The theory of inhomogeneous tunnel junctions

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(Submitted July 22, 1973)
Zh. Eksp. Teor. Fiz. **66**, 1051–1066 (March 1974)

A microscopic theory of tunnel phenomena is developed for inhomogeneous barriers. "Point" junctions of the tunnel type, or a set of such junctions, are considered in the theory as a limiting case. It is shown that a tunnel current from two apertures oscillates as a function of the voltage, with a period that is inversely proportional to the distance between the apertures. This is due to interference phenomena in the tunneling. The oscillations are a macroscopic phenomenon and possess a large period. Other forms of inhomogeneous barriers in both the normal and the superconducting states are studied. Components of the quasiparticle and pair current (Josephson current) are calculated.

1. INTRODUCTION

Tunneling in metal-insulator-metal systems displays interesting features both in the normal[1] 1) and in the superconducting states.[2] Consideration of such effects is usually based on the method of the tunnel Hamiltonian,[3] which describes in first order perturbation theory the transitions between two subsystems ($N \rightleftarrows N$, $N \rightleftarrows S$ or $S \rightleftarrows S$). Recently, after the creation of the microscopic theory of the stationary Josephson current,[2,4-6] interest has developed in systems of "weak coupling," which are different from the classical plane structure S – I – S (I is an insulator homogeneous over the thickness of the layer). Systems of the type S – N – S (N is a layer of normal metal)[7-10], S – C – S bridge structures[11], plane S – Σ – S junctions[12], and so on were investigated. The purpose of the present paper is to study the tunnel mechanism of current flow between two metals separated by an inhomogeneous barrier whose transparency changes from point to point on the transition surface. The treatment is consistently microscopic and allows us to study the effects connected with the inhomogeneities in both normal and superconducting structures.

The inhomogeneity of the barrier may be due, for example, to local thinning out or to defects in the structure of the insulating layer, impurities, surface roughness, and other imperfections, which are unavoidable in real junctions,[13] or which are created deliberately by some method or other.[14] The theory considered below also includes the case of a "point" junction of the tunnel type (a tunnel contact of small area) as a limiting case. Experiments (see[15]) in which observation of oscillations of the tunnel current in systems containing organic insulating coatings was reported, served as the direct stimulus for these investigations. The oscillations of the current (and of its derivatives dI/dV, d^2I/dV^2) took place as a function of the voltage on the junction and were originally connected with the features of the spectra of the organic impurities. A hypothesis was put forth by I. K. Yanson,2) according to which such behavior can be explained by the multiplicity of the junction (the presence of a network of regions of increased transparency), which can lead to interference phenomena in the tunneling of electron waves in the crystal. It will be seen below that this point of view finds its reflection in the formalism of the theory that we have developed. At the same time, the absence of detailed information on the structure of the barrier in[15] did not allow us to make a unique identification of the observed features.

The considered model of the tunnel junction consists of two identical semiinfinite metal half-spaces: $-\infty < z < 0$ and $0 < z < \infty$ (regions 1 and 2) separated by a potential barrier of the form

$$V(\mathbf{r}) = V_0 f(\rho)\delta(z), \qquad (1.1)$$

where $\boldsymbol{\rho}$ is a two-dimensional vector in the plane $z = 0$, $f(\boldsymbol{\rho})$ is a dimensionless function larger than or of the order of unity, normalized, for example, in such a way that $|f|_{min} = 1$ (for a homogeneous barrier, $f = 1$). The δ-function character of the barrier assumed in (1.1) is not an essential limitation—thus, for example, in the case of a superconducting state the thickness of the insulator, $d \sim 20 - 30$ Å, is always small in comparison with the correlation length of superconductivity theory, $\xi \sim 10^{-4}$ cm, and therefore any barrier is virtually a δ-function barrier. For normal metals, the assumption (1.1) is a more serious limitation and does not convey certain features of the tunneling that occur in real structures, for example, the sharp directivity of the effect. Nevertheless, this is not of any essential significance for the phenomena discussed below.

In what follows, V_0 will be assumed to be a large quantity, which guarantees smallness of the coefficient of transmission of the electron through the barrier ($D \sim v_F^2/V_0^2$). The case of a "point" junction can be obtained if we assume the function $f(\boldsymbol{\rho})$ to be equal to unity in the limits of some region $\boldsymbol{\rho} \in S$ and to infinity outside of S ($f = \infty$ for $\boldsymbol{\rho} \overline{\in} S$), or if $f(\boldsymbol{\rho})$ is specified in the form of a Gaussian function

$$f(\rho) = e^{\rho^2/a^2} \qquad (1.2)$$

with sufficiently small a. In Sec. 2 we shall show that the wave functions of the problem considered can be found by expansion of the Schrödinger equation with a potential of the form (1.1) in a series in powers of the reciprocal of V_0 for an arbitrary function $f(\boldsymbol{\rho})$. This allows us to find the tunnel current of an arbitrary inhomogeneous barrier.

A general method is set forth in Sec. 3 for calculation of the tunnel current, based on the Green's function technique (a preliminary communication on this method for the case of homogeneous barriers has been published previously[16]). In the case of normal metals, the present method gives results equivalent to those obtained by the method of Bethe and Sommerfeld[17] with account of the dependence of the transmission coefficient D on the components p_x, p_y, p_z of the momentum of the incident electron that is specific for the problem at hand (the dispersion law is assumed to be quadratic and isotropic).

Interference effects are investigated for normal metals, effects that are due to inhomogeneities and are connected with effects of the type of Friedel oscillations for degenerate Fermi systems.[18] Finally, Secs. 4 and 5 are devoted to the study of similar effects in the superconducting state and to a calculation of the Josephson current for inhomogeneous barriers.

2. WAVE FUNCTIONS OF AN INHOMOGENEOUS BARRIER

The idea of the calculation of tunnel phenomena in a potential field of the type (1.1) is illustrated by the method given below for finding the wave functions. We write the Schrödinger equation in the form

$$\left[-\frac{\nabla^2}{2m}+V_0 f(\rho)\delta(z)\right]\psi=E\psi \tag{2.1}$$

and seek the wave functions ψ_p^1 and ψ_p^2 that correspond to particles with momentum p incident on the barrier from the left and right, respectively ($p_z > 0$ and $p_z < 0$). In first order in $1/V_0$,

$$\psi_p{}^1(\mathbf{r})=\begin{cases} e^{ikz}e^{i\varkappa\rho}-e^{-ikz}e^{i\varkappa\rho}+\dfrac{1}{V_0}\varphi_1^{(-)}(\mathbf{r}), & z<0\\ \dfrac{1}{V_0}\varphi_1^{(+)}(\mathbf{r}), & z>0\end{cases}, \tag{2.2}$$

where $\varphi_1^{(\pm)}$ are functions that are independent of V_0, $\mathbf{p} = (\boldsymbol{\kappa}, k)$. From the boundary condition at the point z = 0, which follows from (2.1).

