The acceleration of atoms by light

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The forces acting on an atom in a resonant field may be of three types. A force (1), due to the scattering of photons with the spontaneous emission rate, acts on an atom in the field of a traveling plane wave. In the field of two oppositely moving waves a gradient force (3), due to stimulated scattering of the quanta from one light beam into the other, acts on the atom. The forces (1) and (3) are well known. In this article it is shown that a force which arises from a certain combination of spontaneous and stimulated transitions in the atom, also acts on an atom in the field of a standing wave. This force is spatially homogeneous and resonantly depends on the velocity of the atom relative to the wave. In magnitude this force occupies an intermediate position between forces (1) and (3). The possibility of using these forces to accelerate atoms is discussed. The investigation of this problem is of interest for applications such as the production of dense, monoenergetic atomic beams with energies up to several keV and for the selection of excited atoms and isotopes. It is shown that an atomic cluster of density 10^{15} cm^{-3} can be accelerated to an energy of 10 keV in intense light beams with energy fluxes of the order of 5×10^9 W/cm^2. In light beams of medium intensity—energy fluxes of the order of 10^2 to 10^5 W/cm^2—atoms can be accelerated and heated up to thermal energies. As an example, the feasibility of selecting He($2\ ^3S$) atoms and producing soft x-ray generation in the $2\ ^1P$-$1\ ^1S$ transition is assessed.

1. INTRODUCTION

The possibility of accelerating atoms by light pressure is investigated in the present article. The light pressure arises as the result of photon scattering by the atom. In this connection the force acting on the atom depends substantially on the spatial structure of the field.

The following force acts on an atom, whose lower level is the ground state (or a metastable state) and whose upper level has a decay rate γ, in the presence of a traveling plane-wave field $E \sim \exp[i\omega t - ikx]$ [1,2]

$$F_{sp} = \hbar k \gamma W, \quad W = \frac{|dE|^2}{((\omega-\omega_0)^2+\gamma^2/4)\hbar^2+2|dE|^2}, \tag{1}$$

where W is the probability that the upper level is occupied, and ω_0 and d are the frequency and dipole moment of the transition. The maximum value of the force F_{sp} is limited by the spontaneous emission rate. $F_{sp} \approx \hbar k \gamma/2$ in a strong field. The characteristic order of magnitude of this quantity is 10^{-4} to 10^{-3} eV/cm.

In the field of oppositely moving waves (with real amplitudes E_1 and E_2)

$$E = E_1 \exp[i(\varphi_1 - \omega_1 t + kx)] + E_2 \exp[i(\varphi_2 - \omega_2 t - kx)] \tag{2}$$

there is a gradient force, due to stimulated transitions, acting on the atom. In the presence of such a field the atom may absorb a quantum with momentum $\hbar k$ and emit a quantum with momentum $-\hbar k$. The frequency of such transitions is of the order of $2\ dE/\hbar$, so the effective force is $F \sim 2\ kdE$. The reverse order of transitions, with the absorption of momentum $-\hbar k$ first and then the emission of momentum $\hbar k$, is also possible. Therefore, the final expression for the force F significantly depends on the phase difference $\varphi = \varphi_1 - \varphi_2 + (\omega_1 - \omega_2)t + 2kx$ between the waves which are moving in opposite directions:

$$F = F_0 \sin\varphi, \quad F_0 = 4kd^2E_1E_2/\hbar\Delta,$$
$$\Delta = {}^1/_2(\omega_1+\omega_2) - \omega_0. \tag{3}$$

Here for simplicity the case of perturbation theory is used, $\hbar\Delta \gg dE$. It is also assumed that $\omega_1 - \omega_2 \ll \Delta$. The exact expression for the gradient force will be derived in Sec. 6.

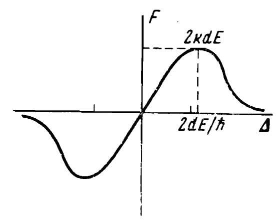

FIG. 1

The force F is schematically shown in Fig. 1 as a function of the detuning; the force reaches its maximum value, $F_{max} = 2kdE$, at $\Delta \sim \Delta_m = 2dE/\hbar$. In strong fields, close to the critical ionization field E $\sim 10^6$ V/cm,[1)] we have the estimate $F_{max} \sim 1$ keV/cm and $\Delta_m \sim 10^{13}$ Hz. In effect the force F can be used to accelerate trapped atoms by varying the frequencies of the oppositely moving waves:[2)][5]

$$\varphi_1 - \varphi_2 = \dot{\Omega} t^2. \tag{4}$$

Such an acceleration method is of interest for the production of monoenergetic atomic beams with energies up to a few keV. The density of atoms, which can be thus accelerated, is estimated in the second section.

Since the force F only acts on resonant atoms, it can be used to spatially separate excited and unexcited atoms. In order to do this, it is sufficient to impart a transverse momentum to the excited atom which is comparable with the thermal momentum. In order to change the velocity to 10^4 or 10^5 cm/sec, it is necessary to give $\sim 10^2$ to 10^3 recoil impulses (each of momentum $\hbar k$) to the atom. One can use the acceleration of trapped atoms mentioned above for this purpose. Other acceleration mechanisms, connected with the violation of the adiabatic approximation and with the passage of the atoms through a modulated light beam, have been previously considered.[6]

The process of resonant transfer of the quanta from one light beam to the other is discussed in Sec. 3, and the possible acceleration effect is estimated.

