Propagation of laser-radiation absorption waves in a transparent solid dielectric

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The self-similar motion of a thermal cascade ionization wave induced by laser radiation in a transparent solid dielectric is considered. Expresssions are obtained for quantities characterizing the front of the absorption wave, including its velocity. The limits of stability of the self-similar solutions are established.

1. It is well known[1,2] that microinclusions that absorb laser radiation may lead to the local heating of a transparent solid dielectric to temperatures $T \sim 10\,000$ deg. The main result of this is the appearance of a local concentration n_e of free electrons which is sufficient to produce further ionization by electron impact.[3] Since the time necessary for a change in the local temperature (for the parameters considered below) is much greater than the characteristic times for ionization and recombination, the electron concentration is determined only by the temperature, i.e., $n_e = n_e(T)$. For temperatures $T \lesssim 10^4$ deg the plasma can, at least qualitatively, be looked upon as ideal and $n_e(T)$ can be estimated from the Saha formula. Since the ionization potential J is much greater than the plasma temperature, we shall retain only the exponential temperature dependence $\exp(-I/T)$ in all the quantities, where $I = J/2$. We note that, since we shall have to know the temperature in the region where it is much higher than the initial temperature of the medium, the latter may be taken to be zero.

In view of the foregoing, the temperature $T(r, t)$ can be determined from the following set of equations

$$\partial T/\partial t=\chi\Delta T+g(T, |\mathbf{E}|^2), \tag{1}$$

$$\Delta \mathbf{E}+(\omega/c)^2\varepsilon(T)\mathbf{E}=0. \tag{2}$$

In these expressions χ is the temperature diffusivity, $g(T, |\mathbf{E}|^2) = (\omega/8\pi C)\operatorname{Im}\epsilon(T)|\mathbf{E}|^2$, ω is the frequency of the laser radiation, C is the heat capacity per unit volume of the dielectric, and $\epsilon(T)$ is the permittivity which, in the collision-time approximation, can be written in the form

$$\varepsilon(T)=1-\frac{\tilde\omega^2 e^{-I/T}}{\omega^2+\nu^2}\left(1-\frac{i\nu}{\omega}\right), \tag{3}$$

where $\tilde\omega \exp(-I/T)$ is the electron plasma frequency and ν is the frequency of electron collisions.[1)]

The wave equation for **E** given by (2) is written with allowance for the fact that the velocity of light c is much greater than the characteristic velocities of propagation of heat.

Equations (1) and (2) describe the propagation of the laser-absorption wave which we assume can develop as a result of cascade thermal ionization of the medium around the absorbing inhomogeneity.[2)] We shall consider time intervals for which the width of the temperature front is much less than its radius of curvature, and will be interested in the region near the intersection of the direction of propagation of light and the absorption wave front. The problem is therefore one-dimensional and, since the function $g(T, |\mathbf{E}|^2)$ is proportional to $\exp(-I/T)$, it resembles the problem of the thermal propagation of plasma.[10-11]

2. By analogy with[4,5] we shall seek the solution of (1) and (2) which is a function of only the combination $\xi = x + vt$, where the positive direction of the x axis coincides with the direction of propagation of light and the origin of ξ is taken to be the point at which the heat source function $g(\xi)$ is a maximum. We shall show below that this solution is stable, and will determine the region of stability.

In accordance with the foregoing, we have

$$I/T_0 \gg 1, \tag{4}$$

where $T_0 = T|_{\xi=0}$. Moreover, we shall suppose, and this is confirmed by the result, that the inequality

$$\frac{T_0}{I}\beta|\xi| \ll 1, \quad \beta=\frac{IT_0'}{T_0^2}, \quad T_0'=\left.\frac{dT}{d\xi}\right|_{\xi=0} \tag{5}$$

is satisfied up to those values of ξ for which the function $g(\xi)$ becomes negligible, i.e.,

