The Luttinger method in the theory of thermomagnetic phenomena

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The technique developed by Luttinger for calculating the kinetic coefficients is generalized to the case of an arbitrary external magnetic field. In contrast to the existing quantum theory of thermomagnetic phenomena, it is not necessary to specially subtract terms due to inhomogeneity from the expressions for the skew fluxes. Relations of the Wiedemann–Franz type, which are valid for arbitrary impurity concentrations and arbitrary magnetic field strengths, are derived for the kinetic coefficients.

The quantum theory of thermomagnetic phenomena in a disordered system meets with certain difficulties connected with the necessity for the accurate determination of the charge and energy fluxes, besides the usual complexities in the determination of the responses of the system to thermal and nonmechanical influences. In fact, the off-diagonal kinetic coefficients calculated before 1964 by a number of authors[1-3] did not satisfy the Onsager-Einstein symmetry relations. In 1964 Obraztsov[4] showed that in computing the nondissipative thermomagnetic currents in quantizing magnetic fields (for which $\hbar\Omega \gg \bar{\epsilon}$ and $\Omega\tau \gg 1$, where Ω is the cyclotron frequency, $\bar{\epsilon}$ the mean electron energy, and τ the interval between collisions with impurities), it is necessary to take into account the fluxes connected with the magnetic moment due to the quantization of the motion of the conduction electrons (Landau diamagnetism). Subsequently, a number of authors generalized and supplemented his results (reviews of these papers are given in[5,6]). General expressions for all the off-diagonal kinetic coefficients of conductors located in an arbitrary magnetic field were derived by Peletminskiĭ[7].

In the present paper we propose a method of computing the off-diagonal kinetic coefficients that generalizes to the case of magnetic fields the Luttinger method[8], in which the equivalent (in virtue of the Einstein relations) influence of the inhomogeneous "gravitational" field and the temperature inhomogeneity on the transport properties of the system is considered. This allows us to find the thermal coefficients by calculating in the usual manner the linear response of the system to a mechanical (gravitational) perturbation without, in contrast to the previous investigations, the use of any additional assumptions about the existence of a local-equilibrium distribution function. In the method proposed here the fluxes connected with the magnetic moment is automatically taken into account, and we need not separately subtract, as was done in[4-7], the terms due to the inhomogeneity (i.e., terms of the type c curl M). The expressions obtained for the off-diagonal kinetic coefficients, which allows us to, for example, use the experimental data on the Hall effect to compute those kinetic quantities (like the transverse thermoelectromotive force or the skew thermal conductivity) that are difficult to measure. Besides this, expressions are obtained for the thermomagnetic coefficients in the case of weak, as well as quantizing magnetic fields.

According to Luttinger[8], the total Hamiltonian of a system of charges in the field of impurities has the form

$$\mathscr{H}=\int h(\mathbf{r})\,d\mathbf{r},\quad h(\mathbf{r})=(1+\varphi_{gr}(\mathbf{r}))\,[h_0(\mathbf{r})+e\varphi_{el}(\mathbf{r})\rho(\mathbf{r})]. \qquad (1)$$

Here $h_0(\mathbf{r})$ is the unperturbed Hamiltonian density in an external magnetic field, $\rho(\mathbf{r})$ is the particle-number density operator, and φ_{el} and $-c^2\varphi_{gr}$ are the potentials of the external electric and gravitational fields.

