_0. For light pulses which are incoherent both over the space and time spectra, we have

$$\langle(\delta\varphi^{NL})^2\rangle^{1/2}\approx\frac{k_0z_0\langle(\delta\varepsilon^{NL})^2\rangle^{1/2}}{2\varepsilon_0}$$

$$\approx\begin{cases}\dfrac{\varepsilon_2k_0^2\rho_0^2\langle|\mathscr{E}|^2\rangle\nu}{2\varepsilon_0}\ \text{for}\ t_0>\tau,\\[2ex]\dfrac{\varepsilon_2k_0^2\rho_0^2\langle|\mathscr{E}|^2\rangle\nu't_0}{2\varepsilon_0\tau}\ \text{for}\ t_0<\tau.\end{cases} \tag{4}$$

In these expressions, t_0 is the pulse length, $\nu=(\tau_0/\tau)^{1/2}$ for $\tau_0<\tau$ and $\nu=1$ for $\tau_0>\tau$; $\nu'=(\tau_0/t_0)^{1/2}$ for $\tau_0<t_0$. The required condition that the additional correlation due to the nonlinear interaction is small will be satisfied if $\langle(\delta\varphi^{NL})^2\rangle^{1/2}\ll 1$ or[1)]

$$P_0\ll P_{\text{кр}}/\nu\ \text{ for }\ t_0>\tau, \tag{5a}$$
$$W_0\ll W_{\text{кр}}/\nu'\ \text{ for }\ t_0<\tau. \tag{5b}$$

In these expressions, $P_0={}^1\!/_4(\rho_0^2/r_0^2)P$ and $W_0=P_0t_0$ are, respectively, the radiation power and energy on the scale of the transverse beam correlation (P is the total power of the incoherent beam), $P_{crit}=\epsilon_0^2v/2k_0^2\epsilon_2$ and $W_{crit}=P_{crit}\tau$ are, respectively, the critical power and energy for self-focusing or self-defocusing.[1,2,20]

3. TRANSFORMATION OF THE TIME SPECTRUM DURING THE SELF-INTERACTION OF INCOHERENT RADIATION

In this section, we consider processes connected with the transformation of radiation energy over the frequency spectrum.[2)] In order to establish the specific features of these processes, we shall confine our attention, for the sake of simplicity, to the transformation of incoherent light beams with a flat envelope. The symmetry of the problem shows that the functions B_{ij} in (3) are independent of the transverse coordinate $\mathbf{r}$. Bearing this in mind, and transforming from the stationary process (i.e., the process of formation of a pulse of length t_0 much greater than the correlation time τ_0 and the relaxation time τ) to correlation functions for the components of the frequency spectrum, we obtain

$$\frac{\partial B_{12}(\omega,z)}{\partial z}=\frac{k_0\varepsilon_2}{2\pi\varepsilon_0\tau}B_{12}(\omega,z)\int_{-\infty}^{+\infty}d\omega'B(\omega',z)\frac{(\omega-\omega')}{(\omega-\omega')^2+\tau^{-2}}, \tag{6}$$

where

$$B_{ij}(\omega,z)=\int_{-\infty}^{+\infty}d\tau\,B_{ij}(\tau,z)e^{i\omega\tau},\qquad B_{ij}(\tau,z)=B_{ij}(\eta_1-\eta_2,z),$$
$$B(\omega,z)=B_{11}(\omega,z)=B_{22}(\omega,z).$$

It is clear from (6) that if at the entrance to the nonlinear medium the correlation function can be written in the form

$$B_{12}(\omega,0)=\frac{2\pi}{v\varepsilon_0}f_{12}(\boldsymbol{\rho})I(\omega,0)$$

