The motion of charged radiating shells in the general theory of relativity and "friedmon" states

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The equations of motion of charged radiating shells are derived in general relativity. It is shown on the basis of this model that during the anticollapse stage there is a possibility for open systems to undergo transitions into semiclosed ones, as well as for transitions of nonfriedmon states into friedmon states, as a result of processes of emission of charge and energy.

1. INTRODUCTION

The determination of interior solutions (i.e., solutions inside a distribution of matter) for the Einstein equations is a relatively difficult problem. The problem is complicated all the more if the matter can lose energy by radiation, since in this case one must take into account the additional radiation reaction forces. There are several related questions for which it is desirable to have an exact interior solution for a radiating system. Thus, in studying the problem of gravitational collapse it was pointed out[1,2] that possibly the emission of energy exerts a crucial influence on the dynamics of the collapse.

Another problem for which it is necessary to know an exact solution of the interior problem is that of the possibility of formation of semi-closed universes from open ones. Semi-closed universes were first considered by Kleĭn, Zel'dovich, and Novikov[3-5] and are structures for which the boundary of matter is situated in the second R-region, connected to the space of an outside observer through a narrow "throat." Semi-closed universes have a geometry that differs strongly from the Euclidean one and are characterized from a physical point of view by a gravitational mass defect comparable to the total mass of the system. Obviously, if the initially open system loses part of its mass as a result of radiation, the remaining matter may be compressed as a result of radiation reaction and the gravitational mass defect may increase. An exact solution of the problem involving radiation is required in order to clarify whether there exist conditions for which a semi-closed universe can be formed as a result of such a process.

Finally, a solution of the equations for a radiating system is necessary also for the discussion of the following problem. It was shown in papers by Markov and the author[6-8] that in the investigation of the problem of self-energy of sources of electric fields, a natural generalization of the concept of electric point charge is the concept of "friedmon," an object which appears as a result of simultaneous solution of the Maxwell-Einstein equations and representing a semiclosed universe for which the external parameters (i.e., the external mass and the size) are completely determined by its charge and are the minimal ones admitted by the theory. An investigation of the equations for a radiating system allows one to answer the question whether such objects can appear in the Universe under definite conditions, i.e., whether on account of emission of energy and charge a system can transform into a friedmon.

In all cases mentioned above it is very convenient to consider a simple model in which the source of the field is a massive charged radiating shell, i.e., a distribution of matter which is concentrated near the surface of a sphere of radius ρ the thickness of the shell being small and the mass of the matter inside the shell finite. A theory of massive shells was developed in[9,10]. In this model the dynamics of the system is described by prescribing the dependence of the radius ρ of the shell on the time τ, and the partial differential equations necessary for the description of a continuous medium here are replaced by an ordinary differential equation for the function $\rho(\tau)$. Israel[9] has shown how this equation can be obtained from the Einstein equation if one knows the metric both inside and outside the shell and specifies the distribution of matter on the surface of the shell. Israel's equations have been used for the derivation of an equation of motion of a spherically-symmetric neutral[9,10] and charged[11] shell and of a shell consisting of radiation[12].

In the present paper we derive the equations of motion of charged radiating shells in a form which is useful for the description both of emission and absorption processes of radiation. These equations are used for establishing a relation between the parameters which characterize the system before and after the radiative process. As a result of an analysis of these relations we show that in the anticollapse stage open systems can go over into semiclosed ones and nonfriedmon states can go over into friedmon states.

We use the following notation: greek indices take on the values 1, 2, 3, 4 and lower-case latin ones, i, j = 2, 3, 4. The tetrad components of tensors have the indices A, B = 1, 2, 3, 4 or X, Y = 2, 3, 4. A prime denotes differentiation with respect to coordinate time z and a dot denotes differentiation with respect to proper time τ. The speed of light is $c = 1$, κ is the gravitational constant. The signature of the metric is (- - - +).

2. THE EXTERNAL METRIC FOR CHARGED RADIATING SYSTEMS (THE CHARGED VAIDYA METRIC)

In the Introduction we have indicated that in order to obtain equations of motion of charged radiating shells it is necessary to know the metric outside the source, i.e., in the region where only radiation is present. The nature of the radiation can be arbitrary, and the radiation itself is described thermodynamically by giving the energy-momentum tensor $T_{\mu\nu} = qk_\mu k_\nu$, where $k_\mu k_\nu = 0$ and $q \geq 0$. The quantity q describes the energy density of the radiation in an appropriately chosen reference frame. For a neutral spherically symmetric body the external metric in the presence of radiation was found by Vaidya[13-15] and is of the form

$$ds^2=2p\,dz\,dr-r^2d\sigma^2+f(z,\ r)\,dz^2, \qquad (1)$$

where

$$f(z,r)=1-\frac{2\varkappa m(z)}{r},\quad d\sigma^2=d\theta^2+\sin^2\theta\, d\varphi^2,\quad k=p\frac{\partial}{\partial r}.$$

for $p = +1$ the z coordinate has the meaning of retarded time u, and from the positivity of q it follows that m(u) is a decreasing function, i.e., the system radiates energy. For $p = -1$ the radiation is absorbed by the system ($m' \geq 0$) and z has the meaning of advanced time.

