Effect of localized states in a barrier on electron tunneling

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The tunnel current passing through a dielectric layer containing impurity centers with energy levels in the forbidden band is calculated. It is shown that the quasistationary electron states arising at such centers lead to an appreciable increase in the tunnel current. The volt-ampere characteristics of metal-dielectric-metal and semiconductor-dielectric-metal systems are investigated and the existence of a current peak is demonstrated. Tunneling through an amorphous dielectric is also considered.

The simplest theories of tunneling in solid tunneling structures consider a layer of dielectric between two conductors as an ideal crystal, neglecting the role of impurities and structure defects. The effects of impurities on tunneling have been taken into account in a number of studies. In the paper of Schmidlin,[1] the role of the impurities was reduced to an increase in the tunnel factor due to local lowerings of the barrier brought about by the fluctuation potential of the impurities. Kane[2] considered a doped p-n junction and showed that the tunnel current can increase (in comparison with the case in which impurities are absent) as a result of transitions between the tails of the density of states near the edges of the bands. Parker and Mead[3] showed that impurity states in the Schottky barrier can considerably ease the tunneling of the electrons. Parker and Mead started out from the fact that under stationary conditions, the number of transitions from the semiconductor to the impurity, N_{si}, is equal to the number of transitions from the impurity to the metal N_{im}. The numbers N_{si} and N_{im} are proportional to

$$f_s(1-f_i)\exp\left(-2\int_0^{z_i} p\,dz\right) \quad \text{and} \quad (1-f_m)f_i\exp\left(-2\int_{z_i}^{d} p\,dz\right),$$

respectively, where 0, d are the turning points, z_i the coordinate of the impurity, p the momentum of the electron, and f_s, f_i and f_m the occupation numbers. It then follows that the probability of a two-step semiconductor-impurity-metal transition is greater than the possibility of direct tunneling by a factor

$$\left[C_1\exp\left(-2\int_0^{z_i} p\,dz\right)+C_2\exp\left(-2\int_{z_i}^{d} p\,dz\right)\right]^{-1}$$

(C_1 and C_2 are constants that do not contain exponentially small quantities).

However, this kinetic treatment does not give an exhaustive description of the phenomenon and is unjustified at a number of points. The basic result of the kinetic method is that the probability of a two-step semiconductor-impurity-metal transition is expressed in terms of the probability of the single-step transitions N_{si} and N_{im}. Calculation of the quantities N_{si}, N_{im} themselves requires, of course, a quantum-mechanical treatment. We emphasize that the problem is a three-dimensional one, since the total potential energy of the electron is equal to the sum of the one-dimensional potential of the barrier and the potentials of the impurity atoms located in it (a simplified one-dimensional model of resonance tunneling was considered in[4,5]). Moreover, applicability of the method of[3] requires satisfaction of the usual conditions for kinetic consideration, which allow us to neglect interference effects in calculation of the probability of a two-step transition. Such conditions can be realized in principle if tunneling is accompanied by some inelastic process (electron-phonon interaction, excitation of impurity atoms, surface states and so on) that rather quickly destroys the phase correlation of the wave functions before and after the interaction with the impurity. In that paper, however, only elastic tunneling is considered, i.e., a situation of the type of elastic resonance scattering. Finally, the kinetic-equation method does not take level broadening due to tunneling into account. In particular, it remains unclear whether the result refers to the value of the transition probability at the maximum or to the transition probability averaged over the energy.

In the present paper, a quantum-mechanical theory of resonance tunneling through a barrier containing randomly distributed impurities is developed. The basic difficulty lies in finding the wave function and the tunnel level width for the three-dimensional problem, which does not allow separation of the variables (in contrast, for example, to the problem of a hydrogen atom in a homogeneous electric field). This difficulty can be overcome if we make the natural assumption that the size of the bound state on the impurity is much smaller than the barrier width.

Moreover, tunneling through an amorphous dielectric is considered. It should be kept in mind that real tunnel dielectric films are frequently amorphous or polycrystalline. The role of the forbidden band in such materials is taken by a range of energies with relatively low level density, in which all the states are localized.[6] It can be expected that the localized states in amorphous dielectrics play the same role in tunneling as impurity states do in crystalline dielectrics.