$$\left(\frac{\partial\psi}{\partial z}\right)_{z=+0}-\left(\frac{\partial\psi}{\partial z}\right)_{z=-0}=2mV_0 f(\rho)\psi(\mathbf{r})_{z=0}, \tag{2.3}$$

we conclude that

$$\varphi_1^{(+)}(\rho, z=0)=\varphi_1^{(-)}(\rho, z=0)=-\frac{ike^{i\varkappa\rho}}{mf(\rho)}, \tag{2.4}$$

whence we get, with (2.1),

$$\varphi_1^{(\pm)}(r)=-\frac{ik}{(2\pi)^2 m}\int d\varkappa'\varphi(\varkappa-\varkappa')e^{i\varkappa'\rho}e^{\pm ik'z}. \tag{2.5}$$

Here we have introduced the function

$$\varphi(\varkappa-\varkappa')=\int\frac{e^{i(\varkappa-\varkappa')\rho}}{f(\rho)}d\rho \tag{2.6}$$

and

$$k'=(\varkappa^2+k^2-\varkappa'^2)^{1/2},\quad 0<\arg k'<\pi/2.$$

By the use of (2.2), (2.5), we easily find the transmission coefficient of the barrier in the corresponding order in V_0^{-1}. Denoting the transmission coefficient by $D_1(p)$ for particles incident on the barrier from the left, and by $D_2(p)$ for particles incident on the right, and introducing the additional term $-\frac{1}{2}eV$ sign z into the Hamiltonian to take into account the difference in the potentials V applied between regions 1 and 2 (Fig. 1), we get

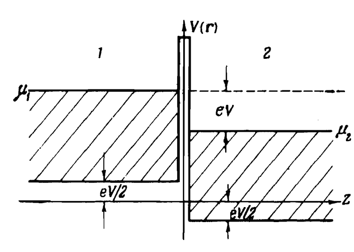

FIG. 1

$$D_{1,2}(\mathbf{p})=\frac{|k|}{(2\pi)^2(mV_0)^2S}\int d\varkappa'|\varphi(\varkappa-\varkappa')|^2\operatorname{Re}k'_\pm, \tag{2.7}$$

where

$$k_\pm'=(\varkappa^2+k^2-\varkappa'^2\pm 2meV)^{1/2}. \tag{2.8}$$

With the help of (2.7), we find the tunnel current between the metals as the difference $I = I_1 - I_2$, where

$$I_1=\int_0^\infty\frac{dk}{\pi}\int\frac{Sd\varkappa}{(2\pi)^2}\frac{ek}{m}n_p(1-n_p{}^+)D_1(\mathbf{p}),$$
$$I_2=\int_{-\infty}^0\frac{dk}{\pi}\int\frac{Sd\varkappa}{(2\pi)^2}\frac{e|k|}{m}n_p(1-n_p{}^-)D_2(\mathbf{p}); \tag{2.9}$$

n_p are the Fermi distribution functions.

$$n_p=\frac{1}{\exp\{(\varepsilon_p-\mu)/T\}+1},\quad n_p{}^\pm=\frac{1}{\exp\{(\varepsilon_p\pm eV-\mu)/T\}+1}. \tag{2.10}$$

For a homogeneous barrier (f = 1), we have $\varphi(\boldsymbol{\kappa}) = (2\pi)^2\delta(\boldsymbol{\kappa})$, which leads, after substitution in (2.7), (2.8), to the expressions

$$D_{1,2}(\mathbf{p})=\frac{|k|k_\pm'}{(mV_0)^2}\qquad k_\pm'=\operatorname{Re}(k^2\pm 2meV)^{1/2}. \tag{2.11}$$

For the case $eV \ll \mu$, $T \ll \mu$, the current is equal to

$$I=\rho_\infty{}^{-1}SV, \tag{2.12}$$

where (see[2]) the conductivity of the homogeneous barrier is

$$\rho_\infty{}^{-1}=2\pi^2V_0{}^2/c^2\mu^2. \tag{2.13}$$

The formula (2.12) is identical with the well-known expression for the tunnel current given in[17].

3. TUNNEL CURRENT IN THE NORMAL STATE

1. We derive a formula for the current in an inhomogeneous junction, using the Green's-function technique. In this section we consider the case of the normal state. A generalization to the case of superconductors is contained in Secs. 4 and 5. We shall assume that the voltage on the barrier is turned on jumpwise at the time t = 0, and calculate the current at times that are much greater than the times of the transient processes that arise on such a turning on. We define the Green's function of the electrons:[19]

$$G(x, x')=-i\operatorname{Sp}\hat\rho T(\bar\psi(x)\bar\psi^+(x')), \tag{3.1}$$

where x = (r, t), the operators $\bar\psi$, $\bar\psi^+$ are the field operators in the Heisenberg representation, and $\hat\rho$ is the density matrix.

$$\hat\rho=e^{-\beta H_0}/\operatorname{Sp}e^{-\beta H_0},\quad \beta=1/T. \tag{3.2}$$

The Hamiltonian of the system $H = H_0 + H_f(t)$, where H_0 is the Hamiltonian of the free electrons in the field of the barrier (1.1):

$$H_0=\int d\mathbf{r}\psi^+(\mathbf{r})\left(\frac{\hat p^2}{2m}+V(\mathbf{r})\right)\psi(\mathbf{r}), \tag{3.3}$$

and $H_f(t)$ is the part connected with the voltage at the barrier:

$$H_f(t)=\int d\mathbf{r}\psi^+(\mathbf{r})W(\mathbf{r})\theta(t)\psi(\mathbf{r}). \tag{3.4}$$

We shall assume that in zeroth order in the transmission, the potential of the electric field is equal to (V is the applied potential difference)

$$W(z)=-\tfrac{1}{2}eV\operatorname{sign}z. \tag{3.5}$$

The Hamiltonian $H_f(t)$ describes the evolution of the system in time and leads to the generation of a current which flows between the metals. We expand the Green's function G(x, x′) in the eigenfunctions of the Hamiltonian H_0, which were defined in Sec. 2:

$$G(x,x')=\sum_{pp',ik}G^{ik}_{pp'}(t,t')\psi_p^i(\mathbf{r})\psi_{p'}^{*k}(\mathbf{r}'),\quad i,k=1,2. \tag{3.6}$$

The current density in the system is

$$j(\mathbf{r})=-\frac{ie}{m}(\hat{p}-\hat{p}')G(x,x')\Big|_{\substack{t'=t+0\\ r'\to r}}. \tag{3.7}$$

