The isotropization of the cosmological expansion owing to particle production

V. N. Lukash and A. A. Starobinskiĭ

Moscow Physico-technical Institute
L. D. Landau Institute for Theoretical Physics, USSR Academy of Sciences
(Submitted December 26, 1973)
Zh. Eksp. Teor. Fiz. **66**, 1515–1527 (May 1974)

We consider the problem of isotropization during the cosmological expansion of a homogeneous Universe with flat comoving space (a model of Bianchi type I) under the action of the gravitation of particles produced near the anisotropic Kasner singularity. The interaction of the produced particles with each other is taken into account. The reaction of the produced particles on the metric near the particle singularities leads to a quick isotropization of the cosmological expansion. We list restrictions on the isotropization instant, starting from the observed isotropy of the microwave background.

At the present time the expansion of the Universe is described with good accuracy by the isotropic and homogeneous Friedmann model. The high degree of isotropy of the microwave background observed today ($\Delta T/T \lesssim 10^{-3}$) bears witness to the fact that the Universe became isotropic already at an extremely early stage of its expansion (it is even possible that the instant of isotropization $t_F \sim t_{Pl} = (G\hbar/c^5)^{1/2} \sim 10^{-43}\,s)$[1]. At the same time the general solution of the Einstein equations is anisotropic and inhomogeneous near a singularity[2]. Therefore one must either consider that the present-day Friedmann state of the Universe is "accidental," i.e., a consequence of a special way of specifying the initial conditions near the singularity (t = 0), or it is necessary to find physical processes which could effectively isotropize the expansion of the Universe during the early stages.

Zel'dovich has advanced the hypothesis that the effect of particle pair production near the anisotropic singularity leads to a rapid transition of the anisotropic expansion into a quasi-isotropic one.[3] For the first time the phenomenon of pair production in strong gravitational fields was considered by Parker[4], but he limited himself to the consideration of the case of isotropic gravitational fields (i.e., of a gravitational field described by the Friedmann metric). Current ideas about properties of elementary particles (in particular the so-called principle of conformal invariance) indicate that in a Friedmann Universe particles with mass $m = 0$ cannot be produced at all[4-7], and the reaction of produced particles with $m \neq 0$ on the metric is everywhere small (including the vicinity of the singularities)[6-7].

The production of pairs in an anisotropic gravitational field was first considered by Zel'dovich and one of the authors[7]. The following important circumstance became clear: the problem of particle pair production from the vacuum under the action of a strong external gravitational field can be solved rigorously only in the case of collapse, when for $t = -\infty$ space-time is flat and one can correctly and uniquely define a "vacuum" state of the system, i.e., a state in which particles of a given kind are absent. If for $t \to -0$ the metric has a singularity of the anisotropic type, then for $t = -0$ it is impossible to define a vacuum state, in distinction from the case of an isotropic singularity. The reason for this is the fact that as $t \to -0$ the energy of the produced particles tends to infinity like t^{-4} (in the sequel we shall always assume $8\pi G = c = \hbar = 1$)[7]. However, for arbitrary $t \neq 0$ the vacuum state of the system can be defined by means of diagonalization of the Hamiltonian of the quantum field, as proposed by Grib and Mamaev[6].

In the present paper we solve the problem of cosmological expansion taking into account particle production near the singularity and their reaction on the metric. For $t \to +0$ let the metric of space-time have an anisotropic singularity of the Kasner type

$$ds^2=dt^2-a^2(t)\,(dx^1)^2-b^2(t)\,(dx^2)^2-c^2(t)\,(dx^3)^2; \qquad (1)$$

where for $t \to 0$

$$a\sim t^{q_1},\quad b\sim t^{q_2},\quad c\sim t^{q_3},\quad \sum_{\alpha=1}^{3} q_\alpha=\sum_{\alpha=1}^{3} q_\alpha^2=1,$$

and all $q_\alpha < 1$ (we exclude the case where one q = 1 for the reason mentioned below). We cannot correctly define a vacuum state for t = 0. On the other hand, the effect of particle production appears necessarily and one cannot switch it off completely. In addition, it is clear that the concept of external classical gravitational field, and consequently the whole analysis carried out earlier[7], are valid only for $t > t_{Pl}$.

We therefore consider the following model: for $t < t_0$, with

$$t_0 \gg t_{Pl}, \qquad (2)$$

let there be no particle production; then for $t = t_0$ when one can define a vacuum state correctly by means of the method of diagonalization of the Hamiltonian of the quantum fields, the particle production is switched on.[1)] For $t > t_0$ the expectation value of the energy-momentum tensor of the quantized fields (e.g., of the electromagnetic field) in this state (this vacuum expectation value is a functional of the classical metric) is substituted into the right-hand side of the Einstein equation in order to obtain a self-consistent solution. Thus, one searches for a solution of the equations

$$R_{ik}-{}^1/_2 g_{ik}R=\langle T_{ik}\rangle \qquad (3)$$

with the initial condition $\langle T_{ik}\rangle = 0$ for $t = t_0$. The tensor $\langle T_{ik}\rangle$ describes the production of pairs of particles and the vacuum polarization[7]. The admissibility of such an approximation is guaranteed by the validity of the condition (2).

