Inhomogeneous states of antiferromagnetic and magnetoexcitonic semiconductors

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A homogeneous state of strongly doped antiferromagnetic semiconductors may be less favored energywise than an inhomogeneous state. Weakly inhomogeneous and strongly inhomogeneous states are possible. The former correspond to a noncollinear antiferromagnetic ordering, where an angle between the sublattice moments varies periodically in space. The second type of states occurs when the crystal can be separated into ferromagnetic and antiferromagnetic domains, the former having an excess of conduction electrons and the latter having a shortage. For relatively low concentrations, the most favorable of the geometries considered is that for which spherical ferromagnetic regions form a periodic structure within the antiferromagnetic matrix (the ferromagnetic part of the crystal is multiply connected). For larger conduction-electron concentrations it is more profitable when the spherical antiferromagnetic regions form a periodic structure within the ferromagnetic matrix (the antiferromagnetic part of the crystal is singly connected). With increase of the concentration, the conductivity sharply rises near a concentration for which the ferromagnetic part of the crystal changes from multiply connected to singly connected. In this case a large negative mangetoresistance should be observed near the field strength at which the topology of the ferromagnetic part of the crystal changes. The same effect should also be observed on increase of temperature. Strongly inhomogeneous states of the same kind with separate ferromagnetic and diamagnetic domains should also be possible in strongly doped magnetoexciton semiconductors.

1. FORMULATION OF PROBLEM AND QUALITATIVE PICTURE

It was established previously[1,2] that the homogeneous state of a magnetic conductor can be unstable against sufficiently large fluctuations of the conduction-electron density (this possibility was also pointed out by Krivoglaz[3]). It was shown in[2], in particular, that the state of a strongly doped antiferromagnetic semiconductor in which it is subdivided into alternating planar antiferromagnetic and ferromagnetic domains with a deficit of conduction electrons in the former and an excess in the latter may turn out to be energywise more favored than its homogeneous state.

We report here a detailed investigation of antiferromagnetic as well as magnetoexcitonic strongly-doped semiconductors. We shall show that both weakly inhomogeneous and strongly inhomogeneous states of crystals are possible. The former can be realized only in antiferromagnets and constitute a two-sublattice structure in which the angle between the moments of the sublattices varies periodically in space (noncollinear antiferromagnet with superstructure).

Strongly-inhomogeneous states are possible both in antiferromagnetic and in magnetoexcitonic semiconductors. In the former, the crystal breaks up into ferromagnetic and antiferromagnetic domains, and in the latter into ferromagnetic and diamagnetic domains. In the ferromagnetic domains the carrier density should be higher than on the average over the crystal. At relatively small carrier densities, the geometry that is most favored among all those considered in the paper is the one in which spherical ferromagnetic regions form a periodic structure within a nonferromagnetic matrix. At large densities, on the other hand, it is more convenient to have spherical nonferromagnetic regions form a periodic structure within a ferromagnetic matrix.

It follows from the foregoing that there exists a density n_T such that the ferromagnetic part of the crystal changes its topology from multiply-connected to singly-connected. With increasing carrier density n, an abrupt jump of the conductivity should occur near n_T. We have also investigated the properties of the inhomogeneous state in a magnetic field. It has been established, in particular, that at $n < n_T$ the topology of the ferromagnetic part can be altered by the field, as a result of which breakdown of the crystal under the influence of the magnetic field should be observed.

The appearance of domains with different types of magnetic ordering in an antiferromagnetic conductor resembles to a certain degree the subdivision of nonmagnetic conductors into paramagnetic and diamagnetic domains in a quantizing magnetic field, but the causes of the two are entirely different.

Owing to the exchange interaction between the carriers and the localized magnetic moments, the carrier energy turns out to depend on the character and degree of magnetic order in the crystal. It is quite obvious (and can be rigorously proven) that the minimal carrier energy is reached for ferromagnetic order. It is quite clear that although one carrier cannot change the type of order in an antiferromagnetic crystal of macroscopic dimensions, under certain conditions an inhomogeneous state of the system wherein antiferromagnetic order gives way to ferromagnetic order in a certain microscopic region may turn out to be energywise favored. This microscopic region is a potential well for the conduction electron, where it gains s-d exchange energy. The electron is localized in this region and thereby makes it stable.

In a paper by one of us[4], where the concept of such a state was introduced, it was designated by the term "magnetic polaron." This term should be regarded as inappropriate, since it has been used frequently by others to denote states of entirely different types, and this may lead to confusion. Carrier states in which the carriers are autolocalized in ferromagnetic microscopic regions which they themselves produce will henceforth be designated by the term "ferron."