The generalization of the Vaidya metric to the case when the system is charged and the radiation can transfer charge, given in[16,17] has the form (1), but in this case

$$f(z,r)=1-\frac{2\varkappa m(z)}{r}+\frac{\varkappa e^2(z)}{r^2}. \qquad (1')$$

For constant m and e this metric coincides with the usual Reissner-Nordstrøm metric and at $e = 0$ it goes over into the well-known Schwarzschild metric.

The boundary of the radiating system is a sphere with the radius r depending on time:

$$r=R(z). \qquad (2)$$

In the case when the source of the field is a massive shell, the boundary of matter coincides with the surface of the shell. In studying the motion of the sources of the field it is important to keep in mind that the coordinates $(r,\ \theta,\ \varphi,\ z)$ under discussion do not cover the whole of spacetime. This is easily checked by noting that the metric in these coordinates is not geodesically complete. The structure of the whole of spacetime depends on the function $f(z,\ r)$, but in its general features it will be similar to the structure of the maximal analytic extension of the Schwarzschild metric obtained by Kruskal[18], and to the extension of the Reissner-Nordstrøm metric found by Graves and Brill[19]. The Penrose diagrams for the appropriate complete spacetimes are shown respectively in Fig. 1 and Fig. 2. We use these diagrams in studying the motion of shells in the case when the emission or absorption occurs during a short proper time interval, so that before and after the emission or absorption the system moves in its own field with constant parameters m and e.

Both figures represent space-time in the coordinates introduced by Penrose[20]. The details of the construction of such diagrams for different metrics can be found, e.g., in the paper of Walker[21].* The notation in Fig. 1 and 2 is analogous to that used in[20,21]. The lines

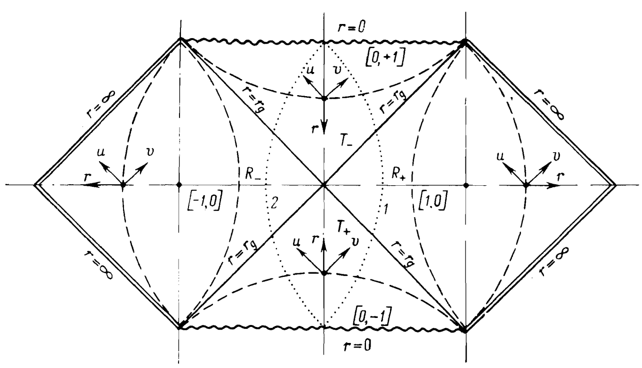

FIG. 1. The Penrose diagram for the maximally extended Schwarzschild-Kruskal spacetime (a section θ = cont, φ = const).*

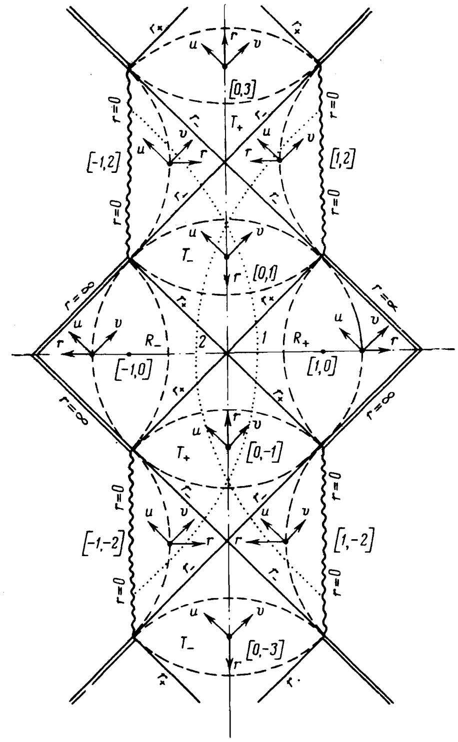

FIG. 2. The Penrose diagram for the maximally extended Reissner-Nordstrøm Graves-Brill spacetime for $m > e\kappa^{-1/2}$ (a section θ = const, φ = const).

$r = 2\kappa m$ for the Schwarzschild metric and the lines $r = r_\pm = \kappa(m \pm (m^2 - e^2/\kappa)^{1/2})$ for the Reissner-Nordstrøm metric partition spacetime into parts. In the scale chosen by us the centers of these parts are at points with integer coordinates n and l. In the Schwarzschild case there are four parts and the Penrose diagram for the Reissner-Nordstrøm spacetime consists of a countable set of parts. The equation u = const (v = const) describes straight lines making an angle of 45° (135°) with the X axis.