The main contribution to $\langle T_{ik}\rangle$ comes from particles with energies $\omega \sim t_0^{-1}$, which were produced at the earliest possible moment, i.e., for $t \sim t_0$. As will be shown below, the produced particles do not start right away to influence the metric, but only at the time $t^* \sim t_0(t_0/t_{Pl})^{3/2} \gg t_0$ (cf. Eq. (17)). Therefore at the beginning, for $t_0 \leq t \ll t^*$ one can consider the production of particles in a given metric, without taking into account their inverse reaction on this metric, as was done in[7]. In the sequel, as the influence of the gravitation of the free particles on the metric becomes important, one may already neglect the quantum effects of pair production and vacuum polarization and the tensor $\langle T_{ik}\rangle$ takes on a purely classical form.

The fundamental result of this paper is the proof that under this strong limitation on particle production, the metric (1) nevertheless becomes isotropic at the time $t_F \sim t_0(t_0/t_{Pl})^2$ (up to logarithmic terms, cf. Eqs. (16), (21), (34)). If, as it should happen in reality, $t_0 \sim t_{Pl}$, then also $t_F \sim t_{Pl}$, in agreement with present-day observational data (cf. Sec. 3 and[1]).

1. THE ENERGY MOMENTUM TENSOR OF THE PRODUCED PARTICLES

It is first necessary to compute the quantity $\langle T_{ik}\rangle$. For this purpose we consider the production of particles in the metric (1) with the condition that for $t = t_0$ the quantum state of the system, consisting of the external classical homogeneous gravitational field plus the free quantized fields[2)], be the vacuum state, i.e. $\langle T_{ik}\rangle = 0$ for all quantized fields.

If all the q_α satisfy the condition $q_\alpha < 1$, the integral

$$\int_0^t \omega(t)\,dt,$$

converges at the lower limit, where

$$\omega^2 = k_1^2/a^2 + k_2^2/b^2 + k_3^2/c^2 + m^2,$$

and ω, k_α and m are respectively the covariant components of the four-momentum and the mass of the particle. Then for a computation of the number of produced particles one can make use of the sudden perturbation approximation (cf.[7] for details). Near an anisotropic singularity one may neglect the rest mass m if one takes into account that for all known elementary particles $Gm^2/hc \ll 1$. Therefore in the sequel we shall consider $m = 0$.

Approximate formulas for the number of produced particles for $t \gg t_0$ have the form:

$$n(\mathbf{k}) = \frac{\pi}{2\omega(t_0)t_0}\theta\left[\frac{\pi}{t_0} - \omega(t_0)\right] \quad \text{for bosons},$$
$$n(\mathbf{k}) = \frac{1}{2}\theta\left[\frac{\pi}{t_0} - \omega(t_0)\right] \quad \text{for fermions}, \tag{4}$$

where

$$\theta(x) = \begin{cases} 1 & x>0 \\ 0 & x<0 \end{cases}, \quad \mathbf{k} = (k_1, k_2, k_3).$$

These formulas describe well the behavior of $n(\mathbf{k})$ for $\omega \to 0$ and the rapid (in reality exponential) decay of $n(\mathbf{k})$ for $\omega(t_0) > \pi/t_0$, i.e., in that region where the wavelength is smaller than the horizon. Similar results were obtained by Berger[8] in computing the production of graviton pairs traveling along the same axis (the production of other gravitons was artificially frozen by Berger[8], which, generally speaking, is impossible).

The distributions (4) are isotropic for $t = t_0$, in reality the distribution of produced particles is anisotropic, however from the results of[7] it follows that although the pressures along different axes at $t \sim t_0$ are different, they are of the same order and therefore one may neglect this initial anisotropy of the particle distribution. The normalization coefficient of n(k) is selected in such a manner that, as shown in[7], the energy of the produced particles should be of order t_0^{-4} for $t \sim t_0$.

It is very important that the quantity $n(\mathbf{k})$ is finite and does not depend on the method of renormalization of the infinite vacuum polarization. This is related to the fact that renormalizable terms in the expectation values $\langle T_{ik}\rangle$ are local, i.e., they depend on the values of the tensor g_{ik} and its derivatives at the given instant of time, whereas the expression $n(\mathbf{k})$ is determined from nonlocal integrals and depends essentially on the behavior of the metric for $t \sim t_0$. The contribution from the instantaneous vacuum polarization is proportional to t^{-4} and the contribution from the already produced particles is proportional to $t^{-2} + p t_0^{-2} - p$, where $p \geq 0$. Therefore, for $t > (3-5)t_0$ one may already neglect the quantum effects and write $\langle T_{ik}\rangle$ for each quantized field in the form corresponding to free classical particles. It follows from the symmetry of the problem that only the diagonal components of $\langle T_{ik}\rangle$ are non-zero:

$$\varepsilon \equiv \langle T_0{}^0\rangle = \frac{1}{(2\pi)^3\gamma^{1/2}}\int d^3k\,\omega(\mathbf{k},t)\,n(\mathbf{k}),$$
$$p_1 \equiv -\langle T_1{}^1\rangle = \frac{1}{(2\pi)^3\gamma^{1/2}}\int \frac{d^3k}{\omega(\mathbf{k},t)}\frac{k_1^2}{a^2(t)}n(\mathbf{k}) \tag{5}$$

etc., where $\gamma^{1/2} = abc$, and the function n(k) is taken from (4).

