Breakdown of gases through laser radiation with high-energy quanta

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We consider the breakdown of gases through laser irradiation with high-energy quanta. We show that for the same energy gain rate of the electrons in the wave field the breakdown conditions change appreciably when the quantum energy changes.

1. The breakdown of gases through laser irradiation, discovered by Meyerand and Haught[1] has recently been discussed in a large number of papers (see the review[2] and also[3-10]). This is connected both with the purely physical interest in studying the processes accompanying laser breakdown and with the practical importance of the determination of the magnitude of threshold fluxes characterizing the breakdown strength of gases and, in particular, of air. However, in all cited papers the study of the breakdown of gases was made on the basis of the classical kinetic equation which describes only the continuous motion of the electron along the energy axis. Recently considerable progress has been made in the construction of lasers with high energy radiation quanta. Lasers have been constructed with molecular nitrogen ($\hbar\omega \approx 4$ eV)[11] and hydrogen ($\hbar\omega \approx 7.5$ eV)[12,13], and lasers with compressed inert gases ($\hbar\omega$ = 6 to 8 eV)[14-17] have been intensely studied. The utilization of the high-frequency range (up to the x-ray band (see the review[18])) is now one of the main problems of quantum electronics. When the quantum energy is comparable to or larger than typical energies characterizing the range of inelastic energy losses of an electron, the classical kinetic equation is no longer applicable to describe the ionization process due to the discrete nature of the gain of energy by the electron in the laser field. Connected with this effect is the possibility of an appreciably easier penetration of the electron through the region where vibrational and electronic levels of a molecule or an atom can be excited and this can appreciably change the picture of the gas breakdown.

In the present paper we give an analytical discussion of the problem of the cascade breakdown of gases through laser irradiation with high-energy quanta. According to the comparison with the numerical solution of the quantal kinetic equation given below the analytical solution is valid at least up to quantum energies of about 5 to 6 eV. We note that the only research, so far as we know, about the breakdown of gases which takes the discrete nature of the absorption of light quanta into account[19] has been done numerically under very restricted conditions ($\hbar\omega$ = 1.8 eV and a radiation flux density of 7.8×10^{10} W/cm^2).

2. The quantal kinetic equation for the electron distribution function $n(\epsilon, t) = n_0 F(\epsilon) e^{\gamma t}$ in the light field is of the form (see[2])

$$\gamma F(\varepsilon) = GN_a\Big\{-a(\varepsilon)F(\varepsilon) - b(\varepsilon)F(\varepsilon) + a(\varepsilon-\hbar\omega)F(\varepsilon-\hbar\omega) + b(\varepsilon+\hbar\omega)F(\varepsilon+\hbar\omega)\Big\}_+ + Q = W + Q, \quad \int_0^\infty F(\varepsilon)\,d\varepsilon = 1, \tag{1}$$

where $n(\epsilon)d\epsilon$ is the number of electrons per cm^3 with energies in the range ϵ to $\epsilon + d\epsilon$; n_0 is the initial electron density; G (cm^{-2}s^{-1}) is the light quanta flux; N_a the molecule density; $a(\epsilon)$ the light absorption coefficient, per electron of energy ϵ and per molecule; $b(\epsilon + \hbar\omega) = \{\epsilon/(\epsilon + \hbar\omega)\}^{1/2} a(\epsilon)$ is the coefficient for induced radiation for an electron of energy $\epsilon + \hbar\omega$. The collision integral Q includes terms connected with the creation of electrons, ionization, the excitation of vibrational and electronic molecular levels, and so on.

Let us expand W in a series in $\hbar\omega$. In that case (1) can be written in the form

$$\gamma F = -\partial J_q/\partial\varepsilon + Q, \tag{2}$$

where J_q is the electron flux in energy space which is connected with the interaction of the electrons with the laser field. If we retain in the expansion of W terms of second order in $\hbar\omega$ we have for J_q the usual classical expression[2]:

$$J_q = \frac{\alpha_0}{3} F - D\frac{\partial F}{\partial\varepsilon}, \tag{3}$$

where $\alpha_0 = (e^2E_0^2/2m\omega^2)\nu_{eff}$ is the rate at which an electron acquires energy in the field E_0 with frequency ω (ν_{eff} is the elastic collision frequency); $D = \frac{2}{3}\epsilon\alpha_0$ is the electron diffusion coefficient in energy space.

