

# Inhomogeneous cyclotron resonance line broadening in semiconductors in a quantizing magnetic field

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A one-dimensional kinetic equation for cyclotron resonance in a quantizing magnetic field is derived on the basis of the equation for the translationally invariant part of the density matrix. A general solution of this equation is obtained for elastic scattering. Inhomogeneous broadening of the cyclotron resonance lines, due to the nonparabolicity of the electron spectrum renormalized as a consequence of interaction with the scatterers, is studied. It is shown that in the case of scattering of nondegenerate electrons an inhomogeneous line broadening arises under conditions when the inelastic parameter for scattering within the Landau zone is  $\beta \equiv \hbar \nu_0 / l T \sim 1$  ( $\nu_0$  is the velocity of sound,  $T$  the temperature in energy units, and  $l$  the magnetic length). This broadening is comparable with the contribution of dissipative processes to the linewidth. On scattering by charged impurities, when  $\lambda \ll r_0$  ( $\lambda$  is the electron thermal wavelength and  $r_0$  the screening range) inhomogeneous broadening also arises, and this leads to an increase in the linewidth by a factor of 4, compared with the dissipative width. Moreover, the line becomes asymmetric and its maximum is shifted toward low frequencies.

## INTRODUCTION

For an analysis of the cyclotron resonance line in a quantizing magnetic field (quantum cyclotron resonance) it is necessary to use a quantum kinetic equation. In principle, it can be obtained by using, for example, the graphic technique described by Konstantinov and Perel<sup>[1]</sup> or the procedure of decoupling the chain of equations for the correlation functions<sup>[2]</sup> in the representation of Landau quantum numbers. However, the form of the equation turns out to be extraordinarily complicated. The point is that the presence in the electron Hamiltonian of a vector potential that depends on the coordinates makes the problem formally translationally noninvariant<sup>[3]</sup> and increases the number of arguments in the single-particle density matrix. Apparently, because of the mathematical difficulties arising in this case, the kinetic equation has not, to the present date, been drawn upon for an analysis of quantum cyclotron resonance. The partial results available at present have been obtained without use of the kinetic equation within the framework of significant simplifications.<sup>[4,5]</sup> The effect of scattering on the resonance frequency has not been considered at all.

However, it is not difficult to see that such an effect can take place. Interaction with scatterers leads to shifts in the electron energy levels in the zeroth and first Landau zones by the amounts  $\Delta \epsilon_0(\epsilon)$  and  $\Delta \epsilon_1(\epsilon)$  (renormalization of the electron spectrum). Here  $\epsilon$  is the longitudinal energy of the electron (the kinetic energy along the magnetic field);  $\epsilon$ ,  $\Delta \epsilon_0$  and  $\Delta \epsilon_1$  are measured in units of  $\hbar \omega_c$ , where  $\omega_c$  is the cyclotron frequency. With account of the shift, the resonance frequency of the electrons with energy  $\epsilon$  (the partial frequency) is  $\omega_p(\epsilon) = \omega_c + \eta(\epsilon)$ , where  $\eta(\epsilon) = \omega_c [\Delta \epsilon_1(\epsilon) - \Delta \epsilon_0(\epsilon)]$  is the partial frequency shift. The absorption line of such electrons has a maximum at the frequency  $\omega_p$  and a half-width  $\delta \omega_p(\epsilon)$  determined by the dissipative processes.

The cyclotron resonance spectrum turns out to be smeared out over some region  $\delta \eta$ . If the dependence  $\eta(\epsilon)$  is weak (the renormalized spectrum remains approximately parabolic), the half-width of the cyclotron resonance line is  $\delta \omega \approx \delta \omega_p(\bar{\epsilon})$ , where  $\bar{\epsilon}$  is the mean value of  $\epsilon$ . In the opposite case, an inhomogeneous line broadening arises, due to the nonparabolicity of the re-

normalized spectrum.  $\Delta \epsilon_0$  and  $\Delta \epsilon_1$  are of the same order in the interaction constant as the line width:  $\Delta \epsilon_0, \Delta \epsilon_1 \sim (\omega_c \tau)^{-1}$  ( $\tau$  is a quantity of the order of the momentum relaxation time). In this sense, the shift is not small at an arbitrarily weak interaction.

In the present paper, the equation for the previously introduced<sup>[6]</sup> translationally-invariant part of the density matrix is used for the consideration of the cyclotron resonance. It is assumed that the interaction between electrons is absent and the interaction with the scatterers is weak. It is also assumed that the electric field can be regarded as homogeneous. This means, in particular, that the concentration of electrons is so small that the depth of the skin layer is much greater than all the characteristic dimensions of the problem. This condition is certainly satisfied in typical semiconductors. The spectrum of the non-interacting electrons is isotropic and quadratic.

A one-dimensional kinetic equation is obtained for the quantum cyclotron resonance. The equation is used for investigation of the effect of the partial frequency shift on the cyclotron resonance line.

## THE KINETIC EQUATION

1. We shall describe the system of electrons by the single particle density matrix

$$\rho(\mathbf{r}_1, \mathbf{r}_2) = \langle \psi^\dagger(\mathbf{r}_2) \psi(\mathbf{r}_1) \rangle,$$

where  $\psi^\dagger(\mathbf{r})$  and  $\psi(\mathbf{r})$  are the creation and annihilation operators of the electron at the point  $\mathbf{r}$ . The change in  $\rho$  with time is determined by the Hamiltonian

$$\begin{aligned} \hat{H} = \int d\mathbf{r} \psi^\dagger(\mathbf{r}) \left[ \frac{1}{2m} \left( -i\hbar \nabla - \frac{e}{c} \mathbf{A} \right)^2 - e\mathbf{r} \cdot \mathbf{E} \right] \psi(\mathbf{r}) + \sum_s \hbar \omega_s \left( b_s^\dagger b_s + \frac{1}{2} \right) \\ + \sum_s (C_s^\dagger b_s^\dagger + C_{-s} b_{-s}) \int e^{-i\mathbf{r} \cdot \mathbf{s}} \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) d\mathbf{r}. \end{aligned}$$

Here the first, second, and third terms on the right are the Hamiltonians of the electrons, phonons, and the electron-phonon interaction, respectively;  $\mathbf{A}$  is the vector potential of the magnetic field  $\mathbf{H} = (0, 0, H)$ , assumed to be constant and homogeneous;  $\mathbf{E}$  is the electric field;  $b_s^\dagger$  and  $b_s$  are the creation and annihilation operators of phonons with wave vector  $\mathbf{s}$  and frequency  $\omega_s$ ;  $C_s$  is a function characterizing the interaction of electrons with phonons. The remaining notation

is standard. (For simplicity,  $C_S$  and  $\omega_S$  are assumed to be independent of the direction of the vector  $\mathbf{s}$ .)

