# Effects of stimulated scattering for an electromagnetic pulse incident on a plasma layer

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An analysis is reported of the self-interaction of an electromagnetic pulse in a plasma layer of finite thickness which is connected with stimulated scattering (parametric instability). The self-interaction can be seen in amplitude modulation effects and in the anomalous absorption of the original signal. The results are given of a numerical calculation and of some analytic estimates of the formation of the plasma turbulence spectra during the development of parametric instability, and of the effects of self-interaction in the pump wave.

Several experiments on the sounding of the ionosphere with high-intensity electromagnetic waves[1-3] have established the presence of nonlinear effects near the point of reflection. In particular, it was established that there was excitation of plasma waves and anomalous absorption of the incident electromagnetic wave, which was probably due to nonlinear stimulated scattering (parametric instability) near the point of reflection.[4-7] 1) In this paper, we shall be concerned with the theory of parametric instability in the case of an electromagnetic pulse of arbitrary length incident on a finite layer of plasma.

In practice, the finite thickness of the plasma layer in which the nonlinear effects appear is due to its inhomogeneity. In inhomogeneous plasma, the excitation of parametric instability is possible only in a relatively thin plasma layer below the point of reflection where weakly attenuated plasma waves with frequency close to the pump frequency can be present. The development of the parametric instability is accompanied by the excitation of plasma turbulence, whose spectrum is formed by the nonlinear transformation of plasma waves toward larger scales and attenuation. The transfer of energy from the pump to the plasma waves may substantially exceed linear attenuation, and is the reason for the anomalous absorption of the original signal.

The necessary condition for the appearance of effects associated with parametric instability is that the corresponding growth rate $\gamma_N$ should exceed the linear attenuation of plasma waves $\gamma_l$ ($\gamma_N > \gamma_l$) and the length $\tau$ of the pump pulse must be greater than the instability development time, i.e., $\tau > \gamma_N^{-1}$. The nonlinear transformation of plasma waves along the spectrum through the individual satellites may lead to a peculiar amplitude modulation of the original electromagnetic pulse.

A quantitative description of the above effects can be achieved by solving the set of analytic equations for the transverse and longitudinal waves which, in the case of weakly ionized plasma in which we are interested, has the form[8]

$$\frac{\partial W_t}{dt} + \frac{\partial(\mathbf{v}_t W_t)}{\partial \mathbf{r}} = -W_t \int w_{tl}(\mathbf{k}_t, \mathbf{k}_1) W_{\mathbf{k}_1} d\mathbf{k}_1 - \gamma_t W_t, \tag{1a}$$

$$\frac{\partial W_{\mathbf{k}}}{\partial t} + \mathbf{v}_l \frac{\partial W_{\mathbf{k}}}{\partial \mathbf{r}} - \frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{r}} \frac{\partial W_{\mathbf{k}}}{\partial \mathbf{k}}$$
$$= w_{tl}(\mathbf{k}_t, \mathbf{k}_l) W_{\mathbf{k}} W_t - \gamma_l W_{\mathbf{k}} - W_{\mathbf{k}} \int w_{ll}(\mathbf{k}, \mathbf{k}_1) W_{\mathbf{k}_1} d\mathbf{k}_1 + a_{\mathbf{k}}, \tag{1b}$$

where $W_t$ and $W_l = \int W_{\mathbf{k}} d\mathbf{k}$ are, respectively, the energy densities in the transverse (t) and longitudinal ($l$) waves per unit volume, $\mathbf{v}_{t,l}$ is the group velocity of the waves, $\mathbf{k}_{t,l}$ is the wave vector, $\gamma_{t,l}$ is the linear damping, and $a_{\mathbf{k}}$ is the emissive power. The interaction kernels have the form

