Theory of a two-component Fermi liquid

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A generalization of Landau's Fermi-liquid theory to the case of a two-component Fermi liquid in which quantum transitions are allowed between the components is proposed. The excitation spectra of a two-component charged Fermi liquid are found. In the general case there are eight branches of collective oscillations in such a system. At least three of these branches have a linear dispersion law. The behavior of the dielectric constant of a two-component Fermi liquid in the resonance frequency regions is studied. It is shown that in metals with overlapping energy bands the resonance phenomena due to quantum transitions can make an appreciable contribution to the energy losses of charged particles.

INTRODUCTION

It is well known that in many metals there are several sorts of carrier; the electron liquid of these metals is, therefore, a many-component Fermi liquid. The Hamiltonian of such a Fermi liquid should contain terms responsible for the interband transitions, of which it is especially important to take account in metals with overlapping energy bands.

The oscillations of a two-component Fermi liquid were considered without taking these terms into account in[1-4]. In such a model, the absence of the degrees of freedom responsible for transitions not only depletes the collective-excitation spectrum of the system, but also distorts it.

We show that in a two-component Fermi liquid in which virtual transitions between the components are allowed, there are in the general case eight branches of collective excitations. One of these corresponds to modified Langmuir oscillations. Another has a linear dispersion law (zero sound). The absence of an activation frequency for the oscillations of this type is connected with the fact that in the long-wave region the total charge density does not change in them (although the densities of the carriers within each of the bands varies, both as a consequence of the motion of the carriers and as a result interband transitions). In the absence of transitions, the zero-sound dispersion law coincides with that obtained in[1]. The third and fourth branches of the collective excitations (isotopic oscillations) are associated entirely with virtual interband transitions and are the analog of Frenkel excitons for metals. In the long-wave limit these branches are degenerate and have an activation frequency of the order of the energy-band hybridization potential. In addition to the types of excitation mentioned, four spin-wave branches, two of which have a linear dispersion law, can exist in a two-component Fermi liquid.

In the last Sections of the article we consider the dielectric constant of a two-component Fermi liquid and show that allowance for the interband transitions makes an appreciable contribution to the energy losses of fast charged particles passing through metals with overlapping bands.

1. TWO-COMPONENT FERMI LIQUID

In order to generalize the Landau theory of the Fermi liquid[5] to the case of a two-component Fermi liquid, in the fundamental relations of this theory

$$\delta\mathscr{E}=\mathrm{Sp}\int\frac{d^3r\,d^3p}{(2\pi)^3}\hat{\varepsilon}(\mathbf{p},\mathbf{r},t)\delta\hat{n}(\mathbf{p},\mathbf{r},t), \tag{1.1}$$

$$\delta\hat{\varepsilon}(\mathbf{p},\mathbf{r},t)=\mathrm{Sp}\int\frac{d^3p'}{(2\pi)^3}\hat{f}(\mathbf{p},\mathbf{p}')\delta n(\mathbf{p},\mathbf{r},t) \tag{1.2}$$

we shall assume $\hat{\epsilon}$ and $\hat{n}$ to be nondiagonal two-row matrices, and the function $\hat{f}$ to be a four-row nondiagonal matrix in the isotopic (band) space ($\mathscr{E}$ is the total energy of the Fermi liquid, $\hat{\epsilon}$ if the quasi-particle energy, $\hat{n}$ is the density matrix, $\hat{f}$ is the Landau correlation function, and $\hbar = 1$). We introduce the notation

$$\langle\lambda'|\hat{\varepsilon}(\mathbf{p},\mathbf{r},t)|\lambda\rangle=\hat{\varepsilon}^{\lambda\lambda'}(\mathbf{p},\mathbf{r},t), \tag{1.3}$$

$$\langle\lambda'|\hat{n}(\mathbf{p},\mathbf{r},t)|\lambda\rangle=\hat{n}^{\lambda\lambda'}(\mathbf{p},\mathbf{r},t), \tag{1.4}$$

$$\langle\lambda'\beta'|\hat{f}(\mathbf{p},\mathbf{p}')|\lambda\beta\rangle=\hat{f}^{\lambda\lambda'\beta\beta'}(\mathbf{p},\mathbf{p}'), \tag{1.5}$$

where λ and β (isotopic indices) indicate the label of the component and take the values 1, 2. The functions (1.3)–(1.5) are matrices in spin space. Taking (1.3)–(1.5) into account, we rewrite the formulas (1.1) and (1.2) in the form

$$\delta\mathscr{E}=\mathrm{Sp}\sum_{\lambda\lambda'}\int\frac{d^3r\,d^3p}{(2\pi)^3}\hat{\varepsilon}^{\lambda\lambda'}(\mathbf{p},\mathbf{r},t)\delta\hat{n}^{\lambda'\lambda}(\mathbf{p},\mathbf{r},t), \tag{1.6}$$

$$\delta\hat{\varepsilon}^{\lambda\lambda'}(\mathbf{p},\mathbf{r},t)=\mathrm{Sp}\sum_{\beta\beta'}\int\frac{d^3p'}{(2\pi)^3}\hat{f}^{\lambda\lambda'\beta\beta'}(\mathbf{p},\mathbf{p}')\delta\hat{n}^{\beta'\beta}(\mathbf{p},\mathbf{r},t), \tag{1.7}$$

where the trace is taken over the spin indices.

