Radiative corrections to e^+e^- pair production by a high-energy photon in the field of an electron or a nucleus

E. A. Vinokurov, É. A. Kuraev, and N. P. Merenkov

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences
(Submitted January 15, 1974)
Zh. Eksp. Teor. Fiz. 66, 1916–1925 (June 1974)

The radiative correction to the cross section for production of e^+e^- pairs by a high-energy photon in the field of an electron is found in analytical form in the Weizsäcker–Williams approximation. Expressions are obtained for the spectra of positrons $d\sigma/d\Delta = f(\Delta)$, $\Delta = \epsilon_+/\omega_0$, and of photons $d\sigma/d\beta = F(\beta)$, $\beta = \omega/\omega_0$, produced in the process $\gamma e^\pm \to \gamma e^+ e^- e^\pm$ [ω_0, e_+, and ω are, respectively, the energies of the initial photon, the positron of the pair, and the final photon in the rest system of the initial electron (positron)]. The case of pair production in the field of a nucleus is discussed. Using the asymmetry of the positron spectrum relative to the point $\Delta = 1/2$, we find for $\rho(\Delta)$ the expression $\rho(\Delta) = f(\Delta) - f(1-\Delta)$, which can be checked experimentally.

The process of pair production by a photon in the field of an electron or a nucleus, and also the radiative corrections to it, have been studied in a large number of experimental and theoretical papers over a period of many years.[1] However, the need for a description of this process of ever increasing accuracy constantly arises in connection with experiments on photoproduction of hadrons from nucleons, where it must be taken into account as background. Inclusion of the radiative correction to the process of pair production by a high-energy photon is necessary, for example, in problems of reconstruction of the initial photon spectrum if the energy resolution of the pair components is less than one percent. In our opinion it is desirable to carry out a program of study of radiative corrections to the production of e^+e^- pairs by photons in nuclei.

At the present time it is possible to set up in a number of different installations, say, experiments on measurement of the energy spectra of the pair fragments with good statistics and quite good energy resolution; in the literature known to the authors there are no indications of such experiments.[2] In addition, the process of pair production by a photon with radiation of an additional photon must be taken into account as a background in experiments to check nonlinear effects in quantum electrodynamics (Delbruck scattering, disintegration of photons in the field of a nucleus). In this connection we should note a recent experiment performed in the German electron synchrotron (DESY) on the disintegration of photons in the field of a nucleus.[3] In this experiment, whose main purpose was to study Delbruck scattering, high-energy photons hit a target (Zn, Au), and a photon radiated at an angle of the order of several milliradians with an energy 87% of the initial photon energy was detected. The authors of the experiment considered all such photons as formed in the disintegration of the initial photon into two photons in the field of a nucleus. However, the cross section measured in this experiment is two orders of magnitude greater than that predicted theoretically.[4] At the same time, good agreement with the results of this experiment [5] is obtained by taking into account the production of a photon together with an electron-positron pair by the initial photon in the field of a nucleus ($\gamma Z \to \gamma e^+e^-Z$) in a single process.[1)] This mechanism of photon production, as follows from analysis of the experimental data,[3] was not considered by the authors.

In Sec. 1 of the present work we present an analytic expression found in the Weizsäcker-Williams (WW) approximation for the photon spectrum produced in the process $\gamma e \to \gamma e^+e^-e$. In Sec. 2 we obtain the spectrum of positrons produced in creation of an e^+e^- pair by a photon in the field of an electon. In the Born approximation the spectrum is determined by the expression

$$\frac{d\sigma_0}{d\Delta} = 4\alpha r_0^2 \left\{ L\left[1 - \frac{4}{3}\Delta(1-\Delta)\right] + O\left(\frac{m_e^2}{s}\right)\right\}, \tag{1}$$

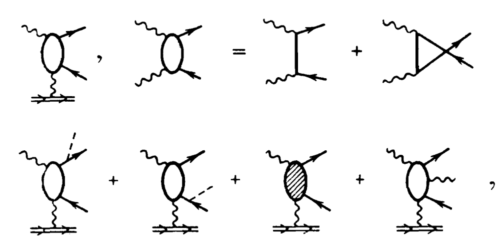
where

$$L = \ln R, \quad R = s\Delta(1-\Delta)/m_e^2 e^{1/2}, \quad s = (k+p)^2, \quad r_0 = \alpha/m_e, \quad e = 2.718,$$

