Stabilization of shock instability at the expense of the inhomgeneity of the medium

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We consider nonlinear wave interactions in which waves of positive and negative energy participate, in a nonequilibrium inhomogeneous plasma. It is shown that the spatial evolution of the shock instability in an inhomogeneous plasma becomes stabilized under certain conditions. The physical reason of the stabilization is that the perturbations are carried out from the region in which instability development is possible.

1. INTRODUCTION

As is well known, highly interesting properties are observed in waves with negative energy [1-4], which exist, for example, in a plasma containing an admixture of particles with inverted level populations [5], in a magnetoactive plasma with a definite deviation from equilibrium [3,4], and in plasma-beam systems [6,7]. A characteristic feature of waves with negative energy is the onset of instabilities in those cases when energy is drawn from the mode with negative energy.

Assume that the energy is withdrawn as a result of three-wave interactions. Then the amplitudes of all the interacting waves can grow simultaneously to quite appreciable values. This qualitative conclusion can be drawn from the results of a number of papers [2-4] in which various model problems have been considered theoretically. Thus, Coppi, Rosenbluth, and Sudan [4] have considered the interaction between a negative-energy wave and positive-energy waves under the following approximations: 1) the plasma was assumed to be uniform and unbounded, 2) only the terms quadratic in the amplitudes were retained in the equations for the amplitudes of the interacting waves, 3) it was assumed that the amplitudes of all the waves depend only on the time (i.e., the terms $V_i \partial C_i/\partial x$ were neglected, where V_i and C_i are the group velocities and amplitudes of the corresponding waves). The following result was obtained for this model problem [4]: the amplitudes of all three interacting waves increase to infinity within a finite time $t_{\exp 1}$, the so-called explosive instability.

We note that these results [4] can easily be reformulated for the case when a negative-energy wave is excited in stationary manner on the boundary of a certain region. In this case it can be assumed that the wave amplitudes depend only on the coordinate x and, if all $V_i > 0$, the simple substitution $C_i \to (C_i')(V_i)^{-1/2}$, $x \to (V_1V_2V_3)^{1/2}t$ reduces the system of equations for the amplitudes of the interacting waves to the system of equations investigated in [4], and consequently the wave amplitudes will increase to infinite values over a finite length. In this case we shall speak of spatial evolution of the explosive instability.

Of course, both the temporal and the spatial models can be close to the experimental conditions. In either case, the model should be improved by taking into account effects that limit the infinite growth of the interacting-wave amplitudes in time and in space. Thus, it was shown [8,9] that saturation of the "temporal explosion" can be due to an off-resonance departure of the resonant interaction of the waves as a result of nonlinear frequency shifts. The explosive instability can also be stabilized by the reaction of the excited waves on the particle orbits and on their distribution function, and consequently also on the dispersion properties of the medium [5,10-12].

If we are interested in the spatial evolution of explosive instability, then it follows from simple physical considerations that an important role is played here by the inhomogeneity of the medium. In fact, in an inhomogeneous medium there is always an outflow of perturbations from the region of resonant interaction where the explosive instability develops. If the inhomogeneity is appreciable, this outflow of the perturbations can be expected to stabilize the spatial "explosion." We note that since this is a linear effect, it can come into play under real conditions earlier than the nonlinear dephasing or the nonlinear change of the dispersion properties of the medium[1)].

It will be shown below that the detuning of the spatial synchronism, which results from the inhomogeneity, leads under certain conditions to stabilization of the spatial development of the explosive instability. The stationary distribution of the intensities of the interacting waves can take the form of damped spatial oscillations of these quantities. We consider the stability of such distributions in the most typical limiting cases: 1) $V_1 \ll V_2, V_3$, 2) $V_1 \gg V_2 \approx V_3$, 3) $V_1 \approx V_2 \approx V_3$, where V_1 is the group velocity of the negative-energy wave and $V_{2,3}$ are the group velocities of the positive-energy waves. The indicated distributions are stable in all the considered cases. These examples, and also the considerations advanced above concerning the dephasing, give grounds for hoping that the obtained distributions are stable in the general case.

In conclusion, to illustrate the general conclusions, examples are given of plasma systems in which negative-energy waves are produced, and conditions are indicated under which the explosive instability becomes stabilized by inhomogeneity in these systems.

