On the theory of relativistic-electron-beam injection into a spatially confined plasma

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The injection of a relativisitic electron beam into a plasma cylinder with a free surface is investigated. The possibility of excitation in such a system of low-frequency surface waves leads to the appearance in the plasma of a slowly varying high-amplitude induced current that considerably exceeds the beam current and significantly affects the dynamics of beam injection into the plasma.

1. INTRODUCTION

Recently there has been a considerable upsurge in interest in the problem of the interaction of high-power relativistic electron beams with a plasma. This is explained by the fact that the limiting electron-beam currents in a dense plasma are considerably higher than in a vacuum[1], which is due, in particular, to the compensation of the magnetic field of the beam current by the field of the current induced in the plasma. The problem of the electrical and magnetic neutralization of the magnetic field of a beam upon its injection into a spatially unbounded homogeneous plasma have already been theoretically investigated[2,3]. The injection of an electron beam into a spatially confined plasma surrounded by a metallic sheath is considered in[4], where it is shown that in such a formulation of the problem the boundedness of the plasma practically does not affect the character of the neutralization of the beam; its effect only amounts to the introduction of small quantitative changes into the results of the theory of electron-beam injection into a spatially unbounded plasma. This was to be expected, since the electromagnetic oscillations of a plasma cylinder in a metallic sheath does not qualitatively differ in nature from the oscillations of the unbounded plasma.

Qualitatively new phenomena should appear in the injection of an electron beam into a plasma cylinder with a free surface, since, in such a plasma, besides the volume electromagnetic oscillations similar to the oscillations in an unbounded plasma, the existence of surface waves is possible. In this case it is very important that surface modes are, under certain conditions, low-frequency oscillations, and, therefore, upon the injection of an electron beam into a plasma cylinder with a free surface they can give rise to induced fields and currents that vary slowly in space and significantly affect the entire character of the injection of the beam into the plasma. In the present paper we investigate quantitatively precisely this problem.

2. THE REACTION OF A BOOUNDED PLASMA SYSTEM WHEN AN ELECTRON BEAM IS INJECTED IN IT

Let a relativistic electron beam of radius r_0 and with a sharp leading edge be injected along the axis into a plasma cylinder with a free surface and of radius $R \geq r_0$. Let us investigate the fields and currents induced by the beam and accompanying it at distances far from the plane of injection. The charge and current densities of the beam under the conditions when its spreading can be neglected can be written in the form

$$\rho_0 = en\eta(-z'), \quad \mathbf{j}_0 = \rho_0 \mathbf{u}, \tag{1}$$

where $z' = z - ut$, n is the (uniform) beam density, and u is the beam velocity.

The beam density n is assumed to be much smaller than the plasma density N, which allows us to consider the beam to be a perturbation that induces in the plasma fields and currents obeying the Maxwell equations:

$$\operatorname{rot}\mathbf{B} = \frac{4\pi}{c}(\mathbf{j}+\mathbf{j}_0) + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$
$$\operatorname{div}\mathbf{B} = 0, \quad \operatorname{div}\mathbf{E} = 4\pi(\rho+\rho_0). \tag{2}$$

Assuming that the sought-for quantities depend only on r and z', we expand them in a Fourier series in z':

$$A(z', r) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk e^{ikz'} A(k, r).$$

Then the system of Maxwell equations for the Fourier components of the fields and currents is easily reduced to a second-order equation for $E_z(k, r)$:

$$\left[\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \varkappa^2\right]E_z(k, r) = -4\pi e\frac{n\varkappa^2}{k^2\varepsilon},$$
$$\varkappa^2 = k^2\left[1 - \frac{u^2}{c^2}\varepsilon\right], \quad \varepsilon = \begin{cases} 1 - \omega_p^2/ku(ku+i\nu) = \varepsilon_1 & r<R \\ 1 = \varepsilon_2 & r>R \end{cases}. \tag{3}$$