The symmetry properties of the matrix $\hat{f}$ permit us to reduce the number of independent elements of the matrix. Besides the Hermiticity condition

$$\hat{f}=\hat{f}^{+} \tag{1.8}$$

the matrix $\hat{f}$, being the second functional derivative of the total energy of the Fermi liquid with respect to the density matrix, satisfies the relation

$$f^{\lambda\lambda'\beta\beta'}_{\tau\tau'\alpha\alpha'}(\mathbf{p},\mathbf{p}')=f^{\beta\beta'\lambda\lambda'}_{\alpha\alpha'\tau\tau'}(\mathbf{p}',\mathbf{p}) \tag{1.9}$$

(the subscripts indicate the spin indices).

Below, we shall assume the Hamiltonian and ground state of the Fermi liquid to be invariant under spin rotations. Then $\hat{f}(\mathbf{p},\mathbf{p}')$ can be represented in the form

$$\hat{f}^{\lambda\lambda'\beta\beta'}(\mathbf{p},\mathbf{p}')=\varphi^{\lambda\lambda'\beta\beta'}(\mathbf{p},\mathbf{p}')+4(\mathbf{s}\hat{\mathbf{s}})\psi^{\lambda\lambda'\beta\beta'}(\mathbf{p},\mathbf{p}'). \tag{1.10}$$

Following Landau[5] we shall neglect the imaginary part of $\hat{f}$, assuming that real acts of scattering of quasiparticles are of low probability. In other words, we shall assume that the quasi-particles are well defined in the ground state (the "bare" bands are hybridized):

$$\varepsilon_0^{\lambda\lambda'}(\mathbf{p},\mathbf{r},t)=\varepsilon_0^{\lambda}(\mathbf{p})\delta_{\lambda\lambda'}, \tag{1.11}$$

$$n_0^{\lambda\lambda'}(\mathbf{p},\mathbf{r},t)=n_0^{\lambda}(\mathbf{p})\delta_{\lambda\lambda'},\quad n_0^{\lambda}(\mathbf{p})=\left[1+\exp\frac{\varepsilon_0^{\lambda}(\mathbf{p})-\mu}{T}\right]^{-1};\quad (1.12)$$

where μ is the chemical potential and T is the temperature of the Fermi liquid.

Thus, if the matrix $\hat{f}$ is assumed for simplicity to be independent of the momenta, it has seven independent elements

$$\begin{gathered}\hat{f}^{1111}\equiv\hat{f}_1,\quad \hat{f}^{2222}\equiv\hat{f}_2,\qquad \hat{f}^{1221}=\hat{f}^{2112}\equiv\hat{f}_3,\quad \hat{f}^{1122}=\hat{f}^{2211}\equiv\hat{f}_4,\\ \hat{f}^{1112}=\hat{f}^{1121}=\hat{f}^{1211}=\hat{f}^{2111}\equiv\hat{f}_5,\\ \hat{f}^{2221}=\hat{f}^{2212}=\hat{f}^{2122}=\hat{f}^{1222}\equiv\hat{f}_6,\\ \hat{f}^{1212}=\hat{f}^{2121}\equiv\hat{f}_7.\end{gathered}\quad (1.13)$$

The diagonal elements $\hat{f}_1$, $\hat{f}_2$ and $\hat{f}_4$ describe scattering processes without transitions; $\hat{f}_3$ corresponds to exchange interband scattering, when the number of particles in the bands does not change; $\hat{f}_5$ and $\hat{f}_6$ correspond to one-particle interband transitions; $\hat{f}_7$ corresponds to two-particle interband transitions. If in the Fermi liquid there is a band-index conservation law, corresponding to isotopic invariance, the elements $\hat{f}_5$, $\hat{f}_6$ and $\hat{f}_7$ violating this invariance are equal to zero.

We shall find the magnetic susceptibility of a two-component Fermi liquid in the ground state. The magnetic moment of the Fermi liquid equals

$$\mathbf{M}=2\mu_0\,\mathrm{Sp}\int\frac{d^3p}{(2\pi)^3}\hat{s}\delta\hat{n}(\mathbf{p}),\quad (1.14)$$

where μ_0 is the Bohr magneton and $\delta\hat{n}(\mathbf{p})$ is the deviation of the density matrix from (1.12) that is induced by an external magnetic field H. We shall assume that the matrix $\gamma^{\lambda\lambda'}(\mathbf{p})$ of the magnetic moment of the quasiparticles is diagonalized in the same representation as is the one-particle energy (i.e., in the hybridized bands). Then the changes of the energy and density matrix of the quasi-particles have the form

$$\delta\hat{\varepsilon}^{\lambda}(\mathbf{p})=-2\gamma^{\lambda}(\mathbf{p})\hat{s}\mathbf{H},\quad \delta\hat{n}^{\lambda}(\mathbf{p})=\frac{\partial n_0^{\lambda}}{\partial\varepsilon_0^{\lambda}}\delta\hat{\varepsilon}^{\lambda}(\mathbf{p}),\quad (1.15)$$

whence

$$\mathbf{M}=2\mu_0\mathbf{H}\sum_{\lambda}\int\frac{d^3p}{(2\pi)^3}\gamma^{\lambda}(\mathbf{p})\delta\{\varepsilon_0^{\lambda}(\mathbf{p})-\mu\}.\quad (1.16)$$