With the help of known methods (see, for example, [2]), we construct an equation for  $\rho$ . It contains the function

$$\rho_s(\mathbf{r}_1, \mathbf{r}_2) = \langle b_s^+ \psi^+(\mathbf{r}_2) \psi(\mathbf{r}_1) \rangle.$$

In the same way, we write down an equation for  $\rho_S$ . We express the correlators that are contained in this equation, of the form  $\langle b_S^+ b_S^+ \psi^+ \psi \rangle$  in terms of single-particle averages, using the usual decoupling rules. [2] In this approximation, the problem reduces to solution of a set of equations for  $\rho$  and  $\rho_S$ . It is essential that the structure of these equations is such that even in homogeneous fields  $\rho$  turns out to be dependent on each of its arguments separately, i.e.,  $\rho$  does not possess translational invariance in the spatially homogeneous situation.

We introduce the translationally-invariant part of the density matrix  $\tilde{\rho}^{[1]}$

$$\rho(\mathbf{r}_1, \mathbf{r}_2) = \tilde{\rho}(\mathbf{r}_1, \mathbf{r}_2) \exp \left[ -\frac{i}{2l^2} (x_1 - x_2) (y_1 + y_2) \right], \quad (1)$$

where  $l^2 = \hbar c / eH$ . (It is assumed that  $\mathbf{A}$  is of the form  $\mathbf{A} = (-yH, 0, 0)$ .) In the case of Boltzmann statistics in a uniform electric field, [6]  $\tilde{\rho}(\mathbf{r}_1, \mathbf{r}_2) = \rho(\mathbf{r}_1 - \mathbf{r}_2)$ . [1] This conclusion, however, is in no way connected with statistics, and remains valid at arbitrary degeneracy. We also introduce  $\tilde{\rho}_S$ , which is connected with  $\rho_S$  by the same relation (1). It is easy to obtain equations for  $\tilde{\rho}$  and  $\tilde{\rho}_S$  from the equations for  $\rho$  and  $\rho_S$ .

We now write down these equations in the Wigner representation for the case of a homogeneous electric field. We introduce the function  $F(\mathbf{k}, \mathbf{r})$ :

$$F(\mathbf{k}, \mathbf{r}) = \int d\mathbf{x} e^{i\mathbf{k}\mathbf{x}} \tilde{\rho} \left( \mathbf{r} - \frac{1}{2}\mathbf{x}, \mathbf{r} + \frac{1}{2}\mathbf{x} \right), \quad (2)$$

and the function  $F_S(\mathbf{k}, \mathbf{r})$  connected with  $\tilde{\rho}_S$  in similar fashion. In the homogeneous field  $\mathbf{E}(t)$ , the equations for  $F(\mathbf{k}, \mathbf{r}) \equiv F(\mathbf{k})$  and  $F_S(\mathbf{k}, \mathbf{r})$  are satisfied by the substitution  $F_S(\mathbf{k}, \mathbf{r}) = F(\mathbf{k}) e^{i\mathbf{s}\mathbf{r}}$ . For  $F(\mathbf{k})$  and  $F_S(\mathbf{k})$ , we obtain the set

$$\left\{ \frac{\partial}{\partial t} + \left( \frac{e}{\hbar} \mathbf{E} + \frac{e}{mc} [\mathbf{kH}] \right) \nabla_{\mathbf{k}} \right\} F(\mathbf{k}) + iF_S(\mathbf{k}) = 0, \quad (3a)$$

$$\begin{aligned} & \left\{ \frac{\partial}{\partial t} + \left( \frac{e}{\hbar} \mathbf{E} + \frac{e}{mc} [\mathbf{kH}] \right) \nabla_{\mathbf{k}} + i \frac{\hbar}{m} \mathbf{k}\mathbf{s} - i\omega_s \right\} F_S(\mathbf{k}) \\ &= \frac{i}{\hbar} C_s \left[ (N_s + 1) F \left( \mathbf{k} + \frac{1}{2}\mathbf{s} \right) - N_s F \left( \mathbf{k} - \frac{1}{2}\mathbf{s} \right) \right] \\ & - \frac{i}{\hbar} C_s \pi^2 \left( \frac{2l}{V^{1/3}} \right)^4 \sum_{\mathbf{k}'_{\perp}, \mathbf{k}''_{\perp}} F \left( \mathbf{k} + \mathbf{k}'_{\perp} + \frac{1}{2}\mathbf{s} \right) F \left( \mathbf{k} - \mathbf{k}''_{\perp} - \frac{1}{2}\mathbf{s} \right) \\ & \quad \times \exp \{ -2il^2 [\mathbf{k}'_{\perp} \mathbf{k}''_{\perp}] \}, \end{aligned} \quad (3b)$$

$$iF_S(\mathbf{k}) = \frac{i}{\hbar} \sum_{\mathbf{s}} C_s \left[ F_s \left( \mathbf{k} + \frac{1}{2}\mathbf{s} \right) - F_s \left( \mathbf{k} - \frac{1}{2}\mathbf{s} \right) \right] + \text{c.c.} \quad (3c)$$

Here  $N_S = \langle b_S^+ b_S \rangle$  is the phonon distribution function, assumed below to be at equilibrium;  $V$  is the normalization volume;  $\mathbf{k}'_{\perp} = (k'_x, k'_y, 0)$ ;  $\mathbf{k}''_{\perp} = (k''_x, k''_y, 0)$ . The system (3) was obtained by Zil'berman by a different method. [8] (The notation in [8] is different.)