The continuity equations relate the Hamiltonian density h and ρ respectively to the energy flux $\mathbf{q}$ and the particle flux $\mathbf{j}$:

$$\partial\rho/\partial t=-\operatorname{div}\mathbf{j},\quad \partial h/\partial t=-\operatorname{div}\mathbf{q}. \qquad (2)$$

We obtain the expressions for the unperturbed quantities ($\varphi_{el}=\varphi_{gr}=0$) h_0 and $\mathbf{q}_0$ in the nonrelativistic limit from the corresponding components of the energy-momentum tensor of an electron in a magnetic field:

$$h_0=\frac{m}{4}[\Psi^+V_-^2\Psi+(V_+^2\Psi^+)\Psi]+U\Psi^+\Psi-\mu\mathbf{H}\Psi^+\boldsymbol{\sigma}\Psi,$$

$$\mathbf{q}_0=\frac{m}{4}[(V_+^2\Psi^+)\mathbf{V}_-\Psi-(\mathbf{V}_+\Psi^+)V_-^2\Psi]+U\mathbf{j}_0-\mu(\mathbf{H}\boldsymbol{\sigma})\mathbf{j}_0$$

$$-\frac{1}{8}\mathbf{p}[(V_+^2\Psi^+)\Psi-\Psi^+V_-^2\Psi]; \qquad (3)$$

$$\mathbf{j}_0=\frac{1}{2}[\Psi^+\mathbf{V}_-\Psi-(\mathbf{V}_+\Psi^+)\Psi],\quad \rho=\Psi^+\Psi,$$

$$\mathbf{V}_\pm=-\frac{i\hbar}{m}\nabla\pm\frac{e}{mc}\mathbf{A}.$$

Here Ψ and Ψ^+ are the operator wave functions of the electron, $\mathbf{A}$ is the vector potential of the external magnetic field H (div A = 0), U is the electron-impurity interaction energy, σ is the Pauli operator, and μ is magnetic moment of the electron.

Let us find the equations of motion for Ψ and Ψ^+:

$$i\hbar\frac{\partial\Psi}{\partial t}=[\Psi,\mathscr{H}]=(1+\varphi_{gr})\mathscr{H}_-\Psi+(\mathbf{p}\varphi_{gr})\frac{1}{2}\mathbf{V}_-\Psi,$$

$$i\hbar\frac{\partial\Psi^+}{\partial t}=-(1+\varphi_{gr})\mathscr{H}_+\Psi^+-(\mathbf{p}\varphi_{gr})\frac{1}{2}\mathbf{V}_+\Psi^+, \qquad (4)$$

$$\mathscr{H}_\pm={}^1/{}_2mV_\pm^2+U-\mu\mathbf{H}\boldsymbol{\sigma}+e\varphi_{el}.$$

Substituting the expressions (4) into the left-hand side of the first equation in (2), we obtain, up to terms of first order in φ_{el} and φ_{gr}, the equation

$$\operatorname{div}\mathbf{j}(\mathbf{r})=\operatorname{div}[\mathbf{j}_0(\mathbf{r})(1+\varphi_{gr}(\mathbf{r}))]. \qquad (5)$$

It is well known that in a magnetic field the electric current differs from the particle flux by the spin current $\mathbf{j}_s=\mu c\,\operatorname{curl}(\Psi^+\sigma\Psi)$. Varying $\mathscr{H}$ with respect to the vector potential[9], we can show that the total electric current in the gravitational and magnetic fields will have the form

$$\mathbf{j}_{el}=e\mathbf{j}_0(1+\varphi_{gr})+\mu c\,\operatorname{rot}[(1+\varphi_{gr})\Psi^+\boldsymbol{\sigma}\Psi]. \qquad (6)$$

It is clear that the second term in the expression (6) does not (in a constant magnetic field) make any contribution to the volume-averaged electric current, and can be discarded. For the same reason, the last term in the expression (3) for $\mathbf{q}_0$ is negligible.