As it should be, ϵ and p_α satisfy the conservation law $\langle T^{ik}\rangle_{;k} = 0$ or

$$-\frac{1}{\gamma^{1/2}}\frac{d}{dt}(\gamma^{1/2}\varepsilon) = \frac{p_1}{a}\frac{da}{dt} + \frac{p_2}{b}\frac{db}{dt} + \frac{p_3}{c}\frac{dc}{dt} \tag{6}$$

for each species of particles separately. Moreover,

$$\langle T_i{}^i\rangle = \varepsilon - \sum_{\alpha=1}^{3} p_\alpha = 0.$$

We note that the characteristic size of the region of localization of the field quanta $\Delta r \sim \omega^{-1}$ is smaller than the horizon for $t > t_0$, therefore one can indeed talk about these quanta as of particles.

In the sequel it will be seen that for $t \sim t_0$ the reaction of the particles on the metric is still small. Therefore, in order to solve the self-consistent problem (3) it suffices to substitute the quantities (5) multiplied by ν, where ν is an effective number of species of "genuinely elementary particles",[3)] into the right-hand side of the Einstein equations (1) for the metric.

2. THE APPROXIMATION OF COMPLETELY FREE PARTICLES

We first consider that the produced particles do not interact with one another, i.e., that the mean free path is larger than the horizon. In this case the problem of the reaction of the gravitation of the particles created near a singularity on the metric reduces simply to the problem of evolution of the Bianchi type I model (cf. (1)), filled with free particles of mass zero, under the assumption that at some time $t = t_0$ the distribution function is isotropic (cf. (4)). Here we briefly describe the method of solution and give the results in detail.[4)]

Since the metric (1) admits of a scaling transformation which allows one to change independently the scales of the functions a, b, c, we shall consider that at the instant $t = t_0 \gg t_{Pl}$

$$a = b = c = a_0 = \left(\frac{2\nu}{3\pi^2}\int_0^\infty n(\omega_0)\,\omega_0{}^3\,d\omega_0\right)^{-1/4} = t_0{}^{1/2}\left(\frac{\varepsilon}{\varepsilon_F}\right)_0^{-1/4}, \tag{7}$$

where $(\epsilon_F)_0 = 3/4t_0^{-2}$ is the density of matter ($p = (1/3)\epsilon$) in the Friedmann model at $t = t_0$.

For the distributions (4) we obtain

$$a_0 \approx \nu^{-1/4} t_0. \tag{8}$$

For simplicity we consider in the sequel the axisymmetric case

$$a = b. \tag{9}$$

Then it follows from (5) that

$$\varepsilon = \frac{3}{8\gamma} a^2 \left[1 + \frac{c^2}{a(a^2-c^2)^{1/2}} \ln \left(\frac{a+(a^2-c^2)^{1/2}}{c} \right) \right], \tag{10}$$

$$p_3 = \frac{3}{8\gamma} \frac{a^4}{a^2-c^2} \left[1 - \frac{c^2}{a(a^2-c^2)^{1/2}} \ln \left(\frac{a+(a^2-c^2)^{1/2}}{c} \right) \right], \tag{11}$$

$$p_1 = p_2 = \frac{3}{16\gamma} \frac{a^2 c^2}{a^2-c^2} \left[-1 + \frac{2a^2-c^2}{a(a^2-c^2)^{1/2}} \ln \left(\frac{a+(a^2-c^2)^{1/2}}{c} \right) \right]; \tag{12}$$

depending on the sign of $a^2 - c^2$ it is convenient to use the identity

$$\frac{1}{(a^2-c^2)^{1/2}} \ln \left(\frac{a+(a^2-c^2)^{1/2}}{c} \right) = \frac{1}{(a^2-c^2)^{1/2}} \operatorname{Arsh} \frac{(a^2-c^2)^{1/2}}{c} = \frac{1}{(c^2-a^2)^{1/2}} \arcsin \frac{(c^2-a^2)^{1/2}}{c}.$$

The field equations have the form

$$\frac{d^2(\ln a)}{d\tau^2} = \gamma p_1, \quad \frac{d^2(\ln c)}{d\tau^2} = \gamma p_3, \tag{13}$$

$$\frac{d(\ln a)}{d\tau} \frac{d(\ln ac^2)}{d\tau} = \gamma\varepsilon, \tag{14}$$

where τ is defined by the condition

$$dt = \gamma^{1/2} d\tau, \quad \tau \in (-\infty, 0), \quad t \in (0, \infty).$$

Equations (13)—(14) can be solved in three regions:

$$1)\ \mu \equiv \ln \frac{a}{c} > 1, \quad 2)\ \mu < -1, \quad 3)\ |\mu| < 1,$$

and the solutions are matched at the points $|\mu| \sim 1$. A good accuracy of the matching is guaranteed in the given case by the fact that the right-hand sides of (13) determine directly the evolution of the second derivatives as a function of the quantities ln a, ln c (Eq. (14) is a first integral of Eq. (13)). In other words, since at the matching point the corrections are of order of one only for the second derivatives of ln a, ln c, the first derivatives, and all the more, the functions ln a, ln c themselves (obtained respectively by integrating once or twice the quantities $d^2(\ln a)/d\tau^2$, $d^2(\ln c)/d\tau^2$) the corresponding corrections are smaller than one.