$$g(\xi) \ll g(0). \tag{6}$$

In that case $\exp(-I/T) \approx \exp(-I/T_0)e^{\beta\xi}$. If we solve (2) with allowance for (3) and the boundary condition corresponding to the fact that, as $\xi \to -\infty$, the incident wave is completely determined by the laser flux density q, equation (1) assumes the form

$$\chi\frac{d^2T}{d\xi^2}-v\frac{dT}{d\xi}+g(\xi)=0, \tag{7}$$

where

$$g(\xi)=\frac{q}{C}\frac{\nu\alpha^2}{c}\frac{4\mu\,\mathrm{sh}(\pi\mu)}{\pi}e^{-\mu\gamma}y^2|K_{i\mu}(\eta y)|^2,$$

$$\alpha^2=\frac{\tilde\omega^2\exp(-I/T_0)}{\omega^2+\nu^2}, \quad \mu=\frac{2\omega}{\beta c}, \quad \eta=\mu\alpha\left(1-\frac{i\nu}{\omega}\right)^{1/2}, \tag{8}$$

$$\gamma=\operatorname{arctg}\frac{\nu}{\omega}, \quad y=e^{\beta\xi/2}.$$

Henceforth we shall suppose that

$$\nu/\omega \ll 1. \tag{9}$$

Equation (7) must be solved subject to the following boundary conditions:

$$T|_{\xi=-\infty}=0, \quad \left.\frac{dT}{d\xi}\right|_{\xi=\infty}=0.$$

The condition for the extremum of the function $g(\xi)$ at the point $\xi = 0$ yields

$$|K_{i\mu}(\eta)|+\eta\partial|K_{i\mu}(\eta)|/\partial\eta=0. \tag{10}$$

In this expression $K_{i\mu}$ is the cylindrical Bessel function.

From (7) we find $T(\xi)$ and $dT(\xi)/d\xi$. Substituting $\xi = 0$ into these expressions, we obtain

$$T_0=\frac{1}{v}\left[\int_{-\infty}^{0}d\xi\, g(\xi)+\int_0^\infty d\xi\exp\left(-\frac{v}{\chi}\xi\right)g(\xi)\right], \tag{11}$$

$$T_0' = \frac{1}{\chi}\int_0^\infty d\xi \exp\left(-\frac{v}{\chi}\xi\right) g(\xi). \qquad (12)$$

Equations (10)—(12) can be used to determine the unknowns T_0, T_0', v and hence the temperature profile, etc.

From (7) it follows that for points ξ which correspond to (6) and lie on the left of $\xi = 0$, we have $T(\xi) = (x/v)T'(\xi)$. Hence

$$y^{2\lambda}\int_y^\infty dx\, x^{1-2\lambda}|K_{i\mu}(\eta x)|^2 \gg \int_0^y dx\, x|K_{i\mu}(\eta x)|^2 \qquad (13)$$

($y = e^{\beta\xi/2} \ll 1$) where $\lambda = v/\beta\chi$ which, together with (4)—(6), determines the range of validity of the results obtained below.

Next we consider two limiting cases, one of which corresponds to the quasiclassical electromagnetic field and the other to the presence of strong reflection from the absorption wave front.

3. The quasiclassical case corresponds to the inequality

$$\mu \gg 1 \qquad (14)$$

and $\alpha|(1 - i\nu/\omega)^{1/2}| < 1$. Instead of the second condition, however, we shall demand that the stronger inequality $\alpha^2|1 - i\nu/\omega| \ll 1$ be satisfied which, together with (9), assumes the form

$$\alpha^2 \ll 1. \qquad (15)$$

Using the asymptotic behavior of $K_{i\mu}(\eta e^{\beta\xi/2})$, and assuming that (14) and (15) are satisfied, we find that equations (10)—(12) assume the form

$$\mu\alpha^2 v/\omega = 2,$$

$$T_0 = \frac{q}{vC}\left[\frac{e-1}{e} + \Gamma(1-\lambda, 1)\right], \quad T_0' = \frac{q}{\chi C}\Gamma(1-\lambda, 1),$$

where

$$\Gamma(1-\lambda, 1) = \int_1^\infty dx\, x^{-\lambda}e^{-x}$$

is the incomplete gamma function.