It is convenient to introduce the following notation:

$$[k,\pm]=\bigcup_n\,[n,\ (2k-1)\pm n],$$

where in each case the union is taken over all n such that the corresponding blocks under the union sign belong to the spacetime diagram. The parts of spacetime which end up in the blocks [k, +] ([k, −]) are completely described in the coordinates (r, u) (or (r, v), respectively). In writing the metric in the form (1) it turns out that $p = +1$ ($p = -1$) in the blocks [21, ±] ([21 − 1, ±]). Finally we note that the coordinate systems used for the description of the different parts of spacetime may have different orientations. We make the convention to consider the coordinate system $(r,\ \theta,\ \varphi,\ t)$ introduced by an external observer at Euclidean infinity as right-handed. Then it is easy to show that the coordinates $(r,\ \theta,\ \varphi,\ u)$ in the blocks [21, +] ([21 + 1, +]) are right (left-) handed. Similarly, the coordinates $(r,\ \theta,\ \varphi,\ v)$ in the blocks [21 − 1, −] ([21, −]) are right- (left-) handed. We associate with each coordinate system the number σ equal to +1 for right-handed systems and to −1 for left-handed ones.

We note that in the blocks [0, 2n + 1] the coordinate r has a timelike character[1)], i.e., in the course of time it varies strictly monotonically. If the boundary of matter is in a T-region one talks about collapse (decrease of r) or anticollapse (increase of r) of matter. In the case of the metric of form (1) one can partition the whole of spacetime into R- and T-regions, however for m and e which depend on z the boundary between the R- and T-regions will no longer be a horizon, as is the case for the Schwarzschild and Reissner-Nordstrøm spaces.

For a general characterization of the physical properties of the considered system it is important to know through which of the two regions $[1, 0]$ or $[-1, 0]$ passes the worldline of the boundary of matter. If the matter passes through $[1, 0]$ the system is called open, in the other case it is called semiclosed. For semiclosed systems at the time of expansion it is characteristic that the areas of the spheres which surround it decrease as one goes away from the matter to the outside, up to the minimal sphere—"throat." In order to describe the property of semiclosedness it is convenient to introduce the invariant quantity

$$\tilde{E}=e_1{}^\alpha\nabla_\alpha r, \tag{3}$$

where e_1 is the unit vector of the external normal to the boundary of matter. The sign of $\tilde{E}$ characterizes the openness ($\tilde{E} > 0$) or semiclosedness ($\tilde{E} < 0$) of the system when it is in an $R_\pm$-region.

3. DERIVATION OF THE EQUATIONS OF MOTION OF A CHARGED RADIATING SHELL

In this section we use a method similar to the one used in[9,11] to derive the equations of motion of a charged radiating shell. In the case considered by us it is convenient to use the tetrad formalism. There exists a natural choice of tetrads (moving ortho-frames) consisting in the following. The boundary of matter or the surface of the shell forms in the total spacetime a hypersurface Σ_0, separating spacetime into an internal and an external parts. In the coordinates (r, θ, φ, z) the hypersurface is described by the equation (2). Together with Σ_0 we consider the hypersurfaces Σ_C defined by the equations

$$r-R(z)=c. \tag{4}$$

On Σ_C one can choose uniquely an orthonormal system consisting of three vectors, e_2, e_3 and e_4, directing the first two along the θ and φ coordinate lines and the last e_4 into the future, determined by orthogonality and normalization. The constructed triad of vectors is completed to form an orthonormal frame by e_1, the external normal to the surface Σ_C. In the σ-coordinates (r, θ, φ, z) these tetrads are given by the following formulas

$$\begin{gathered}\mathbf{e}_1=\sigma(f+2pR')^{-1/2}\left[(f+pR')\frac{\partial}{\partial r}-p\frac{\partial}{\partial z}\right],\\ \mathbf{e}_2=\frac{1}{r}\frac{\partial}{\partial\theta},\qquad \mathbf{e}_3=\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi},\\ \mathbf{e}_4=(f+2pR')^{-1/2}\left(R'\frac{\partial}{\partial r}+\frac{\partial}{\partial z}\right).\end{gathered} \tag{5}$$

The spacetime metric induces on Σ_C two quadratic forms. The first of these is

$$dl^2=-\rho_c{}^2(\tau)\,d\sigma^2+d\tau^2=b_{ij}dx^idx^j, \tag{6}$$

and characterizes the intrinsic geometry of the surface, through b_{ij}, where

$$\rho_c(\tau)=R(z)-c, \tag{7}$$

and τ is the proper time on Σ_C, related to the z coordinate by

$$\dot{z}=(f_c+2pR')^{-1/2}. \tag{8}$$

The second quadratic form related to Σ_C is the external curvature form Ω, the tetrad components of which are given by

$$\Omega_{XY}=-e_X{}^\alpha e_Y{}^\beta\nabla_\alpha e_{1\beta}. \tag{9}$$

Thus, on each hypersurface Σ_C which partitions spacetime into an exterior and an interior region there appear two sets of quadratic forms b and Ω, induced on it respectively by the metric of the exterior (b^+, Ω^+) and the interior (b^-, Ω^-) parts of spacetime. It follows from the Einstein equations that the condition $b_{ij}^- = b_{ij}^+$ must hold. However, in the general case the quantities Ω are not necessarily continuous on Σ_C. Following Israel[9], we introduce the notation

$$k_{XY}=\Omega_{XY}^+-\Omega_{XY}^-,\quad K_{XY}=\frac{1}{2}(\Omega_{XY}^++\Omega_{XY}^-),\quad S_{XY}=-\frac{1}{8\pi\varkappa}(k_{XY}-\eta_{XY}k_Z{}^Z). \tag{10}$$