A solution of (13)—(14) with the energy-momentum tensor (10)—(12) depends on one physically arbitrary constant A, which determines by how many times the density of matter is smaller than the Friedmann density at the initial instant t_0:

$$A = 2^{5/2} a_0^2 / 3t_0 \approx 2(\varepsilon_F/\varepsilon)_0^{1/3} \tag{15}$$

(a_0 is determined from (7)). For the distribution functions (4) we find (cf. (8))

$$A \approx 2\nu^{-1/2} t_0 \sim t_0/t_{Pl}. \tag{16}$$

In the case when $A \gg 1$, the solution is represented in Fig. 1 and exhibits three characteristic stages of evolution:

1. During the portion $t_0 < t < t_D$ of the evolution ($t_D = t_0 A^3 \ln^{3/2} A$) the solution of (13)—(14) is of the form:

$$c = a_0 A^{-1/2} (t/t^*)^{-1/3} \cdot [1 + (t/t^*)^{4/3}], \tag{17}$$

where $t^* = t_0 A^{3/2} \gg t_0$ is the time when the "vacuum"

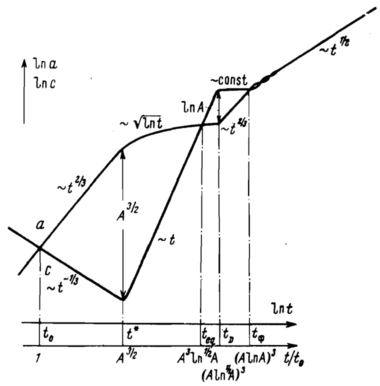

FIG. 1

stage (the Kasner epoch with exponents (2/3, 2/3, −1/3) comes to an end. At that time the influence of the gravitation of the flux of free particles on the evolution becomes important ($p_3 \gg p_1 = p_2$; for $t \sim t^*$, the majority of particles propagates along the X^3 axis, equally in each direction $\pm x^3$). After the instant t^*, for $t > t^*$, the expansion of the Universe occurs in such a way as to damp out the anomalously large pressure in the X^3 direction (cf. (17)):

$$t > t^*, \quad c \sim t, \quad a \sim \ln^{1/2} t, \quad p_3 \sim 1/t. \tag{18}$$

We note that the solution (18) is not the vacuum solution: the gravitation of matter (p_3) affects the metric throughout this whole stage (18) ($t^* < t < t_{eq}$; cf. 2 infra). However on a small portion of the evolution, where one may neglect the variation of the logarithm in (18), the influence of matter is negligibly small and the solution (18) can be considered the vacuum solution, i.e., the Kasner solution with exponents (0, 0, 1).

2. In the region $\mu < -1$ the solution of (13)—(14) is

$$a = a_0 A (2 \ln A)^{1/2} (1 + t/t_D)^{2/3},$$
$$c = 3a_0 A^{-1/3} \frac{t_D}{t^*} \left[1 - \frac{1}{(1+t/t_D)^{1/3}} \right], \tag{19}$$

where $t_D = t^*(A \ln A)^{3/2} = t_0 A^3 \ln^{3/2} A$ is the instant when the "damping" stage of the anisotropy of the deformation tensor starts, a characteristic intermediate aperiodic stage, which directly precedes the Friedmann expansion stage[1,10].

For $t < t_D$ the solution (19) goes over continuously into (17), (18). On the evolution portion $1 > \mu > \mu_D = -\ln \ln A$ ($t_{eq} \lesssim t < t_D$), where $t_{eq} = t^* A^{3/2} (2 \ln A)^{1/2}$ is the instant when the pressures are equal ($p_3 = p_1 = p_2$, $a = c$) the influence of the right-hand side in (13)—(14) is negligibly small, in the leading approximation the solution of (13)—(14) is the vacuum solution (the Kasner solution with exponents (0, 0, 1), cf. (19)) and one may neglect the variation of the logarithm in (17)—(18). Thus the solution (17), (18) (which, strictly speaking has been obtained in the approximation $\mu > 1$) can be extended up to the instant t_D, which guarantees the high accuracy of the matching (17)—(19).

During the "damping" stage, $t_D < t < t_F$, the distribution function n(k) in momentum space (k_1/a, k_2/b, k_3/c) has the form of an oblate spheroid ($p_1 = p_2 \gg p_3$) and in the process of the expansion of the Universe it tends to an isotropic distribution (cf. (19)):

$$t > t_D, \quad c \sim \text{const}, \quad a \sim \left(\frac{t}{t_D} \right)^{2/3}, \quad \frac{p_1}{p_3} \sim \left(\frac{t_D}{t} \right)^{4/3}. \tag{20}$$

3. The damping stage (20) goes on up to the instant $t \sim t_F$ ($\mu \sim -1$), where

$$t_F = t_D (\ln A)^{1/2} = t_0 (A \ln A)^3, \tag{21}$$

followed by an isotropic expansion stage, in which the anisotropy of deformation (and of the energy-momentum tensor (10)—(12) of the particles) is smaller than one and decreases rapidly according to a power law:

$$\mu \approx \left(\frac{t_F}{t}\right)^{1/4} \sin\left[\frac{3}{4}\sqrt{\frac{3}{5}} \ln t + \text{const}\right], \quad \gamma^{1/3} \approx t\left[1 + \frac{8}{45}\left(\frac{t_F}{t}\right)^{1/2}\right],$$
$$\varepsilon \approx \frac{3}{4\gamma^{2/3}}\left[1 + \frac{4}{45}\left(\frac{t_F}{t}\right)^{1/2}\right], \quad \frac{p_3}{p_1} \approx 1 + \frac{8}{5}\mu. \tag{22}$$

At the time $t \sim t_F$ the anisotropy of deformation is of the order of unity:

$$\left|\frac{\Delta H}{H}\right| \sim \left|3\frac{d\mu}{dt} \Big/ \frac{d}{dt}(\ln \gamma)\right| \sim 1;$$

the amplitude of the first oscillation of μ in the Friedmann stage is $\lesssim 1$. A positive addition to the total energy (22) takes into account the excitation energy of the isotropic distribution function of the particles.