[$f_{12}(\rho)$ is the normalized correlation function and $I(\omega, 0)$ is the frequency distribution of the radiation intensity in the $z=0$ plane], then for $z>0$ the solution given by (6) can be written in the form

$$B_{12}(\omega,z)=\frac{2\pi}{v\varepsilon_0}f_{12}(\boldsymbol{\rho})I(\omega,z),$$

where $I(\omega, z)$ satisfies the equation

$$\frac{\partial I(\omega,z)}{\partial z}=\frac{k_0\varepsilon_2}{v\varepsilon_0^2\tau}I(\omega,z)\int_{-\infty}^{+\infty}I(\omega',z)\frac{(\omega-\omega')}{(\omega-\omega')^2+\tau^{-2}}d\omega'. \tag{7}$$

Since the kernel of the integrand in (7) is an odd function, it is readily shown that the integrated radiation intensity is conserved during the propagation of the wave along the z axis:

$$\int_{-\infty}^{+\infty}I(\omega,z)d\omega=\frac{k_0\varepsilon_2}{v\varepsilon_0^2\tau}\iint_{-\infty}^{+\infty}d\omega\,d\omega'\,I(\omega,z)I(\omega',z)\frac{(\omega-\omega')}{(\omega-\omega')^2+\tau^{-2}}=0.$$

Let us now consider the transformation of the frequency spectrum of radiation which, at entry into the nonlinear medium, has a spectral width much greater than τ^{-1}. Equation (7) can then be simplified by replacing $\omega/(\omega^2+\tau^{-2})$ by $\pi\delta(\omega-\tau^{-1})-\pi\delta(\omega+\tau^{-1})$. Using the properties of the δ function and the fact that the width of the spectrum is much greater than τ^{-1}, we have

$$\frac{\partial I(\omega, z)}{\partial z}=\frac{2\pi k_0\varepsilon_2}{v\varepsilon_0{}^2\tau^2}I(\omega, z)\frac{\partial I(\omega, z)}{\partial \omega}. \tag{8}$$

The solution of (8) is known to describe a simple wave transported along the frequency spectrum toward the Stokes region (for $\epsilon_2>0$). The width of the radiation spectrum during the propagation along the z axis remains roughly the same. The effective shift of the maximum in the spectrum is

$$\Delta\omega\approx\frac{2\pi k_0\varepsilon_2 I^m z}{v\varepsilon_0{}^2\tau^2},$$

where I^m is the maximum value of $I(\omega, z)$ in the $z=0$ plane. The shift is compared with the width of the frequency spectrum $\delta\omega_0$ for

$$I^m\approx\frac{v\varepsilon_0{}^2\tau^2\delta\omega_0}{2\pi k_0\varepsilon_2 z}$$

or

$$I\approx I^m\delta\omega_0\approx\frac{v\varepsilon_0{}^2\tau^2(\delta\omega_0)^2}{2\pi k_0\varepsilon_2 z}.$$

For self-focusing of incoherent beams of radiation ($k_0\approx 10^5$ cm^{-1}) with a spectrum width of $\delta\omega_0\approx 10^{12}$ sec^{-1} in liquids with relaxation time $\tau\approx 3\times 10^{-12}$ sec and $\epsilon_2\approx 5\times 10^{-11}$ cgs esu, we find that the integrated intensity which is sufficient for detecting the effect is I[mW/cm^2]$\approx 6\times 10^2$ z^{-1} [cm].

It is interesting to compare the transformation of the time spectrum of the spatially incoherent radiation with a flat envelope and the corresponding transformation of a nonmonochromatic plane wave (and therefore not having random modulation in the transverse direction) incident on a cubic medium. In the latter case, the solution of (1) can be written in the form (see, for example, [2,3])

$$\mathscr{E}=|\mathscr{E}(z=0,\eta)|\exp\Big[i\varphi(z=0,\eta)+\frac{ik_0\varepsilon_2}{2\varepsilon_0\tau}\int_0^\eta d\eta'\exp\Big(-\frac{\eta-\eta'}{\tau}\Big)|\mathscr{E}(z=0,\eta')|^2\Big]. \tag{9}$$