If one uses the Einstein equations one can obtain the following relations between $\Omega^\pm$ and the energy-momentum tensor of matter near Σ_0 (cf.[9]):

$${}^{(3)}R+[K_{XY}K^{XY}-(K_X{}^X)^2+4\pi^2\varkappa^2(S_{XY}S^{XY}-{}^1\!/_2(S_X{}^X)^2)]=-8\pi\varkappa(T_{11}{}^+-T_{11}{}^-), \tag{11}$$

$$K^{XY}S_{XY}=T_{11}{}^+-T_{11}{}^-, \tag{12}$$

$$K_X{}^Y{}_{;\,Y}-K_Y{}^Y{}_{;\,X}=-4\pi\varkappa(T_{1X}{}^++T_{1X}{}^-), \tag{13}$$

$$S_X{}^Y{}_{;\,Y}=T_{1X}{}^+-T_{1X}{}^-, \tag{14}$$

where $T_{AB}^\pm$ are the limits of the tetrad components of the energy-momentum tensor as Σ_0 is approached from the outside and inside and ${}^{(3)}R$ is the scalar curvature computed from the intrinsic metric of the hypersurface Σ_0. The tensor S plays the role of a surface density tensor of energy-momentum on the surface Σ_0 and it was shown in[9] that in normal coordinates $ds^2 = -dq^2 + b_{ij}dx^idx^j$ where the surface Σ_0 has the equation $q = 0$, the tensor S has the following relation to the volume energy-momentum tensor:

$$S_{XY}=\lim_{\varepsilon\to0}\int_{-\varepsilon}^{\varepsilon}T_{XY}\,dq. \tag{15}$$

In the case when the energy-momentum tensor is finite everywhere $(S = 0)$, Eqs. (11)—(14) transform into the usual matching conditions at the separation of two media[23,24] expressed in terms of the geometric characteristics of the separating surface.

In the case in which we are interested of the motion of a charged radiating massive shell, the particles which form the shell move along the radius. Let the total (internal) mass of the shell be $M(\tau)$. Then the relations (15) allows one to conclude that

$$S_{XY}=\frac{M(\tau)}{4\pi\rho^2}\delta_X{}^4\delta_Y{}^4 \tag{16}$$

and therefore

$$k_{XY}=-\frac{\varkappa M(\tau)}{\rho^2}\delta_{XY}. \tag{17}$$

We shall consider that the matter inside the shell is absent and therefore spacetime is flat there.

In order to make use of the relation (11)—(14) for a derivation of the equations of motion of the shell it is necessary to obtain the quantities $\Omega_{XY}^\pm$ making use of the expressions (5). On the basis of the definition (9) one can deduce the following values for the nonvanishing tetrad components of the external curvature tensor[2)]:

$$\Omega_{22}{}^+=\Omega_{33}{}^+=\frac{\sigma(f_0+pR')}{r\sqrt{f_0+2pR'}}$$

$$\Omega_{44}{}^{+}=\sigma\left[\frac{\partial}{\partial z}\left(\frac{p}{\sqrt{f+2pR'}}\right)-\frac{\partial}{\partial r}\left(\frac{pR'+f}{\sqrt{f+2pR'}}\right)\right]_{\Sigma_0}. \qquad (18)$$

The quantities Ω^{-} are obtained from these relations by setting f = 1. Substituting the expressions obtained here into the equations (11)—(14) and taking into account (16) one can obtain a differential equation for the function R(z). It turns out to be more convenient to derive an equation which describes the motion of the shell expressed only in terms of internal characteristics of the shell. For this it is necessary to express R' in terms of $\dot\rho$ and to substitute the corresponding expression into Eq. (18). Making use of the relations (7) and (8) we have

$$R'=\dot\rho(p\dot\rho+E), \quad \dot z=(p\dot\rho+E)^{-1}, \quad E=\sigma\tilde E, \qquad (19)$$

where E is defined by Eq. (3) and a direct calculation shows that $|\tilde E| = (\dot\rho^2+f)^{1/2}$.

The relations (19) allow one to write the following expressions for the nonvanishing tetrad components of the tensor $\Omega^{\pm}_{XY}$:

$$\Omega_{22}{}^{+}=\Omega_{33}{}^{+}=\frac{\sigma E}{\rho}, \quad \Omega_{44}{}^{+}=-\frac{\sigma}{E}\left[\ddot\rho+\frac{1}{2}\frac{\partial f}{\partial\rho}+\frac{p}{2(p\dot\rho+E)^2}\frac{\partial f}{\partial z}\right]_{\Sigma_0};$$
$$\Omega_{22}{}^{-}=\Omega_{33}{}^{-}=\frac{\sqrt{\dot\rho^2+1}}{\rho}, \quad \Omega_{44}{}^{-}=-\frac{\ddot\rho}{\sqrt{\dot\rho^2+1}}. \qquad (20)$$