2. In the linear approximation in the electric field we have  $F = \Phi + f$ ,  $F_S = \Phi_S + f_S$ , where  $\Phi$  and  $\Phi_S$  are equilibrium functions. For  $f$  and  $f_S$  we obtain:

$$\left\{ \frac{\partial}{\partial t} + \frac{e}{mc} [\mathbf{kH}] \nabla_{\mathbf{k}} \right\} f(\mathbf{k}) + i f_S(\mathbf{k}) = -\frac{e}{\hbar} \mathbf{E} \nabla_{\mathbf{k}} \Phi, \quad (4a)$$

$$\begin{aligned} & \left\{ \frac{\partial}{\partial t} + \frac{e}{mc} [\mathbf{kH}] \nabla_{\mathbf{k}} + i \frac{\hbar}{m} \mathbf{k}\mathbf{s} - i\omega_s \right\} f_S(\mathbf{k}) = \frac{i}{\hbar} C_s \left[ (N_s + 1) f \left( \mathbf{k} + \frac{1}{2}\mathbf{s} \right) \right. \\ & \left. - N_s f \left( \mathbf{k} - \frac{1}{2}\mathbf{s} \right) \right] - \frac{i}{\hbar} C_s \pi^2 \left( \frac{2l}{V^{1/3}} \right)^4 \sum_{\mathbf{k}'_{\perp}, \mathbf{k}''_{\perp}} \left[ \Phi \left( \mathbf{k} + \mathbf{k}'_{\perp} + \frac{1}{2}\mathbf{s} \right) \right. \end{aligned}$$

$$\begin{aligned} & \times f \left( \mathbf{k} - \mathbf{k}''_{\perp} - \frac{1}{2}\mathbf{s} \right) + f \left( \mathbf{k} + \mathbf{k}'_{\perp} + \frac{1}{2}\mathbf{s} \right) \Phi \left( \mathbf{k} - \mathbf{k}''_{\perp} - \frac{1}{2}\mathbf{s} \right) \Big] \\ & \quad \times \exp \{ -2il^2 [\mathbf{k}'_{\perp} \mathbf{k}''_{\perp}] \}, \end{aligned} \quad (4b)$$

where  $I f_S(\mathbf{k})$  is given by Eq. (3c) with the replacement of  $F_S$  by  $f_S$ . It is clear from physical considerations that at cyclotron resonance  $f$  is inversely proportional to the square of the coupling constant of electrons with phonons. Therefore, the term  $(e/\hbar) \mathbf{E} \nabla_{\mathbf{k}} \Phi_S$  which is proportional to this constant, is omitted from (4b). One must find  $f_S$  from (4b) and substitute in (4a).

We introduce cylindrical coordinates in the space of the wave vectors, with axis along  $\mathbf{H}$ :

$$\mathbf{k} = (k_{\perp}, \varphi, k_z), \quad \mathbf{s} = (s_{\perp}, \psi, s_z).$$

We assume that the electric field is turned on adiabatically:  $\mathbf{E} \sim \exp(-i\omega t + \omega_c \delta t)$ , ( $\delta \rightarrow +0$ ). With the help of the Green's function of Eq. (4b), a solution periodic in  $\varphi$  can be written, down for this equation in the form

$$f_s(\mathbf{k}) = \frac{1}{2\pi i \omega_c} \sum_n \frac{1}{n - \nu - \nu_s + k_z s_z - i\delta} \int_0^{2\pi} d\varphi \exp(in(\varphi - \psi) - i[\mathbf{k} - \tilde{\mathbf{k}}, \mathbf{s}]) (\dots). \quad (5)$$

One must put in the curly brackets the expression on the right side of (4b), replacing  $\mathbf{k}$  in it by  $\tilde{\mathbf{k}}$ . The dimensionless wave vectors  $l\mathbf{k}$  and  $l\mathbf{s}$  are introduced in (5); these are simply designated as  $\mathbf{k}$  and  $\mathbf{s}$  (dimensioned wave vectors will not be encountered below);

$$\tilde{\mathbf{k}} = (k_{\perp}, \varphi, k_z), \quad \omega_c = |e|H/mc, \quad \nu = \omega/\omega_c, \quad \nu_s = \omega_s/\omega_c.$$

We write down  $f$  in the form of a Fourier series in the angle  $\varphi$ :

$$f(\mathbf{k}) = \sum_n g_n(k_{\perp}, k_z) e^{in\varphi}. \quad (6)$$

We substitute the expressions (5) and (6) in  $I f_S$ . (If the field is expressed in complex form, then the symbol c.c. (complex conjugate) should be replaced by c.c. ( $-\omega$ ), which means that in addition to taking the complex conjugate, we should also change the sign of  $\omega$ .) We shall not write out the subsequent transformations, but show only the main points.

Upon substitution of (5) and (6) in (4a) it turns out that this equation splits into a series of independent equations for each component  $g_n$  separately. Thus, Eqs. (4) do not intermix  $g_n$  having different  $n$ . This fact radically simplifies the calculations; it is a direct consequence of the use of the translationally invariant density matrix.

If  $\mathbf{E} \perp \mathbf{H}$  (cyclotron resonance), the right side of (4a) contains terms proportional to  $e^{\pm i\varphi}$ ; only the functions  $g_{-1}$  and  $g_{+1}$  are different from zero. The equation for  $g_{-1}$  is

$$-i(\nu - 1)g_{-1} + \sum_{\mathbf{k}'} K(k_{\perp} - k'_{\perp}, k_z, k'_z) g_{-1}(k'_{\perp}, k'_z) = -\frac{eEl}{2\hbar\omega_c} \frac{\partial \Phi}{\partial k_{\perp}} \quad (7)$$

Here the summation represents an integral operator which describes the collisions, with kernel  $K(k_{\perp} - k'_{\perp}, k_z, k'_z)$ , which is too cumbersome to write out here.