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Let us now transform the left-hand side of the second equation in (2) to the divergence form, using Eqs. (1), (3), and (4):

$$-\operatorname{div}\mathbf{q}=\mathscr{P}_1+\mathscr{P}_2,$$

$$\mathscr{P}_1=\frac{(1+\varphi_{gr})m}{4i\hbar}[-(1+\varphi_{gr})(\mathscr{H}_+\Psi^+)V_-^2\Psi+\Psi^+V_-^2((1+\varphi_{gr})\mathscr{H}_-\Psi)$$
$$-(V_+^2(1+\varphi_{gr})\mathscr{H}_+\Psi^+)\Psi+(V_+^2\Psi^+)(1+\varphi_{gr})\mathscr{H}_-\Psi+2(1+\varphi_{gr})$$
$$\times(U-\mu\mathbf{H}\sigma+e\varphi_{el})(\Psi^+V_-^2\Psi-(V_+^2\Psi^+)\Psi)]+\frac{(\mathbf{p}\varphi_{gr})m}{2i\hbar}[(\mathscr{H}_+\Psi^+)\mathbf{V}_-\Psi$$
$$-(\mathbf{V}_+\Psi^+)\mathscr{H}_-\Psi]=-\operatorname{div}\{\mathbf{q}_0(1+2\varphi_{gr})+e\varphi_{el}\mathbf{j}_0\},\qquad(7)$$
$$\mathscr{P}_2=\frac{(\mathbf{p}\varphi_{gr})m}{8i\hbar}[\Psi^+V_-^2\mathbf{V}_-\Psi-(V_+^2\Psi^+)\mathbf{V}_-\Psi-(V_+^2\mathbf{V}_+\Psi^+)\Psi$$
$$+(\mathbf{V}_+\Psi^+)V_-^2\Psi].$$

For the transformation of $\mathscr{P}_2$ we shall use the identity

$$(\Delta f)\chi=f\Delta\chi+\Delta(f\chi)-2\operatorname{div}(f\nabla\chi),$$

and we shall also discard the terms that are proportional to total derivatives–terms that do not contribute to the averaged flux:

$$\mathscr{P}_2=\frac{1}{4}p_j\left[\frac{2}{m}p_j(\mathbf{h}\mathbf{j}_0)+((\mathbf{h}\mathbf{V}_+)\Psi^+)V_-^j\Psi-(V_+^j\Psi^+)(\mathbf{h}\mathbf{V}_-)\Psi\right]$$
$$=\frac{i\hbar}{4}h_k\frac{\partial}{\partial x_j}(\Psi^+[V_-^k,V_-^j]\Psi)=\frac{e\hbar^2}{4m^2c}\varepsilon_{jkl}h_kH_l\frac{\partial}{\partial x_j}(\Psi^+\Psi).\qquad(8)$$

Here $c^2\mathbf{h}$ is the gravitational-field strength, $h_k = \partial\varphi_{gr}/\partial x_k$, and ϵ_{jkl} is the antisymmetric unit tensor; summation over repeated indices is implied

Finally, we can write for the total-heat-flux operator the expression

$$q^i=q_0^i(1+2\varphi_{gr})+e\varphi_{el}j_0^i-\frac{e\hbar^2}{4m^2c}\varepsilon_{ikl}h_kH_l\Psi^+\Psi.\qquad(9)$$

The last term in the formula (9) arises as a result of the fact that velocity components in a magnetic field do not commute with each other. Otherwise, our results coincide with Luttinger's results, where, however, the terms proportional to φ_{gr} and φ_{el} in zero magnetic field in the linear–in φ–approximation were thereafter discarded because of the absence in that case of equilibrium currents.

Notice that the expressions for the total fluxes in the absence of a magnetic field can be derived from simple arguments. In fact, let

$$\mathscr{H}=\mathscr{H}_0(1+\varphi_{gr})+e\varphi_{el}$$

be the classical electron Hamiltonian. Then, evidently, we have

$$\mathbf{j}=n_e\partial\mathscr{H}/\partial\mathbf{p}=\mathbf{j}_0(1+\varphi_{gr}),$$
$$\mathbf{q}=\mathbf{j}\mathscr{H}=\mathbf{q}_0(1+2\varphi_{gr})+e\varphi_{el}\mathbf{j}_0.$$

In the case of a variable external magnetic field the energy balance equation will have the form

$$\frac{\partial h}{\partial t}+\operatorname{div}\mathbf{q}+\mu\frac{\partial\mathbf{H}}{\partial t}\Psi^+\sigma\Psi+\frac{e}{c}\frac{\partial\mathbf{A}}{\partial t}\mathbf{j}_0=0.$$

It follows from the existence of the invariant

$$T\sqrt{1-2\varphi_{gr}}=\text{const}$$

(T is the temperature in energy units) for matter in thermal equilibrium in a weak gravitational field[10] that in order to determine the coefficients in the equations for the current and heat flux that are proportional to the temperature gradient, it is necessary to make the substitution $h_k \to -T^{-1}\nabla_k T$ (see[8]).