3. ISOTROPIZATION WITH PARTICLE INTERACTION

We now take into account the interaction of the produced particles with each other. The main contribution to the total number of particles and to the integrals (5) comes from particles with maximal energy ω (equal to t_0^{-1} for $t \sim t_0$), therefore it is necessary to consider just these particles. Here $\omega \gg m$, but $\omega \ll t_{Pl}^{-1}$.

The distributions (4) may change substantially only as a consequence of those interaction processes of particles which are related to a large momentum transfer, i.e., to scattering under a large angle $\theta \sim 1$ in the c.m.s. Using the kinematic invariants s, t, u (cf. [11] for their definition), these conditions can be written as

$$t_{Pl}^{-2} \gg s \sim t \sim u \gg m^2. \tag{23}$$

The analysis of the existing theoretical and experimental data leads to the following results relative to the behavior of the cross sections of various processes in the region of variation of s, t, u under consideration.

I. The cross sections of all processes involving gravitons are much smaller than t_{Pl}^2 (e.g., $\sigma \sim \omega^2 t_{Pl}^4$ for the process $g + g \to N + N$[12]), therefore for gravitons of maximal energy

$$\sigma n t \leqslant \left(\frac{t_{Pl}}{t_0}\right)^2 \frac{t}{\gamma^{1/3}} \frac{\gamma^{1/3}(t_0)}{t_0} < \left(\frac{t_{Pl}}{t_0}\right)^2 \ll 1, \tag{24}$$

where n is the density of gravitons. Thus, the gravitons always remain free.

II. The cross sections of electromagnetic processes in the region under consideration have the form:

$$\sigma \sim \alpha^2/\omega^2, \tag{25}$$

where $\alpha = e^2 = 1/137$. Then in the region (23) the singly logarithmic corrections are small, since

$$\frac{\alpha}{3\pi} \ln\left(\frac{\omega}{m_e}\right)^2 < 0.08$$

for $\omega \ll t_{Pl}^{-1}$, and the contribution from the doubly logarithmic corrections of the form $\alpha \ln^2(\omega/m_e)$ are totally absent (cf. the review article by Gorshkov [13]). According to the Weinberg model the cross sections of the weak interactions are of the same order (here one must interpret m in Eq. (23) as the mass of the W boson).

During the stage $t_0 < t < t_D$ the quantity $\omega \sim 1/c$ for particles moving along the X^3 axis (these particles yield the fundamental contribution to the energy during the anisotropic stage), therefore

$$\sigma n t \sim \alpha^2 \left[\frac{c(t)}{c(t_0)}\right]^2 \frac{t}{t_0} \left(\frac{\gamma(t_0)}{\gamma(t)}\right)^{1/2} \sim \alpha^2 A^2 \quad \text{for } t \sim t_{eq}. \tag{26}$$

Thus, if the quantity $A \gg 1$ is selected such that $A \ll 137$, the leptons and photons, which do not participate in the strong interactions remain free during the anisotropic stage of expansion and thermalize considerably later when the process of isotropization is already completed. For $A \gtrsim 137$ it is necessary to take into account the interaction of the leptons and photons in the anisotropic stage, however the hadrons thermalize considerably earlier.

III. In the case of strong interactions the situation is considerably less clearcut in view of the lack of a dynamical theory of these interactions. However, one may assume that in the region (23) the cross sections have the form

$$\sigma \sim 1/\omega^2, \tag{27}$$

in agreement with the unitarity bound and also with the principle of scale invariance at high energy[5)] (cf., e.g., [14]). Then for hadrons $\sigma n t \sim 1$ for $t \sim t_1$,

$$t_1 = t_0 A^2 \ln^{1/2} A, \quad (a/c)_1 = A, \quad t^* \ll t_1 \ll t_{eq}. \tag{28}$$

Therefore, for $t \sim t_1$ (in the anisotropic stage, when the majority of particles propagates along the X^3 axis) the hadronic component of the produced particles thermalizes and then is described by the equation of state $p = \epsilon/3$, whereas the other particles can still be considered free[6)].

Thus, during the stage $t > t_1$ the energy-momentum tensor of matter has the form

$$\varepsilon = \frac{3}{8}\left[(1-\beta_1)\left(\frac{A}{\gamma}\right)^{2/3} + \beta_1 \frac{a^2}{\gamma}\right],$$
$$p_3 = \frac{3}{8}\left[\frac{1-\beta_1}{3}\left(\frac{A}{\gamma}\right)^{2/3} + \beta_1 \frac{a^2}{\gamma}\right], \tag{29}$$
$$p_1 = p_2 = \frac{1-\beta_1}{8}\left(\frac{A}{\gamma}\right)^{2/3},$$

where β_1 = const is the fraction of free particles ($0 < \beta_1 < 1$).