Equation (3) should be supplemented by boundary conditions at the plasma-vacuum interface:

$$\{E_z\}_{r=R} = 0, \quad \left\{\frac{\varepsilon}{\varkappa^2}\frac{\partial}{\partial r}E_z\right\}_{r=R} = 0. \tag{4}$$

The response of the plasma to the injection into it of an electron beam can be sufficiently well elucidated by analyzing the azimuthal magnetic field B_φ, by whose structure we can judge the distribution of the reverse current in the plasma. The azimuthal magnetic field B_φ is determined in terms of its Fourier transform, which is connected with the Fourier transform of the longitudinal field $E_z(k, r)$ by the relation

$$B_\varphi(k, r) = -i\frac{u}{c}\frac{\varepsilon k}{\varkappa^2}\frac{\partial}{\partial r}E_z(k, r). \tag{5}$$

Solving Eq. (3) with the boundary conditions (4), we obtain for the magnetic field $B_\varphi(r, z')$ the following expressions:

a) in the region occupied by the plasma, i.e., for $r < R$:

$$B_\varphi(r, z') = i\frac{B_0}{2\pi}\int_{-\infty}^{\infty}\frac{dk}{k}e^{ikz'}\{\Psi + \Psi_0 I_1(\varkappa_1 r_0)I_1(\varkappa_1 r)\};$$

$$B_0 = 4\pi enur_0/c,$$
$$\Psi = \begin{cases} I_1(\varkappa_1 r)K_1(\varkappa_1 r_0), & r<r_0 \\ I_1(\varkappa_1 r_0)K_1(\varkappa_1 r), & r>r_0 \end{cases}, \tag{6}$$

$$\Psi_0 = \frac{\varkappa_1\varepsilon_2 K_0(\varkappa_1 R)K_1(\varkappa_2 R) - \varkappa_2\varepsilon_1 K_1(\varkappa_1 R)K_0(\varkappa_2 R)}{\varkappa_1\varepsilon_2 I_0(\varkappa_1 R)K_1(\varkappa_2 R) + \varkappa_2\varepsilon_1 I_1(\varkappa_1 R)K_0(\varkappa_2 R)};$$

b) outside the plasma, i.e., for $r > R$:

$$B_\varphi(r,z')=i\frac{B_0}{2\pi R}\int_{-\infty}^{\infty}\frac{dk}{k}e^{ikz'}\frac{I_1(\varkappa_1 r_0)K_1(\varkappa_2 r)}{\varepsilon_2\varkappa_1 I_0(\varkappa_1 R)K_1(\varkappa_2 R)+\varepsilon_1\varkappa_2 I_1(\varkappa_1 R)K_0(\varkappa_2 R)}. \tag{7}$$

We shall be interested in the structure of the field B_φ at distances $|z'| \gg c/\omega_p$, R/γ from the head of the beam. The dominant contribution to the integrals (6) and (7) is then made by the region $k \lesssim 1/|z'| \ll \omega_p/c$, γ/R. Under these conditions the integrands have poles at the points

$$k_{1,2}=\pm k_0+ik', \quad k_3=0; \tag{8}$$

$$k_0=\frac{1{,}2\gamma}{R}\exp\left\{-(\gamma^2-1)\frac{\lambda}{R}\frac{I_0(R/\lambda)}{I_1(R/\lambda)}\right\}, \tag{9}$$

$$k'=-\frac{1}{2}\frac{\nu}{u}(\gamma^2-1)\left\{1-\frac{I_0^2(R/\lambda)}{I_1^2(R/\lambda)}\right\}, \tag{10}$$

$$\lambda=c/\omega_p, \quad \gamma=[1-u^2/c^2]^{-1/2}.$$

The relation (10) was derived under the condition $|k'| \ll k_0$.

The conditions for the existence of poles of the type (8) impose the following limitations on the parameters of the system:

$$\exp\left[(\gamma^2-1)\frac{\lambda}{R}\frac{I_0(R/\lambda)}{I_1(R/\lambda)}\right] \gg \gamma\frac{\lambda}{R}, 1, \tag{11}$$

which are always possible when $R < \lambda$: the inverse relation $R > \lambda$ is possible only at large $\gamma(\gamma^2 > R/\lambda)$.