We assume that the electric field is linearly polarized. It can be split into two components that are circularly polarized in opposite directions.  $g_{-1}$  describes the absorption of the component rotating in the same direction as the electron.  $g_{+1}$  is connected with the opposite component, which, under the conditions of cyclotron resonance is virtually unabsorbed. Therefore, there is no necessity of writing out the equation for  $g_{+1}$ , ( $|g_{+1}| \sim (\omega_c \tau)^{-1} |g_{-1}| \ll |g_{-1}|$ ).

3. Let us assume that the transitions occur only between states of the two lowest Landau bands. (For this it is necessary to satisfy the condition  $e^\alpha \gg 1$ , and to have for degenerate electrons  $\mu \ll 1$  ( $\alpha = \hbar\omega_c/T$ ,  $T$  is the temperature in energy units,  $\mu\hbar\omega_c$  the chemical potential). Moreover, it is necessary to require that the energy of the phonon be small in comparison with  $\hbar\omega_c$ :  $\nu_S \ll 1$ . (For acoustic phonons this is true if  $(m\nu_0^2/\hbar\omega_c)^{1/2} \ll 1$ , where  $\nu_0$  is the velocity of sound, which is practically always satisfied.)

Under these assumptions, the equilibrium distribution function takes the form<sup>[8]</sup>

$$\begin{aligned} \Phi(\mathbf{k}) &= 2\exp(-k_\perp^2)\zeta(\epsilon), \\ \zeta(\epsilon) &= \{e^{\alpha(\epsilon-\mu)} + 1\}^{-1}. \end{aligned} \quad (8)$$

The dependence of the right side of (7) on  $k_\perp$  is determined by the factor  $k_\perp \exp(-k_\perp^2)$ . We shall seek  $g_{-1}$  in the form

$$g_{-1}(k_\perp, k_z) = 2k_\perp \exp(-k_\perp^2)Z(\epsilon). \quad (9)$$

After substitution of this expression in (7), the variables are separated, the factors that depend on  $k_\perp$  are cancelled, and the following expression is obtained for the determination of  $Z(\epsilon)$ :

$$-i(\nu-1)Z(\epsilon) + A(\epsilon) + B(\epsilon) = \frac{eE\ell}{\hbar\omega_c}\zeta(\epsilon). \quad (10)$$

Here  $A(\epsilon) + B(\epsilon)$  is the collision integral.  $A = A^{(+)} + A^{(-)} + A'$  is the set of arrival terms with emission ( $A^{(+)}$  and absorption ( $A^{(-)}$ ) of a phonon,  $A'$  is the component which takes into account the filling of the finite levels.  $B = B^{(+)} + B^{(-)} + B'$  is the set of analogous departure terms. These terms are of the form

$$\begin{aligned} A^{(\pm)}(\epsilon) &= \frac{i}{\hbar^2\omega_c^2} \sum_{s, k_z'} |C_s|^2 \left(N_s + \frac{1}{2} \pm \frac{1}{2}\right) \left(1 - \frac{1}{2}s_\perp^2\right) \\ &\times \exp\left(-\frac{1}{2}s_\perp^2\right) Z(\epsilon') \left[ \frac{1}{\epsilon' - \epsilon \mp \nu_s - \Delta\nu - i\delta} - \frac{1}{\epsilon' - \epsilon \mp \nu_s + \Delta\nu + i\delta} \right] \delta_{k_z', k_z + s_z}, \\ B^{(\pm)}(\epsilon) &= \frac{-iZ(\epsilon)}{\hbar^2\omega_c^2} \sum_{s, k_z'} |C_s|^2 \left(N_s + \frac{1}{2} \pm \frac{1}{2}\right) \exp\left(-\frac{1}{2}s_\perp^2\right) \\ &\times \left\{ \frac{1 - s_\perp^2 + 1/4 s_\perp^4}{\epsilon' - \epsilon \pm \nu_s - \Delta\nu - i\delta} - \frac{1}{\epsilon' - \epsilon \pm \nu_s + \Delta\nu + i\delta} + \frac{1}{2} \frac{s_\perp^2}{\epsilon' - \epsilon - 1 - i\delta} \right. \\ &\left. - \sum_{n=1}^{\infty} \left(\frac{1}{2}s_\perp^2\right)^n \left[ -\frac{1}{(n-1)!} + \frac{s_\perp^2}{n!} - \frac{s_\perp^4}{4(n+1)!} \right] \frac{1}{\epsilon' - \epsilon + n} \right\} \delta_{k_z', k_z + s_z}, \end{aligned} \quad (11a)$$

$$- \sum_{n=1}^{\infty} \left(\frac{1}{2}s_\perp^2\right)^n \left[ -\frac{1}{(n-1)!} + \frac{s_\perp^2}{n!} - \frac{s_\perp^4}{4(n+1)!} \right] \frac{1}{\epsilon' - \epsilon + n} \delta_{k_z', k_z + s_z}, \quad (11b)$$

$$\begin{aligned} A'(\epsilon) &= \frac{i}{\hbar^2\omega_c^2} \zeta(\epsilon) \sum_{s, k_z'} |C_s|^2 \exp\left(-\frac{1}{2}s_\perp^2\right) \left(1 - \frac{1}{2}s_\perp^2\right) Z(\epsilon') \\ &\times \left[ \frac{1}{\epsilon' - \epsilon + \nu_s - \Delta\nu - i\delta} - \frac{1}{\epsilon' - \epsilon - \nu_s - \Delta\nu - i\delta} \right] \delta_{k_z', k_z + s_z}, \end{aligned} \quad (12a)$$

$$\begin{aligned} B'(\epsilon) &= \frac{i}{\hbar^2\omega_c^2} Z(\epsilon) \sum_{s, k_z'} |C_s|^2 \exp\left(-\frac{1}{2}s_\perp^2\right) \zeta(\epsilon') \\ &\times \left[ \frac{1}{\epsilon' - \epsilon - \nu_s + \Delta\nu + i\delta} - \frac{1}{\epsilon' - \epsilon + \nu_s + \Delta\nu + i\delta} \right] \delta_{k_z', k_z + s_z}. \end{aligned} \quad (12b)$$

The integrals of the expressions containing  $(\epsilon' - \epsilon + n)^{-1}$  are understood in the sense of the principal value;  $\Delta\nu = \nu - 1$ .