It follows from (9) that the phase fluctuations in the plane wave at entry into the nonlinear medium $\varphi(z=0, \eta)$ do not lead to a distortion of the spectrum shape when the light beam propagates in the half-space $z>0$. However, even small amplitude fluctuations in the incident radiation may substantially broaden the signal spectrum for a sufficiently long propagation path.[21] In contrast to this, during the propagation of a spatially incoherent beam of radiation with a flat envelope in a nonlinear medium, there is no substantial broadening of the spectrum but, instead, the energy is transformed toward lower frequencies, as described by (7). This difference from the case of a plane amplitude-modulated wave is that, in the latter case, there is an important contribution due to the interaction between Stokes and anti-Stokes components which eventually leads to the enrichment of the spectrum with harmonics on either side of the main frequency. At the same time, during the transformation of spatially incoherent radiation with $P_0\ll P_{crit}/\nu$, the Stokes and anti-Stokes components are "decoupled" and the interaction between them becomes unimportant. The radiation is then enhanced in the Stokes region and suppressed in the anti-Stokes region.[3)]

When the beam envelope at entry into the nonlinear medium is not flat, then, in addition to the energy conversion processes along the frequency axis discussed in this section, there is also the possible transformation of the radius of the envelope and of the beam correlation radius. However, it will be shown below that these effects can be appreciable only for $P_0\gtrsim P_{crit}$. In the opposite limiting case, the nonlinear change in the spatial field structure does not, in fact, occur and the above transformations of the time spectrum are, in fact, the leading processes. Moreover, the results derived in this section may also be of interest for $P_0>P_{crit}$ and for an envelope restricted in the transverse direction, provided only the pulse length is less than the characteristic scale for a nonlinear change in the beam radius.

4. SELF-FOCUSING OF INCOHERENT BEAMS

We must now consider effects connected with the nonlinear transformation of the spatial correlation function for incoherent light beams. It is clear that, when (5) is satisfied, the corresponding effects are possible only for beams restricted in the transverse direction. When the radiation correlation time τ_0 is small in comparison with the relaxation time τ (or $t_0<\tau$ it is small in comparison with the pulse length t_0), the parameter ν (or, correspondingly, ν') is small in comparison with unity. The conditions given by (5) can then be satisfied for $P_0\gtrsim P_{crit}$ (or $W_0\gtrsim W_{crit}$). We shall bear this in mind in the ensuing analysis. If we are interested in the single-moment correlation function, then from (3) it is readily seen that for $\tau_0\ll\tau$, t_0 the contribution of the second term to the transformation of spatial statistics will be small in comparison with the first term. Therefore, without considering the transformation of energy along the time spectrum, and restricting our attention to the transformation of the statistics for the radiation integrated over the frequencies, we can use for further analysis the equation

$$\frac{\partial B_{12}(\eta)}{\partial z}+\frac{i}{k_0}\nabla_r\nabla_\rho B_{12}(\eta)=\frac{ik_0\varepsilon_2}{2\varepsilon_0\tau}B_{12}(\eta)\int_0^\eta d\eta'\exp\Big\{-\frac{\eta-\eta'}{\tau}\Big\}\times[B_{11}(\eta')-B_{22}(\eta')],\quad B_{ij}(\eta)=B_{ij}(\eta,\nu). \tag{10}$$

The equation obtained for $\tau\to 0$ becomes identical with the equation for the correlation function used in [5,6,8] in the analysis of the propagation of monochromatic waves in a randomly inhomogeneous medium (the equation used in these papers includes a further term corresponding to linear scattering). However, the equation finally obtained in [5,6,8] is valid only when intensity fluctuations in the light beam are small in comparison with the mean beam intensity. It follows from the results given in Sec. 2 that the derivation of (10) does not depend on this restriction. This equation can, for a sufficiently broad time spectrum, describe the transformation of the spatial statistics of radiation with strong intensity fluctuations and a small transverse correlation radius (in comparison with the radius of the beam envelope). The results obtained below provide information on the transformation of both the correlation radius and the radius of the envelope on the multimode beam.