After this procedure, the expressions (6) and (9) for the total fluxes can be written in the following form (cf.[7] and[11]):

$$\mathbf{j}^{el}=\mathbf{j}_0^{el}-\frac{c}{T}[\nabla T\times\mathbf{M}_{orb}],$$

$$\mathbf{q}=\mathbf{q}_0+c[\mathbf{E}\times\mathbf{M}_{orb}]-\frac{2c}{Te}[\nabla T\times\mathbf{L}]+\frac{e\hbar^2n_e}{4m^2cT}[\nabla T\,\mathbf{H}],\qquad(10)$$
$$\mathbf{L}=\frac{1}{V}\sum_\alpha\varepsilon_\alpha n_\alpha(\mathbf{M}_{orb})_{\alpha\alpha}.$$

Let us show this, using as an example the term $j_0^x\varphi_{gr}$, and assuming that $\mathbf{A} = (-Hy, 0, 0)$ and $\varphi_{gr} = -yh_y$:

$$\operatorname{Sp}(j_0^x\varphi_{gr})=\frac{e}{VT}\sum_\alpha n_\alpha(yv_x)_{\alpha\alpha}\nabla_yT=\frac{c\nabla_yT}{VT}\sum_\alpha n_\alpha\frac{\partial\varepsilon_\alpha^{orb}}{\partial H}=-cM_{orb}\frac{\nabla T}{T}.\qquad(11)$$

It is natural that only the orbital, and not the total, magnetic moment enters into the expressions (10); for the spin magnetic moment cannot be represented in the form of a moment of a current. It should be emphasized that the corrections to $\mathbf{q}_0$ and $\mathbf{j}_0$ in (10) make a contribution only for the off-diagonal ($j_x \sim \nabla_y T$) fluxes. Therefore, below our main attention will be focused precisely on the skew $\mathbf{j}$ and $\mathbf{q}$ fluxes.

The quantities $\mathbf{j}_0$ and $\mathbf{q}_0$ can be expressed in terms of the temperature Green function G:

$$j_0^i=\lim\left\{\left[\frac{i\hbar}{2m}(\nabla_i'-\nabla_i)-\frac{eA_i}{mc}\right]G(\mathbf{r},\mathbf{r}',\tau,\tau')\right\},$$
$$q_0^i-\zeta j_0^i=\lim\left[\frac{1}{2}\{\mathscr{H}_0-\zeta,v_i\}_+G(\mathbf{r},\mathbf{r}',\tau,\tau')\right],\qquad(12)$$

for $\mathbf{r}' \to \mathbf{r}$ and $\tau' \to \tau + 0$. Here ζ is the chemical potential, $\mathbf{v}$ the electron-velocity operator in the magnetic field, and $\mathscr{H}_0 = 1/2mv^2 + U - \mu\mathbf{H}\sigma$.

