Behavior of zero-gap semiconductors with a linear dispersion law in strong magnetic fields

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(Submitted January 4, 1974)

Zh. Eksp. Teor. Fiz. **67**, 205–207 (July 1974)

The behavior, in a magnetic field, of substances with a vanishing energy gap and a linear isotropic dispersion law at the point of contact of the bands (zero-gap semiconductors of the first kind) is studied. It is shown that bound states with a sharply anisotropic dispersion law arise.

It was shown by Abrikosov and Beneslavaskiĭ [1] that substances with a band gap equal to zero can exist and the energy spectrum in the vicinity of the point of contact of the valence band and conduction band (the Fermi point) can be linear and isotropic (semiconductors of the first kind). Other forms of dependence of the energy on the quasi-momentum near the Fermi point are also possible, but in a magnetic field the behavior of zero-gap semiconductors of the first kind is the most interesting.

We shall find the energy levels of the electrons in a uniform magnetic field.

The equation for the wavefunctions in the one-electron approximation:

$$(\tilde{E}-\boldsymbol{\sigma}\mathbf{k})\psi(\mathbf{r})=0, \quad \tilde{E}=E/v.$$

was obtained earlier [1] from symmetry arguments, using the methods of group theory. The corresponding equation in a magnetic field has the form

$$\{\tilde{E}-\boldsymbol{\sigma}[\mathbf{k}-e\mathbf{A}(\mathbf{r})]\}\psi(\mathbf{r})=0, \tag{1}$$

where v is the velocity of the electrons, $\mathbf{A} = (0, Hx, 0)$, $\mathbf{k}$ is the quasi-momentum, $\mathbf{A}$ is the vector potential, and $\boldsymbol{\sigma}$ are the Pauli matrices. We shall use a system of units in which $\hbar = c = 1$.

It follows from the solution of Eq. (1) that

$$\tilde{E}^{2}_{n,\mu}=p_z^2+R^{-2}(2n+1-\mu), \qquad R=(eH)^{-1/2}, \quad \mu=\pm1, \quad \mathbf{H}\parallel z. \tag{2}$$

The corresponding wavefunctions have the form

$$\psi_{n,+1}(\mathbf{r})=\frac{\exp\{i(p_y y+p_z z)\}}{[2\tilde{E}_{n,+1}(\tilde{E}_{n,+1}+p_z)]^{1/2}}\begin{pmatrix}(E_{n,+1}+p_z)V_n(\xi)\\ -i(2eHn)^{1/2}V_{n-1}(\xi)\end{pmatrix},$$
$$\psi_{n,-1}(\mathbf{r})=\frac{\exp\{i(p_y y+p_z z)\}}{[2\tilde{E}_{n,-1}(\tilde{E}_{n,-1}-p_z)]^{1/2}}\begin{pmatrix}i[2eH(n+1)]V_{n+1}(\xi)\\ (E_{n,-1}-p_z)V_n(\xi)\end{pmatrix}; \tag{3}$$

where

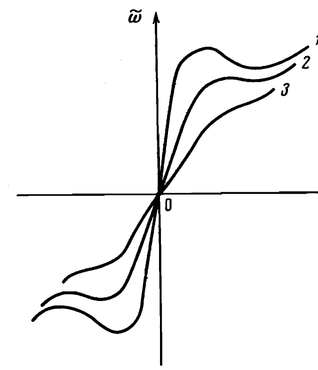
$$V_n(\xi)=\frac{e^{-\xi^2/2}(eH)^{1/4}}{\pi^{1/4}2^{n/2}(n!)^{1/2}}H_n(\xi), \qquad \xi=\frac{x}{R}-Rp_y,$$

and $H_n(\xi)$ is a Hermite polynomial.

Since negative energy values correspond to holes, the corresponding wavefunctions for the holes are obtained by replacing $\tilde{E}_{n,\mu}\to-\tilde{E}_{n,\mu}$.