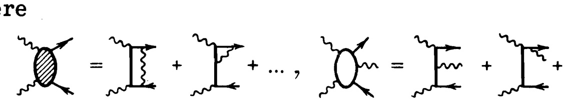
$\Delta = \epsilon_+/\omega_0$ is the fraction of the initial photon energy carried away by the positron, $0 \le \Delta \le 1$; k and p are the 4-momenta respectively of the initial photon and electron; $\omega_0 = s/2m_e$ is the initial photon energy in the electron system. In the next order of perturbation theory the contribution to the spectrum of the positron can be represented in the form

$$\frac{d\sigma}{d\Delta} = \left(\frac{d\sigma}{d\Delta}\right)_v + \left(\frac{d\sigma}{d\Delta}\right)_s + \left(\frac{d\sigma}{d\Delta}\right)_h, \tag{2}$$

where the first two terms describe the correction to the spectrum (1) due to virtual and soft real photons, and the third determines the contribution to the radiative correction due to radiation of a hard photon (see the figure).



where



The contribution of the first two terms in Eq. (2) was first obtained analytically by Mork and Olsen.[7] These authors also estimated the third term by a numerical integration method. Using the value obtained by us recently of the cross section differential in the photon frequency for radiation of two photons by one electron in an e^+e^- collision,[8] and also the substitution rule,[9] we obtain in analytic form the distribution in energy of the positron and photon in the process $\gamma e \to \gamma e^+e^-e$ (see further Eq. (A.1)). Integration of Eq. (A.1) over the photon-energy fraction β for a fixed positron-energy fraction Δ permits us to obtain an analytic expression for the radiative correction to the positron spectrum due to radiation of a hard photon. The total radiative correction to the positron spectrum is determined by Eq. (7).

In Sec. 3 the results are transformed to the case of pair production in the screened field of a nucleus and a correction is found to the total cross section for photon absorption. In formulating an experiment to check the radiation corrections to the process of pair production by a photon in the field of a nucleus, it is necessary to take into account terms which are not asymptotic in the high-energy limit, which originate from the spectrum in the Born approximation (in Eq. (1) the terms $\sim\alpha r_0^2 L(s/m_e^2)^{-1}$). An estimate shows that for photon energies in the rest system of the nucleus $\omega_0 = 1.5$ GeV the contribution of the radiative correction is of the same order as the contribution of nonasymptotic terms and amounts to $\approx 1\%$ of the Born cross section. Thus, there is a definite difficulty in taking into account the background.

In Sec. 4 we show that the possibility exists, however, of separating from the cross section the part responsible for the radiative correction determined by radiation of a hard photon. This possibility is based on the fact that the positron spectrum is not symmetric relative to the point $\Delta = 1/2$, the violation of this symmetry occurring completely as the result of the contribution of the radiative correction taking into account radiation of a hard photon.

1. In a recent article [8] Kuraev et al. obtained in the WW approximation the cross section differential in photon energy for radiation of two photons by one of the electrons in an e^+e^- collision:

$$d^2\sigma^{br}/d\beta_1 d\beta_2 = F^{br}(\Delta, \beta_2), \quad \beta_i = \omega_i/E_0,$$

where ω_i and E_0 are the energies of the photon and initial electron in the c.m.s., $\Delta = 1-\beta_1-\beta_2$. From this spectrum it is possible to obtain by means of the substitution-rule procedure [9] the cross section differential in the final photon and positron energies for production of an e^+e^- pair in the field of an electron with radiation of an additional photon by one of the pair components (see the figure)

$$d^2\sigma^p/d\Delta d\beta = \Phi^p(\Delta, \beta), \quad \Delta = \varepsilon_+/\omega_0, \quad \beta = \omega/\omega_0,$$

where ϵ_+ and ω are the positron and photon energies in the c.m.s. of the initial particles. The functions $F^{br}(\Delta, \beta)$ and $\Phi^p(\Delta, \beta)$ are related by the following expression:

$$\Phi^p(\Delta, \beta) = -F^{br}(-\Delta/\beta, 1/\beta). \tag{3}$$

The function $\Phi^p(\Delta, \beta)$ is given in the Appendix (see Eq. (A.1)). Integration of Eq. (A.1) over the energy fraction of the pair positron Δ between the limits 0 and $1-\beta$ gives the following expression for the spectrum of photons produced in the process $\gamma e \to \gamma e^+e^-e$:

$$\frac{d\sigma}{d\beta} = \frac{2(\alpha r_0)^2}{105\pi}(\ln s_1)(1-\beta)I,$$
$$I = P_1 + P_2\frac{\pi^2}{6} + P_3\ln\gamma + P_4\ln^2\beta + (P_5 + P_6\ln\gamma)\eta\ln\beta + P_7\eta D, \tag{4}$$

where

$$s_1 = \frac{s\beta(1-\beta)}{m_e^2}, \quad \gamma = \frac{(1-\beta)^2}{\beta}, \quad \eta = \frac{1-\beta}{1+\beta}, \quad D = \int_{1-1/\beta}^{1-\beta} dx\frac{\ln(1-x)}{x},$$