Substituting (29) into (13)—(14) we obtain the solution (cf. [10,15]):

$$a = 4D\beta_1^{-1/2}\alpha\left(\int_1^\alpha \frac{e^{\alpha^2}}{\alpha^2} d\alpha + D_0\right),$$
$$\gamma^{1/3} = \frac{16D^2}{A^{2/3}(1-\beta_1)} \alpha e^{\alpha^2}\left(\int_1^\alpha \frac{e^{\alpha^2}}{\alpha^2} d\alpha + D_0\right), \tag{30}$$

where $D > 0$ and D_0 are constants. The function $\alpha = \alpha(t)$ is determined from the equation

$$\frac{d\alpha}{d\tau} = D\alpha\left(\int_1^\alpha \frac{e^{\alpha^2}}{\alpha^2} d\alpha + D_0\right). \tag{31}$$

The solution (30)—(31) must be matched with (17)—(18) for[7)] $t \sim t_1$ ($\alpha \sim \alpha_1$). From the matching conditions it follows that:

$$\alpha_1^2 \approx \frac{2}{3\beta_1} \ln\frac{t_1}{t^*} \approx \frac{\ln A}{3\beta_1} \gg 1, \quad 2\sqrt{2} D D_0 \approx a_0 A \beta_1,$$
$$D_0 \approx \frac{\beta_1}{1-\beta_1} \frac{\exp(\alpha_1^2)}{\alpha_1} \approx \frac{2\ln A}{3(1-\beta_1)} \frac{\exp(\alpha_1^2)}{2\alpha_1^3} \gg \frac{\exp(\alpha_1^2)}{2\alpha_1^3} \gg 1. \tag{32}$$

Thus, the matching is carried out during the stage when the integrals in (30)—(31) are much smaller than D_0 and

$\alpha \gg 1$. In this case we have from (30)–(31)

$$a \sim (\ln t)^{1/2}, \quad c \sim t, \tag{33}$$

corresponding to (18).

In the sequel the quantitative picture of isotropization depends on the magnitude of the parameter β_1. According to current conceptions, $0.1 \lesssim \beta_1 \lesssim 0.5$ (e.g., if one considers as genuinely elementary particles all leptons, the photon, the graviton and the three quarks, then $\beta_1 \approx 0.5$).

The stage (18)–(33) lasts until $t \sim t_F$, $\alpha \sim \alpha_F$, $\Delta H/H \sim 1$, where

$$\frac{\exp\alpha_F^2}{2\alpha_F^3} \approx D_0 \gg 1, \quad t_F \approx t_0 (A \ln A)^2 \gg t_1, \tag{34}$$

after which follows the isotropic stage of expansion, in which the anisotropy of the deformation tensor decreases extremely slowly, proportionally to $\ln^{-1}(t/t_{Pl})$ (cf. (30)–(31), [1,10]):

$$\gamma^{2/3} = \frac{1-\beta_1}{2} A^{2/3} t^2 \left\{1 - \frac{3\beta_1}{\ln[A(t/t_F)^{1.5\beta_1}]}\right\},$$
$$a^2 = 3\left(\frac{1-\beta_1}{2}\right)^{2/3} At/\ln\left[A\left(\frac{t}{t_F}\right)^{1.5\beta_1}\right] = a_0^2 (A\ln A)^2 t/t_F \ln\left[A\left(\frac{t}{t_F}\right)^{1.5\beta_1}\right]. \tag{35}$$

If $1 \ll A \ll 30\beta_1^{3/2} \lesssim 10$, the solution is applicable up to the time

$$\frac{a}{c} \approx A/\ln^{2/3}\left[A\left(\frac{t}{t_F}\right)^{1.5\beta_1}\right], \tag{36}$$

after which the pressure of the free particles in the 1–2 plane becomes important.

The distribution function of the free particles during this stage is isotropic up to small deviations, and the anisotropy of the deformation decreases according to a power law (cf. (22)):

$$\mu \approx (3-5)\left(\frac{t_F}{t}\right)^{1/4} \sin\left[\frac{1}{2}\ln t + \text{const}\right]. \tag{37}$$

In this case the leptons and photons thermalize, having an isotropic distribution function $n(\mathbf{k})$.

We now consider the case of large $A \gg 30\beta_1^{3/2}$ (cf. Fig. 2). At the instant $t \sim t_2$, when the leptons and photons get thermalized, the distribution function of the gravitons is still highly anisotropic (the main fraction of gravitons propagates along the direction $\mathbf{X}^3$);

$$\left(\frac{t_2}{t_F}\right)^{1/2} = \frac{5\cdot 10^4}{(\ln A)^3(1+15\beta_1/\ln A)^2} \gg 1,$$
$$1 \ll \left(\frac{a}{c}\right)_2 = \frac{2A}{(\ln A)^{1/2}(1+15\beta_1/\ln A)^{3/2}} \ll A. \tag{38}$$