The formulas (1.7) and (1.15) give two equations for the determination of the magnetic moments of the quasiparticles:

$$\gamma^{\lambda}(\mathbf{p})+2\,\mathrm{Sp}\sum_{\beta}\int\frac{d^3p'}{(2\pi)^3}\psi^{\lambda\lambda\beta\beta}(\mathbf{p},\mathbf{p}')\gamma^{\beta}(\mathbf{p}')\delta\{\varepsilon_0^{\lambda}(\mathbf{p}')-\mu\}=\mu_0.\quad (1.17)$$

For functions ψ that are independent of the momenta and for isotropic dispersion laws for the carriers, the magnetic susceptibility of the two-component Fermi liquid has the form

$$\chi=\mu_0\sum_{\lambda}g^{\lambda}(\mu)\gamma^{\lambda};\quad (1.18)$$

$$\gamma^{(1)}=\mu_0\zeta^{-1}[1+(\psi_2-\psi_4)g^{(2)}(\mu)],\quad (1.19)$$

$$\begin{gathered}\gamma^{(2)}=\mu_0\zeta^{-1}[1+(\psi_1-\psi_4)g^{(1)}(\mu)];\\ \zeta=[1+\psi_1g^{(1)}(\mu)][1+\psi_2g^{(2)}(\mu)]-\psi_4^2g^{(1)}(\mu)g^{(2)}(\mu),\end{gathered}\quad (1.20)$$

where $g^{\lambda}(\mu)$ is the density of states of the λ-th component of the Fermi liquid at the Fermi level. The condition for ferromagnetism for the two-component Fermi liquid has the form

$$\sum_{\lambda}g^{\lambda}(\mu)\gamma^{\lambda}<0.\quad (1.21)$$

If one of the quantities $g^{\lambda}(\mu)$ equals zero, the expressions (1.18)–(1.21) go over into the well-known formulas for the one-component Fermi liquid. We note that the off-diagonal (in the isotopic indices) elements of the matrix $\hat{f}$ do not appear in the expressions for the static quantities. However, as will be seen below, they play an important role in the determination of the spectrum of the characteristic excitations of a two-component Fermi liquid.

2. DISPERSION EQUATION

A. Kinetic-Equation Method

The Landau-Silin kinetic equation[5,6] in the case of a two-component charged Fermi liquid has the form

$$\frac{\partial\hat{n}}{\partial t}-i\{\hat{\varepsilon},\hat{n}\}_-+\frac{1}{2}\left\{\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}},\frac{\partial\hat{n}}{\partial\mathbf{r}}\right\}_+-\frac{1}{2}\left\{\frac{\partial\hat{\varepsilon}}{\partial\mathbf{r}},\frac{\partial\hat{n}}{\partial\mathbf{p}}\right\}_+-e\frac{\partial\Phi}{\partial\mathbf{r}}\frac{\partial\hat{n}}{\partial\mathbf{p}}=\hat{I},\quad (2.1)$$

where $\hat{\epsilon}$ and $\hat{n}$ are matrices in the isotopic and spin spaces, and $\hat{I}$ is the collision integral; the curly brackets $\{\ldots\}_-$ and $\{\ldots\}_+$ respectively denote the commutators and anticommutators of the operators standing in the brackets; Φ is the potential of the self-consistent Coulomb field, satisfying the equation

$$\Delta\Phi(\mathbf{r},t)=-4\pi e\,\mathrm{Sp}\sum_{\lambda}\int\frac{d^3p}{(2\pi)^3}\delta\hat{n}^{\lambda\lambda}(\mathbf{p},\mathbf{r},t).\quad (2.2)$$

The linearized equation (2.1) in Fourier components is of the form

$$\begin{gathered}\omega\delta\hat{n}+\{\delta\hat{\varepsilon},\hat{n}_0\}_-+\{\hat{\varepsilon}_0,\delta\hat{n}\}_--{}^1\!/_2\mathbf{k}\{\hat{\mathbf{v}}_0,\delta\hat{n}\}_+\\ -{}^1\!/_2\mathbf{k}\{\delta\hat{\varepsilon}-e\Phi,\delta(\hat{\varepsilon}_0-\mu)\hat{\mathbf{v}}_0\}_+=-(i/\tau)\delta\hat{n},\end{gathered}\quad (2.3)$$

where $\mathbf{v}_0^{\lambda}(\mathbf{p})=\partial\epsilon_0^{\lambda}(\mathbf{p})/\partial\mathbf{p}$ is the group velocity of the quasi-particles.