To the degree of accuracy of interest to us, we have

$$G(x,x')=\int G^{(0)}(x,x_1)(\Sigma(x_1)-\mathbf{r}_1e\mathbf{E}^*(x_1))G^{(0)}(x_1,x')dx_1,$$
$$G^{(0)}(x,x')=T\sum_{\alpha,\omega_m}\frac{\psi_\alpha(\mathbf{r})\psi_\alpha^*(\mathbf{r}')}{i\omega_m-\varepsilon_\alpha+\zeta}\exp(-i\omega_m(\tau-\tau')),\qquad(13)$$
$$\Sigma=\varphi_{gr}(\mathscr{H}_0-\zeta)+(\mathbf{p}\varphi_{gr})\frac{\mathbf{v}}{2},\quad \mathbf{E}^*=-\nabla\left(\varphi_{el}+\frac{\zeta}{e}\right),\quad \omega_m=\pi T(2m+1),$$

where ϵ_α and $\psi_\alpha(\mathbf{r})$ are the energy and wave function of an electron in the state $|\alpha\rangle$ in the field of the impurities and in an external magnetic field. For the subsequent concrete calculations we set

$$\mathbf{H}=(0,0,H),\quad \mathbf{E}=(0,Ee^{-i\omega_0\tau},0),\quad \varphi_{gr}=y\frac{\nabla_yT}{T}\exp(-i\omega_0\tau).$$

We shall also take into account the fact that

$$\langle\alpha|\Sigma|\beta\rangle=-i\hbar(2T(\varepsilon_\alpha-\varepsilon_\beta))^{-1}\cdot\langle\alpha|\{\mathscr{H}_0-\zeta,v_y\}_+|\beta\rangle\nabla_yT.$$

Substituting the expressions for $\mathbf{j}_0$ and $\mathbf{q}_0$ from (12) into (10), we obtain the following expressions for the volume-averaged total off-diagonal fluxes ($\omega_0 = 0$, $\delta \to 0$):

$$j^x=\sigma_{xy}E_y^*-(\beta_{xy}^{(1)}+\beta_{xy}^{(2)})\nabla_yT,$$
$$q^x=(\gamma_{xy}^{(1)}+\gamma_{xy}^{(2)})E_y-(\varkappa_{xy}^{(1)}+\varkappa_{xy}^{(2)})\nabla_yT;$$
$$\sigma_{xy}=-\frac{e^2}{V}\sum_{\alpha,\beta}(v_x)_{\beta\alpha}(y)_{\alpha\beta}\frac{n_{\alpha\beta}}{\varepsilon_{\alpha\beta}-i\delta},$$
$$\beta_{xy}^{(1)}=\frac{ie\hbar}{2TV}\sum_{\alpha,\beta}(v_x)_{\beta\alpha}(\{\mathscr{H}_0-\zeta,v_y\}_+)_{\alpha\beta}\frac{1}{\varepsilon_{\alpha\beta}}\frac{n_{\alpha\beta}}{\varepsilon_{\alpha\beta}-i\delta}$$
$$\beta_{xy}^{(2)}=-\frac{e}{VT}\sum_\alpha n_\alpha(yv_x)_{\alpha\alpha}=\frac{1}{T}\gamma_{xy}^{(2)},\qquad(14)$$
$$\gamma_{xy}^{(1)}=-\frac{e}{2V}\sum_{\alpha,\beta}(\{\mathscr{H}_0-\zeta,v_x\}_+)_{\beta\alpha}(y)_{\alpha\beta}\frac{n_{\alpha\beta}}{\varepsilon_{\alpha\beta}-i\delta},$$
$$\varkappa_{xy}^{(1)}=\frac{i\hbar}{4VT}\sum_{\alpha,\beta}\frac{1}{\varepsilon_{\alpha\beta}}(\{\mathscr{H}_0-\zeta,v_x\}_+)_{\beta\alpha}(\{\mathscr{H}_0-\zeta,v_y\}_+)_{\alpha\beta}\frac{n_{\alpha\beta}}{\varepsilon_{\alpha\beta}-i\delta},$$
$$\varkappa_{xy}^{(2)}=\frac{2}{VT}\sum_\alpha n_\alpha(\varepsilon_\alpha-\zeta)(yv_x)_{\alpha\alpha}-\frac{e\hbar n_eH}{4m^2cT},$$
$$\varepsilon_{\alpha\beta}=\varepsilon_\alpha-\varepsilon_\beta,\quad n_{\alpha\beta}=n(\varepsilon_\alpha)-n(\varepsilon_\beta),$$

where $n(\epsilon_\alpha)$ is the energy distribution function, n_e is the electron concentration, and V is the volume of the system.