Writing the current in the form (3) does not take into account the discrete nature of the energy gain by an electron in a variable electromagnetic field and can therefore not describe the process of the "discontinuous" penetration of the electrons through the barriers of the vibrational and electronic molecular energy levels. The quantum nature of the absorption of the light can be taken into account already in the next term in the expansion of W; Planck's constant occurs explicitly in the kinetic equation. This statement is confirmed by the results of the numerical calculation given in Sec. 5 below.

In the approximation which is cubic in $\hbar\omega$ we have

$$J_q = \frac{\alpha}{3}\left[F - 2\varepsilon\frac{\partial F}{\partial\varepsilon} + \frac{(\hbar\omega)^2}{2}\frac{\partial^2 F}{\partial\varepsilon^2}\right], \tag{4}$$

where $\alpha = \alpha_0(1 + \hbar\omega/\epsilon)^{3/2}$ is a more accurate way to describe the rate at which the electron gains energy. The collisional term Q_v which occurs in Q is connected with the excitation of vibrational molecular levels and can also be written in the form (see below)

$$Q_v = -\partial J_v/\partial\varepsilon. \tag{5}$$

The term in Q which is connected with the ionization and with the excitation of the electronic molecular levels can, as in[2], be written in the form

$$Q_{in} = \begin{cases} 0 & \text{if } \varepsilon < \varepsilon_1^* \\ \infty & \text{if } \varepsilon > \varepsilon_1^* \end{cases}, \tag{6}$$

where $\epsilon_1^* \approx$ I corresponds to the energy of excitation of the first electronic level of the molecule.

We can thus write Eq. (1) in the region $\epsilon <$ I in the form

$$\gamma F = -\frac{\partial}{\partial\varepsilon}(J_q + J_v). \tag{7}$$

Introducing now the effective probability k for the penetration of electrons through the region of excitation of electronic molecular levels we can write down the boundary conditions for the current:

$$(J_q+J_v)_{\varepsilon=0}=J_q(0)=J_q(I)\,(1+k)=(J_q+J_v)_{\varepsilon=I}(1+k). \tag{8}$$

Moreover, we have

$$F(I)=0, \tag{8'}$$

$$(\partial F/\partial\varepsilon)_{\varepsilon=I}=0. \tag{8''}$$

Condition (8') means physically that electrons with $\epsilon \approx I$ lose their energy instantaneously in ionization and condition (8") that the region of energies where $F(\epsilon) \approx 0$ must be of the order of the quantum energy $\hbar\omega$ (as a consequence of the discrete nature of the absorption of light).

Integrating Eq. (7) and using (8) we get

$$\gamma=k(J_q+J_v)_{\varepsilon=I}. \tag{9}$$

For small k (which is the case of practical interest) the current $J_q(\epsilon) + J_V(\epsilon) \approx$ constant and the problem of determining γ can be reduced to solving the equation

$$\frac{(\hbar\omega)^2}{2}\frac{\alpha}{3}\frac{\partial^2 F}{\partial\varepsilon^2}-2\varepsilon\frac{\alpha}{3}\frac{\partial F}{\partial\varepsilon}+\frac{\alpha}{3}F+J_v(\varepsilon)=\frac{\gamma}{k}, \tag{10}$$

$$\int_0^{\infty} F(\varepsilon)\,d\varepsilon=1. \tag{10'}$$

We now turn to a determination of the current J_V which is connected with the interaction between electrons and the vibrational molecular levels. The collision integral Q_V has the form

$$Q_v=N_a\sum_{m=0}^{\infty}\left\{\sigma_{0m}(\varepsilon+\hbar\omega_{0m})F(\varepsilon+\hbar\omega_{0m})v(\varepsilon+\hbar\omega_{0m})-\sigma_{0m}(\varepsilon)F(\varepsilon)v(\varepsilon)\right\}, \tag{11}$$