The relaxation time τ, besides being determined by real collisions of electrons with each other, with phonons and with impurities, is also determined by the lifetime of a quasi-particle in a band. Below, we shall omit the collision integral, assuming the frequency to be complex. It is easy to see (cf., e.g.,[7]) that the expressions (1.7) and (2.3) can be written out without taking account of the Coulomb potential Φ in explicit form, if we take into account that $\hat{n}_0$ is diagonal (in λ) and understand by $\hat{f}\to\tilde{\hat{f}}$ the sum of the short-range screening interaction $\hat{f}$ and the long-range Coulomb interaction Q:

$$\tilde{\hat{f}}(\mathbf{p},\mathbf{p}')=\hat{f}(\mathbf{p},\mathbf{p}')+Q,\quad Q\equiv4\pi e^2/k^2.\quad (2.4)$$

Solving Eq. (2.3), we have

$$\delta\hat{n}^{\lambda\lambda'}=\delta\hat{\varepsilon}^{\lambda\lambda'}\frac{n_0^{\lambda'}-n_0^{\lambda}-(\xi^{\lambda}+\xi^{\lambda'})/2}{\varepsilon_0^{\lambda'}-\varepsilon_0^{\lambda}-(\eta^{\lambda}+\eta^{\lambda'})/2},\quad (2.5)$$

$$\xi^{\lambda}(\mathbf{p})=\delta\{\varepsilon_0^{\lambda}(\mathbf{p})-\mu\}\mathbf{k}\mathbf{v}_0^{\lambda}(\mathbf{p}),\quad (2.6)$$

$$\eta^{\lambda}(\mathbf{p})=\omega-\mathbf{k}\mathbf{v}_0^{\lambda}(\mathbf{p}).\quad (2.7)$$

Substituting (2.5) into (1.7), we obtain the integral dispersion equation for the collective excitations of a two-component Fermi liquid:

$$\delta\hat{\varepsilon}^{\lambda\lambda'}=\mathrm{Sp}\sum_{\beta\beta'}\int\frac{d^3p'}{(2\pi)^3}\tilde{\hat{f}}^{\lambda\lambda'\beta\beta'}\delta\hat{\varepsilon}^{\beta'\beta}\frac{n_0^{\beta}-n_0^{\beta'}-(\xi^{\beta}+\xi^{\beta'})/2}{\varepsilon_0^{\beta}-\varepsilon_0^{\beta'}-(\eta^{\beta}+\eta^{\beta'})/2}.\quad (2.8)$$

If the collective excitations are not coupled with the spin-density oscillations, and the matrix φ does not depend on the momenta, the dispersion equation takes the form

$$\det\hat{\Lambda}_Q(\omega,\mathbf{k})=0,\quad (2.9)$$

$$\hat{\Lambda}_Q(\omega,\mathbf{k})=1-2\varphi^{\lambda\lambda'\beta\beta'}\int\frac{d^3p}{(2\pi)^3}\frac{n_0^{\beta}-n_0^{\beta'}-(\xi^{\beta}+\xi^{\beta'})/2}{\varepsilon_0^{\beta}-\varepsilon_0^{\beta'}-(\eta^{\beta}+\eta^{\beta'})/2}.\quad (2.10)$$

B. Random-Phase Approximation

In view of the fact that the specific form of Eq. (2.1), containing commutators and anticommutators of operators, is a postulate in the Landau-Silin theory, it is of interest to obtain the dispersion equation (2.10) by the methods of quantum field theory. (We note that for the one-component Fermi-liquid, unlike the two-component one, the above-mentioned specific features of Eq. (2.1) have no effect on the final results.)

In the random-phase approximation (RPA), the equation for the vertex part of the two-particle interaction in a two-component Fermi liquid has the form

$$\Gamma^{\lambda_1\lambda_2,\lambda_3\lambda_4}_{\gamma_1\gamma_2,\gamma_3\gamma_4}(P_1,P_2;K)=\tilde{\Gamma}^{\lambda_1\lambda_2,\lambda_3\lambda_4}_{\gamma_1\gamma_2,\gamma_3\gamma_4}(P_1,P_2;K) - \frac{i}{(2\pi)^4}\sum_{5,6}\int d^4P\tilde{\Gamma}^{\lambda_1\lambda_6,\lambda_5\lambda_2}_{\gamma_1\gamma_6,\gamma_5\gamma_2}\left(P_1,P+\frac{K}{2};K\right) \times\Gamma^{\lambda_5\lambda_2,\lambda_6\lambda_4}_{\gamma_5\gamma_2,\gamma_6\gamma_4}\left(P-\frac{K}{2},P_2;K\right)G^{\lambda_5}\left(P-\frac{K}{2}\right)G^{\lambda_6}\left(P+\frac{K}{2}\right), \tag{2.11}$$

where $\tilde{\Gamma}$ is the bare interaction, P_1 and P_2 are the four-momenta of the colliding particles, λ is the isotopic index and γ the spin index, $K = (k, \omega)$ is the four-momentum transfer in the collision, and $G^\lambda(P)$ are the diagonal terms of the electron Green function ($P = (p, \epsilon)$):

$$G^{\lambda\lambda'}_{\gamma\gamma'}(P)=G^\lambda(P)\delta^{\lambda\lambda'}_{\gamma\gamma'}, \tag{2.12}$$

$$G^\lambda(P)=[\varepsilon-\mu-\varepsilon_0{}^\lambda(\mathbf{p})+i\delta]^{-1}. \tag{2.13}$$