The ratio β_2 of the quantity of gravitons to the total number of leptons, photons, gravitons, is of the order 0.15 in our model. Thus, during the stage $t > t_2$ the energy-momentum tensor of matter has the form (up to the time $t \sim t_2$ the contribution of the free particles to the total density of matter, ϵ, is negligibly small)

$$\varepsilon = \frac{3}{8}\left[(1-\beta_1)\left(\frac{A}{\gamma}\right)^{2/3} + \beta\frac{a^2}{\gamma}\right], \quad p_3 = \frac{3}{8}\left[\frac{1-\beta_1}{3}\left(\frac{A}{\gamma}\right)^{2/3} + \beta\frac{a^2}{\gamma}\right],$$
$$p_1 = p_2 = \frac{1-\beta_1}{8}\left(\frac{A}{\gamma}\right)^{2/3}, \quad \beta = \beta_1\beta_2, \tag{39}$$

and the solution of the field equations (13)–(14) is analogous to the solution (30)–(31)

$$a \approx a_0 A\left(\frac{t}{t_F}\right)^{1/2}\left[\ln A \Big/ \left(1 + \frac{15(\beta_1-\beta)}{\ln A} + \frac{3\beta\ln(t/t_F)}{2\ln A}\right)\right]^{1/2},$$
$$c \approx a_0 (\ln A)^2 \left(\frac{t}{t_F}\right)^{1/2}\left(1 + \frac{15(\beta_1-\beta)}{\ln A} + \frac{3\beta\ln(t/t_F)}{2\ln A}\right). \tag{40}$$

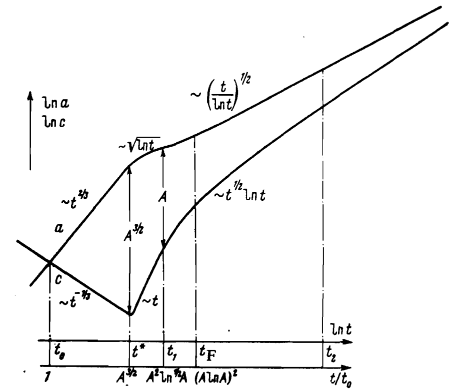

FIG. 2

Thus, the presence of the flux of free gravitons (for $A \gg 1$) leads practically to a freezing of the anisotropy of the deformation at the stage $p = \epsilon/3$, which can cause significant deviations of the microwave background from isotropy up to the present time (cf. [1,16]).

The distribution function of the free particles is isotropic at the instant when $a = c$ (cf. (7)). Therefore if $A^{2/3} \ll 160\beta + 15\beta_1 + 5$ ($(a-c)/c \ll 1$ at the time when the equation of state changes, $t \sim t_c$), then the distribution function of the gravitons is isotropic at the present time and the anisotropy of the background radiation is small. If at the time $t \sim t_c$

$$a \gg c, \quad A^{2/3} \gg 160\beta + 15\beta_1 + 5, \tag{41}$$

then the pressure of the gravitons in the 1–2 plane can be neglected ($p_3 \gg p_1 = p_2$). Then the well-known formulas [1,10] are valid for the description of the evolution. At the stage when the expansion is determined by nonrelativistic matter ($p = 0$, $t > t_c$), $a/c \approx \text{const} \gg 1$, i.e., in this case at the present time there should be a flux of free gravitons.

Thus, the evolution of the directed particle flux in a Bianchi type I metric takes place similarly to the evolution of an isolated standing gravitational wave of small amplitude in a flat nonstationary Universe (a metric of the type VII_0 [16]). The amplitude of the anisotropy of the quadrupole component of the background radiation is determined by the formula (cf. [1,16])

$$\frac{\Delta T}{T} = \frac{1.2\cdot 10^{-3}\beta Z_l}{180\beta + 15\beta_1 + (1-4.5\beta)\ln(t_0/t_{Pl})}, \tag{42}$$

where Z_l is the red shift at the instant of "decoupling" (when the Universe becomes transparent to radiation).

If the instant of "decoupling" coincides with the start of recombination, $Z_l \sim 10^3$ and $\beta_1 \approx 0.5$ ($\beta \approx 0.08$), then the anisotropy of the background radiation computed from Eq. (42) will exceed the observed value. Thus, already from observations of the background radiation, it follows that the Universe expanded isotropically from the very beginning, with (cf. (41), $\beta_1 \approx 0.5$)

$$A \sim t_0/t_{Pl} \ll 100. \tag{43}$$

We have considered above a homogeneous model of the simplest symmetry with a flat comoving space. More complicated models (in particular, models with matter flux) require a special investigation of the process of particle production and evolution of the metric. However, one can already draw some general conclusions, valid for any problem of particle production in cosmology.

1. The influence of the particles produced near the

singularity leads to isotropization of the cosmological expansion (but not to isotropization of the curvature tensor, evening out of inhomogeneities, etc.; thus, this effect is analogous to the first viscosity of fluids), so that in the process of evolution any solution approaches the quasi-isotropic one.

2. In the case of cosmological expansion the gravitation of matter determines the evolution in an essential way, at least starting from the Planck instant of time 10^{-43} s (up to that time classical general relativity is not applicable[17]). Thus, the vacuum solution is impossible in the case of cosmological expansion. The general "eight-functional" vacuum solution[2] can, so it seems, be realized only during the stage of collapse, the duration of the vacuum stage being limited by the Planck value of the curvature of spacetime, since the energy of the particles created at that time is sufficient to change the solution substantially (from the estimates of the paper of Doroshkevich and Novikov[17], no more than 2–3 oscillations are possible in all cases).