where σ_{0m} is the cross-section for the excitation of the m-th vibrational level of the molecule with a quantum energy $\hbar\omega_{0m}$; v is the electron velocity. We can expand Eq. (11) in a power series in $\hbar\omega_{0m}$. This is connected with the fact that the average energy lost by an electron when it excites a molecule, i.e., the average magnitude of the quantum absorbed by the molecule $\langle\hbar\omega_{0m}\rangle \approx 0.1$ eV is appreciably less than that energy of an electron for which vibrational levels are effectively excited (1 to 3 eV). In that case

$$Q_v=-\frac{\partial J_v}{\partial\varepsilon}=\frac{\partial}{\partial\varepsilon}[\alpha^*(\varepsilon)F(\varepsilon)],$$
$$\alpha^*(\varepsilon)=N_a v(\varepsilon)\sum_m \delta_{0m}(\varepsilon)\hbar\omega_{0m}, \tag{12}$$

where $\alpha^*(\epsilon)$ is the rate of electron energy losses through the excitation of vibrational levels.

The region 2Δ in which an effective excitation of the vibrational structure of the molecule occurs is small compared to I. In our calculations we can thus assume that

$$\alpha^*(\varepsilon)=\begin{cases}\alpha^*=\text{const} & \text{if } |\varepsilon-\bar\varepsilon_2|<\Delta\\ 0 & \text{if } |\varepsilon-\bar\varepsilon_2|>\Delta\end{cases}, \tag{13}$$

where $\bar\epsilon_2$ is the average electron energy in that region.

We can solve Eq. (10) separately in three regions:

$$\varepsilon<\bar\varepsilon_2-\Delta, \qquad \bar\varepsilon_2-\Delta<\varepsilon<\bar\varepsilon_2+\Delta, \qquad \varepsilon>\bar\varepsilon_2+\Delta,$$

and afterwards join the functions $F(\epsilon)$ and their derivatives in the points $\epsilon = \bar\epsilon_2 - \Delta$ and $\epsilon = \bar\epsilon_2 + \Delta$. In the first two regions the solution of Eq. (10) has the form

$$F(\varepsilon)=\begin{cases}-\dfrac{3\gamma}{k\alpha_2}\dfrac{1}{3\alpha^*/\alpha_2-1}+C_1e^{\lambda_1\varepsilon}+C_2e^{\lambda_2\varepsilon}, & \bar\varepsilon_2-\Delta<\varepsilon<\bar\varepsilon_2+\Delta\\ \dfrac{3\gamma}{k\alpha_1}+C_3e^{\lambda_3\varepsilon}+C_4e^{\lambda_4\varepsilon}, & \varepsilon<\bar\varepsilon_2-\Delta\end{cases}, \tag{14}$$

where

$$\lambda_{1,2}=\frac{2\bar\varepsilon_2}{(\hbar\omega)^2}\pm\left[\frac{4\bar\varepsilon_2^2}{(\hbar\omega)^4}+\frac{2}{(\hbar\omega)^2}\left(\frac{3\alpha^*}{\alpha_2}-1\right)\right]^{1/2},$$
$$\lambda_{3,4}=\frac{2\bar\varepsilon_1}{(\hbar\omega)^2}\pm\left[\frac{4\bar\varepsilon_1^2}{(\hbar\omega)^4}-\frac{2}{(\hbar\omega)^2}\right]^{1/2},$$
$$\alpha_{1(2)}=\alpha_0(1+\hbar\omega/\bar\varepsilon_{1(2)})^{3/2},$$

while $\bar\epsilon_1$ is the average energy in the region $\epsilon < \bar\epsilon_2 - \Delta$.

In the region $\epsilon > \bar\epsilon_2 + \Delta$ we shall look for the function $F(\epsilon)$ in the form

$$F(\varepsilon)=3\gamma/k\alpha_0+\exp(\sigma(\varepsilon)/(\hbar\omega)^2),$$
$$\sigma(\varepsilon)=\sigma_0(\varepsilon)+\hbar\omega\sigma_1(\varepsilon)+(\hbar\omega)^2\sigma_2(\varepsilon)+\ldots \tag{15}$$