$$P_1 = -\frac{1}{105}(25528\gamma^2 + 116044\gamma + 151556), \quad P_2 = 256\gamma^3 + 1092\gamma^2 + 1260\gamma + 420,$$
$$P_3 = \frac{1}{105}(676\gamma^3 + 9877\gamma^2 + 58415\gamma + 62160),$$
$$P_4 = 64\gamma^3 + 305\gamma^2 + 475\gamma + 269 - 276\gamma^{-1},$$
$$P_5 = \frac{1}{105}(676\gamma^3 + 38109\gamma^2 + 211637\gamma + 266660 - 53632\gamma^{-1}),$$
$$P_6 = 32\gamma^2 + 416\gamma + 1310 + 1184\gamma^{-1},$$
$$P_7 = 128\gamma^3 + 802\gamma^2 + 1082\gamma - 470 - 1184\gamma^{-1}.$$

In the limit of small photon frequencies $\beta \to 0$, $I(\beta) \sim \beta^{-1}(128\pi^2 - 630)$ and for $\beta \to 1$ $I(\beta) \sim 70(\pi^2 - 4)$. The quantity $(1-\beta)I/105\pi$ is given in the table. Note that $R(\beta) = (1-\beta)I$ satisfies the relation $R(\beta) = -\beta R(1/\beta)$, which is the consequence of crossing symmetry.

The spectrum (4) (see Sec. 3) is interesting in that it represents the main elementary (nonshower) mechanism of aging of a photon in matter and must be taken into account in problems involving the propagation of high-energy photons in very rarified media, say, in interstellar matter, where the radiation length of the photon has the order of galactic distances.

2. In this section we will discuss the spectrum of positrons produced as the result of collision of a high-energy photon with an electron. In the Born approximation the spectrum is determined by Eq. (1). The first two terms in the radiative correction to the Born spectrum (see Eq. (2)), which take into account respectively the contributions of virtual and soft real photons, whose frequencies in the c.m.s. do not exceed $\epsilon\sqrt{s}/2$, $\epsilon \ll 1$, were calculated in the WW approximation in ref. 7. We find, however, that it is necessary to present here the results of calculation of these terms with correction of the misprints in ref. 7:

$$\left(\frac{d\sigma}{d\Delta}\right)_v + \left(\frac{d\sigma}{d\Delta}\right)_s = \frac{2(\alpha r_0)^2}{105\pi}(\ln s_1)[G_1 + G_2\ln\varepsilon];$$
$$\gamma_1 = \frac{1}{\Delta(1-\Delta)}, \quad \eta_1 = 1-2\Delta, \quad \xi = \ln\frac{\Delta}{1-\Delta}, \quad D_1 = \int_{1-\Delta} dx\frac{\ln(1-x)}{x}, \tag{5}$$

$$s_1 = \frac{s\Delta(1-\Delta)}{m_e^2}, \quad h(x) = \frac{1}{x}\int_0^x dt\, t\,\mathrm{cth}\, t,$$
$$G_1 = -C_1 - C_2|\eta_1| - a_5\eta_1 D_1 - (C_3 + C_4\ln\gamma_1)\ln\gamma_1 - a_3\xi^2 - [a_9 + C_5\ln\gamma_1 - a_{12}h(\xi/2) + C_6 h(\xi)]\xi\eta_1,$$
$$G_2 = 2[a_2 + a_7\ln\gamma_1 + a_{10}\xi\eta_1]; \tag{5a}$$

$$C_1 = \frac{\pi^2}{6}(16\gamma_1^2 - 518\gamma_1 + 630 + 420\gamma_1^{-1} + 840\gamma_1^{-2}) + 32\gamma_1 + \frac{59941}{105} - \frac{48916}{105}\gamma^{-1}$$
$$C_2 = \frac{\pi^2}{6}(-48\gamma_1^2 + 576\gamma_1 - 1044 + 552\gamma_1^{-1}),$$
$$C_3 = -\frac{11333}{105}\gamma_1 + \frac{144655}{210} - \frac{22260}{105}\gamma_1^{-1},$$
$$C_4 = 8\gamma_1^2 - \frac{763}{4}\gamma_1 + \frac{1085}{4}, \quad C_5 = -96\gamma_1 + \frac{327}{2} - 211\gamma_1^{-1},$$
$$C_6 = -42\gamma_1 + 56 + 448\gamma_1^{-1},$$
$$a_2 = -16\gamma_1 + 200 - 184\gamma_1^{-1}, \tag{5b}$$
$$a_3 = -16\gamma_1^2 + \frac{785}{4}\gamma_1 - \frac{1763}{4} + 424\gamma_1^{-1} - 396\gamma_1^{-2}, \quad a_5 = 315\gamma_1 - 175 - 210\gamma_1^{-1},$$
$$a_7 = 8\gamma_1^2 - 96\gamma_1 + 174 - 92\gamma_1^{-1}, \quad a_9 = -\frac{11333}{105}\gamma_1 + \frac{9443}{210} - \frac{6608}{105}\gamma_1^{-1},$$
$$a_{10} = 8\gamma_1^2 - 96\gamma_1 + 174 - 92\gamma_1^{-1}, \quad a_{12} = 32\gamma_1^2 - 426\gamma_1 + 752 + 80\gamma_1^{-1}.$$

In ref. 7, errors were made in C_2 and C_4.