We shall calculate the total current in the first nonvanishing order in the transmission. It is convenient to carry out the calculation of the current in the plane of the barrier z = 0, since in this case, it is unnecessary to account for the corrections to W(z) due to the finite value of D. We integrate (3.7) over the coordinates in the plane of the barrier and use Eq. (3.6). In the first nonvanishing order in the transmission, we have for the total current:

$$I=-\frac{ie}{m}\sum_{pp'}I^{12}_{p'p}(0)\,[G^{21}_{pp'}(t,t')-G^{12}_{pp'}(t,t')]_{t'=t+0}, \tag{3.8}$$

where

$$I^{ik}_{p'p}(z)=\int d\rho[\psi^{*k}_{p'}(\mathbf{r})\hat{p}\psi_p^i(\mathbf{r})-\psi_p^i(\mathbf{r})\hat{p}\psi^{*k}_{p'}(\mathbf{r})]. \tag{3.9}$$

We now calculate the necessary Green's function. From the equation of motion for G(x, x′),

$$\left[i\frac{\partial}{\partial t}-\frac{\hat{p}^2}{2m}+\mu-V(\mathbf{r})-W(z)\theta(t)\right]G(x,x')=\delta(x-x'), \tag{3.10}$$

we get, using the expansion (3.6),

$$\left(i\frac{\partial}{\partial t}-\xi_p\right)G^{ik}_{pp'}(t,t')-\theta(t)\sum_{p''m}W^{im}_{pp''}G^{mk}_{p''p'}(t,t')=\delta(t-t')\delta_{ik}\delta_{pp'}, \tag{3.11}$$

where $\xi_p = p^2/2m - \mu$. We shall assume $t' > 0$. For solution of the set (3.11), it is necessary to impose boundary conditions on the function G(t, t′). Two relations follow directly from Eq. (3.11):

$$G(t'+0,t')-G(t'-0,t')=-i,\quad G(+0,t')-G(-0,t')=0, \tag{3.12}$$

and a third condition, similar to that obtained in the book of Kadanoff and Baym, [19] follows from the analytic properties of the function G. We have

$$G^<(i\beta,t')=-G^>(0,t'), \tag{3.13}$$

where

$$G^<(t,t')=G(t,t')\ \text{at}\ t<0,$$
$$G^>(t,t')=G(t,t')\ \text{at}\ t>t'. \tag{3.14}$$

We solve Eq. (3.11), using (3.12) and (3.13) by expansion in powers of $1/V_0$. The function G^{21}_{pp} satisfies the equation

$$\left[i\frac{\partial}{\partial t}-\xi_p+\frac{eV}{2}\theta(t)\right]G^{21}_{pp'}(t,t')=W^{21}_{pp'}G^{(0)11}_{p'}(t,t') \tag{3.15}$$

and is $\sim 1/V_0$ ($G^{(0)}$ is the Green's function of the zeroth-order approximation in $1/V_0$). We write down the solution of the last equation:

$$G^{21}_{pp'}(t',t)=\frac{(n_{p'}-n_p)\,[1-\exp(-i(\xi_p-\xi_{p'}-eV)t')]}{i(\xi_p-\xi_{p'}-eV)}W^{21}_{pp'}. \tag{3.16}$$

Here $W^{ik}_{pp'}$ are the matrix elements of W(z). It is easy to establish the fact that G^{12} is obtained from (3.16) by the substitutions $1 \rightleftarrows 2$ and $eV \to -eV$.

We calculate the current by means of Eq. (3.8), using the expressions obtained for G^{12}, G^{21} and the wave functions of Sec. 2. Carrying out the corresponding calculations, we obtain

$$I=\frac{\pi e}{m^4V_0^2}\mathrm{Re}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{p}'}{(2\pi)^3}kk'(k'\tilde{k}'+k\tilde{k}+2kk')$$
$$\times|\varphi(\varkappa-\varkappa')|^2\delta(\xi_p-\xi_{p'}+eV)(n_p-n_{p'}), \tag{3.17}$$

where, as before, $\mathbf{p} = (\boldsymbol{\kappa}, k)$ and the integration is performed over the region $k > 0$, $k' > 0$. In (3.17), we have introduced the notation

$$\tilde{k}=\sqrt{\varkappa^2+k^2-\varkappa'^2},\quad \tilde{k}'=\sqrt{\varkappa'^2+k'^2-\varkappa^2},\quad \mathrm{Re}\,\tilde{k},\ \tilde{k}'>0.$$

Taking into account that $eV \ll \mu$, we can write down (3.17) in simpler form:

$$I=\frac{4\pi e}{m^4V_0^2}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{p}'}{(2\pi)^3}k^2k'^2\theta(k)\theta(k')$$
$$\times|\varphi(\varkappa-\varkappa')|^2(n_p-n_{p'})\delta(\xi_p-\xi_{p'}+eV). \tag{3.18}$$

It is easy to establish the fact that, accurate to within the approximations made, the expression obtained for the current is equivalent to the formulas obtained at the end of Sec. 2. For example, for a homogeneous barrier $\varphi(\boldsymbol{\kappa}-\boldsymbol{\kappa}') = (2\pi)^2\delta(\boldsymbol{\kappa}-\boldsymbol{\kappa}')$, we get Eq. (2.12) (Ohm's law) as before, with ρ_∞ expressed by means of Eq. (2.13). The above method is much more automatic and enables us to carry out a simple generalization to the case of superconductors.

2. We now investigate the expression (3.18) obtained for the current. Integrating over the angles of the vectors $\mathbf{p}$, $\mathbf{p}'$ and using the explicit expression for the function $\varphi(\boldsymbol{\kappa}-\boldsymbol{\kappa}')$ (2.6), we get

$$I=\frac{2e}{(2\pi)^3m^4V_0^2}\int\frac{d\rho_1}{f(\rho_1)}\int\frac{d\rho_2}{f(\rho_2)}\int_0^\infty p^2dp\int_0^\infty p'^2dp'(n_p-n_{p'})$$
$$\times\frac{1}{\rho^4}\left(\frac{\sin p\rho}{p\rho}-\cos p\rho\right)\left(\frac{\sin p'\rho}{p'\rho}-\cos p'\rho\right)\delta(\xi_p-\xi_{p'}+eV), \tag{3.19}$$

where $\rho = |\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|$. As $V \to 0$, the current has the form

$$I=V/R, \tag{3.20}$$

where the quantity R is the resistance of the inhomogeneous junction in the zeroth approximation and is given by the expression (p_0 is the Fermi momentum)

$$R^{-1}=\frac{2e^2p_0^2}{(2\pi)^3m^2V_0^2}\int\frac{d\rho_1}{f(\rho_1)}\int\frac{d\rho_2}{f(\rho_2)}\frac{1}{\rho^4}\left(\frac{\sin p_0\rho}{p_0\rho}-\cos p_0\rho\right)^2. \tag{3.21}$$

In particular for a homogeneous barrier (f = 1), we then obtain $R = R_\infty = \rho_\infty/S$, where ρ_∞ is determined by expression (2.13) and S is the area of the junction. If the junction is a spot of small radius ($a \ll p_0^{-1}$): $f(\boldsymbol{\rho}) = 1$ for $\rho < a$ and $f(\boldsymbol{\rho}) = \infty$ for $\rho > a$, an elementary calculation according to Eq. (3.21) leads to the expression

$$R=9R_\infty/2(ap_0)^2. \tag{3.22}$$

Consequently, the tunnel current is proportional in the given case not to the first but to the second power of the area of the junction ($S = \pi a^2$): $I \propto S^2$. This is a reflection of the diffraction phenomena for apertures of small diameter, similar to those which take place in optics (see [20]).