If individual ferron states of carriers of the type described above are possible in antiferromagnetic crystals with very small carrier densities, then at relatively large carrier densities collective ferron states may become favored in these crystals. In other words, ferromagnetic regions, each of which has an excess of conduction electrons, may become possible in an antiferromagnetic crystal. The conditions for their realization are essentially different from those for individual ferron states. The reason is that the energy per conduction electron required to replace antiferromagnetic order by ferromagnetic order is smaller in the case of collective ferron states than in the case of individual states. However, the formation of collective ferrons, unlike the individual ferrons, is accompanied by an increase of the Coulomb energy of the electrons. Crystals can therefore exist in which individual ferron states are favored but collective states are never favored, and vice versa; collective ferron states may be realized in crystals in which individual ferrons are impossible.

At very high densities, the carriers play the decisive role in the establishment of magnetic order, so that a crystal in the homogeneous state should become ferromagnetic. However, the subdivision of the crystal into domains with ferromagnetic and antiferromagnetic order is possible also under these conditions. The reason is the following: the conduction electrons, being concentrated in the ferromagnetic regions, again acquire a net gain of s-d exchange energy, just as in the homogeneous ferromagnetic state. But the decrease of the electron density in the remainder of the crystal makes it possible to restore the antiferromagnetic order in this remainder, i.e., to obtain a gain in the energy of the direct exchange between the magnetic atoms. This gain is offset to a certain degree by the loss of kinetic energy of the electrons and of the Coulomb energy.

In either case, the homogeneous states, while unstable to large-amplitude fluctuations, remain stable with respect to small fluctuations. There exists, however, a density interval intermediate between the regions of absolute or relative stability of the antiferromagnetic and ferromagnetic states, in which neither state corresponds even to a relative minimum of the energy. It can be shown that in this interval the energy of the noncollinear antiferromagnetic (NCAF) state of the type proposed by de Gennes[5] is lower than that of the collinear states[6]. However, as will be shown later on, the homogeneous NCAF state may also turn out to be unstable, and not only with respect to strong fluctuations but also with respect to small ones. If the instability is caused only by strong fluctuations, then a strongly-inhomogeneous ferro-antiferromagnetic state is produced and leads simultaneously to a gain in the s-d exchange energy in the ferromagnetic domains and in the d-d exchange energy in the antiferromagnetic domains. Under certain conditions, however, it is possible to have in this interval also a weakly-inhomogeneous state with NCAF order, where the angle between the moments of the sublattices and the electron density vary periodically in space.

Inhomogeneous states are also possible in magnetoexcitonic semiconductors. A characteristic of the latter is that in the ground state the electron spin of a partially filled d shell of a magnetic atom is equal to zero. However, the excitation energy of a magnetic atom in a triplet state can be small. If the crystal contains conduction electrons, then their exchange interaction with the excited atoms lowers the energy of the system. At low carrier densities, individual ferron states are also possible here, in which excitons with spins parallel to one another are excited in a certain microscopic region, and the conduction electrons becomes autolocalized in such a microscopic region[7]. It will be shown below that, at sufficiently high carrier densities, collective ferron states are energywise favored here, too, i.e., the crystal in the ground state breaks up into ferromagnetic and diamagnetic domains.

Neutron diffraction investigations have shown that the spectra of the strongly doped antiferromagnetic $LaMnO_3$ semiconductors constitute superpositions of spectra corresponding to the ferromagnetic and antiferromagnetic order[8]. This is usually interpreted in terms of noncollinear antiferromagnetism. The results of the present paper lead to an alternate interpretation of this effect as evidence of the existence of collective ferron states. The subdivision of a strongly doped antiferromagnetic semiconductor into ferromagnetic and antiferromagnetic domains was observed directly by Vitins and Wachter[9] but in their case the phenomenon was complicated by strong spatial fluctuations of the impurity potential owing to the relative smallness of its concentration.