The correction to the spectrum (1) due to radiation of a hard photon is obtained by integration of Eq. (A.1) over the photon energy fraction β from ϵ to $1-\Delta$ for fixed Δ:

$$\left(\frac{d\sigma}{d\Delta}\right)_h = \frac{2(\alpha r_0)^2}{105\pi}(\ln s_1)\Big\{-A_1 - A_2\ln(1-\Delta) - A_3\ln\Delta - A_4\ln^2(1-\Delta)$$

$$-A_5 \ln\Delta \ln(1-\Delta) - A_6 \ln^2\Delta - A_7 \frac{\pi_2}{6} - A_8 \int_0^{1-\Delta} \frac{dx}{x} \ln(1-x) + G_2 \ln\varepsilon \Big\}; \qquad (6)$$

$$A_1 = -\frac{3842}{15}\Delta^3 - \frac{3852}{5}\Delta^2 + \frac{26017}{30}\Delta + \frac{1234}{5} - 144(1-\Delta)^{-1} - 32\Delta^{-1},$$

$$A_2 = \frac{418}{15}\Delta^4 - \frac{1615}{3}\Delta^3 + 1194\Delta^2 - \frac{3245}{3}\Delta + \frac{3154}{3} - 32(1-\Delta)^{-1} - \frac{1426}{5}\Delta^{-1},$$

$$A_3 = -\frac{418}{15}\Delta^4 + \frac{763}{3}\Delta^3 - 230\Delta^2 - 144\Delta - \frac{1365}{2} + 660(1-\Delta)^{-1} - 272(1-\Delta)^{-2},$$

$$A_4 = 368\Delta^3 - 552\Delta^2 + 880\Delta - 628 + 384\Delta^{-1} - 32\Delta^{-2}. \qquad (6a)$$

$$A_5 = -400\Delta^3 + 744\Delta^2 - 954\Delta - 6 + 192\Delta^{-1} - 32\Delta^{-2} + 384(1-\Delta)^{-1} - 32(1-\Delta)^{-2},$$

$$A_6 = 142\Delta^4 - 353\Delta^3 + 435\Delta^2 - 280\Delta + \frac{611}{2} - 354(1-\Delta)^{-1} + 334(1-\Delta)^{-2} - 128(1-\Delta)^{-3},$$

$$A_7 = 284\Delta^4 - 770\Delta^3 + 1518\Delta^2 - 1042\Delta + 587 - 504\Delta^{-1} + 32\Delta^{-2},$$

$$A_8 = -284\Delta^4 + 770\Delta^3 - 1254\Delta^2 + 918\Delta - 253 - 504\Delta^{-1} + 32\Delta^{-2}.$$

Adding Eqs. (5) and (6), we obtained for the total radiative correction to the positron spectrum

$$\frac{d\sigma}{d\Delta} = \frac{2\alpha^2 r_0^2}{105\pi}(\ln s_1)\psi(\Delta), \quad \psi(\Delta) = -B_1 - B_2\frac{\pi^2}{6} - B_3 \ln(1-\Delta) - B_4 \ln\Delta - B_5 \ln^2\Delta - B_6 \ln^2(1-\Delta) - B_7 \ln\Delta \ln(1-\Delta) - B_8 J(\Delta) - B_9 I(\Delta),$$

$$J(\Delta) = \int_0^{1-\Delta} dx \frac{\ln(1-x)}{x}, \quad I(\Delta) = \int_1^{\Delta/(1-\Delta)} dx \frac{\ln x}{x-1}; \qquad (7)$$

$$B_1 = -\frac{3842}{15}\Delta^3 - \frac{4568}{15}\Delta^2 + \frac{12041}{30}\Delta + \frac{2453}{3} - 112(1-\Delta)^{-1},$$

$$B_2 = (-552\Delta^2 + 552\Delta - 1044 + 480(1-\Delta)^{-1} + 480\Delta^{-1} - 48(1-\Delta)^{-2} - 48\Delta^{-2})|1-2\Delta| + 1124\Delta^4 - 2478\Delta^3 + 1980\Delta^2 - 930\Delta + 1364 - 1284\Delta^{-1} + 48\Delta^{-2} - 192(1-\Delta)^{-1} + 16(1-\Delta)^{-2},$$