We consider further a set of apertures of small radius, located at the points ρ_i (i = 1, 2, ..., n). By virtue of (2.6), we have

$$\varphi(\varkappa)=\sum_{i=1}^n S_ie^{i\varkappa\rho_i} \tag{3.23}$$

where S_i is the area of the i-th aperture. Using the relation for the current (3.18), integrating over the angles, and taking into account that the fundamental contribution to the integral for $eV \ll \mu$ is given by the vicinity of the

Fermi surface ($p \approx p_0$, $p' \approx p_0$), we obtain

$$I = I_0 + I^1_{osc} + I^2_{osc}, \tag{3.24}$$

where I_0 is the sum of contributions of the type (3.20), (3.22) from the individual apertures, and I^1_{osc} and I^2_{osc} are oscillating interference terms that have the form ($v_0 = p_0/m$)

$$I^1_{osc} = \rho_\infty^{-1} \frac{V}{\pi p_0^2} \sum_{i \neq j} S_i S_j \frac{\cos(eV|\rho_i - \rho_j|/v_0)}{|\rho_i - \rho_j|^4} \Psi(X_{ij}), \tag{3.25}$$

$$\Psi(x) = \frac{x}{\operatorname{sh} x}, \quad X_{ij} = \frac{\pi eVT}{p_0 v_0^2}|\rho_i - \rho_j|, \quad |\rho_i - \rho_j| p_0 \gg 1; \tag{3.26}$$

$$I^2_{osc} = \rho_\infty^{-1} \frac{1}{\pi emp_0} \sum_{i \neq j} S_i S_j \frac{\sin(eV|\rho_i - \rho_j|/v_0)}{|\rho_i - \rho_j|^5} \cos 2p_0|\rho_i - \rho_j| \Psi(Y_{ij}),$$
$$Y_{ij} = 2\pi T|\rho_i - \rho_j|/v_0. \tag{3.27}$$

We draw attention to the analogy of the resulting temperature dependences to the dependences that take place in the theory of quantum oscillatory phenomena in normal metals [21].

3. Going over to a discussion of the physical meaning of the results obtained, we note that for a pair of apertures the distance between which is equal to d, Eq. (3.27) describes the oscillations of the current as a function of the separation distance (d) of these apertures, oscillations with period $\Delta d_2 \approx \pi/p_0$. The amplitudes of such oscillations fall off rapidly with increase in the temperature and for $T > v_0/d$ they become virtually unobservable. The first expression (formula (3.25)), on the other hand, describes oscillations with a large period $\Delta d_1 \sim \pi\mu/p_0 eV \gg \Delta d_2$, the amplitude of which falls off very slowly with increase in temperature, inasmuch as $X_{ij}/Y_{ij} \sim eV/\mu \ll 1$. Considering the range of temperatures $v_0/d \ll T \ll v_0\mu/deV$, we can neglect the term I^2_{osc} in comparison with I^1_{osc}, and in the calculation of I^1_{osc} we can assume $T = 0$. The oscillating part of the current then takes the form

$$I_{osc} \approx \frac{V\rho_\infty^{-1}}{\pi \rho_0^2} \sum_{i \neq j} S_i S_j \frac{\cos(eV\rho_{ij}/v_0)}{\rho_{ij}^4}, \tag{3.28}$$

where $\rho_{ij} = |\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|$.

The oscillations considered describe interference phenomena in the tunneling of electrons through the apertures and are analogous to Friedel oscillations in metals with degenerate Fermi statistics (see, for example, [18]). Generally speaking, the amplitude of the oscillating terms is small. As follows from a comparison of (3.28) with (3.22), $I_{osc}/I_0 \sim 1/(p_0 d)^4$, where d is the distance between any pair of apertures. At $p_0 d \sim 10$, the effect becomes accessible to experimental observation, in particular, if automatic differentiation of the volt-ampere characteristics is possible; this allows us to detect weak departures from Ohm's law. In the case of metals of the bismuth type with small electron groups and large deBroglie wavelengths, observation of the effect is possible at macroscopic distances between the apertures, $d \gtrsim 10^{-4}$ cm.

The nature of the discussed effect is due to the presence of a preferred momentum p_0 in the distribution of electrons in **p** space and of a maximum transferred momentum $2p_0$ (Fig. 2a). For tunneling of the electron from metal 1 with conservation of energy to metal 2 (see Fig. 1), two preferred values of the momentum appear:

$$p_0 = \sqrt{2m\mu}, \quad p_0' = \sqrt{2m(\mu + eV)}$$

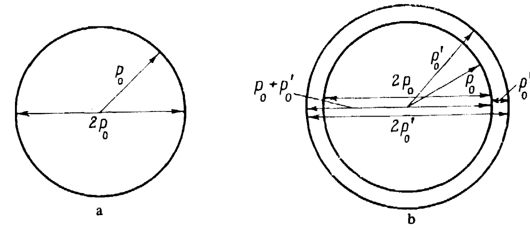

FIG. 2

and the characteristic transferred momenta are (Fig. 2b): $2p_0$, $2p_0'$, $p_0 + p_0'$, $p_0' - p_0$. The first three combinations correspond to oscillations with small period, which, taken together, form the term I^2_{osc} in the current. The last quantity ($p_0' - p_0$) is small and leads to long-period oscillations. Such oscillations are conveniently observed experimentally, inasmuch as the current changes not only as a function of d, but also as a function of the applied voltage V. For $eV \ll \mu$, we get $\Delta p = p_0' - p_0 \approx p_0 eV/2\mu = eV/v_0$, which just corresponds to the period of the oscillations in the term I^1_{osc} (Eq. (3.25)).