A simple and rather effective method of selecting excited atoms is connected with heating the atoms in a field having a broad frequency spectrum. This problem is investigated in Sec. 4. As an example, in Sec. 5 we study the possibility of creating population inversion and obtaining laser generation in helium for the resonant $2\,^1P - 1\,^1S$ transition (wavelength 584 Å) with the aid of light pressure effects.

In the present article we show (see Sec. 6) that, in addition to the forces (1) and (3), in the field of a standing wave there is also another force on the atom, which arises as a result of a certain combination of spontaneous and stimulated transitions occurring in the atom. This force is spatially homogeneous and resonantly depends on the velocity of the atom relative to the wave. In magnitude this force occupies an intermediate position between the forces (1) and (3). In Sec. 7 we estimate the possible effect of using this force to accelerate atoms.

2. THE ACCELERATION OF TRAPPED ATOMS

In the process of accelerating the trapped atoms, one wave is absorbed and the other wave is intensified. Although we consider an optically transparent medium, nevertheless the process of absorption and intensification takes place rather effectively, being proportional to the magnitude of the atoms' acceleration. The mechanism for the transfer of energy from one wave to the other is related to diffraction scattering. In fact, the trapped atoms form a plane diffraction grating with a period $\lambda/2$ between the planes, where λ is the wavelength. In the accelerated reference frame in which the shape of the force F does not depend on the time, the equilibrium position x_0 of the entrapped atoms is determined from the condition

$$Ma=F_0 \sin 2kx_0, \tag{5}$$

where M is the mass of the atom, and the acceleration $a = \dot{\Omega}/k$. If the acceleration is equal to zero, the equilibrium position coincides with the nodes of the standing wave sin 2kx, and there is no scattering of the waves. For $a \neq 0$ the equilibrium position is displaced somewhat from the nodal position, and diffraction scattering appears, which is proportional to a. If we neglect relativistic effects of order $l/tc \ll 1$ (l is the characteristic length and t is the duration of the process), then in the comoving coordinate system we have the time-independent Maxwell equation

$$\frac{d^2E}{dx^2}+k\varepsilon(x)E=0, \quad \varepsilon=1+\frac{4\pi d^2 n(x-x_0)}{\hbar\Delta}, \tag{6}$$

where n(x) is the density of the trapped atoms in the inertial reference frame, this density being a periodic function which only contains harmonics of the form cos 2mkx, where m is an integer.

If the density of atoms is small, the amplitudes in expression (2) are slowly varying functions of the coordinates. Introducing the symbols n_0 and n_1, respectively, for the averages of the quantities n(x) and the first harmonic n(x) cos 2kx over one period, we obtain

$$\frac{dE_{1,2}}{dx}=\frac{2\pi d^2 kn_1}{\hbar\Delta}E_{2,1}\sin 2kx_0. \tag{7}$$

We shall use the equilibrium condition (5). Then we obtain the following expressions for the energies $I_{1,2} = |E_{1,2}|^2/2\pi$ of the light beams:

$$I_1(x)=I_1(0)+{}^1\!/_2 Man_1x, \quad I_2(x)=I_2(l)+{}^1\!/_2 Man_1(x-l). \tag{8}$$

The dependence of the intensity on the coordinate x is shown schematically in Fig. 2: The beam energies are

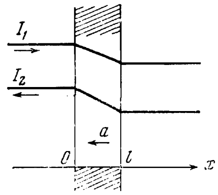

FIG 2

constant outside the layer of thickness l, and they vary linearly inside the layer.

Let us define the permissible thickness l of the gas layer as the distance over which the intensity decreases by a factor of two:

$$I/l=Man_1, \tag{9}$$

which implies that the inertial forces are in equilibrium with the gradient of the electromagnetic pressure. We emphasize that uniformly accelerated motion of trapped atoms is possible only under the condition

$$a<a_c=F_0/M. \tag{10}$$

In the opposite case, no entrapped atoms are present.

As an example, let us estimate the density of atoms in a layer of thickness 1 cm, with the atoms having an acceleration $a = 10^{14}$ cm/sec^2. For $E \sim 10^6$ V/cm and the mass M of the helium atom, we have $n = 2.4 \times 10^{15}$ cm^{-3}. Such a quantity of gas can be accelerated to a velocity of 10^8 cm/sec during a time interval of 10^{-6} sec; here the energy of the atoms will be on the order of 20 keV. We note that in this case $a_c = 3.6 \times 10^{14}$ cm/sec^2, i.e., condition (10) is satisfied with a certain amount of margin. The depth of the potential well in which the particle is found has a magnitude dE $\sim 10^{-2}$ eV, so at room temperatures an appreciable fraction of the resonant atoms can be trapped. In the approximation assumed here, the total energy of the light beams is conserved:

$$I_1(0)+I_2(l)=I_1(l)+I_2(0). \tag{11}$$

This is associated with the fact that the energy of the light beam is very much greater than the energy of the accelerated atoms. Thus, in the indicated example the energy flux in each beam amounts to 5×10^9 W/cm^2. 5×10^3 J of energy pass through 1 cm^2 during the interaction time of 10^{-6} sec. In this connection the atoms, which are located in a volume of 1 cm^3, acquire an energy on the order of 8 J.

Thus, in the present case the efficiency of the acceleration mechanism under consideration amounts to 1.6×10^{-3}. We note that the efficiency increases in proportion to the interaction time.

Variation of the frequency of the field. The acceleration of the trapped atoms is determined by the rate of change of the difference frequency of the field, $a = \dot{\Omega}/k$. For $a = 10^{14}$ cm/sec with $k = 10^5$ cm^{-1}, it is necessary to have $\dot{\Omega} = 10^{19}$ sec^{-2}.