2. INSTABILITY AND WEAKLY INHOMOGENEOUS STATES OF ANTIFERROMAGNETS

We confine ourselves to the case when the electron density $n(\mathbf{r})$ varies little over distances on the order of the lattice constant a. We use the s-d model and assume that the width W of the conduction band is large in comparison with the product of the s-d exchange integral A by the spin S of the magnetic atom. The energy of the system, in first order in AS/W, can be written in the form

$$E=E_l+E_M, \tag{1}$$

$$E_l=\frac{3}{5}\mu_0\int\frac{n^{5/3}(\mathbf{r})}{n_0^{5/3}}d\mathbf{r}+\frac{e^2}{2\varepsilon_0}\int\frac{[n(\mathbf{r})-n_0][n(\mathbf{r}')-n_0]}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}\,d\mathbf{r}'$$
$$-\frac{AS}{2}\int n(\mathbf{r})\cos\theta(\mathbf{r})d\mathbf{r}+D\{n\}, \tag{2}$$

where ϵ_0 is the dielectric constant of the ideal crystal (in the absence of conduction electrons),

$$\mu_0=\frac{\hbar^2}{2m}(6\pi^2n_0)^{2/3},\quad n_0=\frac{1}{V}\int n(\mathbf{r})d\mathbf{r}. \tag{3}$$

The term $D\{n\}$ corresponds to going beyond the Thomas-Fermi approximation. It takes into account effects that are connected with the inhomogeneity of the electron distribution over the crystal. It is impossible to write a general expression for it. For those physical situations considered in the present paper, its structure will be specified below.

The expression for the Fermi energy μ_0 of a homogeneously distributed gas is written out under the assumption that owing to the nonzero magnetic moment of the crystal S cos $\theta(\mathbf{r})$ (per atom), all the electrons are spin-polarized, which requires, in any case, satisfaction of the inequality $\frac{1}{2}AS\cos\theta(\mathbf{r}) > \mu_0$. In the case of an antiferromagnet with two equivalent sublattices at T = 0, the exchange-interaction energy of the magnetic atoms with one another and with the external field H is given by the expression

$$E_M = -\frac{JS}{2a^3}\int \cos 2\theta(\mathbf{r})\,d\mathbf{r} - \frac{HS}{a^3}\int \cos\theta(\mathbf{r})\,d\mathbf{r}, \tag{4}$$

(J < 0, the field is in energy units, $\pm\theta(\mathbf{r})$ is the angle between the magnetic moment of the crystal and the sublattice moment).

The ground-state energy is determined at AS ≪ W by minimizing the sum of (2) and (4) with respect to $n(\mathbf{r})$ and cos $\theta(\mathbf{r})$. One of the extrema of the energy corresponds to a homogeneous state with $n(\mathbf{r}) = n_0$:

$$\cos\theta(\mathbf{r}) = An_0a^3/4|J|. \tag{5}$$

Obviously, if the density n_0 is less than the critical value $n_F = 4|J|/Aa^3$, at which cos $\theta = 1$, NCAF order is realized; if the density exceeds n_F, then a homogeneous ferromagnetic state is realized. The region in which the NCAF ordering energy is extremal is also bounded from below, inasmuch as spin depolarization of the electrons takes place at $\frac{1}{2}AS\cos\theta < \mu_0$, and the effective field with which they act on the spins of the magnetic atoms, and which causes the appearance of the moment of the crystal, is proportional to the degree of their polarization. According to[6], the lower bound of this region is

$$n_A = 4\mu_0 n_F/3AS,\quad n_F = 4|J|/Aa^3. \tag{6}$$

We now investigate the stability of the NCAF state against small oscillations of the electron density $n(\mathbf{r}) - n_0$. According to Hohenberg and Kohn[10], the functional $D\{n\}$ of the electron density in (2) is given in this case by

$$D\{n\} = \int\left[K(\mathbf{r}-\mathbf{r}') - \frac{1}{2}\frac{d\mu_0}{dn_0}\delta(\mathbf{r}-\mathbf{r}')\right][n(\mathbf{r})-n_0][n(\mathbf{r}')-n_0]\,d\mathbf{r}\,d\mathbf{r}'. \tag{7}$$

where the Fourier transform $K(\mathbf{q})$ of the quantity $K(\mathbf{r})$ is connected with the dielectric constant of the conducting crystal $\epsilon(\mathbf{q}, 0)$, calculated at a fixed value of cos θ, by the relation

$$K(q) = \frac{2\pi e^2}{q^2[\varepsilon(\mathbf{q},0)-\varepsilon_0]}. \tag{8}$$