The considerations that have been set forth applied to an impurity-free metal with an infinitely long mean free path l. It is clear that to preserve interference with account of scattering, it is necessary that the condition $l \gtrsim d$ be satisfied. As is clear from the foregoing interpretation, the effect is due to the interference of "hot" electrons, whose energy $\epsilon \sim eV$. The path length for such electrons, which is governed by electron-electron collisions, is equal to $l(\epsilon) \sim p_0^{-1}(\mu/\epsilon)^2$. Assuming $eV \sim v_0/d$, we obtain $l \sim p_0 d^2$, whence the condition for observability of the oscillations (along with a sufficiently long impurity path length) takes the form $p_0 d \gg 1$. Similar discussions in the case of electron-phonon scattering, for which $l(\epsilon) \sim (v_0/\omega_D)(\omega_D/\epsilon)^3$ (ω_D is the Debye energy) lead to the more rigid limitation $p_0 d \gg v_0/s$, where s is the sound velocity. The latter estimate pertains to the region of low energies $\epsilon \ll \omega_D$. For $\epsilon \gtrsim \omega_D$, the path length l_{e-ph} does not depend on the energy, [22,23] and is equal to $\sim v_0/\omega_D$. The condition for the existence of the effect in this case takes the form $p_0 d \ll v_0/s$. Thus the effect becomes unobservable only in the energy range $d \sim d_0 = v_0\omega_D \sim 10^{-5}$ cm. For $d \gg d_0$ or $d \ll d_0$, the damping of the oscillations due to electron-phonon interaction will be small.

Differentiating Eq. (3.28) four times with respect to V, and assuming the factor in front of the cosine to be a slowly changing function of the voltage, we obtain

$$\frac{d^4 I_{osc}}{dV^4} \approx \frac{V\rho_\infty^{-1}}{\pi p_0^2}\left(\frac{e}{v_0}\right)^4 \operatorname{Re} \sum_{i \neq j} S_i S_j \exp\left(\frac{ieV}{v_0}|\rho_i - \rho_j|\right). \tag{3.29}$$

With the aid of this expression, we can consider a number of problems—for example, a system of regularly spaced apertures that forms a two-dimensional or one-dimensional lattice, etc.[3)] For a junction of arbitrary shape (S), with f = 1, Eq. (3.29) can obviously be rewritten as

$$\frac{d^4 I_{osc}}{dV^4} \approx C \cdot \operatorname{Re} \int_{(S)} d\rho \int_{(S)} d\rho' \exp\left(\frac{ieV}{v_0}|\rho - \rho'|\right), \tag{3.30}$$

where C is a constant whose form is obvious from (3.29). For a circular aperture of finite radius ($a \gg p_0^{-1}$) calculation of the integral (3.30) by the stationary-phase method gives the result

$$\frac{d^4 I_{osc}}{dV^4} \approx C \cdot \frac{4\pi^{5/2} a^4}{(eVa/v_0)^{3/2}} \cos\left(\frac{2eVa}{v_0} - \frac{\pi}{4}\right). \tag{3.31}$$

We again obtain current oscillations as a function of the voltage V, the period of which is $\Delta(eV) = 2\pi v_0/d$ ($d = 2a$ is the diameter of the aperture). Estimation of the amplitude of the oscillations, in accord with (3.31), gives (cf. (3.28)):

$$(\Delta V)^4 \frac{d^4 I_{osc}}{dV^4} \sim I_0 \left(\frac{\Delta V}{V}\right)^{3/2} (p_0 d)^{-2}, \qquad (3.32)$$

where $I_0 = V/R_\infty$. It is then seen that the smallness of the effect is determined only by the factor $(a_0/d)^2$, where $a_0 \sim p_0^{-1}$ is the lattice constant.

4. Up to this point, we have considered interference phenomena in the case of point apertures ($a \ll p_0^{-1}$) or finite regions with a sharp boundary, the diffuseness of which is also less than p_0^{-1}. Given satisfaction of these conditions, the oscillations turn out to be undamped as a function of the voltage, see formula (3.28). We now analyze the extent to which diffuseness or "imprecision" of the apertures affects the phenomenon considered. As a model, we use the case of a "Gaussian" aperture, the height of whose potential is described by Eq. (1.2), $1/f(\rho) = e^{-\rho^2/a^2}$, where the quantity a has the meaning of the effective radius of the aperture. For a system of Gaussian apertures, we put

$$f^{-1}(\rho) = \sum_{i=1}^{n} \lambda_i \exp(-|\rho-\rho_i|^2/a_i^2). \qquad (3.33)$$

Substitution of (3.33) in (2.6) gives

$$\varphi(\varkappa) = \sum_{i=1}^{n} S_i e^{i\varkappa\rho_i} \exp\left\{-\frac{1}{4} a_i^2 \varkappa^2\right\}, \qquad (3.34)$$

and $S_i = \pi a_i^2 \lambda_i$ has the meaning of the effective "area" of the i-th aperture (cf. (3.34) with (3.23)). Substituting (3.34) in the formula for the current (3.18) and repeating all the calculations that lead to Eq. (3.28), we obtain

$$I_{osc} \approx \frac{V\rho_\infty^{-1}}{\pi p_0^2} \sum_{i\neq j} S_i S_j \frac{\cos(eV|\rho_i-\rho_j|/v_0)}{|\rho_i-\rho_j|^4} \exp\left\{-\left(\frac{eV}{2v_0}\right)^2 (a_i^2+a_j^2)\right\}. \qquad (3.35)$$

This formula is applicable for $(p_0 a)^2 \ll p_0 d$, where $a = \max(a_i, a_j)$. The difference of (3.35) from the expression (3.28) developed earlier consists in the presence of the additional exponential, which diminishes with increase in V. The oscillations now become damped in their amplitude as a function of V, and the number of observed periods is, in order of magnitude, $\Delta N \sim d/a$.

4. STATIONARY JOSEPHSON EFFECT

We now calculate the current of an inhomogeneous junction in the superconducting state for a zero difference in potentials between the metals (stationary Josephson effect[4]). We carry out the calculation of the current in a way similar to the microscopic calculation for a homogeneous junction.[2,7]

The current density in the superconductor—insulator—superconductor system is determined by the temperature Green's function[22]

$$\mathbf{j}(\mathbf{r}) = -\frac{ie}{m} T \sum_\omega [(\nabla_{r'} - \nabla_r) G_\omega(\mathbf{r}, \mathbf{r}')]_{r'=r}. \qquad (4.1)$$

As in Sec. 3, it is convenient to calculate the total current through the barrier in the plane of the junction z = 0:

$$I = \int j(\rho, z=0)\, d\rho. \qquad (4.2)$$

As was noted in[2,7], the problem is simplified in such a method of calculation, inasmuch as there is no need to calculate the corrections to the parameter of order Δ