Since $\Gamma(1 - \lambda, 1) \approx 1/e(1 + \lambda)$, the above set of equations yields

$$\left(\frac{T_0}{I}\right)^2 = \frac{q}{q_0}e^{I/T_0}, \quad v = \frac{q}{CT_0}, \quad q_0 = \frac{eCIv\chi\tilde{\omega}^2}{c\omega^2}, \qquad (16)$$

and hence

$$T_0 \approx I \Big/ \left(\ln\frac{q_0}{q} - 2\ln\ln\frac{q_0}{q}\right),$$
$$v \approx \frac{q}{IC}\left(\ln\frac{q_0}{q} - 2\ln\ln\frac{q_0}{q}\right), \qquad (17)$$
$$\beta \approx \frac{\nu}{c}\left(\frac{\tilde{\omega}}{\omega}\right)^2 \frac{q}{q_0}\ln^2\frac{q_0}{q}.$$

4. We now consider the case of strong reflection which corresponds to

$$\mu^2 \ll 1, \quad \alpha^2 \gg 1. \qquad (18)$$

The first of these inequalities ensures that for $\xi \le 0$ the function $K_{i\mu}(\mu\alpha e^{\beta\xi/2})$ can be replaced by the first term of its expansion into a series, and if we use (9) we find from (10)—(12) that

$$\mu\alpha = 2e^{-1-C},$$

$$T_0 = \frac{4vq}{Cc\beta v}, \quad T_0' = \frac{4vq}{\chi Cc\beta}\left[1 - \frac{5}{2}(\mu\alpha)^2\right].$$

In this expression C = 0.577... is the Euler constant. Since $\mu = 2\omega/\beta c$, the above set of equations yields

$$T_0 \approx I \Big/ \left(\ln\frac{\tilde{q}_0}{q} - 2\ln\ln\frac{\tilde{q}_0}{q}\right),$$
$$v \approx 6\frac{\chi\hat{\omega}}{c}\left(\frac{q}{\tilde{q}_0}\right)^{1/2}, \quad \beta \approx 5\frac{\tilde{\omega}}{c}\left(\frac{q}{\tilde{q}_0}\right)^{1/2}\ln\frac{\tilde{q}_0}{q}, \qquad (19)$$

where $\tilde{q}_0 \approx 6C\chi I\tilde{\omega}^2/\nu c$.

5. We now consider the stability of the above solutions. To do this, we must, as usual, add a small perturbation to the above solution of (1) and (2), i.e., write

$$T(\xi, t) = T(\xi) + \tilde{T}(\xi)e^{-pt}, \quad \mathbf{E}(\xi, t) = \mathbf{E}(\xi) + \tilde{\mathbf{E}}(\xi)e^{-pt}.$$

The stability problem then reduces to the determination of the range of the parameters of the problem, where the set of equations given by (1) and (2) after linearization in $\tilde{T}(\xi)$, $\tilde{\mathbf{E}}(\xi)$ does not have eigenvalues p whose real part is negative. By eliminating $\tilde{\mathbf{E}}(\xi)$ from the above set of equations and substituting $\tilde{T}(\xi) = \exp\{v\xi/2\chi\}\psi(\xi)$, we obtain

$$p\psi(\xi) = \left(-\chi\frac{d^2}{d\xi^2} + \frac{v^2}{4\chi}\right)\psi(\xi) - \int_{-\infty}^\infty d\xi' F(\xi, \xi')\psi(\xi'). \qquad (20)$$