Thus the kinetic equation for cyclotron resonance in a quantizing magnetic field reduces to a one-dimensional integral equation relative to the function  $Z(\epsilon)$  that determines the nonequilibrium distribution of the electrons over the longitudinal energies.

The individual terms are not difficult to connect with the transitions (real and virtual) between the stationary states in the magnetic field, in accordance with the form of their denominators in (11) and (12). The electric

field transfers the electron from the zeroth Landau band to the first (the transition  $0 \rightarrow 1$ ). As a consequence of the scattering, the electrons execute both intraband transitions,  $0 \rightarrow 0$  and  $1 \rightarrow 1$ , and interband transitions. Thus, the terms in  $A^{(\pm)}$  describe intraband transitions. The same applies to  $A'$  and  $B'$ , and also to the components in  $B^{(\pm)}$  corresponding to the first two terms in the curly brackets of Eq. (11b). The third component in this bracket corresponds to the transition  $1 \rightarrow 0$ . The sum over  $n$  in (11b) corresponds to virtual interband transitions with increase in the band number. (Real transitions of this type yield terms of the order of  $e^{-\alpha}$ ; they were omitted from Eq. (11).)

In the case of elastic scattering,  $A' = B' = 0$ , the form of the collision integral does not depend on the statistics. Since the interband scattering is always elastic in the approximation  $\nu_S \ll 1$ , Eqs. (12) have no terms that describe interband transitions.

4. We limit ourselves below to the case of nondegenerate electrons and omit  $A'$  and  $B'$ . We assume that the collision broadening of the electron energy levels is small in comparison with the longitudinal energy. This condition is excellently satisfied in experiments.<sup>[9]</sup> In this connection, we omit the detuning  $\Delta\nu$ . With the aid of the relation

$$(\pm i\delta)^{-1} = \mp i\pi\delta(x)$$

we write down  $B$  in the form  $B_\delta + B_P$ , where  $B_\delta$  contains  $\delta$  functions and  $B_P$  principal values. (We note that if  $\Delta\nu = 0$  we have  $A_P = 0$ .) We represent  $B_P$  in form

$$B_P(\epsilon) = i\omega_c^{-1}\eta(\epsilon)Z(\epsilon). \quad (13)$$

The quantity  $\eta(\epsilon)$  has an explicit physical meaning: it is identical with the partial frequency shift defined in the Introduction and computed with the help of standard second-order perturbation theory. The presence of  $B_P$  in Eq. (10) means the replacement of the frequency of the bare electron by the renormalized (partial) frequency  $\omega_P(\epsilon) = \omega_c + \eta(\epsilon)$ .

5. Equations (11) are very complicated. They are much simpler in the case  $\alpha \gg 1$ . Then, in the formulas for  $B_P^{(\pm)}$ , we can omit  $\epsilon$  from the denominators of the terms which describe the interband transitions,  $\epsilon' - \epsilon - 1 - i\delta$  and  $\epsilon' - \epsilon + n$ . The shift corresponding to these terms does not depend on  $\epsilon$  and does not affect the line width. It can be included in the determination of the resonant frequency. Therefore we omit the terms in  $B_P^{(\pm)}$  which describe the interband transitions. After all these simplifications, the equations for  $A^{(\pm)}$  and  $B^{(\pm)} = B_\delta^{(\pm)} + B_P^{(\pm)}$  take the following form:

$$\begin{aligned} A^{(\pm)}(\epsilon) &= -\frac{2\pi}{\hbar^2\omega_c^2} \sum_{s, k_z'} |C_s|^2 \left(N_s + \frac{1}{2} \pm \frac{1}{2}\right) \left(1 - \frac{1}{2}s_\perp^2\right) \\ &\times \exp\left(-\frac{1}{2}s_\perp^2\right) Z(\epsilon') \delta(\epsilon' - \epsilon \mp \nu_s) \delta_{k_z', k_z + s_z}, \end{aligned} \quad (14a)$$

$$\begin{aligned} B_\delta^{(\pm)}(\epsilon) &= B_1^{(\pm)}(\epsilon) + B_2^{(\pm)}(\epsilon) + B_3^{(\pm)}(\epsilon) = \frac{2\pi}{\hbar^2\omega_c^2} Z(\epsilon) \\ &\times \sum_{s, k_z'} |C_s|^2 \left(N_s + \frac{1}{2} \pm \frac{1}{2}\right) \exp\left(-\frac{1}{2}s_\perp^2\right) \\ &\times \left\{ \left(1 - \frac{1}{2}s_\perp^2\right) \delta(\epsilon' - \epsilon \pm \nu_s) + \frac{1}{8}s_\perp^4 \delta(\epsilon' - \epsilon \pm \nu_s) + \frac{1}{4}s_\perp^2 \delta(\epsilon' - 1) \right\} \delta_{k_z', k_z + s_z}, \end{aligned} \quad (14b)$$

$$\begin{aligned} B_P^{(\pm)}(\epsilon) &= \frac{i}{\hbar^2\omega_c^2} Z(\epsilon) \sum_{s, k_z'} |C_s|^2 \left(N_s + \frac{1}{2} \pm \frac{1}{2}\right) \\ &\times \exp\left(-\frac{1}{2}s_\perp^2\right) \frac{s_\perp^2 - 1/4 s_\perp^4}{\epsilon' - \epsilon \pm \nu_s} \delta_{k_z', k_z + s_z}. \end{aligned} \quad (14c)$$

The terms  $B_1^{(\pm)}$ ,  $B_2^{(\pm)}$  and  $B_3^{(\pm)}$  correspond to the first,

second and third members in the curly brackets in (14b). We use Eqs. (14) for  $A^{(\pm)}$  and  $B^{(\pm)}$ .