Let us transform the expressions (14), using the relations

$$(v_i)_{\alpha\beta}=\frac{i}{\hbar}\varepsilon_{\alpha\beta}(r_i)_{\alpha\beta},\qquad \frac{1}{x-i\delta}=\mathrm{P}\frac{1}{x}+i\pi\delta(x)$$

and removing, where possible, one summation

$$\sigma_{xy}=-\frac{e^2\pi\hbar}{V}\sum_{\alpha,\beta}(v_x)_{\beta\alpha}(v_y)_{\alpha\beta}\frac{\partial n}{\partial\varepsilon_\alpha}\delta(\varepsilon_\alpha-\varepsilon_\beta),$$
$$\beta_{xy}^{(1)}=-\frac{e\hbar\pi}{VT}\sum_{\alpha,\beta}(\varepsilon_\alpha-\zeta)(v_x)_{\beta\alpha}(v_y)_{\alpha\beta}\frac{\partial n}{\partial\varepsilon}\delta(\varepsilon_{\alpha\beta})+\beta_{xy}^{(3)},\tag{15}$$

$$\beta_{xy}^{(3)}=\frac{e}{VT}\sum_\alpha n_\alpha(yv_x)_{\alpha\alpha}.\tag{16}$$

It can be seen that the Kubo response terms containing the principal value $(\sim(\epsilon_\alpha-\zeta)(v_x)_{\beta\alpha}(v_y)_{\alpha\beta}n_{\alpha\beta}\epsilon_{\alpha\beta}^{-2})$ cancel out in the current the additional term $\sim\beta_{xy}^{(2)}$. The quantities γ_{xy} and κ_{xy} can be transformed using similar procedures. Notice that the transformations carried out above are quite formal and that the precise meaning of the operations performed can be justified only during the passage to definite limits; therefore, it is more convenient in certain concrete calculations to directly use the formulas (14)[2)].

Finally, we obtain the following expressions for the kinetic coefficients:

$$\sigma_{xy}=\frac{-e^2\pi\hbar}{V}\int dE\frac{\partial n}{\partial E}\Phi(E),$$
$$\beta_{xy}=-\frac{e\pi\hbar}{VT}\int dE\frac{\partial n}{\partial E}(E-\zeta)\Phi(E)=\frac{\gamma_{xy}}{T},$$
$$\varkappa_{xy}=\frac{-\pi\hbar}{VT}\int dE\frac{\partial n}{\partial E}(E-\zeta)^2\Phi(E),\tag{17}$$
$$\Phi(E)=\left\langle\sum_{\alpha,\beta}\delta(E-\varepsilon_\alpha)\delta(E-\varepsilon_\beta)(v_x)_{\beta\alpha}(v_y)_{\alpha\beta}\right\rangle.$$

The symbol $\langle\ldots\rangle$ denotes averaging over all the impurity configurations. Expressions similar to ours for σ and β without a magnetic field were first obtained in[12, 13]. Using the fact that $\psi_\alpha(\mathrm{H})=\psi_\alpha^*(-\mathrm{H})$, we can easily verify that all the symmetry requirements for the kinetic coefficients are satisfied. The results obtained indicate the existence between the kinetic coefficients of general relations (analogs of the Wiedemann-Franz relations) that remain valid outside the limits of applicability of the kinetic equation. In particular, we have respectively for a nondegenerate and degenerate gas the relations

$$\beta_{xy}=\frac{1}{e}\left(\frac{\partial\sigma_{xy}T}{\partial T}\right)_\zeta,\quad \varkappa_{xy}=\frac{1}{e^2}\left(\frac{\partial}{\partial T}T^2\left(\frac{\partial\sigma_{xy}T}{\partial T}\right)_\zeta\right)_\zeta$$
$$(T\gg\varepsilon_F),\tag{18}$$
$$\beta_{xy}=\frac{e\hbar\pi^3T}{3}\frac{\partial\Phi}{\partial E}\bigg|_{E=\zeta},\quad \varkappa_{xy}=\frac{\pi^2T}{3e^2}\sigma_{xy},$$
$$\sigma_{xy}=e^2\pi\hbar\Phi(\zeta)\quad(T\ll\varepsilon_F).$$