Using (8') and (8") we then find

$$F(\varepsilon)=\frac{3\gamma}{k\alpha_0}\left\{1-\left(\frac{\varepsilon}{I}\right)^{1/2}\exp\left[\frac{1}{32}\left(\frac{\hbar\omega}{\varepsilon}\right)^2\right]\right\}, \quad \varepsilon>\bar\varepsilon_2+\Delta. \tag{16}$$

We get from (10') for the constant γ which describes the development of the cascade

$$\frac{k}{3\gamma}=\frac{I}{3\alpha_0}+\left(\frac{1}{\alpha_0}+\frac{1}{3\alpha^*-\alpha_2}\right)$$
$$\times\left\{-2\Delta+\frac{1}{a^2-b^2}\left[2a-e^{-2a\Delta}\left(\frac{a^2+b^2}{b}\operatorname{sh}2b\Delta+2a\operatorname{ch}2b\Delta\right)\right]\right\}+\left(\frac{1}{\alpha_1}+\frac{1}{3\alpha^*-\alpha_2}\right)$$
$$\times\left\{\frac{1}{c^2-d^2}\left[2c-e^{-c(\bar\varepsilon_2-\Delta)}\left(\frac{c^2+d^2}{d}\operatorname{sh}d(\bar\varepsilon_2-\Delta)-2c\operatorname{ch}d(\bar\varepsilon_2-\Delta)\right)\right]\right\} \tag{17}$$
$$\times\left[-1+e^{-2a\Delta}\left(\frac{a}{b}\operatorname{sh}2b\Delta+\operatorname{ch}2b\Delta\right)\right]+$$
$$+\frac{1}{c^2-d^2}\left[-1+e^{-c(\bar\varepsilon_2-\Delta)}\left(\frac{c}{d}\operatorname{sh}d(\bar\varepsilon_2-\Delta)+\operatorname{ch}d(\bar\varepsilon_2-\Delta)\right)\right]\frac{a^2-b^2}{b}e^{-2a\Delta}\operatorname{sh}2b\Delta.$$

Here

$$a=\frac{2\bar\varepsilon_2}{(\hbar\omega)^2}, \quad b=\left[\frac{4\bar\varepsilon_2^2}{(\hbar\omega)^4}+\frac{2}{(\hbar\omega)^2}\left(\frac{3\alpha^*}{\alpha_2}-1\right)\right]^{1/2},$$
$$c=\frac{2\bar\varepsilon_1}{(\hbar\omega)^2}, \quad d=\left[\frac{4\bar\varepsilon_1^2}{(\hbar\omega)^4}-\frac{2}{(\hbar\omega)^2}\right]^{1/2}.$$

If $\alpha^*/\alpha_0 \ll 1$, which corresponds to the case of atomic gases, we get from (17)

$$\gamma=k\alpha_0/I, \tag{18}$$

which agrees with the result of [2]. If $\alpha^*/\alpha_0 \gg 1$ and $\hbar\omega \approx 0$ (physically this case corresponds to the breakdown of molecular gases, for instance, the breakdown of air by the radiation from a CO_2 laser)

$$\gamma=\frac{k\alpha_0}{4\bar\varepsilon_1}\exp\left(-\frac{3\Delta\alpha^*}{\bar\varepsilon_2\alpha_0}\right). \tag{19}$$

The same result, reflecting the classical process of the diffusion of electrons through the barrier of vibrational levels, was obtained in [8].

If $\alpha^*/\alpha_0 \gg 1$ and $\hbar\omega$ is sufficiently large ($\hbar\omega > \bar\epsilon_2\sqrt{(\alpha_2/\alpha^*)}$) we can write Eq. (17) in the form

$$\frac{k}{3\gamma}=\frac{1}{\alpha_0}\left(\frac{1}{3}I-2\Delta\right)-\frac{1}{\alpha_0}\frac{\alpha_2}{3\alpha^*}\left[2\bar\varepsilon_2-e^{-\varphi}\left(2\Delta\frac{3\alpha^*}{\alpha_2}\frac{\operatorname{sh}A}{A}+2\bar\varepsilon_2\operatorname{ch}A\right)\right]$$
$$+\frac{1}{\alpha_1}\left\{\left[2\bar\varepsilon_1-e^{-\varphi}\left(-(\bar\varepsilon_2-\Delta)\frac{\sin B}{B}+2\bar\varepsilon_1\cos B\right)\right]\right.$$
$$\times\left[-1+e^{-\varphi}\left(\frac{\psi}{2}\frac{\operatorname{sh}A}{A}+\operatorname{ch}A\right)\right]+\left[1-e^{-\varphi}\left(\varphi\frac{\sin B}{B}+\cos B\right)\right] \tag{20}$$
$$\left.\times 2\Delta\frac{3\alpha^*}{\alpha_2}e^{-\varphi}\frac{\operatorname{sh}A}{A}\right\},$$