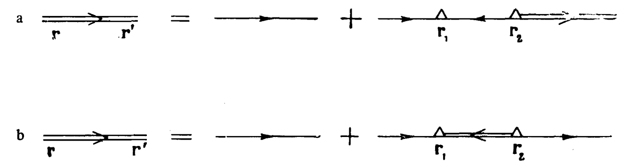

FIG. 3

due to the finite transmission (D). In zeroth order in the transmission, Δ can be chosen in the form

$$\Delta(\mathbf{r}) = \begin{cases} \Delta e^{i\varphi_1}, & z<0 \\ \Delta e^{i\varphi_2}, & z>0 \end{cases}. \qquad (4.3)$$

In the calculation of the Green's function, we shall start out from the Gor'kov equations,[22] which can be represented in integral form:

$$G_\omega(\mathbf{r}, \mathbf{r}') = G_\omega^n(\mathbf{r}, \mathbf{r}') - \int d\mathbf{r}_1 \int d\mathbf{r}_2 G_\omega^n(\mathbf{r}, \mathbf{r}_1) \Delta(\mathbf{r}_1) G_{-\omega}^n(\mathbf{r}_2, \mathbf{r}_1) \Delta^*(\mathbf{r}_2) G_\omega(\mathbf{r}_2, \mathbf{r}'). \qquad (4.4)$$

The diagram Fig. 3a is to be compared with the equation just written. In this case, the single line corresponds to the Green's function of the normal metal (G_ω^n) and the double line to the Green's function of the superconductor. By regrouping the terms of the series which corresponds to Eq. (4.4), we can represent G_ω in another way that is more convenient for calculation of the current. The corresponding equation for G_ω is shown in Fig. 3b or, analytically,

$$G_\omega(\mathbf{r}, \mathbf{r}') = G_\omega^n(\mathbf{r}, \mathbf{r}') - \int d\mathbf{r}_1 \int d\mathbf{r}_2 G_\omega^n(\mathbf{r}, \mathbf{r}_1) \Delta(\mathbf{r}_1) G_{-\omega}(\mathbf{r}_2, \mathbf{r}_1) \Delta^*(\mathbf{r}_2) G_\omega^n(\mathbf{r}_2, \mathbf{r}'). \qquad (4.5)$$

Substituting (4.5) in (4.1) and (4.2), we obtain

$$I = \frac{ie}{m} T \sum_\omega \int d\rho \iint d\mathbf{r}_1 d\mathbf{r}_2 \Delta(\mathbf{r}_1) \Delta^*(\mathbf{r}_2) G_{-\omega}(\mathbf{r}_2, \mathbf{r}_1) \times \left[\left(\frac{\partial}{\partial z'} - \frac{\partial}{\partial z}\right) G_\omega^n(\mathbf{r}, \mathbf{r}_1) G_\omega^n(\mathbf{r}_2, \mathbf{r}')\right]_{r'=r}. \qquad (4.6)$$

Using the equation satisfied by G_ω^n:

$$[i\omega + \mu + \nabla^2/2m - V(\mathbf{r})] G_\omega^n(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \qquad (4.7)$$

and the Green's function, we obtain the following expression for the current[2,4]:

$$I = -ieT \sum_\omega \int d\mathbf{r}_1 \int d\mathbf{r}_2 (\operatorname{sign} z_1 - \operatorname{sign} z_2) \Delta(\mathbf{r}_1) \Delta^*(\mathbf{r}_2) G_\omega^n(\mathbf{r}_1, \mathbf{r}_2) G_{-\omega}(\mathbf{r}_2, \mathbf{r}_1). \qquad (4.8)$$

This formula is a general one and is valid for any inhomogeneous barrier. It follows from (4.8) that in the calculation of the current with accuracy $\sim 1/V_0^2$ it is necessary to use the Green's functions of first approximation G_ω^{n1} and $G_{-\omega}^1$, where the index 1 denotes the first order in $1/V_0$. On the basis of (4.7),

$$G_\omega^n(\mathbf{r}, \mathbf{r}') = G_\omega^{n0}(\mathbf{r} - \mathbf{r}') + V_0 \int d\rho_1 G_\omega^{n0}(\mathbf{r} - \rho_1) f(\rho_1) G_\omega^n(\rho_1, \mathbf{r}'), \qquad (4.9)$$

where G_ω^{n0} is the Green's function of free electrons.[22] Writing down the solution (4.9) in the form of a series in $1/V_0$, we easily obtain, in first order,

$$G_\omega^{n1}(\rho, \mathbf{r}') = -\frac{1}{mV_0 f(\rho)} \frac{d}{d|z'|} G_\omega^{n0}(\rho - \rho', \mathbf{z}'). \qquad (4.10)$$

Substituting this expression in (4.9), we obtain

$$G_\omega^{n\infty}(\mathbf{r}, \mathbf{r}') = G_\omega^{n0}(\mathbf{r} - \mathbf{r}') - G_\omega^{n0}(\rho - \rho', |z| + |z'|) \qquad (4.11)$$

($G_\omega^{n\infty}$ is the Green's function for an infinitely high barrier). Using (4.9), (4.10), we find the expression for G_ω^n with the accuracy necessary for us:

$$G_\omega^n(\mathbf{r},\mathbf{r}')=G_\omega^{n\infty}(\mathbf{r},\mathbf{r}')-\frac{1}{m^2V_0}\frac{d^2}{d|z|d|z'|}\int\frac{d\boldsymbol{\rho}_1}{f(\boldsymbol{\rho}_1)}G_\omega^{n0}(\mathbf{r}-\boldsymbol{\rho}_1)G_\omega^{n0}(\boldsymbol{\rho}_1-\mathbf{r}'). \tag{4.12}$$

We can represent the Green's function of the superconducting state G_ω^1 graphically, as is done in Fig. 4. The light line denotes G_ω^{n1} and the heavy line the Green's function of the superconducting halfspace. In zeroth order in $1/V_0$, we obtain (compare with (4.11))

$$G_\omega^\infty(\mathbf{r},\mathbf{r}')=G_\omega^0(\mathbf{r}-\mathbf{r}')-G_\omega^0(\boldsymbol{\rho}-\boldsymbol{\rho}',\ |z|+|z|'). \tag{4.13}$$