The singularities of the vertex part Γ determine the spectrum of the characteristic oscillations of the Fermi liquid. Assuming for simplicity that the interaction is independent of the momenta of the colliding particles, we obtain the dispersion equation

$$\det\left\{1+\sum_{5,6}\tilde{\Gamma}^{\lambda_1\lambda_2,\lambda_5\lambda_6}_{\gamma_1\gamma_2,\gamma_5\gamma_6}\Pi^{\lambda_5\lambda_2,\lambda_6\lambda_6}_{\gamma_5\gamma_2,\gamma_6\gamma_6}(K)\right\}=0, \tag{2.14}$$

where the polarization operator Π equals

$$\Pi^{\lambda_5\lambda_2,\lambda_5\lambda_6}_{\gamma_5\gamma_2,\gamma_5\gamma_6}(K)=\delta_{\gamma_5\gamma_2}\delta_{\gamma_5\gamma_6}\frac{i}{(2\pi)^4}\int d^4PG^{\lambda_5}\left(P-\frac{K}{2}\right)G^{\lambda_6}\left(P+\frac{K}{2}\right). \tag{2.15}$$

Integrating once in (2.15) gives

$$\int d^4P\,G^{\lambda_5}\left(P-\frac{K}{2}\right)G^{\lambda_6}\left(P+\frac{K}{2}\right)=2\pi i\int d^3p\,\frac{n^{\lambda_5}(\mathbf{p}-\mathbf{k}/2)-n^{\lambda_6}(\mathbf{p}+\mathbf{k}/2)}{\omega+\varepsilon^{\lambda_5}(\mathbf{p}-\mathbf{k}/2)-\varepsilon^{\lambda_6}(\mathbf{p}+\mathbf{k}/2)}. \tag{2.16}$$

Expanding (2.16) to terms of first order in k, we see that in the limit of small k the dispersion equation (2.14) coincides with (2.9) if

$$f^{\lambda_1\lambda_2,\lambda_3\lambda_4}_{\gamma_1\gamma_2,\gamma_3\gamma_4}=\tilde{\Gamma}^{\lambda_1\lambda_2,\lambda_3\lambda_4}_{\gamma_1\gamma_2,\gamma_3\gamma_4}. \tag{2.17}$$

The formula (2.17) establishes the connection between the matrix $\hat{f}$ and the vertex part of the two-particle interaction in a two-component Fermi liquid.

3. COLLECTIVE EXCITATIONS

We now determine the spectra of the characteristic oscillations of a two-component Fermi liquid for isotropic dispersion laws for the carriers. In this case, the expression (2.10) can be written in the form

$$\hat{\Lambda}_Q(K)=1+\tilde{\varphi}^{\lambda\lambda'\beta\beta'}T^{\beta'\beta}, \tag{3.1}$$

$$T^{\beta'\beta}(K)=\frac{1}{2\pi^2}\int p^2\,dp\int_{-1}^{1}dx\,\frac{n_0{}^\beta(p)-n_0{}^{\beta'}(p)-{}^1/_2kx[\delta(p-p_0{}^\beta)+\delta(p-p_0{}^{\beta'})]}{\omega-[\varepsilon_0{}^\beta(p)-\varepsilon_0{}^{\beta'}(p)]-{}^1/_2kx[v_0{}^\beta(p)+v_0{}^{\beta'}(p)]}, \tag{3.2}$$

where p_0^β are the limiting Fermi momenta of the quasi-particles. It is easy to see that

$$T^{\beta'\beta}(K)=T^{\beta\beta'}(-K). \tag{3.3}$$

Using the symmetry properties (1.13) of the matrix $\hat{f}$, we give the explicit form of the dispersion equation for a two-component Fermi liquid:

$$T^{11}T^{22}T^{12}T^{21}\left\{\left[\varphi_7{}^2-\varphi_3{}^2-\left(\frac{1}{T^{12}}+\frac{1}{T^{21}}\right)\varphi_3+\frac{1}{T^{12}T^{21}}\right] \times[(\varphi_4+Q)(A+B)+AB]-2\left[\varphi_7-\varphi_3-\frac{1}{2}\left(\frac{1}{T^{12}}+\frac{1}{T^{21}}\right)\right] \times[(\varphi_4+Q)(\varphi_5-\varphi_6)^2+\varphi_5{}^2A+\varphi_6{}^2B]\right\}=0, \tag{3.4}$$

$$A=\frac{1}{T^{11}}+\varphi_1-\varphi_4,\quad B=\frac{1}{T^{22}}+\varphi_2-\varphi_4. \tag{3.5}$$

The quantities T^{11} and T^{22} can be calculated without knowledge of the quasi-particle dispersion laws:

$$T^{11}=\frac{p_0^{(1)2}}{2\pi^2u}J(v_1),\quad T^{22}=\frac{p_0^{(2)2}}{2\pi^2u}J(v_2), \tag{3.6}$$

$$J(v)=\frac{2}{v}+\frac{1}{v^2}\ln\frac{1-v}{1+v},\quad J(v)<0, \tag{3.7}$$

$$v^\lambda=v_0{}^\lambda/u,\quad u=\omega/k, \tag{3.8}$$

where v_0^λ are the limiting Fermi velocities of the quasi-particles.