$$B_3 = \frac{418}{15}\Delta^4 - \frac{6187}{15}\Delta^3 + \frac{3966}{5}\Delta^2 - \frac{3584}{5}\Delta + \frac{4763}{15} - 32(1-\Delta)^{-1} - \frac{208}{3}\Delta^{-1},$$

$$B_4 = -\frac{418}{15}\Delta^4 + \frac{1927}{15}\Delta^3 - \frac{1266}{5}\Delta^2 - \frac{1273}{15}\Delta - \frac{39791}{30} + \frac{13138}{15}(1-\Delta)^{-1} - 272(1-\Delta)^{-2},$$

$$B_5 = -254\Delta^4 + 453\Delta^3 - 406\Delta^2 + 158\Delta + \frac{265}{2} - 375(1-\Delta)^{-1} + 318(1-\Delta)^{-2} - 128(1-\Delta)^{-3}, \qquad (7a)$$

$$B_6 = -396\Delta^4 + 778\Delta^3 - 799\Delta^2 + 410\Delta - 446 + 171\Delta^{-1} - 32\Delta^{-2} + 192(1-\Delta)^{-1} - 16(1-\Delta)^{-2},$$

$$B_7 = 792\Delta^4 - 1588\Delta^3 + 1790\Delta^2 - 614\Delta + 923 - 780(1-\Delta)^{-1} + 32(1-\Delta)^{-2},$$

$$B_8 = -284\Delta^4 + 714\Delta^3 - 1170\Delta^2 + 302\Delta + 41 - 1092\Delta^{-1} + 32\Delta^{-2} + 588(1-\Delta)^{-1},$$

$$B_9 = 736\Delta^3 - 1104\Delta^2 + 1760\Delta - 696 + 384\Delta^{-1} - 32\Delta^{-2} - 384(1-\Delta)^{-1} + 32(1-\Delta)^{-2}.$$

As expected, the parameter ϵ drops out of the total radiative correction to the spectrum (1). The function $\psi(\Delta)/105\pi$ is given in the table. In the limit of small positron energies $\Delta \to 0$, $\psi(\Delta) \sim (105/2)\ln^2\Delta$, and in the case in which the positron carries away almost all of the photon energy, $\Delta \to 1$, $\psi(\Delta) \sim -210 \ln^2(1-\Delta)$. These limiting cases for $\psi(\Delta)$ have a doubly logarithmic nature: The energy invariant $\tilde{s} = (k_1 + k_2)^2$ for the process $\gamma\gamma \to e^+e^-\gamma$ is proportional to $1/\Delta(1-\Delta)$ and will be large in both cases. The imaginary part of the $\gamma\gamma \to \gamma\gamma$ amplitude at zero angle, which corresponds to the $e^+e^-\gamma$ intermediate state for a fixed positron-energy fraction Δ, is proportional to $(\ln^2\Delta)/\tilde{s}\,\Delta$, $\Delta \to 0$.

x	$(1-x)I/$ $/105\pi$	$\rho_1(x)/105\pi$	$\psi(x)/105\pi$	x	$(1-x)I/$ $/105\pi$	$\rho_1(x)/105\pi$	$\psi(x)/105\pi$
0.01	—	13.1	3.47	0.53	0.78	0,077	2.76
0.05	28.7	3.5	1.87	0,57	0.64	0,178	2.57
0.09	11,5	1.3	1.7	0.61	0.58	0.27	2.41
0.13	9	0,36	1.7	0.65	0,49	0,35	2,28
0.17	5.3	−0.098	1.77	0.69	0,40	0,40	2.15
0,21	4.15	−0.319	1.86	0.73	0.36	0,42	2,02
0,25	2,9	−0.41	1.96	0.77	0.32	0.375	1.85
0.29	2.1	−0.418	2.06	0,81	0.255	0.23	1,62
0.33	1.9	−0,38	2.18	0.85	0,19	−0,09	1,24
0,37	1,45	−0.31	2,31	0.89	0.12	−0.75	0,6
0.41	1.25	−0,23	2.46	0,93	0.09	−2.14	−0.6
0.45	1,04	−0.13	2.64	0,97	0.05	−6.0	−3.9
0.49	0,85	−0.03	2.86	0,99	0,03	−13.1	−9.6

We note that the distribution (6) taking into account the contribution of hard photons and also the total correction to the positron spectrum $\psi(\Delta)$ are not symmetric relative to the point $\Delta = 1/2$ in the absence of the contribution of virtual and soft real photons (Eq. (1) is invariant under the substitution $\Delta \to 1-\Delta$). Violation of this symmetry originates from the fact that in the case of radiation of a hard photon the sum of the energy fractions carried away by the positron and electron will not be equal to unity.