2. FUNDAMENTAL EQUATIONS

We consider a weakly-inhomogeneous medium whose properties depend on the coordinate x. Assume that three waves are excited in it, described in the geometrical-optics approximation by expressions of the type (j = 1, 2, 3)

$$C_j(x,t)\exp\left\{i(\omega_j t - k_j x) - i\int_0^x \Delta k_j(\xi)\,d\xi\right\}, \quad (1)$$

where $C_j(x, t) = a_j(x, t)\exp\{i\varphi_j(x, t)\}$ are complex amplitudes that vary slowly with x and t (owing to the nonlinearity and weak inhomogeneity); $k_j + \Delta k_j(x)$ are local wave vectors satisfying at the point x = 0 the decay conditions

$$\omega_1=\omega_2+\omega_3,\ k_1=k_2+k_3,$$
$$\Delta k_1(0)=\Delta k_2(0)=\Delta k_3(0)=0. \tag{2}$$

For the sake of argument we assume that the wave with frequency ω_1 and with wave vector k_1 is the negative-energy wave, and that the two others have positive energy.

In accordance with the statements made in the Introduction, it is expedient to seek for the inhomogeneous medium a stationary time-independent distribution of the amplitudes of the interacting waves in space. The system of equations describing such distributions with neglect of the linear increments or decrements of the waves takes the form

$$V_1\frac{da_1}{dx}=\beta a_2a_3\cos[\Phi+\int_0^x\Delta k(\xi)d\xi],$$
$$V_2\frac{da_2}{dx}=\beta a_1a_3\cos[\Phi+\int_0^x\Delta k(\xi)d\xi],$$
$$V_3\frac{da_3}{dx}=\beta a_1a_2\cos[\Phi+\int_0^x\Delta k(\xi)d\xi], \tag{3}$$
$$\frac{d\Phi}{dx}=-\beta\left(\frac{a_2a_3}{V_1a_1}+\frac{a_1a_3}{V_2a_2}+\frac{a_1a_2}{V_3a_3}\right)\sin[\Phi+\int_0^x\Delta k(\xi)d\xi],$$

where β is the matrix element of the nonlinear interaction, $\Phi=\varphi_2+\varphi_3-\varphi_1$. The system (3) has integrals

$$V_1n_1-V_2n_2=m_1,\quad V_1n_1-V_3n_3=m_2, \tag{4}$$

where $n_i(x)=a_i^2(x)$.

Assume for the sake of argument that $V_1>0$. A spatial picture analogous to explosive instability arises only in the case when the directions of all group velocities are the same, i.e., $V_2, V_3>0$. (In the case when the group velocity of at least one of the waves is oppositely directed, the amplitudes of all the waves are bounded, as follows even from (4). We shall therefore not consider this case.) Thus, let the group velocities of all the waves be positive (have the same direction). We are interested in the possibility of stabilizing the spatial explosive instability as a result of the inhomogeneity of the medium or as a result of the finite deviation Δk in the homogeneous medium. It follows from (4) that

$$n_1>\frac{m_1}{V_1},\quad\frac{m_2}{V_2},\quad n_{2,3}=\frac{V_1n_1-m_{1,2}}{V_{2,3}}, \tag{5}$$

i.e., the wave amplitudes can, generally speaking, increase without limit.

Taking the conservation laws (4) into account and introducing the dimensionless variables

$$\xi=x/L,\ n(\xi)=n_1(\xi L)/n_1(0),\ \varkappa(\xi)=L\Delta k(\xi L),$$
$$r_1=\frac{m_1}{V_1n_1(0)}\quad r_2=\frac{m_2}{V_2n_2(0)},\quad L=\frac{(V_2V_3)^{1/2}}{\beta n_1^{1/2}(0)}, \tag{6}$$
$$\Theta(\xi)=\Phi(\xi)+\int_0^\xi\varkappa(\eta)d\eta,$$

we can reduce the system (3) to a system of two equations for the relative "number of quasiparticles" $n(\xi)$ of the negative-energy wave and for the relative phase shift of the waves $\Theta(\xi)$:

$$\frac{dn}{d\xi}=2[n(n-r_1)(n-r_2)]^{1/2}\cos\Theta=\frac{\partial\mathscr{H}}{\partial\Theta},$$
$$\frac{d\Theta}{d\xi}=\varkappa(\xi)$$
$$-\left[\left(\frac{(n-r_1)(n-r_2)}{n}\right)^{1/2}+\left(\frac{n(n-r_1)}{n-r_2}\right)^{1/2}+\left(\frac{n(n-r_2)}{n-r_1}\right)^{1/2}\right]\sin\Theta=-\frac{\partial\mathscr{H}}{\partial n}. \tag{7}$$