If we use for $\epsilon(\mathbf{q}, 0)$ the well-known Lindhard expression corresponding to the random-phase approximation, then in the case of long-wave fluctuations expression (1) takes the form

$$E = \frac{1}{2}\sum\lambda_{\mathbf{q}}n_{\mathbf{q}}n_{-\mathbf{q}} - \frac{1}{27}\frac{\mu_0}{n_0^2}\sum{}'n_{\mathbf{q}_1}n_{\mathbf{q}_2}n_{\mathbf{q}_3} + \frac{1}{81}\frac{\mu_0}{n_0^3}\sum{}'n_{\mathbf{q}_1}n_{\mathbf{q}_2}n_{\mathbf{q}_3}n_{\mathbf{q}_4}, \tag{9}$$

$$\lambda_{\mathbf{q}} = \frac{2}{3}\frac{\mu_0}{n_0} + \frac{4\pi e^2}{\varepsilon_0 q^2} + \frac{\hbar^2q^2}{36mn_0} - \frac{AS}{2n_F},$$

where $n_{\mathbf{q}}$ is the Fourier component of the density fluctuations and the prime at the summation sign denotes the momentum conservation law.

The homogeneous state of the system becomes unstable upon vanishing of the smallest of the "frequencies" $\lambda_{\mathbf{q}}$, corresponding to the wave vector

$$q_m^2 = \frac{6}{\hbar}\left(\frac{4\pi e^2 n_0 m}{\varepsilon_0}\right)^{1/2} = \frac{6}{\hbar}m\omega_L \tag{10}$$

(ω_L is the plasma frequency). The frequency $\lambda_{\mathbf{q}m}$ is negative at densities n_0 exceeding the critical value n_c defined by the equation

$$\frac{AS}{2n_F} = \frac{1}{3n_c}(2\mu_c + \hbar\omega_{Lc}),\quad \mu_c \equiv \mu(n_c),\quad \omega_{Lc} \equiv \omega_L(n_c). \tag{11}$$

Thus, at $n_0 > n_c$ the excitation of fluctuations with $q \approx q_m$ is energywise favored. From the momentum conservation law it follows that near the critical concentration the cubic term of (9) introduces an asymptotically small contribution to the energy. Since the coefficient of the fourth-order term in (9) is positive, the concentration phase transition to the inhomogeneous state should, in accord with the Landau theory, be of second order, and as a result of this transition there should be established a periodic structure with period $2\pi q_m^{-1}$. The oscillations of the angle between the moments of the sublattices are connected with the oscillations of the electron density by relation (5).

It must be borne in mind, however, that as the density is increased, a jumplike transition of the crystal to a strongly inhomogeneous state, which will be considered in the next section, is possible even before n_c is reached. At any rate, within the framework of the approximations employed below, we find that the conditions for the existence of strongly inhomogeneous states can be much more stringent than those for weakly inhomogeneous states, so that the latter are seemingly realizable in principle.

3. STRONGLY INHOMOGENEOUS STATES

The essentially nonlinear character of the problem suggests that a variational principle be used for its solution. We confine ourselves to a detailed examination of the inhomogeneous state of a conducting antiferromagnet. It is natural to assume that the crystal should break up into alternating regions with increased and decreased electron densities, forming a periodic structure. The approximations employed below consist in the following: 1) the carrier density in the enriched layers is independent of the coordinate; 2) the carrier density in the depleted regions is assumed to be zero. The first of these assumptions is justified at W ≫ AS by the fact that their Fermi energy greatly exceeds the energy of the Coulomb interaction with the spatially-inhomogeneous charge distribution. The second approximation is fully justified when the condition (11) for the instability of the NCAF structure is satisfied in the entire interval $n_A < n < n_F$ and, in addition, the inequality $\mu \ll AS$ holds. It is then clear that the crystal should break up into microscopic regions with ferromagnetic and antiferromagnetic ordering, since, according to (11), if the NCAF structure is unstable at $n = n_0$, then it is all the more unstable when n lies in the interval $n_0 < n < n_F$. According to (6), the density in the former of these microregions should be much higher than in the latter.

All the numerical estimates that follow will be made for parameters typical of antiferromagnetic rare-earth compounds of the EuSe or EuTe type, namely $T_N = 10^{-3}$ eV (~12°K), $T_N = |J|S$, AS/2 = 0.5 eV, $\epsilon_0 = 20$, $N = 0.4\cdot10^{23}$ cm^{-3}, $N = a^{-3}$, and an effective mass equal to the free-electron mass. The results of the calcula-

tions will be shown in Figs. 1–3 below. For the indicated values of the parameters we obtain from (6) $n_F = 1.6 \times 10^{20}$ cm^{-3} and $n_A = 0.4 \times 10^{19}$ cm^{-3}, so that the ratio of the electron densities in the ferromagnetic and antiferromagnetic regions should exceed 40. Thus, at medium electron densities n_0, from 0.5×10^{20} cm^{-3} upward, the electron density in the antiferromagnetic regions does not exceed 10% of n_0, thus demonstrating the validity of the second approximation.