Let us consider the following mechanism for varying the frequency of the field, based on the use of phase modulation. Let one of the beams, let us say E_1, pass through a transparent dielectric whose dielectric constant $\delta\epsilon = \epsilon - 1$ is changing with time uniformly over its entire volume (because of, for example, optical pumping). In this connection, after passing through the dielectric whose length is L the phase of the field E_1 is given by $\varphi_1 = (1/2)kL\delta\epsilon(t)$. The "switching" of the dielectric must take place according to the law

$$\delta\varepsilon(t) = \delta\varepsilon_0 (t/\tau)^2, \tag{12}$$

from which we obtain

$$\dot{\Omega} = kL\delta\varepsilon_0/\tau^2 \tag{13}$$

for the rate of change of the frequency. This quantity can become large if the length L is large. Thus, the order of magnitude we are interested in, $\dot{\Omega} = 10^{19}$ sec^{-2}, can be obtained for $\delta\epsilon_0 \sim 1$, $\tau \sim 10^{-6}$ sec, and $L \sim 10^2$ cm.

Thus, in the present model the time during which the atoms interact with the field is determined by the characteristic "switching time" of the dielectric according to the law (12).

3. TRANSFER PROCESSES

Now let us investigate the possibility of using modulated light beams to accelerate the atoms. In this case the question can only involve small energies of acceleration, when the atom changes its momentum by 10^2 to 10^3 $\hbar k$. The adiabatic approximation is violated in the interaction of an atom with a strongly modulated light beam, and instead of formula (3) for the force it is necessary to use the general expression

$$F = \hbar\left(p\frac{d}{dx}V^* + \text{c.c.}\right), \quad V = \frac{dE}{\hbar}, \tag{14}$$

where p is the dipole moment (measured in units of d) induced by the field.

If a traveling pulse of light, contained within the time interval between t_1 and t_2, falls on an atom, after its passage the momentum of the atom is changed by the following amount:

$$\int_{t_1}^{t_2} dt\, F(t) = {}^1/_2\hbar k[q(t_2) - q(t_1)], \tag{15}$$

where q(t) denotes the difference between the populations of the upper and lower levels. This expression is obtained from formulas (14) and (19) (see below). It is clear from it that, under the influence of a π pulse[7] an atom, which is originally in its ground state, goes into the excited state and changes its momentum by $\hbar k$. If a π pulse traveling in the opposite direction again acts on the atom, the atom returns to the ground state and its momentum again changes by $\hbar k$. The total change of the atom's momentum per cycle is equal to $2\hbar k$.

It is important that π pulses, which are traveling in opposite directions, act on the atom in strictly alternating order. Such conditions can be approximately realized in a field of the following special type. Let us consider a field E_1, generated by a laser in the self-locking mode, in the form of a periodic sequence of rectangular π pulses:

$$E_1(t) = \begin{cases} E_1 = \text{const} \neq 0, & 0 < t < \tau_+ \\ 0 & \tau_+ < t < \tau_+ + \tau_- \end{cases}, \tag{16}$$

where $\tau = \tau_+ + \tau_-$ is the period of the function $E_1(t)$ and, in addition, the condition $\tau_+ \ll \tau_-$ is satisfied. The frequency of the field $\omega_1 = \omega_0$, and the amplitude satisfies the condition for a π pulse, namely, $2dE_1\tau_+ = \pi$.

We shall assume the amplitude of the field E_2 to be constant and to also satisfy the π pulse condition, $2dE_2\tau_- = \pi$, and the frequency $\omega_2 = \omega_0$. Thus, the atom is excited and absorbs a quantum $\hbar k$ during the small time interval τ_+, and under the influence of the weak field $E_2 = E_1\tau_+/\tau_- \ll E_1$ the atom undergoes a transition to the ground state and emits a quantum $-\hbar k$. As a result a force acts on the atom; after averaging over the period τ this force takes the form

$$\bar{F}(t_n) = 2\hbar k q(t_n)/\tau, \tag{17}$$

where $t_n = n\tau$ and n is an integer.

Since each of the pulses, which transfer the atom to the ground state or to the excited state, differs somewhat from a π pulse, q(t) varies slowly with the passage of time. The deviation from a π pulse determines the maximum number of periods N during which the force (17) keeps the same sign. In order to estimate N we introduce the small parameters ϵ_+ and ϵ_- which take the deviation from π pulses into account:

$$\begin{gathered} 2dE_2\tau_- = \pi + \varepsilon_-, \\ 2d|E_1 e^{i\varphi_1 + ikx} + E_2 e^{i\varphi_2 - ikx}|\tau_+ = \pi + \varepsilon_+. \end{gathered} \tag{18}$$

If ϵ_+ and ϵ_- were constant quantities which did not depend on the number of the pulse, then a change in the sign of $q(t_m)$ would occur during the time interval τ/ϵ. The corresponding estimate for the number of periods is $N \sim \epsilon^{-1}$, where $\epsilon \sim \epsilon_+ \sim \epsilon_-$.

In the other limiting case when the signs of $\epsilon_\pm$ change randomly from pulse to pulse, the characteristic value is $N \sim \epsilon^{-2}$.

In our case, according to (18) $\epsilon_\pm$ contains both a constant component, due to the inexact choice of the amplitudes and frequencies of the fields E_1 and E_2, and an alternating part associated with the interference between the fields E_1 and E_2. The question is asked, what about N: Is it of order ϵ^{-1} or ϵ^{-2}? The following calculations amount to finding the criteria for the case $N \sim \epsilon^{-2}$.