6. For elastic scattering  $\nu_S = 0$ , it is easy to solve Eq. (10). We can replace  $Z(\epsilon')$  by  $Z(\epsilon)$  in the expression for  $A^{(\pm)}$  by virtue of the presence of the  $\delta$  function. Then  $A^{(\pm)}$  and  $B_1^{(\pm)}$  cancel each other. Inasmuch as  $\epsilon \ll 1$  and  $\epsilon' \ll 1$  in the case of intraband scattering, we can put  $s = [s_{\perp}^2 + (k_{z'} - k_z)^2]^{1/2} \approx s_{\perp}$  in the terms that describe the intraband transitions. All the principal values vanish in this case:  $\eta(\epsilon) = 0$ , i.e., there is no partial shift in elastic scattering and inhomogeneous broadening does not arise (concerning the scattering from charged impurities, see below). Gathering together the nonvanishing components of the collision integral, we get

$$A+B=Z(\epsilon)/\omega_c\tau(\epsilon), \quad (15)$$

where the relaxation time  $\tau(\epsilon)$  is equal to

$$\frac{1}{\tau(\epsilon)} = \frac{2\pi}{\hbar^2\omega_c} \sum_{s,s'} |C_s|^2 \left(N_s + \frac{1}{2}\right) \exp\left(-\frac{1}{2}s_{\perp}^2\right) \left[\frac{1}{4}s_{\perp}^4\delta(\epsilon'-\epsilon) + \frac{1}{2}s_{\perp}^2\delta(\epsilon'-1)\right] \delta_{k_s, k_{s'} + k}. \quad (16)$$

Equation (16) is identical with the expression for the relaxation time obtained by Kawabata,<sup>[4]</sup> written down in the approximation  $\alpha \gg 1$ . For what follows, we note that the ratio of the term which describes the interband transitions to the term which describes the intraband transitions in (16) is of the order of the ratio of the densities of the final states for the corresponding transitions, i.e., of the order of  $\alpha^{-1/2}$ .

From (10), we obtain

$$Z(\epsilon) = \frac{eEl}{\hbar\omega_c} \zeta(\epsilon) \left[-i(\nu-1) + \frac{1}{\omega_c\tau(\epsilon)}\right]^{-1}. \quad (17)$$

Equations (16) and (17) are in fact the solution of Eq. (10) for the case of arbitrary degeneracy, inasmuch as we have  $A' = A'' = 0$  for  $\nu_S = 0$ . In the nondegenerate case,  $N_S \approx (\alpha\nu_S)^{-1} \gg 1$ . In the case of degeneracy, both cases are possible:  $N_S \gg 1$  and  $N_S \ll 1$ . For impurities, we must replace  $|C_S|^2(N_S + 1/2)$  in (16) by  $1/2NV|u_S|^2$ , where  $N$  is the concentration of impurities,  $u_S$  the Fourier component of the impurity center.

It is necessary to note the following in connection with these formulas. It is seen from (16) that as  $\tau^{-1} \rightarrow \infty$   $\epsilon \rightarrow 0$  as  $\epsilon^{-1/2}$ . This is connected with the inapplicability of perturbation theory for small  $\epsilon$ : the condition of decoupling  $\omega_c\tau \gg 1$  is violated. However, this is not important, if the noted inequality is satisfied for the mean values: since the integral

$$\int_0^{\infty} \epsilon^{-1/2} \operatorname{Re} Z d\epsilon,$$

which determines the absorption, converges, the region of small  $\epsilon$  gives a small contribution.

The half-width of the line is  $\delta\omega = \tau^{-1}(\bar{\epsilon})$  with accuracy to within a factor close to unity.

## ACOUSTIC SCATTERING

This situation occurs in pure Ge and Si.<sup>[9]</sup> For acoustic phonons,  $|C_S|^2 = CsI^{-1}V^{-1}$ .<sup>[10]</sup> The energy of the phonon which interacts with the electrons is of the order of  $\hbar\nu_0/l$ . For  $\beta \equiv \hbar\nu_0/lT \ll 1$ , the intraband scattering is elastic, and for  $\beta \gtrsim 1$ , inelastic. The interband scattering is always elastic. For the case  $\beta \ll 1$ , the number of phonons  $N_S \approx (\alpha\nu_S)^{-1}$ ; inhomogeneous broadening is absent. It is easy to obtain the formula for  $\tau$  from (16). We shall not present it, since this case has already been studied previously.<sup>[4,5]</sup>

We now investigate the case  $\beta \gg 1$ . For  $\beta \gg 1$ , the intraband scattering is possible only by virtue of interaction with phonons with energies  $\hbar\omega_S \gtrsim T \ll \hbar\nu_0/l$ . As  $\beta \rightarrow \infty$  the region  $s_{\perp}$  of the plane that is effective for intraband scattering contracts to a point. Therefore, only the term  $B_3^{(*)}$  need be left in  $A + B_{\delta}$ , a term which describes the interband transitions and in which one must set  $N_S = 0$ . One can neglect  $\epsilon$  in  $B_{\delta}$ ; in comparison with  $\nu_S$ : the frequency shift turns out to be independent of  $\epsilon$  and it can be discarded. The collision integral can again be written with the help of the relaxation time  $\tau_{\infty}$ , where

$$\tau_{\infty}^{-1} = 0.85 Cm^2\omega_c/\pi\hbar^4. \quad (18)$$

Equation (18) is identical with the expression found in<sup>[5]</sup>. What is new is the fact that there is no inhomogeneous broadening if  $\beta \gg 1$ ; the line turns out to be Lorentzian:  $\delta\omega = \delta\omega_{\infty} = \tau_{\infty}^{-1}$ .

It remains to consider the case  $\beta \sim 1$ . The component  $B_{\delta}^{(*)}$  vanishes for arbitrary  $\beta$ , inasmuch as the principal value of  $(\epsilon' - \epsilon - \nu_{S\perp})^{-1}$  is equal to zero. The partial frequency shift can be written down as

$$\eta(\epsilon) = \alpha^{1/2}\tau_{\infty}^{-1}p(\beta, \alpha\epsilon), \quad (19)$$

where the function  $p(\beta, x)$  depends on  $\beta$  as a parameter. The expression for  $p(\beta, x)$  is easily found from (13) by substituting  $B_{\delta}^{(*)}$  everywhere in place of  $B_{\delta}$ .