The dispersion equation (3.4) has solutions in two regions of values of the phase velocity of the oscillations: 1) $u \gtrsim v_0^\lambda$, 2) $u \gg v_0^\lambda$. In the first region, the solution of (3.4) for not too large k is zero sound. In this region,

$$T^{12}=T^{21}\equiv T_0=-\frac{1}{\pi^2}\int dp\,p^2\frac{n_0^{(2)}(p)-n_0^{(1)}(p)}{\varepsilon_0^{(2)}(p)-\varepsilon_0^{(1)}(p)}. \tag{3.9}$$

Recognizing that for small k we have $Q \gg \varphi$, and neglecting terms of order $(ak)^2$ (a is the average distance between the electrons), we obtain from (3.4) a transcendental equation for the phase velocity s of the zero sound:

$$\frac{1}{T^{11}}+\frac{1}{T^{22}}=-(\varphi_1+\varphi_2)+2\left[\varphi_4+\frac{(\varphi_5-\varphi_6)^2}{\varphi_3+\varphi_7+1/T_0}\right]. \tag{3.10}$$

The contribution of the interband transitions to the zero sound is, generally speaking, of the same order as the contribution from the diagonal elements of the interaction.

In the region of large phase velocities, neglecting terms of order $(ak)^2$ we have

$$\Delta_L\Delta_i=0, \tag{3.11}$$

$$\Delta_L=Q\left(\frac{1}{T^{11}}+\frac{1}{T^{22}}\right)+\frac{1}{T^{11}T^{22}}, \tag{3.12}$$

$$\Delta_i=\varphi_7{}^2-\varphi_3{}^2-\left(\frac{1}{T^{12}}+\frac{1}{T^{21}}\right)\varphi_3+\frac{1}{T^{12}T^{21}}. \tag{3.13}$$

The equation $\Delta_L = 0$ determines the spectrum of the modified Langmuir oscillations:

$$\omega_L{}^2=\frac{4\pi e^2N_1}{m_1}+\frac{4\pi e^2N_2}{m_2} \tag{3.14}$$

where $N_{1,2}$ are the densities of the different types of carrier, and $m_{1,2}$ are their effective masses ($m\lambda = p_0^\lambda/v_0^\lambda$).

In order to find the frequencies of the oscillations corresponding to the dispersion equation $\Delta_i = 0$, we shall assume that the difference between the energies of the quasi-particles of the different components of the Fermi liquid is not great, and varies weakly near the Fermi level, i.e., for $p \sim p_0$ the following relation holds:

$$\varepsilon_0^{(1)}(p)-\varepsilon_0^{(2)}(p)=V, \quad V\ll\mu. \tag{3.15}$$

This relation is valid, in particular, when the hybridization potentials are not great, which is the case, e.g., in the transition metals ($V \sim 1$ eV). Under the condition (3.15),

$$T^{12}=\frac{q_0^2}{2\pi^2\tilde{u}}\{J(\bar{v})+d[\bar{v}J(\bar{v})-2]\}, \tag{3.16}$$

$$q_0^2=\frac{p_0^{(1)2}+p_0^{(2)2}}{2}, \quad \bar{v}=\frac{v_0}{\tilde{u}}, \quad v_0=\frac{v_0^{(1)}+v_0^{(2)}}{2}, \quad \tilde{u}=\frac{\omega-V}{k}, \tag{3.17}$$

$$d=\pi^2\Delta N/q_0^2k, \quad \Delta N\equiv N_2-N_1. \tag{3.18}$$

The quantity T^{21} is determined by the condition (3.3). In the limit $k \to 0$ the frequencies of the two branches of isotopic oscillations coincide:

$$\omega_0^2=[V+\Delta N\varphi_3]^2-(\Delta N\varphi_7)^2. \tag{3.19}$$

If the expression in the right-hand side of (3.19) is negative, one of the solutions increases with time. Therefore, the condition

$$|V+\Delta N\varphi_3|>|\Delta N\varphi_7| \tag{3.20}$$

is a necessary condition for the stability of the ground state of a two-component Fermi liquid.

At first glance (see (3.19)), it might appear that the isotopic oscillations can exist even in the absence of Fermi-liquid effects ($\varphi_3 = \varphi_7 = 0$). However, this is not so. In fact, firstly, for $\varphi_3 = \varphi_7 = 0$ the dispersion equation (3.4) has no solutions of the type (3.19); secondly, for

$$|\omega-V|<{}^1/_2k(v_0^{(1)}+v_0^{(2)}) \tag{3.21}$$

going round the pole in the integrand in (3.2) gives damping of the oscillations.