It can be hoped that owing to the relatively weak sensitivity of the variational functional to the form of the trial function, this approximation can give reasonable results even if the inequality $\mu < AS$ is not too strong. The same also pertains to the opposite case $W \ll AS$, when the Coulomb energy is not small in comparison with the Fermi energy.

As to the geometry of the inhomogeneous state, three possibilities are considered: 1) the system breaks up into alternating ferromagnetic and antiferromagnetic layers (the same geometry as in[2]); 2) the ferromagnetic microscopic regions are spherical in shape and are equidistant from one another inside the antiferromagnet; 3) the inverse situation, when antiferromagnetic microscopic regions of spherical shape are situated inside a ferromagnet.

The limitation of electron motion by the ferromagnetic region increases the electron energy (spatial quantization of the orbits). We confine ourselves to regions that are large from the microscopic point of view, when the role of the spatial quantization can be taken into account by perturbation theory. The small parameter is the reciprocal of the product of the Fermi momentum by the characteristic dimension of the system. When the electron energy is calculated in first order in this parameter, it is necessary to replace the density of states of the free electron ρ_0 by

$$\rho(E) = \rho_0(E)\left[1 - \frac{\pi}{4(2mE)^{1/2}}\frac{S_F}{V_F}\right], \qquad (12)$$

where V_F is the volume of the ferromagnetic region and S_F is the area of the surface that separates the ferromagnetic and antiferromagnetic regions[11] (the outer surface of the crystal can be disregarded, since its area is small in comparison with the area of the ferromagnet-antiferromagnet interface).

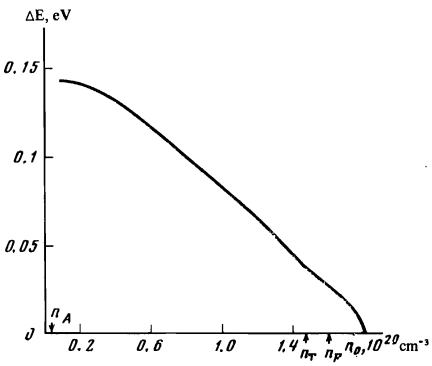

FIG. 1. Dependence of the gain of the electron energy of the inhomogeneous state on the density n_0 ($n_T = 1.47 \times 10^{20}$ cm^{-3}).

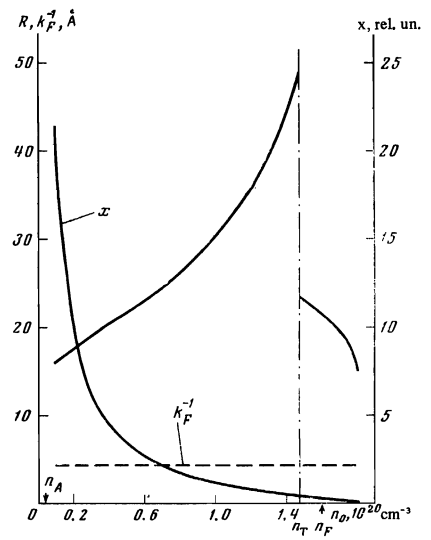

FIG. 2. Dependence of the radii R of ferromagnetic (to the left of n_T) and antiferromagnetic (to the right of n_T) drops, of the relative increase x of the carrier density, and of the reciprocal Fermi momentum on the density n_0; $k_F(n) = (1 + x)^{1/3} k_F(n_0)$.

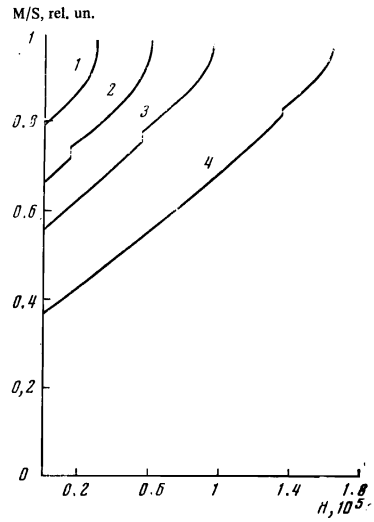

FIG. 3. Dependence of the reduced magnetic moment M/S on the field in the inhomogeneous state: 1) $n_0 = 1.6 \times 10^{20}$ cm^{-3}; 2) $n_0 = 1.4 \times 10^{20}$ cm^{-3}; 3) $n_0 = 1.2 \times 10^{20}$ cm^{-3}; 4) $n_0 = 0.8 \times 10^{20}$ cm^{-3}.