Let us compute from Eqs. (10) the corresponding quantities K, k and S, and substituting these quantities into the equations (11)—(14) we take into account that $T^{-}_{AB}=0$. Then the relation (13) for X = 2, 3 yields $T^{+}_{12}=T^{+}_{13}=0$, Eq. (14) is verified identically for X = 2, 3 and for X = 1 Eqs. (13) and (14) lead to

$$\frac{d}{d\tau}(\rho K_{22})+\dot\rho K_{44}=-2\pi\varkappa\rho T_{14}{}^{+}, \qquad (21)$$

$$T_{14}{}^{+}=\frac{1}{\rho^2}\frac{d}{d\tau}(\rho^2 S_{44})=\frac{1}{4\pi\rho^2}\dot M. \qquad (22)$$

Substituting the corresponding values for K_{22} and K_{44} after some manipulations we obtain from these two relations

$$(p\dot\rho+E)^{-2}\left(\frac{\partial f}{\partial z}\right)_{\Sigma_0}=-\frac{2\varkappa\sigma\dot M}{\rho}. \qquad (23)$$

Making use now of the equality $k_{22}=-(\kappa M/\rho^2)$ with the appropriate value of k_{22} we have

$$\tilde E=\sigma E=\sqrt{\dot\rho^2+1}-\frac{\varkappa M}{\rho}. \qquad (24)$$

One can verify that the remaining equations are simple consequences of these relations.

Remembering that in the case which interests us f has the form (1′) one can rewrite (23) in the form

$$\dot m=\sigma(p\dot\rho+E)\dot M+\frac{e\dot e}{\rho}. \qquad (25)$$

Since $|\tilde E| = (\dot\rho^2+f_0)^{1/2}$, we have, solving (24) with respect to m

$$m=M\sqrt{\dot\rho^2+1}-\frac{\varkappa M^2-e^2}{2\rho}. \qquad (26)$$

If we now differentiate (26) with respect to τ and eliminate $\dot m$ with the help of (25), we finally obtain the required equation of motion of the radiating charged shell

$$\frac{\ddot\rho}{\sqrt{1+\dot\rho^2}}+\frac{\varkappa M^2-e^2}{2M\rho^2}-p\sigma\frac{\dot M}{M}=0. \qquad (27)$$

We recall that $p\sigma$ equals 1 for emission and equals −1 for absorption of radiation. Equations (24), (26), and (27) allow one to investigate the motion of a radiating shell and determine the relation between external and internal parameters. We note that these equations contain only the intrinsic characteristics of the shell, such as M, e, ρ, τ. For the description of the motion of the shell in the variables (r, z) one can make use of the relations (19).

The relations (24) and (26) obtained above allows us, in particular to find out for what relation between the parameters the system is open, and for which it is semiclosed. For this it suffices to determine the value of $\tilde E_0$ at the instant of time-symmetry ($\dot\rho = 0$). Making use of (24) and (26) we obtain for $\tilde E_0$ the following expression:

$$\tilde E_0=\frac{2\varkappa Mm-\varkappa M^2-e^2}{\varkappa M^2-e^2}. \qquad (28)$$

In the sequel we restrict our attention to systems for which $\kappa M^2 > e^2$. In this case the relation (28) proves that $\tilde E_0 > 0$ (the universe is open) for $m \le M < m + (m^2 - e^2/\kappa)^{1/2}$ and $\tilde E_0 < 0$ (the universe is semiclosed) for $M > m + (m^2 - e^2/\kappa)^{1/2}$. In particular, a semiclosed neutral universe has a gravitational mass defect which exceeds half of the total (internal) mass of the system.

4. PULSEWISE EMITTING SHELLS

It is in general difficult to solve the nonlinear differential equation (27). In order to study the properties of radiating shells we proceed in the following manner. We assume that up to a certain time the shell moves in its own field without radiating and has the parameters M_0, e_0 and m_0. Then for a short interval of proper time it radiates or absorbs part of its energy and charge acquiring the parameters M_1, e_1, and m_1. We are interested in the relation between the parameters before and after this process. In order to clarify this relation we note that (27) can be rewritten in the form of Lagrange equations

$$\partial\mathscr{L}/\partial\dot\rho=\pi, \qquad d\pi/d\tau=\partial\mathscr{L}/\partial\rho, \qquad (29)$$

corresponding to the following Lagrangian

$$\mathscr{L}=\dot\rho\ln(\dot\rho+\sqrt{\dot\rho^2+1})-p\sigma\dot\rho\ln M-\sqrt{\dot\rho^2+1}+(\varkappa M^2-e^2)/2\rho M. \qquad (30)$$

This fact can be proved by simple substitution of (30) into (29). For a finite change of M and e the quantity $\partial\mathscr{L}/\partial\rho$ undergoes a bounded change. Therefore in integrating (29) along an infinitesimal proper time interval $(\tau_0-\epsilon, \tau_0+\epsilon)$ containing the instant of emission τ_0, and letting ϵ go to zero we obtain the following law of momentum conservation:

$$\pi_0=\pi_1, \qquad (31)$$

where π_0 and π_1 are the corresponding momenta for $\tau_0 - 0$ and $\tau_0 + 0$. Differentiating (30) with respect to $\dot\rho$ we obtain the following expression for π:

$$\pi=\ln(\dot\rho+\sqrt{\dot\rho^2+1})-p\sigma\ln M. \qquad (32)$$

After a simple transformation (31) can be rewritten in the form[3]

$$M_0(\sqrt{\dot\rho_0^2+1}-p\sigma\dot\rho_0)=M_1(\sqrt{\dot\rho_1^2+1}-p\sigma\dot\rho_1). \qquad (33)$$