The boundary conditions take the form:

$$n(0)=1,\ \Theta(0)=\Phi(0)=\Phi_0,\ r_1\leqslant1,\ r_2\leqslant1.$$

The function $\mathscr{H}$ introduced by us is the Hamiltonian of our system, which is specified, as always, accurate to within a constant:

$$\mathscr{H}=2\sin\Theta[n(n-r_1)(n-r_2)]^{1/2}-\varkappa n. \tag{8}$$

3. STABILIZATION OF EXPLOSIVE INSTABILITY AS A RESULT OF SPATIAL DEPHASING

We consider first the case of a constant spatial phase deviation $\varkappa$ = const $\neq 0$ (this case can also be realized in a homogeneous medium). A general investigation of the influence of the constant phase deviation on the spatial evolution of the explosive instability is not so much of interest in itself as it is necessary for understanding of the stabilizing action of the alternating (as a result of the inhomogeneity) phase deviation, although these cases also differ qualitatively (see below).

Thus, let $\varkappa$ = const; then $\mathscr{H}$ is an integral of the motion, and its use leads readily to the following equation for n:

$$d^2n/d\xi^2=2[n(n-r_1)+n(n-r_2)+(n-r_1)(n-r_2)]-\varkappa(\mathscr{H}+\varkappa n) \tag{9}$$

with boundary conditions

$$n(0)=1,\quad\frac{dn}{d\xi}(0)=2\cos\Phi_0[(1-r_1)(1-r_2)]^{1/2}.$$

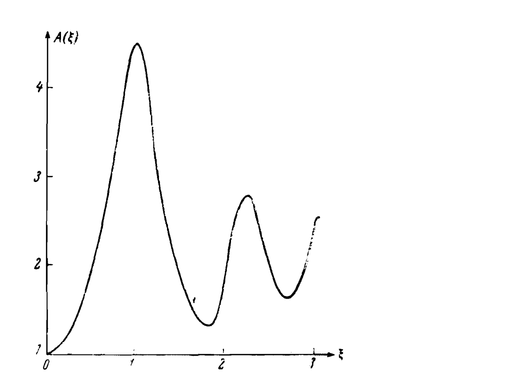
Equation (9) is analogous to the equation of motion of a material point in a potential field U(n) (the role of the coordinate is played by n and the role of the time by ξ), and has an integral

$$\tfrac{1}{2}n'^2+U(n)=W=-\tfrac{1}{2}\mathscr{H}^2=\text{const},$$
$$U(n)=\tfrac{1}{2}\varkappa^2n^2+2(r_1+r_2)n^2-2n^3+(\varkappa\mathscr{H}-2r_1r_2)n.$$

It is now easy to analyze the solutions of (9) qualitatively. It can be assumed here that $r_{1,2}\geq0$ (for negative $r_{1,2}$, we can renumber the waves and reduce the problem to the case $r_{1,2}\geq0$). By investigating different cases it can be shown that the necessary and sufficient condition for the existence of limited stationary distributions of n are the conditions

$$\varkappa\mathscr{H}<2r_1r_2,\quad[\varkappa^2+4(r_1+r_2)]^2>24(2r_1r_2-\varkappa\mathscr{H}). \tag{10}$$

In this case the minimum of the potential energy occurs at positive finite n_m, with $-\mathscr{H}^2/2>U(n_m)$, i.e., the equation has finite periodic solutions. To satisfy the inequalities (10), it suffices to have $\varkappa^2>2|\varkappa|$, $\varkappa^4>24\varkappa^2+2|\varkappa|$. Both inequalities are satisfied at $|\varkappa|>6$ or, in the original notation



$$\gamma_d \equiv \beta n_1^{1/2}(0) < {}^1/_6 |\Delta k| (V_2 V_3)^{1/2}. \qquad (11)$$

These conditions are analogous to the condition for the stabilization of an explosive instability in time (in this case $|\Delta k|(V_2V_3)^{1/2}$ should be replaced by $\Delta\omega$). For certain particular cases, a condition analogous to (11) was obtained in[4].

We proceed now to the most interesting case, when Δk depends on x, with $\Delta k(0) = 0$. In this case the solution of the system (7) can be obtained in analytic form if it is assumed that $\kappa(\xi)$ varies sufficiently slowly in comparison with $n(\xi)$, so that the quantity $\mathscr{H}$ can be regarded as before as an integral of Eqs. (7).