3. In the case of pair production in the field of a nucleus it is necessary to take into account screening of the Coulomb field of the nucleus by atomic electrons, and also corrections resulting from multiple interaction with the nucleus. The Born expression for the positron spectrum (1) in the case of partial screening goes over to the form

$$d\sigma_0/d\Delta \to Z^2\alpha r_0^2[(\Delta^2 + (1-\Delta)^2)\psi_1 + {}^2/_3\Delta(1-\Delta)\psi_2],$$

$$\psi_{1,2} = \Phi_{1,2} - \frac{4}{3}\ln Z - 4f(Z), \quad f(Z) = (Z\alpha)^2 \sum_1^\infty \frac{1}{n(n^2 + Z^2\alpha^2)}. \qquad (8)$$

The functions $\Phi_{1,2}(\omega, Z, \Delta)$ have been tabulated by Bethe and Heitler (see, for example, ref. 10).

In the limit of high initial-photon energies, the case of complete screening is realized:

$$d\sigma_0/d\Delta \to 4Z^2\alpha r_0^2\{[\ln(183Z^{-1/3}) - f(z)](1 - {}^4/_3\Delta(1-\Delta)) - {}^1/_9\Delta(1-\Delta)\}. \qquad (8a)$$

The spectrum of positrons with inclusion of the radiative correction takes the form

$$\frac{d\sigma}{d\Delta} = \frac{d\sigma_0}{d\Delta}\left[1 + \frac{\alpha}{105\pi}\frac{\psi(\Delta)}{a(\Delta)}\right], \quad a(\Delta) = 2 - \frac{8}{3}\Delta(1-\Delta), \qquad (9)$$

$d\sigma_0/d\Delta$ and $\psi(\Delta)$ are given respectively by Eqs. (8), (8a) and (7), (7a). In the case of complete screening the spectrum of photons produced in the process $\gamma Z \to \gamma e^+e^-Z$ is obtained from Eq. (4) by replacement of the factor $2\alpha^2 r_0^2 \ln s_1$ by $2\alpha^2 Z^2 r_0^2 \ln(183Z^{-1/3})$.

We will now find the change in the total cross section for pair production by a photon in the field of a nucleus due to the radiative correction. Integration of Eq. (5) over Δ from 0 to 1 gives for the contribution of the first two terms in Eq. (2) to the total cross section

$$\sigma_1 = \frac{2(\alpha Z r_0)^2}{105\pi}\ln(183Z^{-1/3})\left\{(128\pi^2 - 630)\ln\varepsilon + 3012\xi(3) + \frac{4424}{5}\xi(2) - \frac{6283}{2}\right\}, \qquad (10)$$

$$\xi(2) = \sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6} = 1{,}645, \quad \xi(3) = \sum_1^\infty \frac{1}{n^3} \approx 1.202.$$

This expression is identical to that given in ref. 7 if we take into account that

$$L_3(2) = \int_0^2 \frac{dx}{x}\ln x \ln|1-x| = \frac{7}{8}\xi(3) + \frac{1}{4}\pi^2 \ln 2.$$

The contribution of the hard part is obtained by integration of Eq. (6):

$$\sigma_2 = \frac{2(\alpha Z r_0)^2}{105\pi}\ln(183Z^{-1/3})\left\{(-128\pi^2 + 630)\ln\varepsilon + 372\xi(3) - \frac{4424}{5}\xi(2) - 344\right\}. \qquad (10a)$$

The total cross section for absorption of a photon by a completely screened nuclear field is determined by the sum of Eqs. (10) and (10a):

$$\sigma=\frac{28}{9}\alpha Z^2 r_0^2\left[\ln(183Z^{-1/3})-\frac{1}{42}-f(Z)\right](1+\delta^p),\qquad \delta^p=\frac{9\alpha}{14\pi}\left(\frac{1128}{35}\xi(3)-\frac{6971}{210}\right)\sim 0.009. \tag{11}$$

4. As noted above, in the problem of experimentally checking radiative corrections to the cross section for pair production by a photon, difficulty arises in taking into account asymptotically unimportant terms from the Born cross section. These terms are a background which exceeds the contribution of the correction up to photon energies of 1.5 GeV in the laboratory system. There is, however, the possibility of measuring the part of the radiative correction corresponding to radiation of a hard photon.[2)] This possibility is based on the fact that the contribution of the nonasymtotic terms of the Born approximation ($\sim\alpha^3 e^{-1}\ln(s/m_e^2)$ or $\sim\alpha^3 s^{-1}\ln(183Z^{-1/3})$) to the positron spectrum is symmetric with respect to the substitution $\Delta\to 1-\Delta$, since the sum of the electron and positron energy fractions for events with an e^+e^- pair in the final state (for the condition that the state of the target does not change) is equal to unity. In this connection we note that the number of pair-production events (without taking into account bremsstrahlung) in which this energy balance is not satisfied amounts to m_e^2/sZ of the total number of pair-production events (the bremsstrahlung mechanism of pair production). It is therefore proposed to measure the quantity