$$w_{tl,ll} = \frac{\omega_{0e}^2}{\omega} \frac{\cos^2(\alpha_l, \alpha_{t,l})}{N_e T_e} F\left(x_{t,l}; \frac{T_e}{T_i}\right);$$
$$F\left(x, \frac{T_e}{T_i}\right) = \frac{T_e}{T_i} \frac{a_2}{(1+a_1 T_e/T_i)^2 + (a_2 T_e/T_i)^2},$$
$$a_1 = 1 - 2x\exp\{-x^2\} \int_0^x \exp\{t^2\} dt, \qquad a_2 = \pi^{1/2} x \exp\{-x^2\}, \tag{2}$$
$$x_t = \frac{\omega_t - \omega(\mathbf{k})}{|\mathbf{k}_t - \mathbf{k}| v_{Ti}}, \qquad x_l = \frac{\omega(\mathbf{k}) - \omega(\mathbf{k}_1)}{|\mathbf{k} - \mathbf{k}_1| v_{Ti}},$$

where $v_{Ti} = (2T_i/M)^{1/2}$ is the thermal velocity of ions, $T_{e,i}$ is the temperature of the plasma electrons and ions, $\omega_{0e} = (4\pi e^2 N/m_e)^{1/2}$ is the electron Langmuir frequency, $\boldsymbol{\alpha}_{t,l}$ is the polarization vector, and $\omega(\mathbf{k})$ is the frequency of plasma waves which is related to $\mathbf{k}$ through the dispersion relation. The nonlinear growth rate is given by $\gamma_N = w_{tl} W_t$.

Equations (1a) and (1b) describe the parametric instability when most of the plasma wave energy is localized on the scales [8]

$$(m/M)^{1/4} k_d > k > (m/M)^{1/2} k_d, \tag{3}$$

where $k_d^{-1} = v_{Te}/\sqrt{2}\omega_{0e}$ is the Debye radius, $m/M$ is the ratio of electron and ion masses, and the kinetic stage of instability is being realized:[10]

$$\gamma_N < k v_{Ti}. \tag{4}$$

In our further analysis of (1), we shall simplify the problem by assuming the formation of one-dimensional spectra of plasma waves. In fact, the kernel of (1) is a maximum when the electric fields of the interacting waves are parallel. The presence of the maximum for a large ratio of the plasma-wave energy density to the background level facilitates this simplification. Quasilinear spectra (established plateau on the distribution function for the scattering particles) are unimportant in the case of scattering by ions for a sufficiently large number of collisions when[11]

$$\frac{\gamma_l}{\omega} > \left(\frac{W_l}{N_e T_i}\right)^2 \left(\frac{T_i m}{T_e M}\right)^{1/2} \left(1 + \frac{T_e}{T_i}\right)^{-2} \frac{k_0 v_{Ti}}{\Delta\omega}, \tag{5}$$

where $\Delta\omega$ is the width of the plasma wave spectrum and $k_0$ is a characteristic wave number in the spectrum. This inequality is satisfied with a substantial margin, at least under the conditions prevailing in ionospheric plasma.

To elucidate the main features of the solution of (1) and its possible simplification, let us begin by considering the case of a given pump, i.e., $W_t = \text{const}$. Numerical calculations based on (1b) for uniform plasma show that the plasma wave spectrum produced as a result of nonlinear action has narrow intensity peaks, $W_{ks}$, separated from one another by $\sim k v_{Ti}$ (see also [6,9]). The width of the plasma wave spectrum near the peaks is

$$\Delta\omega_s \approx k v_{Ti}\left(\frac{1-\gamma_l/\gamma_N}{2\ln(W_{ks}/W_{k0})}\right)^{1/2} \ll k v_{Ti}, \tag{6}$$

where $W_{k0} = a_k/\gamma_l$ is the initial plasma-wave level.

Therefore, the plasma wave spectrum can, in the first approximation, be represented by individual monochromatic lines (satellites) whose intensity decreases with increasing satellite number. This enables us to write (1) in the form of a set of equations for a finite number of satellites (see also [6]):

$$\begin{aligned}
&\partial u_t/\partial\tau + \partial u_t/\partial\xi = -\delta u_t u_1 - u_t;\\
&\partial u_1/\partial\tau = \delta(u_t u_1 f(\xi) - u_1 u_2) - u_1 + u_0,\\
&\partial u_2/\partial\tau = \delta(u_1 u_2 - u_2 u_3) - u_2 + u_0,\\
&\quad\cdots\cdots\cdots\cdots\\
&\partial u_s/\partial\tau = \delta(u_{s-1}u_s - u_s u_{s+1}) - u_s + u_0,
\end{aligned} \tag{7}$$