The solution takes the simplest form under the following boundary conditions: $\Theta_0 = \Phi_0 = 0$, $r_1 = r_2 = 0$. Then $\mathscr{H} = 0$ and, consequently, $\sin\Theta(\xi) = \kappa(\xi)/2n^{1/2}$. From the second equation of (7) we obtain the phase

$$\Theta(\xi) = -\int_0^\xi \frac{\varkappa(\xi)}{2} d\xi. \qquad (12)$$

From this and from the first equation of the system (3) we obtain for the amplitude

$$a_1(x) = a_1(0)\left(1 - \int_0^{x/L} d\xi \cos \int_0^\xi \frac{\varkappa(\eta)}{2} d\eta\right)^{-1}. \qquad (13)$$

In the case when $\kappa(\xi) = \kappa'\xi$, the solution (13) can be expressed in terms of a Fresnel integral

$$a_1(x) = a_1(0)\left[1 - \left(\frac{2\pi}{\varkappa'}\right)^{1/2} C\left(\frac{x}{L}\left(\frac{\varkappa'}{2\pi}\right)^{1/2}\right)\right]^{-1}, \qquad (14)$$

where

$$C(x) = \int_0^x \cos\frac{\pi\xi^2}{2} d\xi.$$

The maximum value of the Fresnel integral is $C(\xi_m) \sim 1$, where $\xi_m \sim 1$.

The condition for the absence of spatial explosion then takes the form $\kappa' > 2\pi$, or

$$\gamma_d^2 \equiv \beta^2 n_1(0) < \frac{1}{2\pi}\left|\frac{d(\Delta k)}{dx}\right| V_2 V_3. \qquad (15)$$

It is seen from (14) that when the inequality (15) is satisfied, the amplitude increases with increasing x, from an initial value $a_1(0)$ to a value on the order of $a_1(0)/[1 - (2\pi/\kappa')^{1/2}]$ at $x \sim (2\pi/\kappa')^{1/2}L$, and then, after attenuating slightly, it oscillates and tends to $a_1(0)/[1 - (\pi/2\kappa')^{1/2}]$ asymptotically as $x \to \infty$ (see the figure, which shows a typical stationary distribution of the relative amplitude $A(\xi) = a_1(\xi L)/a_1(0)$ at $\kappa' = 2\pi$). The characteristic width of the region where the amplitudes still increase is

$$\Delta x \sim \left(\frac{d\varkappa}{d\xi}\right)^{-1/2} L = \left(\frac{d(\Delta k)}{dx}\right)^{-1/2}. \qquad (16)$$

If the inequality inverse to (15) is satisfied, the denominator of (14) vanishes at a certain point x_k and the amplitude increases without limit as $x \to x_k$. Thus, in interaction of waves of sufficiently large amplitude, the inhomogeneity of the medium does not stabilize the spatial explosion, and to find the maximum level to which the amplitudes can grow it is necessary to take the nonlinear effects into account.

At $\kappa' \ll 2\pi$ it is permissible to retain only the first term in the expansion of the Fresnel integral. We then find from (14) that, independently of the value of κ', the "explosion length" is $x_k \sim L$, i.e., the results of the theory of homogeneous plasma are valid in the case of very weak inhomogeneity. Allowance for the next higher terms of the expansion yields $x_k > L$, i.e., the explosion length is minimal in a homogeneous plasma.

We show next that the obtained estimates and conclusions also do not change qualitatively in the case of arbitrary initial amplitudes and phases of the interacting waves. (Without loss of generality, we can assume, as before, that $r_1 \geq 0$ and $r_2 \geq 0$). We note first that at $\xi \geq \xi_0 = (48)^{1/3}/\kappa'$ the "potential energy" U(n) takes the form of a well with a minimum at the point $n = n_m$, where

$$n_m = {}^1/_{12}[\varkappa^2 + 4(r_1 + r_2) - J^{1/2}],$$
$$J \equiv \varkappa^4 + 8(r_1 + r_2)\varkappa^2 + 16[(r_1 - r_2)^2 + r_1 r_2] + 48\varkappa \sin\Theta_0 [(1 - r_1)(1 - r_2)]^{1/2}.$$