A similar formula also holds for the Gor'kov functions $F_\omega^\infty(\mathbf{r},\mathbf{r}')$. Carrying out further operations, we obtain the following expression for G_ω^1:

$$G_\omega^1(\mathbf{r},\mathbf{r}')=-\frac{1}{m^2V_0}\frac{d^2}{d|z|d|z'|}\int\frac{d\boldsymbol{\rho}_1}{f(\boldsymbol{\rho}_1)}G_\omega^0(\mathbf{r}-\boldsymbol{\rho}_1)G_\omega^0(\boldsymbol{\rho}_1-\mathbf{r}')$$
$$+e^{i(\varphi_1-\varphi_2)\operatorname{sign} z}\frac{1}{m^2V_0}\frac{d^2}{d|z|d|z'|}\int\frac{d\boldsymbol{\rho}_1}{f(\boldsymbol{\rho}_1)}F_\omega^0(\mathbf{r}-\boldsymbol{\rho}_1)F_\omega^{0+}(\boldsymbol{\rho}_1-\mathbf{r}'). \tag{4.14}$$

This expression differs from Eq. (4.12) only in the presence of a second term which contains the F functions of the superconductor.[22] Nevertheless, the structure of the phase factors in the second term of (4.14) is such that this term turns out not to make a contribution to the current. Substituting (4.14) in the formula (4.8), we finally obtain

$$I=I_c\sin(\varphi_1-\varphi_2), \tag{4.15}$$

$$I_c=\frac{\Delta^2e}{V_0^2}T\sum_\omega\int\frac{d\boldsymbol{\rho}_1}{f(\boldsymbol{\rho}_1)}\int\frac{d\boldsymbol{\rho}_2}{f(\boldsymbol{\rho}_2)}\Phi_\omega^2(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2), \tag{4.16}$$

where $\Phi_\omega(\rho)$ is the following function:

$$\Phi_\omega(\boldsymbol{\rho})=\int\frac{d\boldsymbol{\varkappa}}{(2\pi)^2}e^{i\varkappa\rho}\left[\left(1+\frac{\omega}{\sqrt{\omega^2+\Delta^2}}\right)\frac{1}{\lambda_\omega+\lambda_\omega^+}+\left(1-\frac{\omega}{\sqrt{\omega^2+\Delta^2}}\right)\frac{1}{\lambda_\omega+\lambda_\omega^-}\right], \tag{4.17}$$

$$\lambda_\omega=\sqrt{2m(\xi-i\omega)},\quad \lambda_\omega^\pm=[2m(\xi\pm i\sqrt{\omega^2+\Delta^2})]^{1/2},\quad \xi=\varkappa^2/2m-\mu \tag{4.18}$$

(that branch of the root is chosen for which the real part is positive). Carrying out integration over and summation over ω, we obtain the following simple expression, with account of the condition $\omega\ll\mu$:

$$I_c=\frac{\pi}{2}\frac{\Delta}{eR}\operatorname{th}\frac{\Delta}{2T}. \tag{4.19}$$

Here the role of R is played by the dynamic resistance of the inhomogeneous junction for the case of a zero voltage, which was calculated in Sec. 3 and is expressed by the relation (3.21). Formula (4.18) agrees with the well-known result of Ambegaokar-Baratoff.[24] It follows from the derivation just given that this formula holds for any inhomogeneous barrier.[4)]

5. NONSTATIONARY JOSEPHSON EFFECT FOR INHOMOGENEOUS JUNCTIONS

1. The scheme of calculation of tunnel phenomena for inhomogeneous systems developed in Sec. 3 can be transferred to the case of superconductors. Introducing the

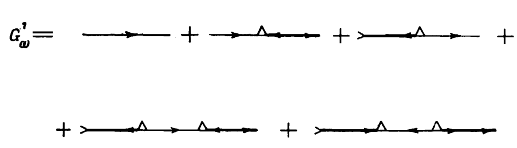

FIG. 4

time-dependent Green's function of the type (3.1), we obtain

$$\left[i\frac{\partial}{\partial t}-H_0-W(z)\theta(t)\right]G(x,x')+\Delta(x)F^+(x,x')=\delta(x-x'),$$
$$\left[i\frac{\partial}{\partial t}+H_0+W(z)\theta(t)\right]F^+(x,x')+\Delta^*(x)G(x,x')=0, \tag{5.1}$$

where

$$F^+(x,x')=i\operatorname{Sp}\hat\rho T(\bar\psi_\uparrow^+(x)\bar\psi_\downarrow^+(x')), \tag{5.2}$$

$$\Delta^*(x)=iF^+(x,x). \tag{5.3}$$

Boundary conditions similar to those discussed in Sec. 3 should be added to Eqs. (5.1). For G(x, x'), these conditions are identical with (3.13), (3.14). In a similar way, we get for the F functions

$$F^+(t'+0,t')-F^+(t'-0,t')=0,\quad F^+(+0,t')-F^+(-0,t')=0,$$
$$F^{+<}(i\beta,t')=-F^{+>}(0,t'). \tag{5.4}$$

Further calculations are entirely analogous to those given in Sec. 3, although they are very cumbersome. The current is finally represented in the form of a sum:

$$I=I_1+I_2, \tag{5.5}$$

where I_1 is the single-particle (or quasiparticle) current, I_2 the current of Cooper pairs (the Josephson superconducting current). The expressions for I_1, I_2 have the following form:

$$I_1=\frac{\pi e}{m^4V_0^2}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{p}'}{(2\pi)^3}A_{\mathbf{pp}'}\left\{\left(1+\frac{\xi_p\xi_{p'}}{\varepsilon_p\varepsilon_{p'}}\right)(n_p-n_{p'})\right.$$
$$\times[\delta(\varepsilon_p-\varepsilon_{p'}-eV)-\delta(\varepsilon_p-\varepsilon_{p'}+eV)]+\left(1-\frac{\xi_p\xi_{p'}}{\varepsilon_p\varepsilon_{p'}}\right)(1-n_p-n_{p'})$$
$$\left.\times[\delta(\varepsilon_p+\varepsilon_{p'}+eV)-\delta(\varepsilon_p+\varepsilon_{p'}-eV)]\right\}, \tag{5.6}$$

$$I_2=\frac{e}{m^4V_0^2}\operatorname{Im}e^{i\varphi(t)}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{p}'}{(2\pi)^3}A_{\mathbf{pp}'}\frac{\Delta^2}{\varepsilon_p\varepsilon_{p'}}$$
$$\times\left\{(n_p-n_{p'})\left(\frac{1}{\varepsilon_p-\varepsilon_{p'}+eV+i\delta}+\frac{1}{\varepsilon_p-\varepsilon_{p'}-eV-i\delta}\right)\right.$$
$$\left.+(1-n_p-n_{p'})\left(\frac{1}{\varepsilon_p+\varepsilon_{p'}+eV+i\delta}+\frac{1}{\varepsilon_p+\varepsilon_{p'}-eV-i\delta}\right)\right\}, \tag{5.7}$$

where

$$\varphi(t)=\varphi_1-\varphi_2-2eVt,\quad \varepsilon_p=\sqrt{\xi^2+\Delta^2}, \tag{5.8}$$

$$A_{\mathbf{pp}'}=k^2k'^2|\varphi(\boldsymbol{\varkappa}-\boldsymbol{\varkappa}')|^2\theta(k)\theta(k'),\quad n_p=(e^{\beta\varepsilon_p}+1)^{-1}. \tag{5.9}$$