We now consider the form of  $p(\beta, x)$ . We begin with  $\beta \sim 1$ . The corrections to the energy of the electron in the zeroth and first Landau bands  $\Delta\epsilon_0(\epsilon)$  and  $\Delta\epsilon_1(\epsilon)$  (see the Introduction) are negative and have minima at the values  $\epsilon^{(0)}$  and  $\epsilon^{(1)}$ , respectively.  $\epsilon^{(0)}$ ,  $\epsilon^{(1)} \sim \alpha^{-1}$ , with  $\epsilon^{(0)} < \epsilon^{(1)}$ . (The latter is connected with the different form of the wave functions in the considered bands.) In this connection, their difference  $\eta(\epsilon)$ , meaning  $p(\beta, \alpha\epsilon)$ , depends on  $\epsilon$  in oscillating fashion. Plots of  $p(\beta, x)$  at  $\beta = 0.5$  and  $1.5$  are shown in Fig. 1. As  $\beta \rightarrow 0$ , the oscillations shift to the region  $x \ll 1$  and vanish as  $\beta \rightarrow \infty$ .

An important contribution is made to the absorption by electrons with  $x \sim 1$ . When  $x$  varies in the region  $x \sim 1$ , the function  $p(\beta, x)$  runs through some range of values  $\delta p$ . The spectrum of the electron turns out to be smeared out over the interval  $\delta\eta \sim \alpha^{1/2}\tau_{\infty}^{-1/2}\delta p$ . For  $\beta \ll 1$  and  $\beta \gg 1$ , the function  $p$  depends very weakly on  $x$  in the case  $x \sim 1$ : inhomogeneous broadening, as has already been pointed out above, is absent. On the other hand, for  $\beta \sim 1$ ,  $p$  oscillates in the range  $x \sim 1$ :

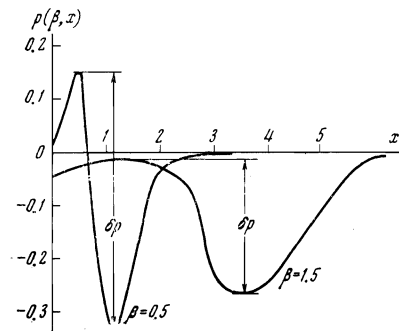


FIG. 1

$\delta\rho \sim 1$  (see Fig. 1). Thus, inhomogeneous broadening does take place:  $\delta\eta \sim \alpha^{1/2} \tau_0^{-1}$ .

For the study of the effect of inhomogeneous broadening on the cyclotron resonance line, it is necessary to take into account the line half-width  $\delta\omega_p(\epsilon)$  due to dissipative processes. However, for  $\beta \sim 1$ , the integral equation (10) is not simplified and can be solved only numerically. We therefore confine ourselves to estimates. It turns out that the contribution of the real transitions is also of the order of  $\alpha^{1/2} \tau_0^{-1}$ . Thus, it can be stated that at  $\beta \sim 1$  we get inhomogeneous broadening that is comparable with the dissipative line broadening.

## SCATTERING BY CHARGED PARTICLES

For the impurities, we set  $\nu_S = 0$ , and replace  $|C_S|^2(N_S + 1/2)$  by  $1/2 NV |u_S|^2$  in  $A^{(+)} + A^{(-)}$  and  $B^{(+)} + B^{(-)}$ . For centers of finite radius, the relaxation time is computed from Eq. (16). The charged impurities ( $u_S = 4\pi e^2 l^2 / V \kappa_0 \epsilon^2$ ,  $\kappa_0$  is the dielectric constant of the crystal) require special consideration.

From (16) we obtain (for simplicity, we set  $\alpha^{1/2} \gg 1$  and omit the component which describes the interband transitions):

$$\frac{1}{\tau(\epsilon)} = \frac{1}{\tau_1} (\alpha\epsilon)^{-\eta} = \frac{2^{1/2} \pi e^4 \alpha^{1/2} N}{\hbar^2 \omega_c \kappa_0 \epsilon^2} (\alpha\epsilon)^{-\eta}. \quad (20)$$

This is a well-known formula, obtained by Kawabata.<sup>[4]</sup> The conditions of applicability of (20) were essentially not formulated in<sup>[4]</sup>.

However, this result is not correct in the general case. In the derivation of (16) it was tacitly assumed that the integrals in (14a) and (14b) converge. At  $|u_S|^2 \sim s^{-4}$  this is not the case. For example, in the expressions for A and B<sub>1</sub>, which cancel one another at  $\nu_S = 0$ , the integrals over  $s_{\perp}$  diverge at zero after replacement of  $s = (s_{\perp}^2 + s_z^2)^{1/2}$  by  $s_{\perp}$ . The divergence as  $s_z \rightarrow 0$  means that the fundamental role is played by the more remote part of the Coulomb potential, which gives rise to transitions with small change in  $k_z$ . Of the two types of transitions  $k_z \rightarrow k'_z = k_z$  and  $k_z \rightarrow k'_z = -k_z$  allowed by the law of energy conservation, the dangerous ones are the first-forward scattering. To obtain finite expressions, we introduce the screening radius  $r_0$ , replacing the quantity  $s^2$  in  $u_S$  by  $s^2 + l^2 r_0^{-2}$ . Moreover, the detuning  $\Delta\nu$  should be preserved in A and B. The problem consists of studying of the shape of the collision integral as a function of the quantity  $r_0$ .