Allowance for the first nonvanishing correction in k to (3.19) gives

$$\omega_1^2=\omega_0^2+2\alpha v_0^2k^2, \tag{3.22}$$

where $\alpha \sim 1$. Without giving the relevant expressions, we note that the degeneracy between the frequencies (3.22) of the two oscillations is lifted only when the terms proportional to $(ak)^4$ are taken into account.

The expressions (3.10) and (3.22) for the spectra of the collective oscillations are valid in the region of small k,

$$d(k)\gg 1. \tag{3.23}$$

In the opposite limiting case, when $d(k) \ll 1$, besides the Langmuir branch there are three zero-sound branches in a two-component Fermi liquid. The phase velocity of one of these satisfies the equation

$$\frac{q_0^2}{2\pi^2u}J\left(\frac{v_0}{u}\right)=(\varphi_7-\varphi_3)^{-1}. \tag{3.24}$$

The phase velocity of the other two branches is determined by the equation

$$\left[\frac{2\pi^2u}{q_0^2}J^{-1}\left(\frac{v_0}{u}\right)+\varphi_7+\varphi_3\right](A+B)=2(\varphi_5-\varphi_6)^2. \tag{3.25}$$

The absence of positive imaginary parts in the oscillation frequencies defines the stability condition for the ground state of the Fermi liquid. (Of course, in the region of large k the instability competes with the damping proportional to $(ak)^2$.)

For $d \to 0$, charge-density oscillations are completely absent in the oscillations with the dispersion equation (3.24). In the other branches (apart from (3.14)), the charge-density oscillations are asymptotically small (proportional to $(ak)^2$).

In addition to the oscillations investigated above, spin waves can propagate in a two-component Fermi liquid. Their dispersion equation can be obtained from (3.4) by making the replacement $\varphi \to \psi$ in it (cf.[5]) and putting $Q = 0$. It is easy to see that, in the general case, in a two-component Fermi liquid there are, for small k ($d \gg 1$), a doubly degenerate spin mode with dispersion law (3.19) and two spin sounds. For large k ($d \ll 1$), four spin waves with a linear dispersion law can exist in a two-component Fermi liquid.

4. DIELECTRIC CONSTANT

The expression for the current in a two-component Fermi liquid is of the form

$$\mathbf{j}(\mathbf{r},t)=e\,\mathrm{Sp}\int\frac{d^3p}{(2\pi)^3}\hat{\mathbf{v}}(\mathbf{p})\delta\hat{n}(\mathbf{p},\mathbf{r},t), \tag{4.1}$$

where the trace is taken over the isotopic and spin indices. Noting that (2.8) can be represented in the form

$$\sum_{\beta\beta'}\Lambda_0^{\lambda\lambda'\beta\beta'}\delta\varepsilon^{\beta'\beta}=i\delta_{\lambda\lambda'}\frac{\boldsymbol{\varepsilon}}{k^2}\mathbf{k}\mathbf{E}, \tag{4.2}$$

$$\hat{\Lambda}_0=\hat{\Lambda}_Q|_{Q=0}, \tag{4.3}$$

with the aid of formula (2.4) we find from (4.1) an expression for the longitudinal conductivity of a two-component Fermi liquid:

$$\sigma_L(\omega,k)=-\frac{e^2u}{k}\sum_{\lambda\beta}Y^{\lambda\lambda\beta\beta}(\omega,k)T^{\lambda\lambda}(\omega,k), \tag{4.4}$$

$$\hat{Y}=\hat{\Lambda}_0^{-1}. \tag{4.5}$$

Substituting (4.4) into the formula for the longitudinal dielectric constant:

$$\varepsilon_L=1+\frac{4\pi i}{\omega}\sigma_L, \tag{4.6}$$

we obtain the following simple expression:

$$\varepsilon_L=\frac{\det\Lambda_Q}{\det\Lambda_0}. \tag{4.7}$$

Thus, all the collective excitations of a two-component Fermi liquid that have been investigated above are zeros of the longitudinal dielectric constant. The excitations (3.24) in the limit $d \to 0$ are an exception. This is connected with the fact that, as already stated above, the charge density will not oscillate in this mode in the limit $d \to 0$.

Near the Langmuir frequency, the dielectric constant (4.7) takes the usual form

$$\varepsilon_L=1-\omega_L^2/\omega^2. \tag{4.8}$$

Near the zero-sound frequency ω_S (the solution of Eqs. (3.10)), the dielectric constant is of the form

$$\varepsilon_L=L_s(ak)^{-2}(1-\omega_s^2/\omega^2), \quad \omega_s=sk, \tag{4.9}$$

$$L_s=-4\pi^3s(ea)^2C\frac{(\varphi_5-\varphi_6)^2}{[\varphi_5A+\varphi_6B]^2}, \tag{4.10}$$

$$C=\sum_\lambda[p_0^\lambda J(v^\lambda)]^{-2}\frac{d}{dv^\lambda}[v^\lambda J(v^\lambda)], \quad C<0. \tag{4.11}$$

Near the frequency (3.22) we have

$$\varepsilon_L=L_i(ak)^{-2}(1-\omega_i^2/\omega^2), \tag{4.12}$$

$$L_i=\left(\frac{a}{2\pi}\right)^2\frac{(\omega_L^2-\omega_i^2)^2[V+\Delta N(\varphi_3+\varphi_7)]}{\pi e^2\Delta N[\varphi_5(N_2/m_2)+\varphi_6(N_1/m_1)]^2}. \tag{4.13}$$

It follows from (4.10) and (4.13) that $L_S > 0$, and the sign of L_i is determined by the sign of the quantity $\varphi_3 + \varphi_7 + (V/\Delta N)$. In order of magnitude, $L_S \sim L_i \sim 1$.