For all $\xi \geq \xi_0$, the quantity J is positive at arbitrary initial conditions. We assume now that the condition (15) or $\kappa' > 2\pi$ is satisfied. Then $\xi_0 \lesssim 1/2$. On the segment $(0, \xi_0)$, the solution of (9) is bounded, since the minimum length of the explosion is unity when expressed in terms of the variables ξ. We shall show that the solution is also bounded at $\xi \geq \xi_0$. To this end, we make the substitutions $N = n - n_m$, $y = \xi - \xi_0$. In terms of the new variables, Eq. (9) becomes

$$d^2N/dy^2 + J^{1/2}N - 6N^2 = 0 \qquad (17)$$

with corresponding boundary conditions. Equation (17) differs from (9) only in that κ is replaced by $J^{1/4}$. Thus, the solution is in fact bounded if $dJ^{1/4}/dy \sim \kappa' > 2\pi$. Thus, the condition for the stabilization of the explosive instability (15) remains approximately in force for arbitrary initial conditions.

The condition (15) can be interpreted qualitatively in the following manner. The size of the region in which spatial synchronism is not yet violated can be determined from the condition $\Delta x \Delta k \sim 1$. On the other hand, the frequency deviation increases with increasing distance: $\Delta k \sim \Delta x\, d(\Delta k)/dx$, whence

$$\Delta x \sim \left(\frac{d(\Delta k)}{dx}\right)^{-1/2}. \qquad (18)$$

One can expect the instability to become stabilized if the "time of outflow of the perturbations from the resonant region" is smaller than the "time of nonlinear interaction":

$$\frac{\Delta x}{V_{av}} \sim \left(\frac{d(\Delta k)}{dx}\right)^{-1/2} \frac{1}{V_{av}} < \frac{1}{\beta n_1^{1/2}(0)} = \frac{1}{\gamma_d}. \qquad (19)$$

This estimate is in qualitative agreement with (15) (cf. also formulas (18) and (16) for the "nonlinear-interaction length"). If Δk = const, it is easy to derive an analog of formula (11) from similar considerations.

4. STABILITY OF SOLUTIONS STABILIZED BY INHOMOGENEITY. STABILIZATION CRITERIA FOR CONCRETE PLASMA SYSTEMS

We thus obtain a complete set of stationary solutions for the amplitudes and phases of three interacting waves; this set depends on six arbitrary parameters—initial or boundary values of the amplitudes and phases. Analysis shows that if condition (15) is satisfied for ξ larger than a certain ξ_0, then the expressions for the amplitudes and phases of the interacting waves approach, with increasing ξ, the corresponding expressions for the case

$$r_1 = r_2 = \varphi_1(0) = \varphi_2(0) = \varphi_3(0) = 0. \qquad (20)$$

(with increasing ξ we have $J^{1/4} \to \kappa$, $n_m \to 0$, and the initial values of the phases become immaterial.)

In order to make the deduced spatial stabilization of the explosive instability more rigorous, it is necessary to investigate the stability of the stationary solutions. By virtue of the foregoing, we confine ourselves to an investigation of the stability in stationary solutions that satisfy the simplest boundary conditions (20) (the initial amplitude $a_1(0)$ of the first wave is then assumed to be arbitrary). Linearizing the initial system of equations for complex amplitudes $C_j(x, t)$ against the background of the obtained stationary solution, we obtain the system

$$\frac{\partial C_1'}{\partial t}+V_1\frac{\partial C_1'}{\partial x}=\beta a_1(x)\exp\left(i\int_0^x\frac{\Delta k}{2}d\xi\right)\left[\left(\frac{V_1}{V_3}\right)^{1/2}C_2'+\left(\frac{V_1}{V_2}\right)^{1/2}C_3'\right],$$

$$\frac{\partial C_{2,3}'}{\partial t}+V_{2,3}\frac{\partial C_{2,3}'}{\partial x}=\beta a_1(x)\exp\left(-i\int_0^x\frac{\Delta k}{2}d\xi\right)\left[\left(\frac{V_1}{V_{2,3}}\right)^{1/2}C_1'+C_{3,2}'^{*}\right], \quad (21)$$

where C_i' are the perturbations of the amplitudes, and $a_1(x)$ is determined by formula (13).

Taking the Laplace transform with respect to time with the parameter p, we reduce the stability problem to the problem as to whether the differential equation has eigenvalues without Re $p > 0$. If regular solutions of this equation with Re $p > 0$ exist, then the stationary solution is unstable, and in the opposite case it is stable. In the general case it is difficult to carry out this analysis, since it is necessary to investigate a sixth-order differential equation with inhomogeneous coefficients. We shall therefore discuss only the most typical particular cases.