When the ratio of the correction $\sim S_F/V_F$ to the principal term in (12) is numerically estimated, it must be recognized that the electron density n in the ferromagnetic region is higher than the value n_0 averaged over the crystal. In the second term it is therefore necessary to take as the electron energy E the Fermi energy of electrons with density n, rather than n_0. Estimates show that for the data given in Figs. 1 and 2 below, this ratio is equal to $\frac{1}{2}$ at $n_0 = 0.2 \times 10^{20}$ cm^{-3} and $\frac{1}{3}$ at $n_0 = 10^{20}$ cm^{-3}. The situation is more favorable in the case of antiferromagnetic spheres, when it is equal to $\frac{1}{4}$.

According to the results of Balian and Bloch[11], the ratio of the third term of the expansion (which is not written out in (12)) to the second is approximately equal to $(6\pi n^{1/3} V_F/S_F)^{-1}$, i.e., ~10% at the parameter values indicated above. This justifies the applicability of the expansion (12) in the indicated concentration interval. These are precisely the concentrations of the donor electrons in the collectivized states in real strongly doped semiconductors, so that the approximations employed are fully justified for them. When the density is lowered, the conditions for the applicability of the expansion (12) become worse.

We denote by R the thickness of the ferromagnetic

domain in the planar case and the radius of the ferromagnetic or antiferromagnetic sphere, and by $x = (n - n_0)/n_0$ the relative increase of the carrier density in the ferromagnetic part of the crystal. These two quantities play the role of the variational parameters. The expression (1) for the energy per unit volume of the crystal can be rewritten in the form

$$E=E_c+D+E_q+E_M, \qquad E_c=\tfrac{3}{5}\mu_0 n_0(1+x)^{5/3}. \tag{13}$$

The term $\{n\}$, which takes into account the inhomogeneity of the electron gas, can be represented, using formula (12), in the form

$$D=\beta\left(\frac{\pi}{6}\right)^{1/3}\frac{5}{16}\frac{E_c}{n_0^{1/3}(1+x)^{1/3}R}, \tag{14}$$

where the coefficient β is equal to 2 for planar layers, to 3 for ferromagnetic spheres, and to 3x for antiferromagnetic spheres.

Obviously, the term D, which is connected with spatial quantization of the electron orbits, plays the role of an effective surface energy that depends on the electron density n and on the variation parameter x. Were this term to be disregarded, the energy of the system would not have a minimum with respect to R in the considered model.

The Coulomb energy E_q for spherical geometry is calculated by subdividing the crystal into Wigner cells. In the principal approximation in $(Rn^{1/3})^{-1}$, it is given by the expressions

$$E_q=\pi e^2 n_0^2 R^2 x^2/6\varepsilon_0 \tag{15a}$$

for planar layers,

$$E_q=\frac{2\pi}{5\varepsilon_0}n_0^2e^2R^2[2x+3-3(1+x)^{2/3}] \tag{15b}$$

for ferromagnetic spheres, and

$$E_q=\frac{2\pi}{5\varepsilon_0}n_0^2e^2R^2x[3x+2-3x^{1/3}(1+x)^{2/3}] \tag{15c}$$

for antiferromagnetic spheres.

The magnetic energy E_M and the magnetic moment M (per atom) take the form

$$E_M=-\left[\frac{AS}{2}n_0-\frac{|J|S}{(1+x)a^3}+\frac{H^2S}{4|J|a^3}\frac{x}{1+x}+\frac{HS}{(1+x)a^3}\right], \tag{16}$$

$$M=\frac{S}{1+x}\left[1+\frac{Hx}{2|J|}\right]. \tag{17}$$

Expressions (13)—(16) are also valid for magnetoexciton conductors if $|J|S/2$ in (16) is replaced by the exciton frequency, and if we put H = 0.

Since the variational approach overestimates the energy of the inhomogeneous states, the region of their stability should be even broader than that obtained below. The stabilization of the inhomogeneous states should be helped also by elastic stresses that develop in the system[1)].