In this equation

$$F(\xi, \xi') = g(\xi)\frac{I}{T^2(\xi)}\left\{\delta(\xi - \xi') - 4\frac{\omega^2}{c^2\beta}\alpha^2\left(\frac{T(\xi)}{T(\xi')}\right)^2\right.$$
$$\times \exp\left[\beta\xi' - \frac{v}{2\chi}(\xi - \xi')\right]\mathrm{Re}\left[\frac{K_{i\mu}(\eta y')}{K_{i\mu}(\eta y)}[I_{-i\mu}(\eta y)K_{i\mu}(\eta y')\theta(\xi' - \xi)\right.$$
$$\left.\left. + K_{i\mu}(\eta y)I_{-i\mu}(\eta y')\theta(\xi - \xi')]\right]\right\},$$

where $g(\xi)$ is given by (8), $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$. Equation (20) resembles the Schroedinger equation with a nonlocal, non-Hermitian potential. The stability of the 'self-similar' state may be violated only by bound states. It will be confirmed by the ensuing calculations that (20) corresponds to the case of a small well in which only one bound state can be present. Proceeding as in the case of the local potential,[12] we obtain the following expression for the bound-state eigenvalue:

$$p = \frac{v^2}{4\chi} - \frac{1}{4\chi}\left[\int_{-\infty}^\infty d\xi \int_{-\infty}^\infty d\xi' F(\xi, \xi')\right]^2. \qquad (21)$$

Let us begin by considering the semiclassical case. Using the quasiclassical expressions for the Bessel functions in the kernel $F(\xi, \xi')$ of (21), and recalling that the function $g(\xi)$ has a sharp maximum at $\xi = 0$, we obtain

$$p \approx \frac{v^2}{4\chi}\left[1 - \frac{e^2}{4}\left(1 - \frac{\alpha^2}{e}\frac{I}{T_0}\right)^2\right]. \qquad (22)$$

It follows from (21) that eigenvalues $p < 0$ are absent and, consequently, the above state is stable for light flux intensities greater than

$$q_{\text{crit}} \approx (e-2)\left(\frac{\omega}{\tilde{\omega}}\right)^2 q_0 \Big/ \ln^3\left[\frac{(\tilde{\omega}/\omega)^2}{e-2}\ln\frac{(\tilde{\omega}/\omega)^2}{e-2}\right], \qquad (23)$$

which is obtained from the condition $p = 0$. In deriving (23) we used the expressions given by (17).

We note that for light flux intensities for which reflection becomes important, the above state can be shown to be stable, as before.

In conclusion, we estimate the critical intensity q_{crit} and the characteristic values of T_0, v and β. In (16) we let $C \sim 1$ cal-deg^{-1}cm^{-3}, $I \approx 40\,000$ deg, $\nu \sim 10^{14}$ sec^{-1}, $\chi \sim 0.01$ cm^2 sec^{-1}, and $\tilde{\omega}/\omega \sim 10$. Equation (23) then yields $q_{\text{crit}} \sim 10^5$ W/cm^3. From (17) we have for $q \sim q_{\text{crit}}$, $T_0 \approx 6000$ deg, $v \sim 10$ cm/sec, and $\beta \sim 1000$ cm^{-1}.

In the case of strong reflection and $q \sim 10^8$ W/cm^2, $\tilde{\omega} \sim 10^{16}$ sec^{-1}, and $\omega/\nu \sim 10$, we find from (19) with the above values of the parameters that $T_0 \approx 10\,000$ deg, $v \sim 1000$ cm/sec, and $\beta \sim 10^5$ cm^{-1}.

We note that, for the above values of the parameters, the inequalities (4)–(6), (13)–(15), and (18), whose validity was assumed in the solution to the problem, are in fact fully satisfied.

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[1)]In general, the permittivity $\epsilon(T)$ will also depend on the self-focusing field. However, we shall consider values of E and times of laser operation for which the phenomenon of self-focusing is usually not observed.

[2)]A similar problem for the gaseous phase was considered in [4–6]. We note also that different regimes of propagation of the laser-absorption wave in gases have been investigated by many workers [7–9].

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