The change of sign of the dielectric constant at the frequencies ω_s and ω_i in metals with overlapping bands can be detected in infrared optics by measuring the coefficient of reflection of an electromagnetic wave from the metal surface. The requirement here that V be small compared with μ for an anisotropic Fermi surface can be satisfied close to certain directions (cf., e.g.,[8]).

5. PASSAGE OF CHARGED PARTICLES

The differential cross-section for scattering of fast charged particles is determined, as is well known, by the charge-density fluctuations in the scattering system:

$$d\sigma = \frac{1}{vN_0}\left(\frac{4\pi q}{k^2}\right)^2 \langle \rho^2 \rangle_{\omega \mathbf{k}} \frac{d^3p'}{(2\pi)^3}, \tag{5.1}$$

where v and q are the velocity and charge of the particle, N_0 is the density of the metal atoms, p′ is the momentum of the scattered particle, $\langle \rho^2 \rangle_{\omega \mathbf{k}}$ is the Fourier component of the correlator of the charge-density fluctuations, and ω and k are the energy and momentum transferred to the particle in the scattering ($d\sigma$ relates to one atom of the metal). By means of the fluctuation-dissipation theorem[9], the fluctuations in a dispersive medium can be expressed in terms of its dielectric properties. We cite the expression for the correlators of the charge-density fluctuations at frequencies ω_s and ω_i:

$$\langle \rho^2 \rangle_{\omega \mathbf{k}}^{s,i} = \frac{1}{2}\left|\frac{N_\omega + 1}{L_{s,i}}\right| (ak)^2 k^2 \omega^2 \delta(\omega^2 - \omega_{s,i}^2), \tag{5.2}$$

where N_ω is the Planck distribution function. For comparison, we give the well-known expression for the charge-density fluctuations in a Langmuir wave:

$$\langle \rho^2 \rangle_{\omega \mathbf{k}}^{L} = {}^1/_2 |N_\omega + 1| k^2 \omega^2 \delta(\omega^2 - \omega_L^2). \tag{5.3}$$

In order of magnitude, the level of the charge-density fluctuations in the isotopic modes and zero-sound modes is $(ak)^{-2}$ times smaller than in the Langmuir wave.

The formulas (5.1)—(5.3) point to the existence of δ-function maxima in the differential cross-section for scattering of charged particles, corresponding to emission ($\omega > 0$) or absorption ($\omega < 0$) of collective excitations of the system by the particle.

In order to determine the true height of the maxima in the scattering cross-section, it is necessary to take into account the damping of the oscillations. For this it is sufficient to make the replacement

$$\delta(\omega^2 - \omega_k^2) \to 2\pi^{-1}\gamma_k \omega_k^2 \{(\omega^2 - \omega_k^2)^2 + (2\gamma_k \omega_k^2)^2\}^{-1}, \tag{5.4}$$

in formulas (5.2) and (5.3), where γ_k is the relative damping constant. For zero sound, $\gamma_k \sim (ak)^2$ for $sk \gg T$[5].

The kinematics of the scattering of particles by oscillations of the zero-sound type was investigated in detail in the paper[10]. The total energy losses of a particle along unit path are determined by the expression

$$d\varepsilon/dx = N_0 \int \omega \, d\sigma. \tag{5.5}$$

Substituting (5.1) into (5.5) and performing the integration, we find the loss of energy of a charged particle to emission of zero sound and of isotopic oscillations in a two-component Fermi liquid:

$$\frac{d\varepsilon^s}{dx} = \frac{1}{2L_s}\left[\frac{q\omega_{s\,max}(ak_{max})}{v}\right]^2, \tag{5.6}$$

$$\frac{d\varepsilon^i}{dx} = \frac{1}{2|L_i|}\left[\frac{q\omega_i(ak_{max})}{v}\right]^2. \tag{5.7}$$

Comparing (5.6) and (5.7) with the energy losses of the particle to emission of plasmons

$$\frac{d\varepsilon^L}{dx} = \left(\frac{q\omega_L}{v}\right)^2 \ln\frac{k_{max}v}{\omega_L}, \tag{5.8}$$

we see that, generally speaking all the losses can have the same order of magnitude.

The collective Fermi-liquid effects in metals with overlapping energy bands that have been investigated in this paper can be detected, evidently, in the study of the characteristic losses of electrons with energies of the order of 10^3—10^4 electron volts, and also by the methods of infrared optics.

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235