As seen from (13)—(16), only the effective surface energy D and the Coulomb energy E_q are sensitive to the geometry of the ferromagnetic part of the sample. Minimization of the sum $Q = D + E_q$ with respect to R (at fixed x) leads to the following expressions for the various geometries:

$$Q_F/n_0\approx 0.7\gamma x^{2/3}(1+x)^{2/3} \tag{18a}$$

for planar domains,

$$Q_F/n_0\approx 1.2\gamma(2x+3-3(1+x)^{2/3}]^{1/2}(1+x)^{2/3} \tag{18b}$$

for ferromagnetic drops, and

$$Q_A/n_0\approx 1.2\gamma x[3x+2-3x^{1/3}(1+x)^{2/3}]^{1/2}(1+x)^{2/3} \tag{18c}$$

for antiferromagnetic drops. We use here the notation

$$\gamma=(\mu_0 e^2 n_0^{1/3}/\varepsilon_0)^{1/2}.$$

Expressions (18) enable us to determine the particular geometry that ensures a minimum of the system energy. It must be borne in mind that the Wigner-cell method does not yield high accuracy in the case when the matrix volume is considerably smaller than the volume of the drops having the other type of magnetic order and contained in the matrix. Therefore expression (18b) yields a reliable value of Q_F only at sufficiently large x, while expression (18c) yields a reliable value of Q_A at sufficiently small x (we recall that, by definition, x is the ratio of the volumes of the antiferromagnetic and ferromagnetic regions in the crystal).

At low densities, $n_0 \ll n_F$, the value of x can be large, and therefore, according to (18) and (18b), the energy minimum is reached in the case of ferromagnetic spherical domains in an antiferromagnetic matrix. At densities n_0 exceeding n_F, the value of x should be small. Therefore the minimum of the energy is reached in the case of antiferromagnetic spherical domains in the interior of the ferromagnetic matrix. Planar domains give a higher energy than spherical ones in all cases.

An analysis of Eqs. (13)—(18) shows that within the framework of the assumed model, the inhomogeneous states are energywise more favored than the homogeneous state at arbitrarily small n_0 if the following inequality holds

$$AS/2>2^{1/3}[\tfrac{2}{5}\mu_F+\gamma_F], \tag{19}$$

where the subscript F denotes that the corresponding quantities are calculated at $n_0 = n_F$ (see Fig. 1). (The criterion (19) is obtained by calculating the energy of the state with ferromagnetic drops in the limit $x^{1/3} \gg 1$ at H = 0.) Of course, it is meaningful in practice only down to concentrations at which $\mu \gtrsim e^2 n_0^{1/3}/\epsilon_0$. At lower concentrations, spatial fluctuations of the impurity potential, which lead to transitions of electrons into currentless states, become significant. The limitation (19), generally speaking, is stronger than (11) near n_F.

On the high-concentration side, the region where inhomogeneous states exist is bounded by the limiting density $n = n_C$, which is determined from the condition $\partial E/\partial x = 0$ at x = 0. When the definitions (13), (16), and (18c) are taken into account, it takes the form

$$0.4\mu_c+1.51\gamma_c=\frac{|J|S}{n_c a^3}\left[1-\frac{H}{2|J|}\right]^2. \tag{20}$$

when this concentration is approached from below (see Fig. 2), the fraction x of the volume occupied by the antiferromagnetic phase tends to zero (we recall that $V_A = x/(1 + x)$), but the radius of each antiferromagnetic drop remains finite and tends to

$$R_c n_c^{1/3}=\left(\frac{5\varepsilon_0\mu_c}{16\pi e^2 n_c^{1/3}}\right)^{1/3}. \tag{21}$$

The critical density may turn out to be smaller if the condition for the spin polarization of the electrons is violated.

We now discuss the effect of an external magnetic field on the inhomogeneous state. As follows from (17), the magnetic moment of the sample increases with the field, both as a result of the appearance of magnetization in the antiferromagnetic part of the sample, and as a result of an increase in the dimensions of the ferromagnetic part of the sample. Therefore the rate of growth of the moment with the field is higher here than in an undoped antiferromagnet, and the dependence of the moment on the field is essentially nonlinear (see Fig. 3).

Depending on the parameter ratios, two behaviors of the magnet in the field are possible. If in the absence of the field the homogeneous NCAF structure is unstable with respect to small fluctuations of the electron density, then the magnetic field cannot stabilize the structure. This follows from the condition that the energies (2) and (4) be extremal with respect to cos $\theta(\mathbf{r})$ and that the number of particles in the system be constant (the magnetic field adds to the electron energy only a term that does not depend on the electron density and a term that depends on its mean value). Therefore the inhomogeneous state is preserved until total ferromagnetic order is established in a crystal. The critical field at which this takes place is determined from formula (20), given the density n_0.