To write (33) in a more compact form, we introduce in place of $\dot\rho$ the quantity η, related to $\dot\rho$ by means of the following relation: $\dot\rho = \sinh\eta$. Then the condition (33) can be rewritten in the form

$$M_0e^{-p\sigma\eta_0}=M_1e^{-p\sigma\eta_1}. \qquad (34)$$

It is easy to verify that if the system moves without radiating ($\dot{M} = 0$) (27) admits a constant of the motion coinciding with (26), which we rewrite in the following form

$$\frac{1}{2\rho_a} = (\varkappa M_a^2 - e_a^2)^{-1} (M_a \operatorname{ch} \eta_a - m_a), \tag{35}$$

($a = 0$ corresponds to the motion up to the radiation process (or absorption process) and $a = 1$ corresponds to the subsequent motion). At the instant of emission of absorption $\dot{\rho}$ undergoes a jump and ρ is continuous. Therefore at that instant the following condition must be satisfied

$$\frac{M_0 \operatorname{ch} \eta_0 - m_0}{\varkappa M_0^2 - e_0^2} = \frac{M_1 \operatorname{ch} \eta_1 - m_1}{\varkappa M_1^2 - e_1^2}. \tag{36}$$

The relations (34)—(36) establish a connection between the parameters of the system before and after the emission or absorption process. They allow one to determine the change in external mass ($m_1 - m_0$) after the emission of the energy ($M_0 - M_1$) at different stages of the motion of the shell. In the following sections we analyze various situations which can arise under these circumstances.

5. NEUTRAL RADIATING SYSTEMS. THE POSSIBILITY OF TRANSITION OF OPEN UNIVERSES INTO SEMICLOSED UNIVERSES

In the case of neutral ($e = 0$) shells which emit or absorb radiation the analysis of the relations derived in the preceding sections is not complicated. Assume that up to the instant of emission the system moved in its own field and has the parameters m_0 and M_0. The shell initially expands from a point ($\eta_0 = +\infty$) (anticollapse) gets into an R_+- or R_--region and then after the instant of maximal expansion ($\eta_0 = 0$) begins to contract and, getting into a T_--region, it collapses to a point ($\eta_0 \to -\infty$). The possible trajectories for open and semiclosed shells are represented in Fig. 1 respectively by the lines 1 and 2.

If the process of emission or absorption occurs at a time when the velocity of the shell is $\dot{\rho}_0 = \sinh \eta_0$ and as a result of this process the mass of the system becomes equal to M_1, Eqs. (34)—(36) allow one to determine the external mass:

$$m_1 = {}^1\!/_2 M_0 (1-\alpha^2) e^{-p\sigma\eta_0} + m_0 \alpha^2, \tag{37}$$

where $\alpha = M_1/M_0$. We discuss separately the cases of emission and absorption of radiation.

A. Neutral emitting shell ($p\sigma = 1$, $M_0 > M_1$, $\alpha < 1$). For fixed parameters M_0, M_1 and m_0 the minimally possible value of m_1 turns out to be equal to

$$m_1 = \alpha^2 m_0 \tag{38}$$

for emission occurring at the beginning of the anticollapse ($\eta_0 = +\infty$). If the emission occurs at later stages, then m_1 will be larger. If the emission occurs at the instant of maximal expansion ($\eta_0 = 0$) one can conclude from Eq. (37) that the mass of an open system ($\frac{1}{2}M_0 < m_0$) diminishes whereas the mass of a semiclosed system ($m_0 < \frac{1}{2}M_0$) increases. In both cases the mass m_1 tends to the value $\frac{1}{2}M_0$ as $\alpha \to 0$. If the emission occurs in the stage of collapse ($\eta_0 \to -\infty$) m always increases.

These results admit the following interpretation. For emission in a T_+-region the radiation reaction decreases the velocity of the motion and hence the kinetic energy. In addition, part of the internal mass is radiated away. All this leads to a decrease of m. In the case when the emission occurs in a T_--region the radiation reaction increases the speed of motion of the shell and consequently its kinetic energy. If this increase turns out to be larger than the emitted energy, m increases. In addition, one should keep in mind that as the internal mass decreases so does the gravitational mass defect, and as a consequence the external mass of the system can increase. This is the effect which occurs when the emitting sphere is in an R_- region, where the gravitational mass defect is very large.

The relations listed above allow us to show that an open system ($m_0 > \frac{1}{2}M_0$) in the anticollapse stage can go over into a semi-closed state ($m_1 < \frac{1}{2}M_1$) on account of emission of energy. In order to prove this it suffices to rewrite the relation (38) in the form

$$m_1 = \frac{M_1 m_0}{M_0^2} M_1. \tag{39}$$

If after the act of emission the system has a mass $M_1 < M_0^2/2m_0$, (39) shows that the appropriate universe is semi-closed.