where

$$\psi=\frac{4\Delta\varepsilon_2}{(\hbar\omega)^2},\qquad \varphi=\frac{2\varepsilon_1(\varepsilon_2-\Delta)}{(\hbar\omega)^2},$$

$$A=\frac{2\Delta}{\hbar\omega}\sqrt{\frac{6\alpha^*}{\alpha_2}},\qquad B=\sqrt{2}\,\frac{\varepsilon_2-\Delta}{\hbar\omega}.$$

For large $\hbar\omega$ we get for γ expression (18); in that case the conditions for breakdown of molecular gases approach those for atomic gases. In other words, the transmission coefficient of the vibrational level barrier tends for sufficiently large $\hbar\omega$ to unity.

3. The probability k for the penetration of an electron through the region of excitation of electronic molecular levels $\Delta_1 = I - \epsilon_1^*$ also depends on the energy of the quantum. Let us estimate that effect.

As Δ_1 is small compared to I the electron distribution in the excitation region is almost stationary. For the same reason we may assume the probability $1/\tau^*$ for the excitation to be energy-independent and also in J_q put $\epsilon = \langle\epsilon\rangle \approx I$ in the coefficient of $\partial F/\partial\epsilon$ [2] ($\langle\epsilon\rangle$ is the average energy in the region Δ_1). The kinetic equation for the electrons in the region Δ_1 then will be of the form

$$\frac{(\hbar\omega)^2}{2}\frac{\alpha_0}{3}\frac{\partial^3F}{\partial\varepsilon^3}-2I\frac{\alpha_0}{3}\frac{\partial^2F}{\partial\varepsilon^2}+\frac{\alpha_0}{3}\frac{\partial F}{\partial\varepsilon}+\frac{F}{\tau^*}=0, \tag{21}$$

$$F(I)=0,\qquad (\partial F/\partial\varepsilon)_{\varepsilon=I}=0. \tag{21'}$$

The probability for an electron to penetrate through the excitation region is, clearly, equal to the ratio of the currents, $k = J_q(I)/J_q(\epsilon_1^*)$. The solution of Eq. (21) which satisfies the boundary conditions (21') is

$$F=\text{const}\left\{\exp(\lambda_1\varepsilon)-\frac{\lambda_1-\lambda_3}{\lambda_2-\lambda_3}\exp[\lambda_2\varepsilon+(\lambda_1-\lambda_2)I]+\frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_3}\exp[\lambda_3\varepsilon+(\lambda_1-\lambda_3)I]\right\}. \tag{22}$$

where

$$\lambda_1=\beta+y,\quad \lambda_2=\lambda_3^*=\beta-\frac{1}{2}y+\frac{\sqrt{3}}{2}iz,$$

$$y=(\beta^3-\eta^3+\sqrt{\eta^6-\delta^6})^{1/3}+(\beta^3-\eta^3-\sqrt{\eta^6-\delta^6})^{1/3},$$

$$z=(\beta^3-\eta^3+\sqrt{\eta^6-\delta^6})^{1/3}-(\beta^3-\eta^3-\sqrt{\eta^6-\delta^6})^{1/3},$$

$$\beta=\frac{4}{3}\frac{I}{(\hbar\omega)^2},\quad \eta=\left(\frac{3}{\tau^*\alpha_0(\hbar\omega)^2}\right)^{1/3},\quad \delta=\left(\frac{128}{9}\frac{I^3}{\tau^*\alpha_0(\hbar\omega)^8}\right)^{1/4}.$$