where $u_t = W_t v_t/W_{t0} v_{t0}$, $u_s = W_s/W_{t0}$ are the normalized energy densities of the transverse wave and the s-th satellite, $u_0$ is the emissive power, $\tau = \nu t$, $\gamma = \gamma_l = \gamma_t$,

$$\xi = \int \frac{\nu\, dz}{v_t}, \qquad \delta = \frac{\omega_{0e}^2}{\nu\omega}\frac{W_{t0}}{N_e T_e} F_m,$$

$F_m$ is the maximum value of the function $F(x, T_e/T_i)$, $f(\xi) = v_{t0}/v_t$, and $W_{t0}$ and $v_{t0}$ are, respectively, the energy density and group velocity of the pump wave at the beginning of the layer ($\xi = 0$).

In the set of equations given by (7), we have taken into account only the most rapid dependence (near the point of reflection) of the group velocity $v_t$ on the coordinates, and have neglected the spatial transport of the plasma-wave energy density. A numerical solution of the set of equations was carried out for the homogeneous case $[f(\xi) = 1]$, subject to the initial conditions $u_t = 1$, $u_s = u_0$ for $\tau = 0$, and the boundary condition $u_t = 1$ for $\xi = 0$. The nature of the solution depends on the parameters $\delta$ and $u_0$. Figure 1 shows the results of the numerical calculation for $\delta = 4$, $u_0 = 0.003$. The ordinate axis gives the normalized energy flux $u_t$ in the transverse wave and the energy density $u_s$ of the satellites for a number of fixed values of the coordinate $\xi$. The behavior of the transverse wave in time is characterized by damped oscillations and an eventual quasi-stationary value. The number of oscillations decreases with increasing $\xi$. The period of the oscillations is determined by the period of the oscillations in the first satellite at the beginning of the layer $T = \gamma_N^{-1} \ln u_0^{-1}$. In the first approximation the behavior of the satellites is the same as that of the satellites for a given pump: the period of the intensity oscillations increases with increasing $\xi$ because there is a reduction in the intensity of the transverse wave and, therefore, in the growth rate $\gamma_N$. Naturally, the number of satellites decreases with increasing $\xi$.

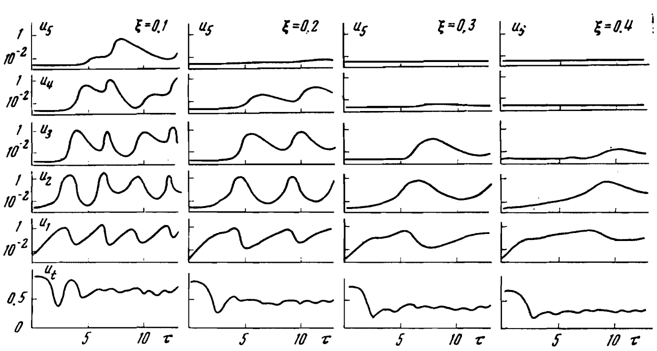


FIG. 1. The intensity of the transverse wave $u_t$ and of the plasma-wave satellites $u_s$ (s = 1-5) as functions of time for different values of $\xi$ and $\delta = 4$, $u_0 = 0.003$ (result of numerical calculation).

The pump-wave intensity decreases both as a result of linear absorption and because of the nonlinear transformation of energy into plasma oscillations. One would expect that the damping of the transverse-wave oscillations and the eventual stationary state are consequences of the averaging of the oscillations in space. In fact, at the beginning of the process, the plasma-wave intensity increases roughly in the same way in the entire space of $\xi$, which determines the first minimum of the transverse wave $u_t$. Subsequently, because of the difference between the periods of satellite intensity oscillations for different values of $\xi$, the phases of the oscillations $u_s$ become incoherent and this leads to the averaging of the nonlinear transformation of the transverse-wave energy into plasma oscillations. This enables us to conclude that the space-averaged plasma-wave intensities and the transverse-wave intensity have quasistationary values after a certain time $\sim T$.