B. Neutral absorbing shell ($p\sigma = -1$, $M_1 > M_0$, $\alpha > 1$). Analyzing the relation (37) it is easy to establish that if radiation is absorbed during early stages of anticollapse ($\eta_0 \to +\infty$), even for small values of the absorbed energy $M_1 - M_0$ the universe may close up ($m_1 = 0$). In this case however, there will be no external space which is flat at infinity. If the absorption occurs at the instant of maximal expansion ($\eta_0 = 0$) the mass of an open system increases, whereas the mass of a semiclosed one decreases. Finally, for collapse ($\eta_0 \to -\infty$) $m_1 = \alpha^2 m_0$ and any absorption of energy increases the mass of the shell.

6. CHARGED RADIATING SYSTEMS. THE POSSIBILITY OF FORMATION OF FRIEDMONS FROM NONFRIEDMONIC STATES

We now consider the case when the shell has a change e. Up to the emission process the system moves in its own field with constant m_0 and e_0, having internal mass M_0 ($M_0 \geq e_0 \kappa^{-1/2}$). In this case the possible trajectories for the open and semi-closed shell are represented in Fig. 2 by lines 1 and 2, respectively. We call attention to the fact that an observer situated at Euclidean infinity will obtain information about all stages of the initial expansion of the open shell, whereas for the semiclosed universe he will be able to observe only phenomena occurring in a T_+-region. In the general case the analysis of the relations (34)—(36) is difficult, therefore we limit our attention to the discussion of two limiting situations: the case when the charged system emits energy, but does not lose charge, and the case when the system emits charge with an unchanged internal mass. The second situation is physically reasonable for the description of processes of emission by a system of charged ultrarelativistic particles the mass μ of which is small compared to their charge ϵ: $\mu \ll \epsilon\kappa^{-1/2}$. For realistic charged particles this relation is valid.

A. Charged radiating shell with constant charge ($p\sigma = 1$, $M_0 > M_1$, $\alpha < 1$, $e_1 = e_0 = e$). In this case one can derive from the relations (34)—(36)

$$m_1 = \frac{M_0(1-\alpha^2)}{2(1-\beta^2)} (\beta^2 e^{\eta_0} + e^{-\eta_0}) + m_0 \frac{\alpha^2 - \beta^2}{1-\beta^2}, \tag{40}$$

where $\beta = e/\kappa^{1/2} M_0$. For fixed values of e, m_0, M_0 and

M_1 the minimal value of m_1 is attained for $\eta_0 = -\ln\beta$ and equals

$$m_1 = \left(m_0 - \frac{e}{\sqrt{\varkappa}}\right)\frac{\alpha^2-\beta^2}{1-\beta^2} + \frac{e}{\sqrt{\varkappa}}. \qquad (41)$$

It is easy to see that if the charge of the shell is small (e.g. $e\kappa^{-1/2} < (1/10)M_0(1-\beta^2)$), then after radiating away a sufficiently large fraction of its energy ($M_1 \le M_0^2(1-\beta^2)4m_0^{-1}$) the initially open universe converts into a semiclosed one[4)]. Thus, in the presence of a small charge an anticollapsing open system can also convert into a semiclosed one after radiating. The consideration of possible types of radiation during the early stages of motion of the shell is carried out in the same manner as in the preceding section.

B. Shells that emit charge ($p\sigma = 1$, $M_0 = M_1$). In spite of the fact that internal mass M remains unchanged in this case, the external mass m will undergo a change. This change of m is related to a lowering of the electrostatic energy and, making use of (34) and (35) it is easy to show that

$$m_1 = m_0 - (e_0^2 - e_1^2)/2\rho. \qquad (42)$$

We make use of this equation to explain the possibility of appearance of friedmon states[6-8] as the final result of such a radiation process. In the preceding section we have already shown that a charged open universe may go over into a semi-closed universe. Let us therefore consider a semi-closed charged universe which emits charge during the anticollapse stage. As the charge e_1 decreases so does the mass m_1. It turns out that m_1 is minimal for fixed values of e_0 and e_1 if the emission takes place at $\rho = r_-$ (the minimal value of ρ in the T_+-region). Since $r_- = \kappa(m_0 - (m_0^2 - e_0^2/\kappa)^{1/2})$ we obtain for the minimal value of m_1 the expression

$$m_1 = m_0\left[1 - \frac{\delta_0^2 - \delta_1^2}{2(1-\sqrt{1-\delta_0^2})}\right], \qquad (43)$$

where $\delta_a = e_a/\kappa^{1/2}m_0$. For friedmon states the characteristic equation is $m_1 = e_1\kappa^{-1/2}$[7]. From the relation (43) it follows that this equality will be satisfied after the emission, if as a result of the radiation process e_1 becomes $e_1 = \kappa^{1/2}(m_0 - (m_0^2 - e_0^2/\kappa)^{1/2})$. One can also see that an initially semiclosed universe remains semiclosed. Thus, summarizing what was said above, we can assert that it is in principle possible that anticollapsing charged systems go over into friedmon states on account of emission of charge and energy.