$$\rho(\Delta)=\frac{d\sigma}{d\Delta}(\Delta)-\frac{d\sigma}{d\Delta}(1-\Delta),$$

which consists of the difference in the numbers of events with positron-energy fractions Δ and $1-\Delta$. Since the initial-photon energy in this case may be not very large (of the order of hundreds of MeV), the case of partial screening is realized ($d\sigma_0/d\Delta$ is given by Eq. (8)):

$$\rho(\Delta)=\frac{d\sigma_0}{d\Delta}\frac{\alpha}{a(\Delta)}\frac{\rho_1(\Delta)}{105\pi},$$
$$\rho_1(\Delta)=\eta_1\left[\tilde P_1-\tilde P_2\ln\gamma_1+\tilde P_3\frac{\pi^2}{6}+\tilde P_4\ln^2\gamma_1+\tilde P_5\ln\Delta\ln(1-\Delta)\right]+\xi[\tilde P_6-\tilde P_7\ln\gamma_1]-\tilde P_8 D_1, \tag{12}$$

$$\begin{aligned}
\tilde P_1&=\frac{3842}{15}\gamma_1^{-1}-\frac{1593}{10}-112\gamma_1,\\
\tilde P_2&=142\gamma_1^{-1}-\frac{1637}{6}+\frac{2283}{5}\gamma_1-136\gamma_1^2,\\
\tilde P_3&=303\gamma_1^{-1}-85+252\gamma_1-16\gamma_1^2,\\
\tilde P_4&=-\frac{299}{2}\gamma_1^{-1}+320-369\gamma_1+247\gamma_1^2-64\gamma_1^3,\\
\tilde P_5&=598\gamma_1^{-1}-1175+1182\gamma_1-510\gamma_1^2+128\gamma_1^3,\\
\tilde P_6&=\frac{836}{15}\gamma_1^{-2}-\frac{5197}{15}\gamma_1^{-1}+\frac{24128}{15}-\frac{3803}{5}\gamma_1+136\gamma_1^2,\\
\tilde P_7&=-142\gamma_1^{-2}+\frac{379}{2}\gamma_1^{-1}-\frac{1115}{2}+735\gamma_1-375\gamma_1^2+64\gamma_1^3,\\
\tilde P_8&=284\gamma_1^{-2}-667\gamma_1^{-1}+178+284\gamma_1-16\gamma_1^2,
\end{aligned} \tag{12a}$$

$a(\Delta)$, ξ, η_1, γ_1, and D_1 are determined by Eqs. (9) and (5a); the function $\rho_1(\Delta)/105\pi$ is given in the table.

We note, however, that the practical use of Eq. (12) encounters difficulty lying in the fact that experimentally it is necessary to measure the energy fractions of the pair positron and electron with an error not exceeding 1%, since $\rho(\Delta)$, for Δ not close to 0 or 1, is roughly 100 times smaller than $d\sigma/d\Delta$, which is determined by Eq. (9). The requirement on energy resolution is relaxed somewhat for $\Delta\to 0$, since $\rho_1(\Delta)\approx(525/2)\ln^2\Delta$, $\Delta\to 0$.

In conclusion we express our gratitude to A. I. Akhiezer, V. N. Baĭer, D. V. Volkov, L. N. Lipatov, and V. S. Fadin for their attention and interest in this work.

APPENDIX

Using Eq. (8) from ref. 7 and expression (3), we obtain

$$\frac{1}{\ln(s/m_e^2)}\Phi^p(\Delta,\beta)=\frac{2(\alpha r_0)^2}{105\pi}\left\{-\tilde A_1-\tilde A_2\ln\frac{\beta}{(1-\Delta)(\Delta+\beta)}-\tilde A_3\ln\frac{(1-\Delta-\beta)\Delta}{(\Delta+\beta)(1-\Delta)}-(\hat I+\hat R_1)(\hat I+\hat R_2)\tilde A_4\ln\frac{(\Delta+\beta)(1-\beta)}{\Delta}\right\}, \tag{A.1}$$