In order to effectively regard the phase $\varphi = \varphi_1 - \varphi_2$ as a random quantity, it is necessary to assume that φ depends on the time. In other words, we introduce a certain small amount of detuning between the frequencies of the fields E_1 and E_2. The problem consists in finding the dependence q(t). In order to solve this problem it is necessary to use the equations of motion for p(t) and q(t):[3)]

$$\dot{p} = iVq, \quad \dot{q} = 2ipV^* + \text{c.c.} \tag{19}$$

In the model of rectangular pulses which we have adopted, it is easy to find the transition matrix from the state of the atom at t_n to the state at t_{n+1}. In the linear approximation with respect to $\epsilon_\pm$ we find

$$\begin{gathered} p(t_{n+1}) = \exp(-i\psi_n)p(t_n) + i\varepsilon_n q(t_n), \\ q(t_{n+1}) = q(t_n) + 2i\exp(-i\psi_n)\varepsilon_n^* p(t_n) + \text{c.c.}, \\ \psi_n = 4kx(t_n) + 2(\varphi_1(t_n) - \varphi_2(t_n)), \\ \varepsilon_n = \exp[i\varphi_2(t_n)/2](\varepsilon_-(t_n) + \varepsilon_+(t_n)\exp[-i\psi_n/2]). \end{gathered} \tag{20}$$

The excess population changes very little with a change of n by unity, but the phase of the dipole moment generally changes substantially. Changing to the interaction representation

$$p(t_n) = r(t_n)\exp\left(-i\sum^n \psi_k\right) \tag{21}$$

and taking into consideration that $r(t_n)$ is a slowly varying function, Eq. (20) can be represented in the following differential form:

$$\dot{r} = i\mu(t)q, \quad \dot{q} = 2i\mu^* r + \text{c.c.} \tag{22}$$

The function

$$\mu(t) = \frac{\varepsilon(t)}{\tau}\exp\left(i\int_0^t \frac{dt}{\tau}\psi(t)\right)$$

which appears here can be approximated in the following way:

$$\mu(t) = \frac{|\varepsilon|}{\tau} e^{i\dot{\omega}(t-t_0)^2}, \; \dot{\omega} = \frac{\dot{\varphi} + 2kv}{\tau}, \tag{23}$$

where t_0 is a slowly varying function of the time and of the initial phases (the explicit form of t_0 is unimportant).

Thus, Eqs. (22) describe the "passage" of the atom through the resonance at $t = t_0$. In this connection the sign of $q(t)$ does not change if the condition for rapid passage through the resonance, $\dot{\omega} \gg (\epsilon/\tau)^2$, is satisfied.[7] Since we are interested in the limit of small velocities, the latter inequality implies the following condition on the frequency detuning of the fields E_1 and E_2:

$$\varepsilon^2 \ll \tau\dot{\varphi} \ll 1. \tag{24}$$

Under this condition the force $\overline{F}(t)$ will have a fixed sign in the ϵ-approximation. A change in the sign of $q(t)$ can only occur during a time interval of order τ/ϵ^2, so that

$$N \sim \varepsilon^{-2}. \tag{25}$$

We note that theoretically ϵ_- may be a small quantity whereas $\epsilon_+ \gtrsim \tau_+/\tau_-$. Thereby we have proved that the maximum number of transmitted quanta is given by

$$N_{max} \sim (\tau_-/\tau_+)^2. \tag{26}$$

In the self-locked mode of laser generation the period of the field τ is determined by the length of the resonator, and the parameter τ_+/τ_- is the number of locked modes.[8] For $N \sim 10^2$ not less than 10 locked modes are necessary.

Comparing $\overline{F}$ with the maximum possible value of the force F_{sp} (given by Eq. (1)) we obtain

$$\overline{F} = 4F_{sp}/\gamma\tau.$$

For $\tau \sim 3 \times 10^{-9}$ sec (the length of the resonator is 50 cm) and $\gamma \sim 10^7$ Hz, we find $\overline{F}/F_{sp} \sim 120$. In this case the energy flux necessary for the π pulse of the strong field with $\tau_+ = \tau/10$ and $d \sim 1$ D amounts to 1.2×10^4 W/cm^2, and the energy flux of the weak field is on the order of 1.2×10^2 W/cm^2.

4. HEATING OF THE ATOMS

Up to now we have only considered coherent mechanisms of acceleration, when the change of the atom's momentum will be proportional to the number of quanta scattered or to t, and the change of energy is of order t^2. Now let us consider a stochastic mechanism of acceleration in which the energy is proportional to t^2. In this case the conditions necessary for the acceleration of atoms are simpler than for the case of coherent acceleration.

Thus, we shall assume that the oppositely moving waves in Eq. (2) have rather broad frequency spectra. Here the force F is assumed to vary stochastically with the time. If the width τ^{-1} of the frequency spectrum satisfies the conditions

$$\tau^2 kF_0/M \ll 1, \quad kv\tau \ll 1, \tag{27}$$

then during the time τ the atom moves a distance which is small compared to the wavelength. Therefore, the random force F can be regarded as spatially uniform.

The change of energy δT, associated with the acceleration of the atoms along the direction of the force F, has the form

$$\delta T = t \int_0^\infty dt \langle F(xt) F(x0) \rangle / M \approx t\tau \langle F^2 \rangle / M. \tag{28}$$

Consequently the displacement δx in the indicated direction can be expressed as

$$\langle \delta x^2 \rangle \sim t^3 \tau \langle F^2 \rangle / M^2. \tag{29}$$

We note that such an acceleration mechanism cannot act for an unlimited period of time. As soon as the second inequality in (27) is violated, the atom begins to effectively interact with only one of the waves and the acceleration slows down. However, for small velocities of the order of thermal velocities, this limitation may turn out to be unimportant.