This fact is of particular interest in relation to the following. The electric potential φ produced by the charge e_0 on the surface of a sphere of radius ρ equals $\varphi = e_0/\rho$. In a T-region, where the minimal value of ρ is $\rho = r_- = \kappa(m_0 - (m_0^2 - e_0^2/\kappa)^{1/2})$, a maximal value is attained for the potential equal to

$$\varphi_{max} = \frac{\delta_0}{\sqrt{\varkappa}(1-\sqrt{1-\delta_0^2})}. \qquad (44)$$

Since $0 \le \delta_0 \le 1$, it is easy to see that $\varphi_{\max} \ge \kappa^{-1/2}$. If the system has a sufficiently large charge e_0 then in a T_+-region the vacuum is unstable with respect to pair production and polarization processes and therefore such an anticollapsing charged system will emit energy and charge. Without a detailed analysis of the dynamical description of these processes one cannot determine the values of the final parameters M_1, e_1 and m_1 of the system. However, in principle, it is possible to expect in some cases that friedmon states may appear as the final states.

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1)Following the terminology proposed by Novikov in [22] such regions of spacetime are called T-regions. In the general case they are defined invariantly as sets of points were $N = g^{\alpha\beta}\nabla_\alpha r \nabla_\beta r > 0$. The points where $N < 0$ form an R-region. The boundaries between T- and R-regions are defined by the equation $N = 0$.

2)The calculations simplify considerably it one notes that $\Omega_{XY} = -\Gamma_{1XY}$, where Γ_{ABC} are the Ricci rotation coefficients for the tetrad e_A. One can therefore write $\Gamma_{ABC} = \Gamma_{A[BC]} + \Gamma_{B[CA]} - \Gamma_{C[AB]}$, where $\Gamma_{A[BC]} = \frac{1}{2}e^\alpha_B e^\beta_C(e_{A\alpha,\beta} - e_{A\beta,\alpha})$ (cf. also [25]).

3)We note that for $\dot\rho/c \ll 1$ in the first approximation (33) yields $M_0\dot\rho_0 = M_1\dot\rho_1 + p\sigma(M_0 - M_1)$. This equation can be simply interpreted as the equality of the momentum before the emission $(M_0\dot\rho)$ to the sum of the momentum after the emission $(M_1\dot\rho_1)$ and the momentum carried off by radiation $p\sigma(M_0 - M_1)$; $p\sigma = -1$ corresponds to radiation incident on the system, $p\sigma = 1$ to emission of radiation.

4)In order to prove this fact it suffices to make use of the expression (28) for $\tilde{E}_0$ and to verify that after the emission of radiation $\tilde{E}_0$ becomes negative.

*For details on Penrose diagrams cf., e.g., S. W. Hawking and G. F. R. Ellis, The Large-Scale Structure of Space-Time, Cambridge U. Press, 1973 (Translator's note).

1 C. W. Misner, Phys. Rev. 137B, 1360 (1965).
2 P. C. Vaidya, Astrophys. J. 144, 943 (1966).
3 O. Klein, in "Werner Heisenberg und die Physik unserer Zeit," Vieweg, Braunschweig, 1961, p. 58.
4 Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. 42, 1667 (1962) [Sov. Phys.-JETP 14, 1158 (1962)].
5 I. D. Novikov, Vestnik MGU, ser. 3, No. 6, 61 (1962).
6 M. A. Markov, Preprint D2-4534, JINR, Dubna 1969.
7 M. A. Markov and V. P. Frolov, Teor. Matem. Fiz. 3, 1 (1970).
8 M. A. Markov and V. P. Frolov, Teor. Matem. Fiz. 13, 41 (1972).
9 W. Israel, Nuovo Cimento 44B, 1 (1966); 48B, 463 (1967).
10 A. Papapetrou and A. Hamoui, Ann. Inst. H. Poincare, 9, 179 (1968).
11 V. de la Cruz and W. Israel, Nuovo Cimento 51A, 744 (1967).
12 W. Israel, Proc. Roy. Soc. A248, 404 (1968).
13 P. C. Vaidya, Nature 171, 260 (1953).
14 P. C. Vaidya, Proc. Ind. Acad. Sci., A33, 264 (1951).
15 P. C. Vaidya, Phys. Rev., 83, 10 (1951).
16 J. Plebanski and J. Stachel, J. Math. Phys. 9, 269 (1968).
17 W. B. Bonnor and P. C. Vaidya, Gen. Rel. Grav. 1, 127 (1970).
18 M. Kruskal, Phys. Rev. 119, 1743 (1960).
19 J. C. Graves and D. R. Brill, Phys. Rev. 120, 1507 (1960).
20 R. Penrose, "Structure of Spacetime," in Batelle Rencontres 1967, ed. by C. M. de Witt and J. A. Wheeler, N. Y. 1968 (Russ. Transl., 1972).
21 M. Walker, J. Math. Phys. 11, 2280 (1970).
22 I. D. Novikov, Communication of the Shternberg State Astr. Observatory, No. 132, 1964, p. 3 and 43.
23 J. L. Synge, Relativity, the General Theory, North-Holland, 1960 [Russ. Transl. 1963].
24 S. O'Brien and J. L. Synge, Comm. Dublin Inst. Adv. St. Ser. A No. 9, 1952.
25 V. P. Frolov, Preprint FIAN, 1973.

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