In the case of extremely strong fields, since the spacing between the levels $\sim H^{1/2}$ we can neglect the effect of the higher levels and consider only the level $\tilde{E}_{0,+1}$. It follows from (3) that in this case $\tilde{E}_{0,+1}\equiv\epsilon=p_z$. There is no branch with $\epsilon=-p_z$. The direction singled out in momentum space is associated with the magnetic field. In this case, if we take (2) and (3) into account, the Green function

$$C(\mathbf{r}_1,\mathbf{r}_2;\tilde{\omega})=\sum_{\lambda}\frac{\psi_\lambda(\mathbf{r}_1)\psi_\lambda^*(\mathbf{r}_2)}{\tilde{\omega}-\tilde{\omega}_\lambda}, \tag{4}$$



Dependence of $\tilde{\omega}$ on q_z for different $q_\perp$ ($q_\perp^{(1)}<q_\perp^{(2)}<q_\perp^{(3)}$).

takes the form

$$C(\mathbf{r}_1,\mathbf{r}_2;\tilde{\omega})=\frac{1}{R\pi^{1/2}}\cdot\times\int\exp\left\{-\frac{\xi_1^2+\xi_2^2}{2}\right\}\cdot\times\frac{\exp\{i\mathbf{p}_\perp\boldsymbol{\rho}_\perp\}}{\tilde{\omega}-p_z}dp_y\,dp_z, \tag{5}$$

where

$$\mathbf{p}_\perp=(p_y p_z), \quad \boldsymbol{\rho}_\perp=(y_1-y_2;\ z_1-z_2), \quad \xi_{1,2}=x_{1,2}/R-Rk_y.$$

We shall study one of the phenomena arising in zero-gap semiconductors of the first kind in magnetic fields. We shall consider the simplest electric loop

$$\Pi(\mathbf{r}_1,\mathbf{r}_2;\tilde{\omega}) =$$ 

$$=-i\,\mathrm{Sp}\int C(\mathbf{r}_1,\mathbf{r}_2;\tilde{\omega}_1)C_1(\mathbf{r}_2,\mathbf{r}_1;\tilde{\omega}+\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{2\pi}. \tag{6}$$

In momentum space,

$$\Pi(\mathbf{q}_1,\mathbf{q}_2;\tilde{\omega})=\int d\mathbf{r}_1\,d\mathbf{r}_2\exp\{i\mathbf{q}_1\mathbf{r}_1-i\mathbf{q}_2\mathbf{r}_2\}\Pi(\mathbf{r}_1,\mathbf{r}_2;\tilde{\omega}). \tag{7}$$

Substituting (5) and (6) into (7), we obtain

$$\Pi(\mathbf{q})=\frac{2}{vR^2}\frac{q_z}{\tilde{\omega}-q_z}\exp\left(-\frac{1}{4}R^2q_\perp^2\right); \quad q_\perp^2=q_x^2+q_y^2. \tag{8}$$

It should be noted that the inclusion of $\exp(-\frac{1}{4}R^2q_\perp^2)$ is an excess of accuracy, since for $H\to\infty$, i.e., $R\to0$, this leads to terms of order unity in Π. It can be shown that terms of the same order arise from summation of the upper levels.

The renormalized Coulomb interaction has the form [2]

$$D(\tilde{\omega},\mathbf{q})=\frac{4\pi e^2}{\varepsilon_0 q^2-4\pi e^2\Pi(\mathbf{q},\omega)}, \tag{9}$$

where ϵ_0 is the dielectric constant. The pole of the D-function determines the energy of the bound states. From (8) and (9) we obtain the energy spectrum of these excitations:

$$\tilde{\omega}=q_z+\frac{8\pi e^2}{R^2\varepsilon_0 v}\frac{q_z}{q_z^2+q_\perp^2}. \tag{10}$$

A qualitative graph of $\tilde{\omega}=\tilde{\omega}(q_z)$ is shown in the

Figure. It is interesting to note that $\tilde{\omega}(q_z)$ is an odd function of the momentum q_z. As was mentioned above, this is associated with the magnetic field. On change of sign of the magnetic field ($\mathbf{H} \to -\mathbf{H}$) the frequency will change sign, so that, in the general case,

$$\tilde{\omega} = \frac{\mathbf{qH}}{|\mathbf{H}|}\left(1 + \frac{8\pi e^2}{R^2 \varepsilon_0 v}\frac{1}{\mathbf{q}^2}\right). \qquad (11)$$

In conclusion I should like to express my gratitude to S. D. Beneslavskiĭ for suggesting the topic and to A. A. Migdal for supervising the work.

[1] A. A. Abrikosov and S. D. Beneslavskiĭ, Zh. Eksp. Teor. Fiz. **59**, 1280 (1970) [Sov. Phys.-JETP **32**, 699 (1971)].
[2] A. A. Abrikosov, L. P. Gor'kov and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, M., 1962 (English translation published by Pergamon Press, Oxford, 1965).

Translated by P. J. Shepherd
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