It should be noted that the poles $k_{1,2}$ correspond to surface electromagnetic waves in the plasma cylinder: to wit, the quantity $k_0 u$ determines the frequency, while $k'u$ determines the damping constant of the waves.

The expressions (6) and (7) are integrated by the method of contour deformation into the complex k plane. In this case it is necessary to take into account the conditions $\mathrm{Re}\,\varkappa_{1,2} > 0$, which guarantee the boundedness of the integrands for $r \to \infty$. From these same conditions follows the necessity for branch cuts along the imaginary axis. Performing the integration over k with allowance for the foregoing, we obtain expressions for the field behind the leading edge of the beam at distances $|z'| \gg c/\omega_p{}' R/\gamma$ in the limit of a collisionless plasma, i.e., in the limit when $\nu = 0$ (or, which is the same thing, at distances $|z'| \ll u/\nu$).

a) In the plasma region, i.e., for $r < R$

$$B_\varphi(r,z')=-B_0 I_1(r/\lambda)I_1(r_0/\lambda)\frac{K_1(R/\lambda)}{I_1(R/\lambda)}$$
$$+2(\gamma^2-1)B_0\frac{\lambda^2}{R^2}\frac{I_1(r/\lambda)I_1(r_0/\lambda)}{I_1^2(R/\lambda)}[e^{-k'z'}\cos k_0 z'-\varphi(k_0|z'|)] \tag{12}$$
$$+B_0\begin{cases}I_1(r/\lambda)K_1(r_0/\lambda), & r<r_0\\ I_1(r_0/\lambda)K_1(r/\lambda), & r>r_0\end{cases},$$

where the function

$$\varphi(x)=\frac{1}{4}\int_{-\infty}^{\infty}dt\frac{\exp(-xe^t)}{t^2+\pi^2/4}\approx\frac{1}{4}-\frac{1}{2\pi}\operatorname{arctg}\frac{\ln x}{\pi/2} \tag{13}$$

varies from ½ to 0 as x varies from 0 to ∞.

b) Outside the plasma, i.e., for $r \gtrsim R$

$$B_\varphi(r,z')=2B_0(\gamma^2-1)\frac{\lambda^2}{Rr}\frac{I_1(r_0/\lambda)}{I_1(R/\lambda)}[e^{-k'z'}\cos k_0 z'-\varphi(k_0|z'|)]. \tag{14}$$

3. ANALYSIS OF THE EXPRESSIONS FOR THE INDUCED MAGNETIC FIELD

Only the last term in the expression (12) does not depend on the radius R of the plasma cylinder. It determines the value of the magnetic field at the distances $|z'| \ll u/\nu$ under consideration in an unbounded plasma $(R \to \infty)$[2,3]. Radiuswise, the strongest limitation on the plasma is manifested in the case when the radius of the plasma is smaller than the characteristic skin depth $\lambda = c/\omega_p$. The magnetic field (12) in the plasma is then largely determined by the second term, and may considerably exceed the self-magnetic field of the beam:

$$B_\varphi\approx 2B_0(\gamma^2-1)\lambda^2 rr_0 R^{-4}[e^{-k'z'}\cos k_0 z'-\varphi(k_0|z'|)]. \tag{15}$$

It is evident from this formula that the internal field B_φ takes on the maximum value $\sim B_0(\gamma^2-1)\lambda^2 rr_0/R^4$, which exceeds the self-field $\sim B_0$ of the beam when $R^2 < \gamma\lambda r_0$ and has the same sign as the latter field at distances close to the leading edge of the beam, i.e., for $k_0|z'| \ll 1$. Farther away, i.e., at distances such that $k_0|z'| > 1$, the magnetic field is made up of a monotonically and slowly decreasing - with distance - part $(\sim\varphi(k_0|z'|))$ and an oscillating part of frequency $\omega_0 = k_0 u$. The latter attenuates exponentially at distances $z_\nu \gtrsim 1/|k'|$, where $z_\nu < u/\nu$ when $R < \lambda$.