The directly observable quantities are not β_{ik} and κ_{ik}, but the thermoelectromotive force α_{ik} and the specific heat λ_{ik}. In the general case it is quite tedious to express them in terms of the kinetic coefficients computed by us, but under conditions when only the skew fluxes are important, it is easy to derive the following simple relations:

$$\alpha_{xx}=\frac{T}{e}\left(\frac{\partial\ln(\sigma_{xy}T)}{\partial T}\right)_\zeta,\quad \lambda_{xy}=\frac{T\sigma_{xy}}{e^2}\left(\frac{\partial}{\partial T}T^2\left(\frac{\partial\ln(\sigma_{xy}T)}{\partial T}\right)_\zeta\right)_\zeta$$
$$\alpha_{xx}=\frac{\pi^2T}{3e}\frac{\partial\ln\Phi(E)}{\partial E}\bigg|_{E=\zeta},\quad \lambda_{xy}=\frac{\pi^2T}{3e^2}\sigma_{xy}\quad(T\ll\varepsilon_F).\tag{19}$$

The relations (19) can be used to determine α_{xx} and λ_{xy} from the measured temperature dependences of σ_{xy}, in the same way as was done in[14] for the thermoelectromotive force and the conductivity of cesium in the absence of a magnetic field. It is necessary to take into account the fact that the relations (18) and (19) are applicable if the averaging over the impurities does not depend on temperature, or if different temperatures are introduced for the carriers and the impurities.

Let us now consider two limiting cases–weak and quantizing magnetic fields. The results of the calculations of σ_{xy} in a weak field H are given by two of the present authors (V.K. and W. Z.) in[15]. The other kinetic coefficients can be computed by a similar method. Here we give only the final formulas, omitting all the simple but tedious transformations:

$$\sigma_{xy}=\frac{i\pi^2e^3H\hbar}{Vc}\int dE\frac{\partial n}{\partial E}\Phi_H(E),$$
$$\beta_{xy}=\frac{i\pi^2e^2H\hbar}{VcT}\int dE(E-\zeta)\frac{\partial n}{\partial E}\Phi_H(E),$$
$$\varkappa_{xy}=\frac{i\pi^2eH\hbar}{VcT}\int dE(E-\zeta)^2\frac{\partial n}{\partial E}\Phi_H(E),\tag{20}$$
$$\Phi_H(E)=\left\langle\frac{i\hbar^3}{m^3}\sum_{\alpha,\beta,\gamma}\delta(E-\varepsilon_\alpha)\delta(E-\varepsilon_\beta)\delta(E-\varepsilon_\gamma)(\nabla_x)_{\gamma\alpha}[(\nabla_y)_{\alpha\beta}(x\nabla_y)_{\beta\gamma}-(x\nabla_y)_{\alpha\beta}(\nabla_y)_{\beta\gamma}]\right\rangle.$$

In these formulas α, β, and γ are electron states in the field of the impurities in the absence of a magnetic field.