We shall suppose that, at entry into the nonlinear medium, the correlation function $B_{12}(\eta)_{z=0}$ is a Gaussian function of $\mathbf{r}$ and $\boldsymbol{\rho}$:[4)]

$$B_{12}(\eta)|_{z=0}=B_0(\eta)\exp\left(-\frac{\rho^2}{\rho_0^2}-\frac{\mathbf{r}^2}{r_0^2}\right).$$

The solution of (10) will be sought in the form

$$B_{12}(\eta)=B_0(\eta)\exp\left[-\frac{\rho^2}{\rho_0^2(z,\eta)}-\frac{\mathbf{r}^2}{r_0^2(z,\eta)}+i\alpha(z,\eta)\mathbf{r}\boldsymbol{\rho}+\varphi(z,\eta)\right]. \quad (11)$$

The function $B(\rho)\sim\exp[-\rho^2/\rho_0(z,\eta)]$ indicates that the correlation radius is independent of the transverse coordinate. In reality, it is obvious that this assumption is valid only for $\mathbf{r}$ corresponding to points near the beam axis, i.e., $|\mathbf{r}|\ll r_0$. For $|\mathbf{r}|\gtrsim r_0$, the zero-aberration approximation (12) used below is unsuitable and the solution in the form given by (11) is invalid: the beam correlation radius for $|\mathbf{r}|\sim r_0$ is a function of the distance from the beam center.

We now consider the transformation of the beam near its axis. In the zero-aberration approximation,[22] we can replace the nonlinear term $B_{11}(\eta')-B_{22}(\eta')$ by its parabolic approximation in terms of the transverse coordinates for $|\mathbf{r}|\ll r_0$:

$$B_{11}(\eta')-B_{22}(\eta')=-B_0(\eta')e^{\varphi(\eta',z)}\frac{2\mathbf{r}\boldsymbol{\rho}}{r_0^2(z,\eta')}. \quad (12)$$

Substituting (11) and (12) in (10), and equating to zero the coefficients in front of $\mathbf{r}^n\boldsymbol{\rho}^m$ $(n, m=0, 1, 2)$, we obtain a set of differential equations for the functions $r_0(z,\eta)$, $\rho_0(z,\eta)$, $\alpha(z,\eta)$, $\varphi(z,\eta)$. Eliminating $\alpha(z,\eta)$ and $\varphi(z,\eta)$ from this system, we obtain the equations for the radius $r_0(z,\eta)$ of the beam envelope and the correlation radius $\rho_0(z,\eta)$:[5)]

$$\frac{\partial^2 r_0(z,\eta)}{\partial z^2}=\frac{N}{k_0^2 r_0^3(z,\eta)}-r_0(z,\eta)\left(\frac{v\varepsilon_0 r_0^2}{2k_0^2 P_{\text{crit}}\tau}\right)\cdot\int_0^{\eta}\exp\left\{-\frac{\eta-\eta'}{\tau}\right\}\frac{B_0(\eta')d\eta'}{r_0^4(z,\eta')}, \quad (13)$$

$$\frac{r_0^2(z,\eta)}{\rho_0^2(z,\eta)}=\frac{N}{4}=\text{const}(z,\eta). \quad (14)$$

In these expressions, $N=\pi r_0^2/\frac{1}{4}\pi\rho_0^2$ is the number of inhomogeneities over the cross section of the beam. The condition $N=\text{const}(z,\eta)$ [see (14)] means that the degree of incoherence of the beam is conserved as it propagates through the nonlinear medium. Since, moreover, the total beam power is also conserved [Eq. (1) has the first integral $P=\int|\mathcal{E}|^2d^2r_\perp=\text{const}(z)$], we find that the power $P_0(z,\eta)=\text{const}(z)$ is also conserved. We note that when $N=1$, equation (13) becomes identical with the well-known expression for the radius of a Gaussian beam in a cubic medium in the zero-aberration approximation (see, for example,[1,2,22,23]). Hence to analyze (13) for $N\neq 1$, we can use the results given in these papers in which the analogous equation is examined. We must now consider separately the stationary and nonstationary self-focusing processes. When the pulse length t_0 is much greater than the relaxation time τ, the second term on the right-hand side of (13) can be simplified by taking out from under the integral sign the quantities $B_0(\eta')$ and $r_0^4(z,\eta')$ at the point $\eta'=\eta$. Equation (13) then reduces to an ordinary differential equation whose solution is