Thus, a radially bounded plasma system can induce a magnetic field and, consequently, a current in the plasma that considerably exceed the self-field and current of the beam.

In the case when the radius of the plasma is large, i.e., when $R \gg \lambda$, the effect of the boundedness of the plasma system weakens appreciably. If in this case $\lambda \gg r_0$, r, then the dominant term on the right-hand side of the formula (6) is the last term, which corresponds to the self-magnetic field of the beam:

$$B_\varphi\approx B_0\begin{cases}r/r_0, & r<r_0\\ r_0/r, & r>r_0\end{cases}. \tag{16}$$

Thus, as in the case of the unbounded plasma, there is no magnetic neutralization of the beam when $r_0 \ll \lambda$.

Finally, if the beam radius is sufficiently large, i.e., if r_0, $r \gg \lambda$, then the expression (12) assumes the form

$$B_\varphi\approx B_0\left\{\frac{\lambda}{2(rr_0)^{1/2}}\left[\exp\left(-\frac{|r-r_0|}{\lambda}\right)-\exp\left(-\frac{2R-r_0-r}{\lambda}\right)\right]\right.$$
$$\left.+2\gamma^2\frac{\lambda^2}{R(rr_0)^{1/2}}\exp\left(-\frac{2R-r_0-r}{\lambda}\right)[e^{-k'z'}\cos k_0 z'-\varphi(k_0|z'|)]\right\}. \tag{17}$$

It follows from this that the magnetic field is concentrated mainly in a thin layer of thickness $\lambda \ll r_0$ near the lateral surface of the beam, i.e., there is neutralization of the magnetic field and, consequently, of the current of the beam over almost its entire cross section. If then the plasma radius is not close in magnitude to the beam radius, i.e., if $R - r_0 \gg \lambda$, then the field B_φ in the skin layer has the same form as the field in the unbounded plasma: $B_\varphi \sim B_0\lambda/r_0 \ll B_0$. In the opposite case when $R - r_0 \lesssim \lambda$ the boundedness of the plasma can significantly affect the magnitude of the field in the skin layer. Thus, for example, when $\gamma^2 > Rr_0/\lambda^2$ the field B_φ can become considerable and even exceed the field B_0 of the beam.

The magnetic field outside the plasma, like the internal field, attains its maximum value when $R \ll \lambda$, when it may considerably exceed the beam's field:

$$B_\varphi=2B_0(\gamma^2-1)\frac{r_0\lambda^2}{rR^2}[e^{-k'z'}\cos k_0 z'-\varphi(k_0|z'|)]. \tag{18}$$

At distances close to the beam head, i.e., for $k_0|z'| \ll 1$, the field attains a value $\sim B_0(\gamma^2-1)r_0\lambda^2/rR^2$, while further away, at distances such that $k_0|z'| > 1$, the external field, like the internal field, is made up of a monotonically decreasing and an oscillating part. If the plasma and beam radii exceed the characteristic skin depth λ, then the formula (14) assumes the form

$$B_\varphi=2B_0\gamma^2\frac{\lambda^2}{rR}\left(\frac{R}{r_0}\right)^{1/2}\exp\left(-\frac{R-r_0}{\lambda}\right)(e^{-k'z'}\cos k_0z'-\varphi(k_0|z'|)], \tag{19}$$

i.e., the external field attains its maximum value when the plasma and beam radii coincide, and falls off exponentially with increasing value of the difference $R - r_0$.