If the homogeneous state is stable at H = 0 with respect to small electron-density fluctuations, then it can become stable also with respect to large fluctuations in sufficiently strong fields. This means that a sufficiently strong field destroys the inhomogeneous state and the NCAF state is established. This situation is perfectly analogous to the destruction of an individual ferron state by an external field in an antiferromagnet, a situation investigated earlier[12].

The described dependence of the character of the inhomogeneous state on the carrier density and on the magnetic field leads to singularities in the conductivity of such samples. As already indicated, at relatively small densities it is more profitable energywise to have an inhomogeneous structure in which the ferromagnetic part of the crystal is multiply connected, and at large densities it is more profitable to have it singly connected. Obviously, an abrupt increase of conductivity should occur near the density at which the energies of these states become commensurate and the ferromagnetic part of the crystal changes from multiply connected to singly connected (a transition from an insulating to a conducting state should have occurred within the framework of the assumed approximation). This effect resembles to a certain degree the onset of percolation in defect-containing conductors.

If at H = 0 it is energywise more profitable to have the ferromagnetic part of the crystal multiply connected, it becomes singly connected in a sufficiently strong magnetic field, or else the crystal goes over to a homogeneous state. In the former case a giant negative magnetoresistance should be observed near that value of the field at which the topology of the ferromagnetic part of the crystal changes. If the crystal goes over into a homogeneous state, then a jump of the magnetic moment should occur together with a jump of the conductivity.

The fields at which a change in topology takes place correspond in Fig. 3 to the small discontinuity on the plot of the magnetic moment, this being due to the use of trial states with different geometries on either side of the discontinuity. At the same time, the smallness of the discontinuity indicates that the employed approximations are consistent with each other. As seen from Fig. 3, the critical fields increase with decreasing carrier density, from dozens to hundreds of kOe.

It should be noted that at $n < n_T$ the conductivity should also increase abruptly with increasing temperature, since the crystal should go over in this case to a homogeneous state. This should also be valid for relatively small impurity concentrations close to those at which collectivization of the donor electrons with formation of an impurity band would take place, if the antiferromagnetic order did not give way to ferromagnetic order in the vicinity of the defects. The formation of such localized ferrons leads to a decrease of the radius of the donor states and consequently to a decrease of the overlap of the orbits of the neighboring donors. As a result, no collectivization of the donor electrons takes place at T = 0, but the collectivization can occur at higher temperatures, since the ferron states are destroyed in that case.

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[1] E. L. Nagaev and A. P. Grigin, Phys. Lett., 38A, 469 (1972).
[2] É. L. Nagaev, ZhETF Pis. Red. 16, 558 (1972) [JETP Lett. 16, 394 (1972)].
[3] M. A. Krivoglaz, Zh. Eksp. Teor. Fiz. 63, 670 (1972) [Sov. Phys.-JETP 36, 354 (1973)].
[4] É. L. Nagaev, ZhETF Pis. Red. 6, 484 (1967) [JETP Lett. 6, 18 (1967)]; Zh. Eksp. Teor. Fiz. 54, 228 (1968) [Sov. Phys.-JETP 27, 122 (1968)].
[5] P. De Gennes, Phys. Rev., 118, 141 (1960).
[6] É. L. Nagaev, Fiz. Tverd. Tela 14, 773 (1972) [Sov. Phys.-Solid State 14, 658 (1972)].
[7] É. L. Nagaev, Zh. Eksp. Teor. Fiz. 59, 1215 (1970) [Sov. Phys.-JETP 32, 664 (1971)].
[8] E. Wollan and W. Koehler, Phys. Rev., 100, 545 (1955).
[9] J. Vitins and P. Wachter, Programma i tezisy mezhdunarodnoĭ konferentsii po magnetizmu, SSSR, Moskva, 22-28 avgusta (Program and Abstracts of Papers at International Conference of Magnetism, Moscow, USSR, 22-28 August 1973), Nauka, 1973, p. 79
[10] P. Hohenberg and W. Kohn, Phys. Rev., 136B, 864 (1964).
[11] R. Balian and C. Bloch, Ann. of Phys., 60, 401 (1970).
[12] G. L. Lazarev and É. L. Nagaev, Fiz. Tverd. Tela 15, 1635 (1973) [Sov. Phys.-Solid State 15, 1101 (1973)].

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