Let us estimate the effect of the acceleration of metastable helium atoms for the following case. We take $E = 3 \times 10^3$ V/cm, $\tau \sim 10^{-10}$ sec, and $t \sim 10^{-4}$ sec. Then the energy of the atoms is $\delta T \sim 3 \times 10^{-2}$ eV, and the displacement $\delta x \sim 10$ cm. Thus, He atoms in the $2\,^3$S state can be effectively extracted from an atomic beam by using two, oppositely moving light beams with an energy flux of 5×10^4 W/cm^2. Here $\Delta_m \sim 1.2 \times 10^{11}$ Hz.

5. THE PRODUCTION OF AN INVERTED POPULATION

Now let us consider the possibility of using light pressure to select excited and unexcited atoms. In order to extract excited atoms from an atomic beam, one can use the acceleration of trapped atoms. For this purpose it is sufficient to have a relatively small rate of change of the frequency: $\dot{\Omega} \sim 10^{16}$ sec^{-2}. Such a method is of interest since here the active atoms form a diffraction grating. Therefore, in this case effectively all of the atoms can radiate, and the small factor $\gamma/k_r v$ is not present in the gain. However, for this to happen it is necessary that the radiated wavelength $2\pi/k_r$ must satisfy the condition $\lambda_r \lesssim \lambda/2$. In the case $\lambda_r \ll \lambda$ the effective Debye-Waller factor becomes small. It is precisely the last case which occurs in the transition from the metastable state of helium to the ground state. Therefore, here the diffraction grating turns out to be ineffective.

The other possibility is related to heating of the atoms. Here the rate of extracting excited atoms from the beam may be very large, of the order of 10^{22} cm^{-3}-sec^{-1}. This implies that the density of the active atoms is determined in practice by the density of these atoms in the beam. A density of He atoms in the $2\,^3$S state of the order of 10^{13} cm^{-3} was obtained in the work by Fugol' and Pakhomov.[9]

Let us estimate the possible gain α associated with the transition from the metastable state to the ground state. The $2\,^3P - 1\,^1S$ transition takes place during 10^{-2} sec, but the $2\,^3S - 1\,^1S$ transition occurs during a much longer time interval. In this connection the factor $\gamma/k_r v \sim 10^{-9}$ or much smaller, so that an appreciable gain can be obtained only at very large densities.

A more realistic possibility consists in the utilization of the two-step transition $2\,^3S - 2\,^1P$ and $2\,^1P - 1\,^1S$. In this connection the intercombination transition $2\,^3S - 2\,^1P$ (wavelength 8720 Å) must occur during a time interval of the order of or smaller than the lifetime $1/\gamma$ of the $2\,^1P$ state, where $\gamma = 2 \times 10^9$ Hz. We can determine the required intensity of the 8720 Å radiation from the condition $2\,dE \sim \hbar\gamma$. Using $d \sim 2 \times 10^{-3}$ D as the dipole moment of the intercombination transition,[10] we obtain

$E \sim 1.5 \times 10^5$ V/cm or an energy flux of the order of 1.2×10^8 W/cm^2. Thus, one can obtain an inverted population of the 2 ^{1}P level relative to the ground state. In terms of the intensity of the 2 ^{1}P – 1 ^{1}S transition the gain has the form

$$\alpha = 0.4\lambda_r^2\gamma n/k_r v \sim 2.7\cdot 10^{-13}\, n, \tag{30}$$

where n is the density of active atoms. Substituting here $n \sim 10^{13}$ cm^{-3}, we obtain $\alpha = 2.7$ cm^{-1}. If we assume that the factor limiting the density of metastable helium atoms is the Penning process, which has a cross section of the order of 10^{-14} cm^2, [11] then for the lifetimes $t \sim 10^{-5}$ sec of interest to us we have $n \sim 10^{14}$ cm^{-3}. Consequently $\alpha \sim 27$ cm^{-1}. Perhaps it would be more promising to use helium molecules in order to obtain the maximum gain.

One can also consider the same scheme of transitions for the metastable states of the Li II and Be III ions.

6. THE FORCE ACTING ON AN ATOM IN A RESONANT FIELD

In this section we study in more detail the force acting on an atom in the presence of a resonant field of the form $E(x)e^{-i\Delta t}$. If the induced dipole moment p of the atom is able to instantaneously follow the field, i.e., if $p = Vf(|V|^2)$, then we are dealing with a gradient force $F = \nabla\langle H\rangle$, where $\langle H\rangle$ is the average energy of the atom in the presence of the external field. However, if a certain amount of retardation appears, this leads to the appearance of a force component which is constant in space. The retardation, which is associated with resonance phenomena in the presence of a nonmonochromatic field, has been previously investigated.[6] The effect of retardation due to dissipative processes is studied below.

Let us consider the case of a quasistationary field $V(x)e^{-i\Delta t}$ in detail:

$$\dot{V} \ll \Delta V. \tag{31}$$

With relaxation taken into account, the equations of motion for the dipole moment p and the difference q of the populations have the form

$$\dot{p} + (\gamma_\perp - i\Delta)p = iVq, \tag{32}$$

$$\dot{q} + \gamma q = 2i(V^* p - \text{c.c.}). \tag{33}$$

The relaxation frequency of the dipole moment is denoted by $\gamma_\perp$. For the free atom (in the absence of collisions) we have $\gamma_\perp = \gamma/2$. Here the distinction between $\gamma_\perp$ and $\gamma/2$ is preserved in order to emphasize certain characteristic features of the retardation effects in weak fields.