4. CONCLUSION

In the preceding sections we have described the process of cosmological expansion of a homogeneous Universe filled with matter consisting of particles which are spontaneously created near the Kasner singularity. The conditions of the problem (production of particles is switched on at the time $t_0 \gg t_{Pl}$) allow one to avoid unknown effects, related to the quantization of spacetime, and to define correctly the vacuum state of the system. It is shown that even for such strong restrictions on the production of particles the expansion nevertheless becomes isotropic at the time

$$t_F \approx t_0 \left(\frac{t_0}{t_{Pl}} \ln \frac{t_0}{t_{Pl}}\right)^2;$$

We recall that in the approximation of completely free particles (cf. Sec. 2, (21))—this happens at the time

$$t_F \approx t_0 \left(\frac{t_0}{t_{Pl}} \ln \frac{t_0}{t_{Pl}}\right)^3.$$

If, as it should be in reality, $t_0 \sim t_{Pl}$, then $t_F \sim t_0 \sim t_{Pl}$. On the other hand, a restriction on the instant of isotropization t_F can be obtained from observations of the microwave background (cf. (43)

$$t_F \ll t_{Pl} 10^7 \sim 10^{-36} \text{ sec.}$$

The authors are grateful to Ya. B. Zel'dovich, for appealing to them to analyze this problem and for discussions, and to I. D. Novikov for discussions and useful remarks.

1)We stress the fact that the condition (2) allows one to make the results of this paper independent of unknown effects related to the quantization of spacetime itself.

2)Among the quantized fields one may also include the inhomogeneous part of the gravitational field which fluctuates on a small scale, and which, when quantized, gives rise to gravitons.

3)It is obvious that ν is finite, probably $\nu \sim 10-15$.

4)Such a problem was first considered in the paper of Doroshkevich et al. [9] In the present paper the matching is carried out more accurately and a solution is constructed which is valid throughout the whole time interval.

5)A constant hadron-hadron scattering cross section or one which increases logarithmically with the energy is obtained on account of scattering under angles which are close to zero and π in the c.m.s., i.e., on account of the regions $t \lesssim m^2 \ll s$ and $u \lesssim m^2 \ll s$.

6)The quick thermalization occurs due to the fact that the interaction cross section (27) increases with the decrease of the energy of the interacting particles. In this case a particle which leaves the flux (along the X^3 axis) as a result of scattering will continue to be scattered on the main mass of particles, although its relative velocity is reduced.

7)At the matching point ($t \sim t_1$) the second derivatives of the functions ln a and ln c experience a discontinuity of the first kind, whereas the first derivatives and the functions a and c themselves are continuous.

1 A. G. Doroshkevich, V. N. Lukash and I. D. Novikov, Zh. Eksp. Teor. Fiz. **64**, 1457 (1973) [Sov. Phys.-JETP **37**, 739 (1973)].

2 V. A. Belinskiĭ, E. M. Lifshitz and I. M. Khalatnikov, Usp. Fiz. Nauk **102**, 463 (1970); Zh. Eksp. Teor. Fiz. **62**, 1606 (1972) [Sov. Phys.-Usp. **13**, 745 (1973); Sov. Phys.-JETP **35**, 838 (1972)].

3 Ya. B. Zel'dovich, ZhETF Pis. Red. **12**, 443 (1970) [JETP Lett. **12**, 307 (1970)]; Preprint, Inst. Appl. Math. No. 66, Moscow 1973, Proc. EGA IAU, Poland, 1973.

4 L. Parker, Phys. Rev. Lett. **21**, 562 (1968); Phys. Rev. **183**, 1057 (1969); Phys. Rev. **D3**, 346 (1971).

5 K. A. Bronnikov and E. A. Tagirov, Preprint P-2-4151 JINR, 1968.

6 A. A. Grib and S. G. Mamaev, Yad. Fiz. **10**, 1276 (1969); **14**, 800 (1971) [Sov. J. Nucl. Phys. **10**, 722 (1970); **14**, 450 (1972)].

7 Ya. B. Zel'dovich and A. A. Starobinskiĭ, Zh. Eksp. Teor. Fiz. **61**, 2161 (1971) [Sov. Phys.-JETP **34**, 1159 (1972)].

8 B. K. Berger, U. of Maryland, Technical Report 73-024, 1972.

9 A. G. Doroshkevich, Ya. B. Zel'dovich and I. D. Novikov, Zh. Eksp. Teor. Fiz. **53**, 644 (1967) [Sov. Phys.-JETP **26**, 408 (1968)].

10 V. N. Lukash, Astrn. Zh. **51**, 281 (1974) [Sov. Astr. **18**, 164 (1974)].

11 V. B. Berestetskiĭ, E. M. Lifshitz and L. P. Pitaevskiĭ, Relyativistskaya kvantovaya teoriya (Relativistic Quantum Theory), p. 1, Nauka 1968, Sec. 68 [Engl. Transl. Pergamon and Addison-Wesley, 1970].

12 Yu. S. Vladimirov, Zh. Eksp. Teor. Fiz. **45**, 251 (1963) [Sov. Phys.-JETP **18**, 176 (1964)].

13 V. G. Gorshkov, Usp. Fiz. Nauk **110**, 45 (1973) [Sov. Phys.-Uspekhi **16**, 322 (1973)].

14 V. V. Ezhela, A. A. Logunov, M. A. Mestvirishvili and V. A. Petrov, Teor. Mat. Fiz. **15**, 153 (1973).

15 I. S. Shikin, Zh. Eksp. Teor. Fiz. **63**, 1529 (1972) [Sov. Phys.-JETP **36**, 811 (1973)].

16 A. G. Doroshkevich, V. N. Lukash and I. D. Novikov, Preprint, Inst. Appl. Math. Moscow, 1974.

17 A. G. Doroshkevich and I. D. Novikov, Astron. Zh. **47**, 948 (1970) [Sov. Astron. **14**, 673 (1971)].

Translated by M. E. Mayer
156