Thus, the stationary transverse-wave intensity for a sufficiently large number of satellites $[\delta u_t f(\xi) \gg 1]$ is determined by the equation [see (7) for $\partial/\partial\tau = 0$]:

$$\frac{\partial u_t}{\partial\xi} = -\delta f(\xi) u_t^2 - u_t. \tag{8}$$

The solution of (8) with given intensity $u_{t0} = 1$ at the beginning of the layer is

$$u_t = e^{-\xi} \Big/ \left\{ 1 + \int_0^{\xi} \delta f(\xi') e^{-\xi'} d\xi' \right\}. \tag{9}$$

The values of $u_t$ calculated from (8) for a homogeneous layer $[f(\xi) = 1]$ and those obtained from a numerical calculation for $\delta = 4$ are shown in Fig. 2. The good agreement between the results enables us to conclude that the stationary solutions which are readily obtainable in analytic form can be used to calculate the nonlinear damping of the transverse wave.

We note that, in the linear approximation ($\delta = 0$), the absorption of the transverse wave is determined by the expression $u_t = e^{-\xi}$. Therefore, the integral with respect to $\xi'$ in (9) determines the anomalous absorption of the transverse wave, which is connected with parametric instability.

We now summarize the above results. During the

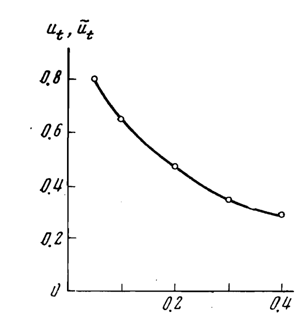


FIG. 2. Numerical calculation of the quasistationary values of the intensity of the transverse wave $\tilde{u}_t$ and the analytic function $u_t(\xi)$ based on (8) for $\delta = 4$ (open circles correspond to $\tilde{u}_t$, solid line corresponds to $u_t$).

developed stage of parametric instability, the plasma-wave spectrum can be represented by a set of individual narrow lines (satellites). The intensity of the individual satellites oscillates in time with a period $\sim\gamma_N^{-1} \ln u_0^{-1}$. The behavior of the transverse wave in the plasma layer is characterized by damped intensity oscillations and an eventual quasistationary value of the intensity. The self-interaction of the transverse wave in the stationary approximation appears as an anomalous absorption which, in weakly inhomogeneous plasma, is taken into account in (9).

[1)]It is assumed below that $T_e \approx T_i$ and, therefore, the decay interaction of waves is not considered.


[1]W. F. Utlant and R. Cohen, Science, **174**, 245 (1971).
[2]A. J. Wong and R. J. Taylor, Phys. Rev. Lett. **27**, 644 (1971).
[3]H. C. Carlson, W. E. Gordon, and R. L. Showen, J. Geophys. Res. **77**, 1242 (1972).
[4]D. F. Valeo, C. Oberman, and F. W. Perkins, Phys. Rev. Lett. **28**, 340 (1972).
[5]W. L. Kruer and E. J. Valeo, Princeton University Report, MATT-919, 1972.
[6]N. A. Mityakov, V. O. Rapoport, and V. Yu. Trakhtengerts, Geomagn. aeron. **14**, 1 (1974).
[7]Yu-Yun Kuo and J. A. Fejer, Phys. Rev. Lett. **29**, 1667 (1972).
[8]V. N. Tsytovich, Teoriya turbulentnoĭ plazmy (Theory of Turbulent Plasma), Atomizdat, 1971.
[9]A. S. Kingsep and L. I. Rudakov, Zh. Eksp. Teor. Fiz. **58**, 582 (1970) [Sov. Phys.-JETP **31**, 332 (1970)].
[10]A. G. Litvak and V. Yu. Trekhtengerts, Zh. Eksp. Teor. Fiz. 60, 1702 (1971) [Sov. Phys.-JETP **33**, 921 (1971)].
[11]V. N. Tsytovich and A. S. Chikhachev, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **12**, 26 (1969).


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