$$r_0^2(z,\eta)=r_0^2\left[1+\frac{z^2}{z_d^2}\left(1-\frac{P_0}{P_{\text{crit}}}\right)\right],\quad \rho_0(z,\eta)=\frac{\rho_0}{r_0}r_0(z,\eta), \quad (15)$$

where $z_d=\frac{1}{2}k_0\rho_0 r_0$ is the diffraction broadening of the incoherent beam in a linear medium. It is clear from (15) that, when $P_0\ll P_{\text{crit}}$, the nonlinear effects do not lead to an additional change in the correlation radius or the radius of the beam envelope.[6)] However, for $P_0\gtrsim P_{\text{crit}}$, the nonlinear effects become important. Self-focusing of a beam of incoherent light is impossible. The crossing point is given by

$$z_f=z_d/\sqrt{P_0/P_{\text{crit}}-1}. \quad (16)$$

For a fixed mean radiation intensity, the self-focusing length increases with increasing radius r_0 of the envelope and decreasing correlation radius ρ_0.

We must now consider briefly the conditions which will ensure that the corresponding effects will be observed experimentally. If, for example, the source of radiation is a multimode beam of light with spectrum width $\delta\nu_0=\delta\omega_0/2\pi c\approx 50\,\text{cm}^{-1}$ and divergence $\theta_0\approx 5\times 10^{-3}$ rad, then for envelope radius $r_0\approx 1$ mm the diffraction spreading is $z_d=\frac{1}{2}k_0\rho_0 r_0\approx$ 20 cm. For pulses with total power $P\approx 100$ MW ($P_0\approx 20$ kW) and length t_0 greater than the relaxation time τ, we find that Kerr-type self-focusing in nonlinear liquids ($P_{\text{crit}}=10$ kW) with isotropically polarized molecules (carbon disulfide, benzene, etc.) leads to the contraction of the beam over a length $z_f\approx 20$ cm. To estimate the validity of (5a), we consider the correlation time $\tau_0\approx 1/\delta\omega_0\approx 10^{-13}$ sec. Since the relaxation time in these liquids is of the order of a few picoseconds, we have $\nu=1/6$. Finally, since $\nu P_0/P_{\text{crit}}\approx 1/3$, it follows that condition (5a) can be regarded as roughly satisfied.

For nonstationary self-focusing ($t_0<\tau$), equation (13) has no analytic solutions. A numerical solution of this type of equation was investigated in [23]. Here, we shall confine our attention to the initial stage of nonstationary self-focusing. For pulses of length $t_0\ll\tau$, we can write

$$\int_0^{\eta}\frac{B_0(\eta')}{r_0^4(z,\eta')}d\eta'\approx\frac{2\pi}{v\varepsilon_0}\frac{w_0}{r_0^4(z,\eta)}$$

where w_0 is the energy density. It is then readily shown that the radius of the beam envelope varies in accordance with the formula

$$r_0^2(z,\eta)=r_0^2\left[1+\frac{z^2}{z_d^2}\left(1-\frac{W_0}{W_{\text{crit}}}\right)\right]. \quad (17)$$

The self-focusing condition is $W_0>W_{\text{crit}}$ so that it follows that if $W_0\ll W_{\text{crit}}$, then even for $W\gtrsim W_{\text{crit}}$ (W is the energy of the entire pulse) there is no appreciable distortion of the spatial statistics for radiation with sufficiently broad spectrum.