All the kinetic coefficients in quantizing magnetic fields in the absence of impurities have been computed by Bar'yakhtar and Peletminskiĭ[11]. As has already been noted, the methods used in this and other similar papers allow us to obtain correct answers only after the effects connected with the presence of the diamagnetic currents have been individually taken into account: for the electric current we must subtract the terms ~curl **M**, for the heat flux the terms ~ curl **L**, etc. In the formalism proposed by us the additional subtractions are not required, and all the results can be obtained either from the formulas (14), using the well known expressions for the eigenfunctions and energies of a free electron in a constant magnetic field[9], or, which is considerably simpler, by computing only σ_{xy} and using the formulas (18) and (19). Notice that in virtue of the specific degeneracy of the levels in a strong magnetic field, in computing terms of the type (11), it is necessary to correctly take into account the presence of boundaries–in the same way as was done in[16]. With the aid of the first formula in (18) it is easy to show that β_{xy} for a nondegenerate gas is proportional to the entropy S of the system (see, for example,[6]):

$$\beta_{xy}=\frac{1}{e}\left(\frac{\partial\sigma_{xy}T}{\partial T}\right)_\zeta=\frac{c}{H}\frac{\partial n_eT}{\partial T}=-\frac{c}{HV}\frac{\partial\Omega}{\partial T}=\frac{cS}{HV}.$$

Let us give here, for reference, the expressions for the kinetic coefficients of the nondegenerate electron gas in quantizing magnetic fields with allowance for the spin splitting[3)]:

$$\sigma_{xy}=\frac{n_eec}{H},\quad \beta_{xy}=\frac{n_ec}{H}\left[\frac{3}{2}+\nu\,\mathrm{cth}\,\nu-\tilde\mu\,\mathrm{th}\,\tilde\mu-\tilde\zeta\right],$$
$$\varkappa_{xy}=\frac{n_ecT}{eH}\left[\frac{15}{4}+3(\nu\,\mathrm{cth}\,\nu-\tilde\mu\,\mathrm{th}\,\tilde\mu-\tilde\zeta)+\frac{\nu^2(1+\mathrm{ch}^2\nu)}{\mathrm{sh}^2\nu}+\tilde\mu^2-2(\tilde\zeta\nu\,\mathrm{cth}\,\nu-\tilde\zeta\tilde\mu\,\mathrm{th}\,\tilde\mu+\nu\tilde\mu\,\mathrm{cth}\,\nu\,\mathrm{th}\,\tilde\mu)+\tilde\zeta^2\right],\tag{21}$$
$$\nu=\hbar\Omega/2T,\ \tilde\mu=\mu/T,\ \tilde\zeta=\zeta/T.$$

Notice that the quantity κ_{xy} contains terms proportional to H, but as we have already indicated, the quantity that has physical meaning is the quantity λ_{xy}, for which, being interested in only the off-diagonal components, we can obtain, using the formulas (21), the following expression:

$$\lambda_{xy}=\varkappa_{xy}-\frac{T\beta_{xy}^2}{\sigma_{xy}}=\frac{n_e cT}{eH}\left(\frac{3}{2}+\frac{\nu^2}{\mathrm{sh}^2\nu}+\frac{\tilde{\mu}^2}{\mathrm{ch}^2\tilde{\mu}}\right). \quad (22)$$

In the classical limit ($\tilde{\mu}$, $\nu \ll 1$) the expression (22) yields the well-known expression for the skew heat flux[17]:

$$\mathbf{q}_\perp=-\frac{5}{2}\frac{cn_eT}{eH^2}[\nabla T\times\mathbf{H}]. \quad (23)$$

The causes of the discrepancy between the theoretical value of the transverse thermoelectromotive force $\alpha_{xx} = \beta_{xy}/\sigma_{xy}$ of the nondegenerate electron gas and the experimentally measured—in quantizing magnetic fields—values[18] remain unclear.

For the computation of the diagonal thermomagnetic coefficients, the Luttinger method has no advantages over the other methods, since in this case the ordinary Kubo formulas are valid. In this case there exist relations of the type (18) between the diagonal components of the tensors σ, β, and κ.

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1) Graduate student of Rostock University, Rostock, East Germany.
2) In the case of quantizing magnetic fields and in the absence of impurities the contribution to the off-diagonal kinetic coefficients is made precisely by the principal values. Concrete calculations show, however, that the general relations between the kinetic coefficients (see, further, the Eqs. (18)) remain valid.
3) There is a small error in the answer for κ_{xy} given in [11].

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228