Notice that allowance for the finiteness of the electron-electron collision rate ($\nu \neq 0$) for the plasma in evaluating the integral (6) at distances $|z'| < u/\nu$ leads to a small additional contribution to (12) given by

$$\delta B_\varphi\approx B_0\frac{\gamma^2-1}{\lambda^2R^2}\int\limits_0^\infty\frac{d\varkappa J_1(\varkappa r)J_1(\varkappa r_0)}{\varkappa^3(\varkappa^2+\lambda^{-2})} \times\left\{\left[\frac{\gamma^2-1}{\varkappa R}J_0(\varkappa R)+J_1(\varkappa R)\ln\frac{1{,}2u\gamma}{\nu R}\left(1+\frac{1}{\varkappa^2\lambda^2}\right)\right]^2+\frac{\pi^2}{4}J_1^2(\varkappa R)\right\}^{-1}. \tag{20}$$

It is quite difficult to estimate this integral. Let us point out that the quantity δB_φ can, generally speaking, be of the order of the self-magnetic field B_0, but small compared to the induced field

$$\sim B_0(\gamma^2-1)\frac{\lambda^2}{R^2}\frac{I_1(r/\lambda)I_1(r_0/\lambda)}{I_1^2(R/\lambda)},$$

if the value of $k_0|z'|$ is not very high. The last requirement is easily fulfilled at practically admissible values of γ and ω_0/ν.

Allowance for the finiteness of the collision rate in computing the integral (7), however, leads to the following result:

$$B_\varphi(r\geqslant R,z')=2B_0\left\{\frac{r_0}{4R}\left[1+\frac{\nu}{u}\frac{\lambda^2}{R}(\gamma^2-1)\int\limits_0^{\nu R/u}dy\exp\left(-y\frac{|z'|}{R}\right)\times\left\{\left(a+y\ln\frac{1{,}2\gamma}{y}\right)^2+\frac{\pi^2}{4}y^2\right\}^{-1}\right]+\frac{\lambda^2}{Rr}(\gamma^2-1)\frac{I_1(r_0/\lambda)}{I_1(R/\lambda)}[e^{-k'z'}\cos k_0z'-\varphi(k_0|z'|)+\delta(\nu)]\right\}; \tag{21}$$

$$a=2\frac{\nu}{u}\frac{\lambda^2}{R}(\gamma^2-1),\qquad \delta(\nu)=\frac{1}{4}\int\limits_{|\ln\frac{\nu}{k_0u}|}^{\infty}dt\frac{\exp(-k_0|z'|e^{-t})}{t^2+\pi^2/4} \leqslant\frac{1}{4}\left(1-\frac{2}{\pi}\operatorname{arctg}\frac{\ln(\nu/k_0u)}{\pi/2}\right).$$

At distances $|z'| \ll u/\nu$, the expression (21) differs little from (14), which was derived for $\nu = 0$.

4. DISCUSSION OF THE RESULTS AND CONCLUSIONS

Thus, the effect of magnetic neutralization of an electron beam injected into a plasma bounded by a vacuum is possible only under the condition when the radius of the beam exceeds the characteristic skin depth $\lambda = c/\omega_p$, just as obtains in the case of the unbounded plasma. In this case the finiteness of the transverse dimension of the plasma can affect only the magnitude of the magnetic field inside a thin skin layer of depth $\sim\lambda$ and then only under the conditions when the plasma and beam radii are very nearly equal (i.e., when $R - r_0 \lesssim \lambda \ll r_0$).

The foregoing analysis of the perturbation of an unmagnetized plasma by an electron beam for different geometries[2-5] allows us to draw the following conclusion: the magnetic (current) neutralization is most easily accomplished in (radially) unbounded plasma. For this purpose, it is only sufficient that the condition $r_0 \gg c/\omega_p$ be fulfilled. The limitation on the dimension of the plasma can lead only to the weakening of the magnetic neutralization effect and, thereby, to a greater resistance to the passage of larger-than-limiting currents in the plasma. Since the two opposite limiting cases when the plasma is bounded by a vacuum and when it is bounded by a perfect conductor are the cases investigated, we can assert that the influence of the medium surrounding the plasma on the induction in the plasma of fields and currents by an electron beam weakens as we vary the permittivity of the medium from $\epsilon = 1$ (for a vacuum) to $\epsilon = \infty$ (for a perfect conductor). If the character of the perturbation of the plasma by the beam is not significantly influenced by the presence of a metallic sheath (when completely filled by the plasma), it is significantly affected by the presence of a plasma-vacuum interface. Thus, in the case when $R < c/\omega_p$, such a system can function as a unique exciter of surface oscillations, amplifying the reverse current and the related magnetic field in the plasma—the effect of beam overcompensation. This phenomenon can be used for a more effective—in comparison with the case of the unbounded plasma (but with the same values of r_0, n, and N)—heating of a plasma through the dissipation of the reverse current.