Simple analysis leads to the following results. We introduce the free path L of the electron along H, which corresponds to the mean distance at which relaxation of the transverse momentum takes place:  $L = \tau(\bar{\epsilon}) \times (\hbar \omega_c \bar{\epsilon} / m)^{1/2}$ . The condition for quantization and the condition of smallness of the collision width of the energy level can be written down in the form of the double inequality

$$l^2 \ll \lambda^2 \ll L^2, \quad (21)$$

where  $\lambda$  is the mean longitudinal wavelength of the electron,  $\lambda = l(2\bar{\epsilon})^{-1/2}$ . For  $r_0^2 \ll l^2$ , we have only the point interaction:

$$\frac{1}{\tau} = \frac{8\sqrt{2} \pi e^4 r_0^4 \alpha^{1/2}}{\hbar^2 \omega_c \kappa_0 l^3} N (\alpha\epsilon)^{-\eta}, \quad (22)$$

$\eta = 0$ —the inhomogeneous broadening is lacking. For  $l^2 \ll r_0^2 \ll \lambda^2$ , the value of  $\tau$  is given by Eq. (20) and again  $\eta = 0$ . For  $\lambda^2 \ll r_0^2 < L^2$ , the contribution of the

dissipative processes is described as before by the relaxation time (20), but there arises a partial shift in the frequency, connected with this time by the relation

$$\eta(\epsilon) = -2\pi i / \tau(\epsilon). \quad (23)$$

Thus, in scattering by screened Coulomb centers in the case in which  $\lambda^2 \ll r_0^2 \ll L^2$ , inhomogeneous line broadening should take place. Figure 2 shows the relative absorption P(x) (the ratio of the absorption at the given frequency to the absorption at its maximum), where  $x = (\omega - \omega_c) \tau_1$ , calculated for  $l^2 \ll r_0^2 \ll \lambda^2$  (dashed) and  $\lambda^2 \ll r_0^2 \ll L^2$  (solid line). The absorption was computed as

$$\int_0^{\infty} \epsilon^{-\eta} \operatorname{Re} Z d\epsilon.$$

For Z at  $r_0^2 \ll \lambda^2$  one must use Eq. (17); at  $\lambda^2 \ll r_0^2$  we must replace  $\tau^{-1}$  in (17) by  $\tau^{-1} + \eta = (1 - 2\pi i) \tau^{-1}$ ;  $\tau$  is given by Eq. (20);  $\xi(\epsilon) \sim e^{-\alpha\epsilon}$ . (We note that at  $r_0^2 \ll \lambda^2$  we can express P(x) in terms of the integral exponential function:  $P(x) = -x^{-3} \exp(x^{-2}) \operatorname{Ei}(x^{-2})$ .)

In the absence of inhomogeneous broadening we have  $\delta\omega \approx 1.3 \tau_1^{-1}$ . It is seen from Fig. 2 that the inhomogeneous broadening leads, in the first place, to an increase in the width of the line by a factor of four. Second, the line becomes asymmetric: its low-frequency wing at its half maximum has about twice the width of the high-frequency wing. Finally, the maximum of the line is shifted in frequency by an amount  $\Delta = -6.9 \tau_1^{-1}$ .

So far as the last possibility  $L^2 \ll r_0^2$  is concerned, in this case, the collision integral as a whole turns out to be finite. However, calculation of the absorption with the help of a distribution function that is not dependent on the coordinates (arranged over the location of the impurities) is valid only for the case  $r_0 < L$ . In the opposite case, relaxation of the momentum takes place within the range of action of a single center. The decoupling procedure that leads to Eq. (10) (this procedure as applied to impurities, has been described previously<sup>[6]</sup>) no longer holds for  $L < r_0$ ; the equation itself loses meaning. Consideration of this case goes beyond the scope of the present paper.

Observation of effects connected with inhomogeneous broadening is possible, for example, on the electrons of InSb at a frequency of  $\sim 10^{13}$  Hz at helium temperatures for samples with impurity concentrations of  $\sim 10^{14}$  cm<sup>-3</sup>. Here the conditions of applicability of the theory,  $\lambda^2$ ,  $r_0^2 \ll L^2$ , are satisfied and the ratio  $r_0^2 / \lambda^2$  can be both larger and smaller than unity.

(Cyclotron resonance has been observed by Appel and Poehler in a similar situation.<sup>[11]</sup> The values obtained for  $\delta\omega$  are sufficiently close to those found above. Unfortunately, the information on the samples used in<sup>[11]</sup> is insufficient for detailed comparison.)

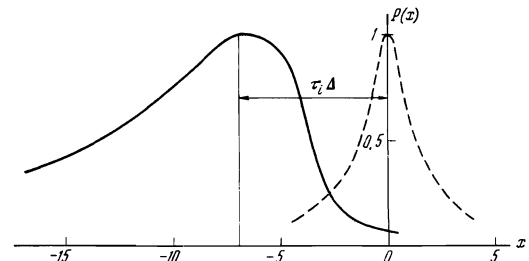


Рис. 2

For observation of the shift, it is necessary to carry out measurements at an identical renormalized frequency  $\omega_c$  (i.e., for given T and H). The situations  $r_0^2 \ll \lambda^2$  and  $\lambda^2 \ll r_0^2$  can occur if we use samples with different impurity concentrations.

In principle, there is no difficulty in generalizing the results obtained here to the case of degenerate electrons. For strong degeneracy, inhomogeneous broadening is always absent, since the contribution to the absorption is made only by those electrons with energy  $\epsilon = \mu$ ; at  $\lambda_\mu^2 \ll r_0^2$  ( $\lambda_\mu$  is the longitudinal wavelength of an electron with energy  $\mu$ ) a frequency shift develops that is equal to the value  $\eta(\epsilon)$  for the case  $\epsilon = \mu$ .

The author is grateful to I. B. Levinson for discussion of the present research and to M. D. Cherepanov for help in the calculations.

$$*[\mathbf{kH}] \equiv \mathbf{k} \times \mathbf{H}.$$

<sup>1)</sup>As has been noted, the matrix  $\tilde{\rho}$  was first introduced by Scher and Holstein, [7] who showed that  $\tilde{\rho}(\mathbf{r}_1, \mathbf{r}_2) = \tilde{\rho}(\mathbf{r}_1 - \mathbf{r}_2)$  (in the absence of an electric field).

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70