Taking the ratio of the currents and making a few straightforward transformations we get

$$k^{-1}=\frac{e^{-(\beta+y)\Delta_1}}{{}^1/_8(\hbar\omega)^2(9y^2+3z^2)}\left(\frac{(\hbar\omega)^2}{2}\times(\beta+y)^2-2I(\beta+y)+1+e^{{}^3/_2y\Delta_1}\left\{\left[\frac{(\hbar\omega)^2}{2}\times\frac{(3\beta^2y-3\beta y^2+{}^3/_4y^3-{}^3/_4z^2y-3z^2\beta)}{z\sqrt{3}}-2I\frac{3\beta y-{}^3/_2y^2-{}^3/_2z^2}{z\sqrt{3}}+\frac{3y}{z\sqrt{3}}\right]\times\sin\frac{\sqrt{3}}{2}z\Delta_1-\left[\frac{(\hbar\omega)^2}{2}\left(\beta^2+2\beta y-\frac{5}{4}y^2-\frac{3}{4}z^2\right)-2I(\beta+y)+1\right]\times\cos\frac{\sqrt{3}}{2}z\Delta_1\right\}\right). \tag{23}$$

Let us consider the most interesting limiting cases.

a) In the case when $\hbar\omega \approx 0$ we have

$$k=\exp(-\sqrt{T^*/\tau^*}), \tag{24}$$

which is the same as the result obtained by Raĭzer [2] taking into account terms quadratic in $\hbar\omega$ in the kinetic equation. Here $T^* = \Delta_1^2/D = 3\Delta_1^2/2I\alpha_0$ is the time for the diffusion of the electron through the excitation region Δ_1;

b) in the case of large quanta $\hbar\omega > \Delta_1$ we get for k the following expression:

$$k=\frac{3e^{-\xi}}{1+2\exp(-{}^3/_2\xi)\cos(3^{1/2}\xi/2)},\qquad \xi=\left(\frac{6\Delta_1^3}{\tau^*\alpha_0(\hbar\omega)^2}\right)^{1/3}. \tag{25}$$

We see easily from (25) that the transmission coefficient for the barrier of the electronic levels tends to unity as $\hbar\omega$ increases.

4. Let us consider a concrete example: we shall estimate the magnitude q^* of the threshold radiation flux density in the case of the breakdown of molecular nitrogen and compare it with results obtained using the approximation (3). For an estimate we can simplify (20) to read

$$\frac{k}{3\gamma}\approx\frac{I}{3\alpha_0}+\frac{2\varepsilon_1}{\alpha_1}\left[\operatorname{ch}\frac{2\Delta}{\hbar\omega}\left(\frac{6\alpha^*}{\alpha_2}\right)^{1/3}-1\right]. \tag{26}$$

From the breakdown criterion $\gamma\tau = 40$ we get from (26) the following expression to determine q^* (in W/cm^2):

$$2.3\cdot10^{-13}\frac{k\nu_{eff}\tau}{(\hbar\omega)_{eV}^2}q^*=\frac{I_{eV}}{3}+\frac{2\varepsilon_{1eV}}{(1+\hbar\omega/\varepsilon_1)^{1/2}}\left(\operatorname{ch}\left[\frac{3\cdot10^{13}(2\Delta)_{eV}^2\langle\hbar\omega_{0m}\rangle_{eV}\Sigma\sigma_{0m}}{\sigma_{tr}(1+\hbar\omega/\varepsilon_2)^{3/2}q^*}\right]^{1/2}-1\right). \tag{27}$$

According to [20] for molecular nitrogen $\Sigma\sigma_{0m} = 3\times10^{-16}$ cm^2, $\sigma_{tr} = 1.2\times10^{-15}$ cm^2, $\langle\hbar\omega_{0m}\rangle = 0.5$ eV. We give in Fig. 1 the threshold radiation flux densities q_1^*, q_2^*, and q_3^*, obtained, respectively, from Eqs. (18), (19), and (27), as functions of $\hbar\omega$ for $\tau = 10^{-6}$ s, k ≈ 0.1, p = 1 atmos, $\nu_{eff} \approx 3\times10^{12}$ s^{-1}. This figure illustrates the effect of the vibrational levels of nitrogen on the conditions for optical breakdown for different magnitudes of the quanta $\hbar\omega$. From the results given here it is clear that the quantum nature of the absorption of radiation by electrons begins to affect the results when $\hbar\omega \approx 1$ eV and when $\hbar\omega \gtrsim 4$ eV the damping of the electrons by vibrational levels ceases to play an important role and nitrogen behaves like an atomic gas.