As has already been mentioned in the Introduction, we are interested in the regime of strong fields and large detuning:

$$V \sim \Delta \gg \gamma. \tag{34}$$

But even upon the fulfillment of this condition, it is generally impossible to neglect relaxation in Eqs. (31) and (32) since there is one more parameter kv having the dimension of a frequency (v is the velocity of the atom relative to the wave), which may be of the order of γ.

According to conditions (31) and (34) the approximate solution of Eq. (32) can be represented in the form of an expansion in powers of $1/\Delta$:

$$p = -\frac{Vq}{\Delta} + i\left(\frac{d}{dt} + \gamma_\perp\right)\frac{Vq}{\Delta^2}. \tag{35}$$

The second term in this expression is small, but it is the only one which gives a contribution to the equation for q(t):

$$\dot{q}[1+\varepsilon(t)] + [\dot{\varepsilon}(t) + \gamma + \gamma_\perp\varepsilon(t)]q = -\gamma. \tag{36}$$

Here we have introduced the dimensionless intensity of the field, $\epsilon(t) = 4|V(t)|^2/\Delta^2$.

The solution of Eq. (36) has the following form:

$$q(t) = -\gamma\eta(t)\int_{-\infty}^{t} dt'\eta(t')\exp\left(-\int_{t'}^{t} d\tau\Gamma(\tau)\right),$$
$$\eta(t) = \frac{1}{(1+\varepsilon(t))^{1/2}}, \quad \Gamma(t) = \frac{\gamma + \gamma_\perp\varepsilon(t)}{1+\varepsilon(t)}. \tag{37}$$

In order to reduce the solution to simple, final formulas, let us investigate the two limiting cases of weak and strong modulation $\epsilon(t)$. We note that in the rest frame of the wave we have

$$\varepsilon(x) = \varepsilon_0 + \varepsilon_1(x), \quad \varepsilon_0 = \frac{4}{\Delta^2}(V_1^2 + V_2^2) = \text{const};$$
$$\varepsilon_1(x) = \varepsilon_{10}\cos 2kx, \quad \varepsilon_{10} = 8V_1V_2/\Delta^2, \tag{38}$$

so that the depth of the modulation is determined by the parameter ϵ_{10}/ϵ_0.

The Case of Weak Modulation

First let us consider the limit $\epsilon_1 \ll \epsilon_0$. In this case in the linear approximation with respect to ϵ_1 we find

$$q(t) = q_0 + A\varepsilon_1(x(t)) + \gamma B\int_0^\infty d\tau\exp(-\Gamma_0\tau)\,\varepsilon_1(x(t-\tau)); \tag{39}$$

$$q_0 = -\frac{\gamma}{\Gamma_0(1+\varepsilon_0)}, \quad \Gamma_0 = \frac{\gamma + \gamma_\perp\varepsilon_0}{1+\varepsilon_0},$$
$$A = -\frac{q_0}{2(1+\varepsilon_0)^{1/2}}, \quad B = \frac{(2+\varepsilon_0)\gamma_\perp - \gamma}{\Gamma_0(1+\varepsilon_0)^3}. \tag{40}$$

The last term in the expansion (39) with respect to ϵ_1 describes the retardation effect of interest to us.

Let us turn our attention to the behavior of the coefficient B in a weak radiation field. When $\epsilon_0 \ll 1$ we have either $B \approx 2\gamma_\perp/\gamma - 1$ for $\gamma_\perp > \gamma/2$ or $B \approx \epsilon_0/2$ for $\gamma_\perp = \gamma/2$. Such an external field dependence of B can be understood from the following considerations.

The change in the population difference is proportional to the energy radiated by the atom per unit time. The radiated energy consists of coherent and incoherent parts. The coherent component in resonance fluorescence is proportional to the intensity of the external field and locally depends on $\epsilon(x)$; it corresponds to the second term in Eq. (39).

For $\gamma_\perp = \gamma/2$ (in this case the single-photon incoherent scattering is forbidden by the law of energy conservation) the incoherent component is of the order of the square of the external field intensity and is a nonlocal function of $\epsilon(x)$. For $\gamma_\perp > \gamma/2$ the prohibition against single-photon scattering is lifted, and the incoherent component becomes of first order in the external field intensity (for more details see, for example,[12]).

Calculating the force F with the aid of Eqs. (14) and (39), we assume that the atom is infinitely heavy and moves with constant velocity so that x = vt. As a result we obtain

$$F(x) = f(x) + \bar{f}, \quad f(x) = -{}^1/_4\hbar\Delta\, q_0\frac{d}{dx}\varepsilon_1(x); \tag{41}$$

$$\bar{f} = f_0\frac{2\Gamma_0 kv}{4(kv)^2 + \Gamma_0^2}, \quad f_0 = {}^1/_4\hbar k\Delta(1+\varepsilon_0)B\varepsilon_{10}^2, \tag{42}$$

Here f(x) is the usual gradient force, obtained in the linear approximation with respect to ϵ_{10}, and $\bar{f}$ is the spatially constant component which is related to the retardation and of order ϵ_{10}^2 (the upper bar indicates averaging over the spatial oscillations). The velocity dependence has a resonant behavior of exactly the same type as the real part of the atomic susceptibility. In a weak field we have $\bar{f} \sim \epsilon_{10}^2 \epsilon_0 \sim \epsilon^3$ for $\gamma_\perp = \gamma/2$ and $\bar{f} \sim \epsilon^2$ for $\gamma_\perp > \gamma/2$.