In the case of a homogeneous junction, Eqs. (5.6), (5.7) give the well-known results obtained by the method of the tunnel Hamiltonian.[25,26] At T = 0, formulas (5.6), (5.7) take the form

$$I_1=\frac{\pi e}{m^4V_0^2}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{p}'}{(2\pi)^3}A_{\mathbf{pp}'}\left(1-\frac{\xi_p\xi_{p'}}{\varepsilon_p\varepsilon_{p'}}\right)$$
$$\times[\delta(\varepsilon_p+\varepsilon_{p'}-eV)-\delta(\varepsilon_p+\varepsilon_{p'}+eV)], \tag{5.10}$$

$$I_2=\frac{e}{m^4V_0^2}\operatorname{Im}\left\{e^{i\varphi(t)}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{p}'}{(2\pi)^3}A_{\mathbf{pp}'}\frac{\Delta^2}{\varepsilon_p\varepsilon_{p'}}\right.$$
$$\left.\times\left(\frac{1}{\varepsilon_p+\varepsilon_{p'}+eV+i\delta}+\frac{1}{\varepsilon_p+\varepsilon_{p'}-eV-i\delta}\right)\right\}. \tag{5.11}$$

2. We now find the oscillating increment to the quasiparticle current for the system of point junctions (3.23). We shall consider only the oscillating current terms of the type (3.25), assuming that the "fast" oscillations analogous to Eqs. (3.27) have small amplitude and can be neglected (see the discussion in Sec. 3). We have, by virtue of (5.10),

$$I_{\rm osc}^1\approx\frac{\rho_\infty^{-1}}{4\pi ep_0^2}\operatorname{Re}\sum_{i\neq j}\frac{S_iS_j}{|\boldsymbol{\rho}_i-\boldsymbol{\rho}_j|^4}\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}d\xi'\left(1-\frac{\xi\xi'}{\sqrt{\Delta^2+\xi^2}\sqrt{\Delta^2+\xi'^2}}\right)$$

$$\times \exp\left\{\frac{i(\xi-\xi')|\rho_i-\rho_j|}{v_0}\right\}\delta\left(\sqrt{\Delta^2+\xi^2}+\sqrt{\Delta^2+\xi'^2}-eV\right). \quad (5.12)$$

Calculating the integral asymptotically for $eVd/v_0 \gg 1$, we get, for a pair of apertures located at a distance d from one another, an oscillating term of the form

$$I^1_{\rm osc}=\frac{\rm const}{(p_0d)^4}\left(\frac{v_0}{eVd}\right)^{1/2}\cos\left[\sqrt{(eV)^2-(2\Delta)^2}\frac{d}{v_0}-\frac{\pi}{4}\right] \quad (5.13)$$

where the constant depends on the ratio $v = eV/2\Delta$ and becomes infinite as $v \to 1$. The result of the expression (5.13) refers to the case $eV > 2\Delta$, inasmuch as the current I_1 vanishes at $eV < 2\Delta$, in accord with (5.12) (it is understood that this is a consequence of the condition $T = 0$; for nonzero temperatures but $T \ll \Delta$, the current I_1 will be exponentially small at $eV < 2\Delta$). The equation (5.13) is valid for $eV \gtrsim 2\Delta$; therefore it does not go over into (3.25) in the limit $\Delta \to 0$ (at the same time, expression (5.12) has the correct limiting form for the normal state, $\Delta \to 0$).

The oscillation effect (5.13) has the same origin as the current oscillations in the normal state, discussed in Sec. 3, with the added account of the specifics of the superconductor spectrum. By virtue of the presence of a gap, the difference $p_1 - p_2$ in the momenta of the electrons in the two metals (see Fig. 5) is determined by the equations (at $eV > 2\Delta$)

$$\mu_1-\varepsilon_{p_1}=\mu_2+\varepsilon_{p_2},$$
$$\mu_1-\mu_2=eV. \quad (5.14)$$

The oscillation term in the current will be $\sim\cos(p_1 - p_2)d$; it is determined by the maximum value of $p_1 - p_2$ for the condition

$$\sqrt{\Delta^2+\xi_{p_1}^2}+\sqrt{\Delta^2+\xi_{p_2}^2}=eV, \quad \xi_p=v_0(p-p_0). \quad (5.15)$$

It is easy to see that the maximum of $p_1 - p_2$ is achieved at $\xi_{p1} = -\xi_{p2}$, and is equal to

$$(p_1-p_2)_{max}=v_0^{-1}[(eV)^2-(2\Delta)^2]^{1/2}, \quad (5.16)$$

which corresponds exactly to formula (5.13). Equation (5.13) is similar to the expression for oscillations in the Tomasch effect[27,28], which are due, however, to another factor, namely, the singularities of the density of states due to the so-called Andreev reflection of the excitations at the interface from a perturbation of the parameter of the order of Δ.[29]

3. In conclusion, we consider the oscillations of the Josephson component of the current I_2. The oscillations as a function of V in Eq. (5.11) appear only in terms containing δ functions and describing the dissipative part of the current I_2' (sometimes called the current of interference of quasiparticles and pairs, see, for example,[30]). The current component of interest to us has the form

$$I^2_{\rm osc}=\frac{({\rm const})'}{(p_0d)^4}\left(\frac{v_0}{eVd}\right)^{1/2}\cos(\varphi-2eVt)\sin\left[\sqrt{(eV)^2-(2\Delta)^2}\frac{d}{v_0}+\frac{\pi}{4}\right]. \quad (5.17)$$

In conclusion, the authors thank I. K. Yanson for his considerable help in setting up the study and for discussion of the results obtained.

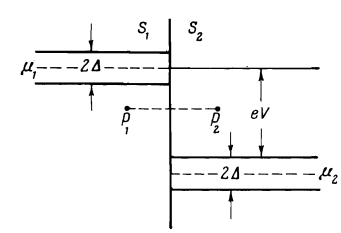

FIG. 5

1)Reference is made to the quasiparticle current.

2)Private communication.

3)Upon replacement of transparent apertures by impenetrable ones (spots), located against the background of a transparent screen, a principle similar to the Babinet principle in optics holds for the interference contribution (see [20]).

4)In the case of several apertures, the quantity R oscillates as a function of their mutual separation distance. However, the period of these oscillations $\sim p^{-1}$; therefore, observation of the effect is not of interest (even without mentioning the fact that in experiment the quantity d for the junction is fixed and the junctions themselves are not strictly point junctions).

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Translated by R. T. Beyer
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