5. To check the applicability of the expansion (4) for the description of the quantum nature of the energy gain by the electrons in the field of a light wave we solved the kinetic equation (1) for the distribution function numerically on an M-220 digital computer. In the calculation we took into account the excitation of the vibrational and electronic levels of the molecule, and also ionization. We took the cross-sections for these proces-

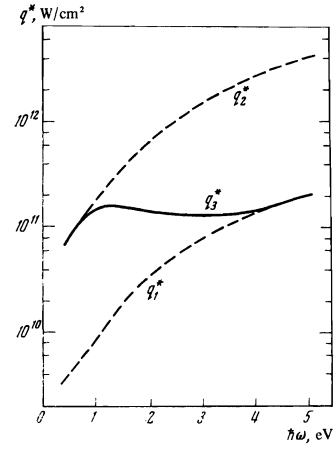

FIG. 1. Threshold radiation flux densities q_1^*, q_1^*, and q_3^*, obtained, respectively, from Eqs. (18), (19), and (27) for nitrogen as functions of $\hbar\omega$ for $\tau = 10^{-6}$ s, k = 0.1, and p = 1 atm.

ses from[21,22] for the nitrogen molecule which is of practical interest. We evaluated for the calculated distribution function the rate of excitation of the electronic levels (ν_x) and the rate of ionization (ν_i). In Fig. 2 we give ν_x and ν_i as functions of the magnitude of the laser radiation quantum for $q/(\hbar\omega)^2 = 1.5 \times 10^{11}$ W/cm^2 eV^2. For comparison we show the results of the solution of the classical kinetic equation. When the quantum of the incident radiation increases the quantities ν_x and ν_i increase appreciably as compared to their classical magnitude. This is connected with the "jump" of electrons through the barrier of the vibrational levels.

In Fig. 3 we give the threshold radiation flux as function of $\hbar\omega$ obtained from a numerical calculation. The breakdown criterion was $\gamma\tau = 40$ ($\gamma = \nu_i$). We obtained then for the transmission coefficient for the barrier of the electronic levels of nitrogen the magnitude $k = \nu_i/\nu_x \sim 10^{-3}$ to 10^{-4}. For comparison we have given in Fig. 3 the results of an analytical calculation using Eq. (27) with $k = 10^{-3}$, $\tau = 10^{-5}$ s.

The good agreement between the results of the analytical and the numerical calculations confirm the adequacy of retaining in the approximate kinetic equation the term which is cubic in $\hbar\omega/\epsilon$, at least for $\hbar\omega \sim 4$ to 5 eV.

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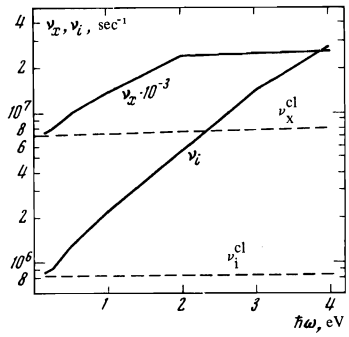

FIG. 2

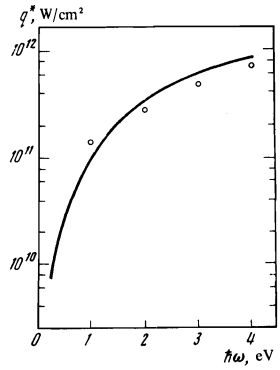

FIG. 3

FIG. 2. Rate of excitation of electronic levels ν_x and rate of ionization ν_i for nitrogen as functions of $\hbar\omega$ for $q/(\hbar\omega)^2 = 1.5 \times 10^{11}$ W/cm^2 eV^2.

FIG. 3. Results of a numerical calculation of the $\hbar\omega$-dependence of the threshold radiation flux density for $k = 10^{-3}$ to 10^{-4} and $\tau = 10^{-5}$ s. The dots indicate the results of a calculation using Eq. (27) for $k = 10^{-3}$ and $\tau = 10^{-5}$ s.

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