1) $V_1 \ll V_2, V_3$. In place of the system (21) it then suffices to investigate a system of two equations for C_2' and $C_3'^*$:

$$\frac{\partial C_2'}{\partial t}+V_2\frac{\partial C_2'}{\partial x}=\beta a_1\exp\left(-i\int_0^x\frac{\Delta k}{2}d\xi\right)C_3'^*,$$

$$\frac{\partial C_3'^*}{\partial t}+V_3\frac{\partial C_3'^*}{\partial x}=\beta a_1\exp\left(i\int_0^x\frac{\Delta k}{2}d\xi\right)C_2'. \quad (22)$$

This system was investigated by Rosenbluth [15], who proved its stability.

2) $V_1 \gg V_2 \approx V_3$. We put $C_2' + C_3' = C$. For the variables C and C_1' we obtain the system

$$\frac{\partial C_1'}{\partial t}+V_1\frac{\partial C_1'}{\partial x}=\beta a_1\left(\frac{V_1}{V_2}\right)^{1/2}\exp\left(i\int_0^x\frac{\Delta k}{2}d\xi\right)C,$$

$$\frac{\partial C}{\partial t}+V_2\frac{\partial C}{\partial x}=2\beta a_1\left(\frac{V_1}{V_2}\right)^{1/2}\exp\left(-i\int_0^x\frac{\Delta k}{2}d\xi\right)C_1.$$

It can be proved, in analogy with the preceding case, that this system has no regular solutions that increase with time.

3) $V_1 \approx V_2 \approx V_3 = V$. Adding the equations (21) and putting $C_1'^* + C_2' + C_3' = C$, we get

$$\frac{\partial C}{\partial t}+V\frac{\partial C}{\partial x}=2\beta a_1\exp\left\{-i\int_0^x\frac{\Delta k}{2}d\xi\right\}C^*.$$

We thus obtain for C and C* a system that coincides with (22) apart from the notation and from the coefficient 2, and the stability of which has been proved. The physical cause of the stability is that the perturbations produced against the background of the stationary waves are also dephased as a result of the inhomogeneity. This deduction seems to be valid for arbitrary values of the group velocities, if the condition (15) is satisfied.

Thus, the condition (15) ensures spatial stabilization of the explosive instability. If it is satisfied with enough margin so that the amplitudes are not too large and remain in the range where perturbation theory is valid, the inhomogeneity is the principal effect that stabilizes the spatial explosion.

We present in conclusion estimates for certain concrete examples of evolution of explosive instability in a plasma, when the group velocities of the interacting waves can all have the same direction and the plasma is stable in the linear approximation. Thus, for cone instability of a magnetoactive plasma [3], the characteristic increment of the nonlinear coherent interaction is $\gamma_d \sim (\epsilon/nT_i)^{1/2}\omega_i$ (ϵ is the energy density of the wave, n is the plasma density, T_i is the ion temperature, and ω_i is the ion cyclotron frequency). According to the estimates of [3], $\omega \sim kv_{Ti}$. Consequently, Eq. (15) takes the form

$$\frac{\varepsilon}{nT_i}\omega_i^2<\frac{1}{6}\frac{d(\Delta k)}{dx}v_{Ti}^2\sim\frac{k}{a}v_{Ti}^2,$$

or

$$ka<k^2r_{di}^2\left(\frac{nT_i}{\varepsilon}\right), \quad (23)$$

where a is the characteristic dimension of the inhomogeneity.

The second example will be presented for the case of ion-beam instability [5]. At sufficiently high beam velocities v_0 in comparison with $v_s = (T_e/m_i)^{1/2}$, this system is linearly stable. For the case $n_b/n_i = 0.3$ and $\gamma = 1$ (n_b is the beam density, n_i is the ion density, and γ is the adiabatic exponent), we use the value of γ_d calculated in [5], namely $\gamma_d = e\varphi kv_s/2T_e$, where φ is the potential of the wave. Calculating the group velocities of the interacting waves, we obtain the stability condition in the form

$$\left(\frac{e\varphi}{T_e}\right)^2k^2v_s^2<\frac{k}{a}v_0v_s,$$

or

$$ka<\frac{v_0}{v_s}\left(\frac{T_e}{e\varphi}\right)^2. \quad (24)$$

We see that the conditions (23) and (24) do not contradict the initial assumption of sufficiently weak inhomogeneity of the plasma ($ka \ll 1$), if the interacting-wave energy is low enough.

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