The Case of Strong Modulation

Now let us proceed to the case of strong modulation, when $\epsilon_{10} \sim \epsilon_0 \sim 1$. Here we shall assume the atom's velocity to be large:

$$kv \gg \gamma. \tag{43}$$

Then the formula for the force $\bar{f}$ takes the following form:

$$\bar{f} = CdE\gamma/v, \quad E = (E_1^2 + E_2^2)^{1/2} \cdot 2^{-1/2}. \tag{44}$$

The dimensionless factor C which enters here depends on the average energy ϵ_0 of the field (or, for a fixed value of the field, on the detuning Δ) and on the modulation depth ϵ_{10}/ϵ_0.

In order to find C it is necessary to evaluate the integral (37) with inequality (43) taken into consideration. In this connection it is convenient to separate the functions $\eta(t)$ and $\Gamma(t)$ into their average values $\bar{\eta}$ and $\bar{\Gamma}$ plus the variable parts $\eta(t) - \bar{\eta}$ and $\Gamma(t) - \bar{\Gamma}$. The integrals of the latter quantities are small; therefore, formula (37) can be substantially simplified, and we finally obtain

$$C = \frac{\hbar\Delta}{2dE}\left[\frac{\bar{\eta}}{\bar{\Gamma}}\overline{\left(\frac{\Gamma}{\eta}\right)} - 1\right]. \tag{45}$$

Thus, the force $\bar{f}$ can be expressed in terms of the correlation between the fluctuations of the parameters η and Γ, which describe the effects of saturation and attenuation in the atom.

The result of averaging in Eq. (45) can be expressed in terms of Legendre functions (see the Appendix). A plot of C as a function of Δ for a fixed energy of the external field and for $\gamma_\perp = \gamma/2$ is shown in Fig. 3. Curve 1 refers to the case $E_2 = E_1$, and curve 2 corresponds to the case $E_2 = (1/2)E_1$. It is clear from the graph that the coefficient C may increase substantially with increasing depth of the field modulation. The maxima of the curves are noticeably displaced towards the side corresponding to small values of the detuning. In the case of curve 1 we have $C \sim C_{max} \approx 0.1$ for $\Delta \approx 0.2\ dE/\hbar$. Using this value of C, let us estimate the order of magnitude of the force $\bar{f}$. The maximum force is reached for $v \sim \gamma/2k$,

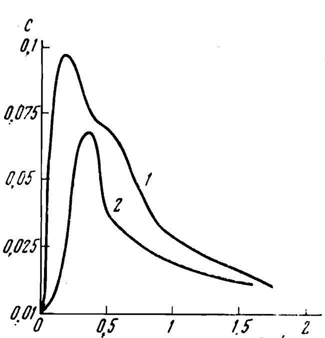

FIG. 3

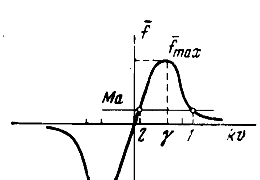

FIG. 4

and $\bar{f}_{max} \sim CdkE$. Thus, we have $\bar{f}_{max} \sim 10^2$ eV/cm for the resonance transition in He($2\,^3S$) with $E \sim 10^6$ V/cm. However, in actual fact the force $\bar{f}$ can be utilized only for velocities much greater than $\gamma/2k$ (see Sec. 7).

In concluding this section we again present the exact expression for the gradient force in a field of arbitrary intensity. Such an expression was previously investigated in [5]; there, however, relaxation processes were not taken into consideration. The potential U(x) of the gradient force depends on the atom's velocity. This dependence simplifies in the limit of small and large velocities.

In the case $kv \gg \gamma$ we have

$$U(x) = \hbar\Delta\gamma\bar{\eta}/2\bar{\Gamma}\eta(x). \tag{46}$$

The function $\eta^{-1}(x)$ determines the magnitude of the Stark splitting in the external field.

For small velocities, $kv \ll \gamma$, the potential has the form

$$U(x) = \frac{\hbar\Delta\gamma_\perp}{4\gamma}\ln\left(1 + \frac{\gamma}{\gamma_\perp}\varepsilon(x)\right). \tag{47}$$

The two potentials (46) and (47) agree for small radiation energies.

7. THE ACCELERATION OF ATOMS

Thus, an average force $\bar{f}$ acts on an atom moving in the field of a standing wave; its sign is determined by the detuning: For $\Delta < 0$ the atom is decelerated by the wave, and for $\Delta > 0$ it is accelerated. The possible acceleration effect is estimated below. According to Eq. (44), under steady-state conditions the atom's energy is a linear function of the time:

$$\tfrac{1}{2}Mv^2 = CdE\gamma t. \tag{48}$$

In such a regime the atom gains energy too slowly: In the extremely strong fields mentioned above, an energy of 1 eV is attained only over distances $\sim 10^2$ cm.

The acceleration effect can be substantially increased if the frequency of one of the oppositely moving waves changes linearly with the time, so that the atom is in the field of a uniformly accelerated wave. Qualitatively the picture of the acceleration in this case can be understood from Fig. 4. In the rest system of the wave the force $\bar{f}(v)$ is balanced by the inertial force Ma on the atom (a denotes the acceleration of the traveling wave). In this connection there are two points of equilibrium, 1 and 2. Point 2 is unstable with respect to small oscillations, but point 1 is stable. The magnitude of the force $\bar{f}$ is determined by the average equilibrium velocity v_1 of the atom relative to the wave.