$$\begin{aligned}
\tilde A_1={}&35\beta[(1-\Delta)^{-2}+(\beta+\Delta)^{-2}]-280(1+\beta)[(1-\Delta)^{-1}+(\beta+\Delta)^{-1}]\\
&+96\beta^2+110\beta+642+110\beta^{-1}+96\beta^{-2}+\Delta[-60\beta^{-2}-604\beta^{-1}+604+60\beta]\\
&+\Delta^2[92\beta^{-2}+600\beta^{-1}+92]+\Delta^3(-64\beta^{-2}+64\beta^{-1})+32\Delta^4\beta^{-2},\\
\tilde A_2={}&[(1-\beta-\Delta)^{-1}+\Delta^{-1}](16\beta^{-2}-96\beta^{-1}+243+33\beta+96\beta^2-16\beta^3-\\
&-276(1-\beta)^{-1})+[(1-\Delta)^{-1}+(\Delta+\beta)^{-1}](-16\beta^{-2}-96\beta^{-1}+33\beta-243\\
&-96\beta^2-16\beta^3+276(1+\beta)^{-1})+280(\beta+\beta^{-1}),\\
\tilde A_3={}&[(\Delta+\beta)^{-1}+(1-\Delta)^{-1}](16\beta^{-2}+96\beta^{-1}+243-33\beta+96\beta^2+16\beta^3\\
&-276(1+\beta)^{-1})-16(\beta^{-3}+\beta^3)-42(\beta^{-2}+\beta^2)-343(\beta^{-1}+\beta)+210\\
&+\Delta[68(\beta^{-3}-\beta^2)-70(\beta^{-2}-\beta)+462(\beta^{-1}-1)]\\
&+\Delta^2[-128(\beta^{-3}+\beta)+210(\beta^{-2}+1)-756\beta^{-1}]\\
&+\Delta^3[152(\beta^{-3}-1)-392(\beta^{-2}-\beta)]+\Delta^4[-156(\beta^{-3}+\beta^{-1})+280\beta^{-2}]\\
&+96\Delta^5(\beta^{-3}-\beta^{-2})-32\Delta^6\beta^{-3},\\
\tilde A_4={}&(1-\beta-\Delta)^{-1}[-128\beta^{-3}+334\beta^{-2}-492\beta^{-1}+453-247\beta+96\beta^2-16\beta^3]\\
&+(\Delta+\beta)^{-1}[16\beta^{-2}+96\beta^{-1}+37-37\beta-96\beta^2-16\beta^3]+112\beta^{-3}-232\beta^{-2}+74\beta^{-1}\\
&+80-34\beta+\Delta[196\beta^{-3}-132\beta^{-2}+276\beta^{-1}-440]\\
&+\Delta^2[276\beta^{-2}-276\beta^{-1}]-184\Delta^3\beta^{-2},
\end{aligned}$$

where Δ and β are the respective energy fractions of the positron and final photon in the process $\gamma e\to\gamma e^+e^-e$. In Eq. (A.1) we have introduced the following operations: $\hat I$ is the identity substitution $\beta\to\beta$, $\Delta\to\Delta$; $\hat R_1$ is the substitution $\beta\to\beta$, $\Delta\to 1-\beta-\Delta$, $\hat R_2$ is the substitution $\beta\to 1/\beta$, $\Delta\to-(1-\beta-\Delta)/\beta$.

1) The necessity of taking into account the process $\gamma Z\to\gamma e^+e^-Z$ in studying the disintegration of a photon in the field of a nucleus $\gamma Z\to\gamma\gamma Z$ was pointed out in 1964 by Shklyarevskiĭ [6] but the quantitative evaluation of the cross section $d^3\sigma/d^3k$ found in ref. 6 is incorrect as the result of errors made by him.

2) We thank L. N. Lipatov for pointing out this possibility.

1. J. W. Motz, H. A. Olsen, and H. W. Koch, Rev. Mod. Phys. 41, 1 (1969).
2. L. C. Maximon, Rev. Mod. Phys. 41, 193 (1969).
3. G. Jarlskog, L. Jonsson, et al., Preprint, DESY, 73/4, 1973.
4. V. N. Baĭer, V. M. Katkov, E. A. Kuraev, and V. S. Fadin, Phys. Lett. (to be publ.).
5. R. M. Dzhilkibaev, É. A. Kuraev, V. S. Fadin, and V. A. Khoze, ZhÉTF Pis. Red. 19, 73 (1974) [JETP Lett. 19, 47 (1974)].
6. G. M. Shklyarevskiĭ, Zh. Éksp. Teor. Fiz. 46, 690 (1964) [Sov. Phys. JETP 19, 470 (1964)].
7. K. Mork and H. Olsen, Phys. Rev. 140, 1661 (1965).
8. É. A. Kuraev, L. N. Lipatov, and N. P. Merenkov, Yad. Fiz. 18, 1075 (1973) [Sov. J. Nucl. Phys. 18, 554 (1974)].
9. É. A. Kuraev, L. N. Lipatov, and M. I. Strikman, Yad. Fiz. 18, 1270 (1973) [Sov. J. Nucl. Phys. 18, 652 (1974)].
10. A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika, Nauka, 1969, equations (30.2.5) and (28.5.5); English transl. of 1959 ed., Quantum Electrodynamics, Interscience Publishers, New York, 1965. V. N. Baĭer, V. M. Katkov, and V. S. Fadin, Izluchenie relyativistskikh élektronov (Radiation of Relativistic Electrons), Nauka, 1973, Sec. 19.

Translated by C. S. Robinson
197