Special investigations have shown that when the metallic sheath is not completely filled with the plasma the system can be treated as a plasma with a free surface if[6]

$$\frac{R_{sh}}{R}>\exp\left[(\gamma^2-1)\frac{\lambda}{R}\frac{I_0(R/\lambda)}{I_1(R/\lambda)}\right]\gg 1. \tag{22}$$

When this condition is violated, the dispersion properties of the system change abruptly and the system becomes similar to a plasma confined by a metallic sheath, i.e., the influence of the vacuum layer practically completely disappears.

It should be pointed out that under the conditions of magnetic overcompensation of the beam the motion of the beam electrons along the axis of the system is possibly impeded. The Lorentz force, which is due to the action of the azimuthal magnetic field $B_\varphi \gg B_0$ of the plasma current, will tend to push the beam electrons away from the axis in those regions where the sign of the induced magnetic field is opposite to that of the beam's self-field, i.e., where the induced current flows in the direction opposite to the direction of flow of the beam current, and drive them towards the axis in the regions where these signs coincide. The pinching effect, which oscillates in strength along the length of the system, will be insignificant if the profile of the beam is sufficiently smooth, or, more precisely, if the characteristic beam-current growth time exceeds the period $2\pi/\omega_0$ of the surface oscillations. In this case the contribution from the oscillatory part in (12) will decrease markedly, owing to the averaging over the oscillations, while the contribution from the nonoscillating part will, in order of magnitude, remain the same. The induced current in such a system will be a reverse current only, and its magnetic field will scatter the beam electrons along the axis. The weakened electron flux of the beam (with a reduced density) will, in its turn, induce a weaker reverse current in the corresponding region of z'.

Thus, it can be expected that under the conditions when the excitation of a high-intensity surface wave ($R \ll \lambda$) by a beam with a smooth profile is possible, the beam will be scattered as it moves away from the injector. For $u \sim c$ and $R \sim r_0$ the characteristic scattering length l_z of the beam can be estimated to be l_z

$\sim c/\Omega$, where $\Omega = eB_\varphi/\gamma mc$ is the electron gyrofrequency in the resultant magnetic field. Noting that $B_\varphi \sim \mu B_0$, where $\mu = \gamma^2\lambda^2/R^2$, we obtain $l_z \sim RN/\gamma n$. The magnetic field of the surface wave significantly exceeds the beam field only at distances $|z'| \lesssim 1/k_0 \sim Re^{2\mu/\gamma}$; therefore, there will be effective scattering if $l_z \lesssim 1/k_0$, or

$$\mu > \ln(N/n) \gg 1. \tag{23}$$

When the condition (23) is fulfilled, the beam will be scattered at a distance $|z'| \gtrsim l_z$, so that a strong (in comparison with the case when the beam is injected into an unbounded plasma, other conditions being equal, and $\lambda > R$) induced plasma current (and, therefore, the effective heating of the plasma during the dissipation of this current) will exist only along the corresponding scattering length l_z. We can increase l_z by applying an external longitudinal magnetic field; this field should, however, not be too strong. In the opposite case, owing to the change in the dispersion relation for the plasma, the spectrum of the surface waves can also change, so that the treatment presented in this paper may no longer be valid.

In conclusion, we must note that although the reverse current induced by the surface wave considerably exceeds, for $\mu \gg 1$, the beam current, it is nevertheless weak compared to the limiting Alfvén current $J_A = e^{-1}mc^2\gamma u$. Indeed, noting that the ratio of the beam current J_0 to J_A (for $r_0 \sim R$) $J_0/J_A \sim \gamma n/\mu N$, and that the reverse current J is of the order of μJ_0, we have

$$J/J_A \sim \gamma n/N < 1, \tag{24}$$

if γ is not too high.

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