The above process of nonstationary self-focusing is of interest in connection with the thermal self-interaction of incoherent radiation of moderate duration. Let us suppose, for example, that a multimode beam of light with divergence $\theta_0\approx 10^{-3}$ rad, pulse length $t_0\approx 10^{-4}$ sec, spectrum width $\delta\nu_0\approx 0.1\,\text{cm}^{-1}$ ($\tau_0\approx 5\times 10^{-11}$ sec), and envelope radius $r_0\approx 0.5$ cm is incident on an absorbing medium (liquid) in which thermal self-focusing is possible. If we suppose that $W_{\text{crit}}\approx 0.01$ J, then condition (5b) will be satisfied in a sufficiently broad range of values of W_0. Self-focusing in this case leads to additional broadening of the envelope if $W_0>0.01$ J or $W>25$ J. In the opposite case ($W<25$ J) there is no distortion in the spatial spectrum of the radiation.

In conclusion, let us compare the results obtained in this section with data on self-focusing in beams with regular inhomogeneities.[2,4,24] It is clear from

(15) that when condition (5a) is satisfied and $r_0 \to \infty$, the radius of the envelope and the beam correlation radius for incoherent radiation remain exactly the same for $z>0$ as for $z=0$. This means that there is practically no change in the spatial statistics of incoherent radiation with a flat envelope in a nonlinear medium. At the same time, it follows from the theory of self-interaction of regular waves that an instability develops during the propagation of a flat [2,24] sufficiently broad monochromatic beam [4] in a cubic medium with $\epsilon_2>0$, which is modulated by a weak signal which may eventually lead to the decay of the beam into individual fibers and to an appreciable broadening of its spatial spectrum. This difference is explained by the fact that, in the case of a plane or quasiplane monochromatic wave, the instability which leads to the change in the spatial spectrum develops due to the interaction of the field with permittivity fluctuations induced as a result of interference between the main wave and the signal superimposed upon it. In contrast to this, when an incoherent wave with a broad space-time spectrum (i.e., waves with small correlation radius ρ_0 and correlation time τ_0) propagates through the medium, the permittivity fluctuations do not have a substantial influence on the transformation of the spatial statistics of the radiation when condition (5a) is satisfied. In the last case, the variation in the correlation radius and in the radius of the beam envelope is determined only by the mean profile $\langle\epsilon^{NL}\rangle$.

[1)]When condition (5) is satisfied on $z = 0$, it is also satisfied for $z \neq 0$ (including $z \gg z_0$) since the power P_0 and the width of the time spectrum of the radiation remain practically unaltered during the nonlinear transformation (see Secs. 3 and 4).

[2)]The self-focusing of coherent light pulses with complex amplitude which is independent of the transverse coordinates in the medium with a relaxation nonlinearity leading to the distortion of the frequency spectrum is considered in [21].

[3)]This is confirmed, in particular, by direct studies of the stability of a monochromatic spatially incoherent wave (power $P_0 \ll P_{crit}$) against small perturbations with frequencies differing by $\pm\Omega$ from the frequency of the main wave. In a medium with $\epsilon_2 > 0$ the growth rate for the Stokes component with frequency $\omega_0 - \Omega$ is positive, and for the anti-Stokes component with frequency $\omega_0 + \Omega$ it is negative.

[4)]It is important to note that for real multimode beams the correlation radius ρ_0 is, in general, a function of the transverse coordinate: with increasing distance from the center of the beam, there is an increase in the correlation radius. [18] However, near the axis of the beam, i.e., in the region where nonlinear conversion occurs most effectively, the quantity ρ_0 can be regarded as constant.

[5)]In the defocusing medium, the power P_{crit} must be replaced by the corresponding value taken with the sign reversed, i.e., the sign in front of the second term on the right-hand side of (13) is positive.

[6)]An analogous conclusion with regard to the self-focusing of monochromatic spatially incoherent beams of light can also be deduced from the results given in [7].

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49