Up to now we have regarded the atom as infinitely heavy and moving along a straight line trajectory. However, in actual fact the velocity of the atom, which has a finite mass, may be significantly modulated in the presence of a strong field due to the gradient force. Here the depth δv of the modulation may turn out to be much greater than $\gamma/2k$—the width of the maximum of the force $\bar{f}(v)$. It is obvious that the condition $v_1 \gtrsim \delta v$ must be satisfied. Hence we obtain the following restriction on v_1:

$$v_1 \gtrsim v_{10} = (2\delta U/M)^{1/2}, \tag{49}$$

where δU is the characteristic variation in the potential of the gradient force. For $\hbar\Delta \sim 0.2$ dE (at the point corresponding to the maximum value of the force $\bar{f}$) we have $\delta U \sim 0.2$ dE. Thus, we arrive at the following estimate for the largest possible accelerating force $\bar{f}_M$ for an atom with mass M:

$$\bar{f}_M = \bar{f}(v_{01}) \approx 0{,}2\gamma (dEM)^{1/2}. \tag{50}$$

In a resonant field, close to the critical field for He($2\,^3S$), we have the estimate $\bar{f}_M \sim 0.4$ eV/cm. The ratio of $\bar{f}_M$ to the maximum force f_{max} for an infinitely heavy atom is equal to $\gamma/2kv_1 \sim 4 \times 10^{-3}$ (for the present example).

We note that the parameter $\gamma/2kv_1$ increases as the mass of the atom increases. In the limit of a macroscopic solid particle with dimensions greater than a wavelength, the gradient force f(x) tends to zero, but the force $\bar{f}$ remains and may have the value $\bar{f}_{max}$. Comparing expressions (23) and (1) for the force, we find

$$\bar{f}_M / F_{sp} \approx 0{,}4 (dEM)^{1/2} / \hbar k, \tag{51}$$

i.e., we obtain the square root of the atom's energy dE in the wave divided by the recoil energy $(\hbar k)^2/2M$. The latter quantity is extremely small (usually it is considerably smaller than the linewidth $\hbar\gamma$). Therefore, one can state that the maximum possible force $\bar{f}_M$ becomes larger than F_{sp} as soon as the amplitude of the field becomes bigger than the linewidth $\hbar\gamma$.

8. CONCLUSION

Thus, we may state that the forces acting on an atom in a resonant field may be of three types.

In the field of a plane traveling wave, a force F_{sp} acts on the atom, this force being associated with the absorption of a photon from the atomic beam and the emission of a spherical wave. The rate of such a process is determined by the spontaneous emission rate γ.

In the presence of a standing wave a gradient force, associated with the transfer of quanta from one light beam to the other, acts on the atom. This process occurs with the stimulated transition frequency $2dE/\hbar$.

Finally, a force due to a certain combination of spontaneous and stimulated transitions also acts on an atom in the field of a standing wave. This force is spatially homogeneous and resonantly depends on the atom's velocity relative to the wave. All three types of forces can be used to accelerate atoms. The investigation of various acceleration mechanisms is of interest for the following applications: 1) The production of dense, monoenergetic atomic beams with energies up to several keV; 2) the extraction of excited atoms from atomic beams and the production of an inverted population with respect to the ground state; 3) isotope separation.

In the first case we are talking about the utilization of very intense light beams, of the order of 10^9 to 5×10^9 W/cm^2. The selection of excited atoms and isotopes can be achieved in beams of average energy flux, $\sim 10^2$ to 10^5 W/cm^2. By extracting He atoms and ions of Li II and Be III in a metastable state from an atomic beam, one can obtain laser generation in the soft x-ray region. One can utilize the resonant properties of the force (1) for isotope separation.[2] In those cases when the isotopic shift is rather large, the rate of selection can be substantially increased by using one of the acceleration mechanisms considered in Secs. 3 and 4.

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APPENDIX

The dependence of C on ϵ_0 and ϵ_{10} can be expressed with the aid of the Legendre functions $P_{\pm 1/2}(z)$ of the first kind with half-integer subscripts:

$$C = \left(\frac{2}{\varepsilon_0}\right)^{1/2} \frac{P_{1/2}(z) P_{-1/2}(z) - 1 + \xi (P_{-1/2}^2(z) - 1)}{1+\xi},$$

$$\xi = \frac{z(\gamma/\gamma_\perp - 1)}{1+\varepsilon_0}, \quad z = \left[1 - \frac{\varepsilon_{10}^2}{(1+\varepsilon_0)^2}\right]^{-1/2}, \quad z > 1.$$

The parameter z determines the dependence on the average energy of the field and on the depth of the modulation.

Let us present asymptotic expressions for the function $C(\epsilon_0, z)$. For small energies, $\epsilon_0 \ll 1$ and $z - 1 \ll 1$, we find $C \approx (1/4)(2/\epsilon_0)^{1/2}(z-1)(\epsilon_0 + \gamma/\gamma_\perp - 2)$; for large energies and for $\epsilon_{10} = \epsilon_0$ we have

$$C \approx \left(\frac{2}{\varepsilon_0}\right)^{1/2} \left[\left(\frac{2}{\pi}\right)^2 \ln(4(2\varepsilon_0)^{1/2}) - 1\right], \quad \varepsilon_0 \gg 1.$$

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1) He (2^3S) has roughly such a value for the critical field at frequencies close to the resonance frequency [3].

2) The acceleration of charged particles by nonstationary radiofrequency fields has previously been investigated by Gaponov and Miller [4].

3) Here we omit the relaxation constants, having in mind the case when the time of interaction between the atom and the field is small.

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