1. THE WAVE FUNCTION IN THE RESONANCE APPROXIMATION

Tunneling of an electron with an energy E close to some level E_0 of the impurity atom is considered. It is assumed that the radius of the impurity potential and the size of the bound state on the impurity are much less than the barrier width. We shall assume the wave functions of the bound state of the electron on the impurity, $\varphi(\mathbf{r})$, to be known, as well as the Green's function of the electron at the potential barrier without the impurity, $g(\mathbf{r}\mathbf{r}')$. We introduce the impurity scattering operator $\hat{T}$, which satisfies the equation

$$\hat{T}=\hat{U}+\hat{U}\hat{g}\hat{T} \tag{1}$$

($U(\mathbf{r})$ is the impurity potential, $\hat{U} = U(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')$). Then the wave function can be expressed in terms of $\hat{T}$ and

the wave function in the absence of impurities, ψ_0:

$$\psi=\psi_0+\hat{g}\hat{T}\psi_0. \tag{2}$$

The operator for scattering by the impurity in the barrier, $\hat{T}$, is conveniently expressed in terms of the impurity scattering operator in the unbounded dielectric $\hat{T}_0$, which satisfies the equation

$$\hat{T}_0=\hat{U}+\hat{U}\hat{g}_0\hat{T}_0, \tag{3}$$

where $\hat{g}_0$ is the Green's operator of the unbounded dielectric. Finding $\hat{U}=\hat{T}_0(1+\hat{g}_0\hat{T}_0)^{-1}$ from (3) and substituting it in (1), we get

$$\hat{T}=\hat{T}_0+\hat{T}_0(\hat{g}-\hat{g}_0)\hat{T}. \tag{4}$$

Next, we find $\hat{T}_0$ in the resonance approximation. The Green's function of the electron in the field of the impurity in an unbounded dielectric is approximately equal to

$$G_E(\mathbf{rr}')=\frac{\varphi(\mathbf{r})\varphi(\mathbf{r}')}{E-E_0}, \tag{5}$$

Expression (5) is valid if $E-E_0$ is much less than the separation of energy levels of the impurity atom. The impurity scattering operator $\hat{T}_0$ is expressed in terms of $\hat{G}$ by the obvious relation

$$\hat{T}_0=\hat{g}_0^{-1}(\hat{G}-\hat{g}_0)\hat{g}_0^{-1}.$$

In the resonance approximation we have

$$\begin{aligned}\hat{T}_0(\mathbf{rr}')&=\int\hat{g}_0^{-1}(\mathbf{rr}_1)\varphi(\mathbf{r}_1)d\mathbf{r}_1\int\hat{g}_0^{-1}(\mathbf{r}'\mathbf{r}_2)\varphi(\mathbf{r}_2)d\mathbf{r}_2(E-E_0)^{-1}\\&=\frac{U(\mathbf{r})\varphi(\mathbf{r})U(\mathbf{r}')\varphi(\mathbf{r}')}{E-E_0}\end{aligned} \tag{6}$$

inasmuch as $\varphi(\mathbf{r})$ satisfies the equation $[\hat{g}_0^{-1}-U(\mathbf{r})]\varphi(\mathbf{r})=0$.

It is seen from (5) that $\hat{T}_0$ has a multiplicative structure in the sense of the dependence on $\mathbf{r}$ and $\mathbf{r}'$. This allows us to solve Eq. (4) easily. We seek $\hat{T}$ in the form

$$\hat{T}(\mathbf{rr}')=\varphi(\mathbf{r})U(\mathbf{r})\varphi(\mathbf{r}')U(\mathbf{r}')t.$$

Then

$$t=\left[E-E_0-\int U(\mathbf{r})\varphi(\mathbf{r})[\hat{g}-\hat{g}_0]U(\mathbf{r}')\varphi(\mathbf{r}')d\mathbf{r}\,d\mathbf{r}'\right]^{-1}. \tag{7}$$

The last term in the brackets describes the shift in the impurity level due to the barrier field, and the damping Γ due to transitions to the continuous spectrum. The level shift, which is unimportant for us, can be included in the value of E_0. The desired expression for the wave function follows from Eq. (2):

$$\psi(\mathbf{r})-\psi_0(\mathbf{r})=(E-E_0-i\Gamma/2)^{-1}\int g(\mathbf{rr}_1)U(\mathbf{r}_1)\varphi(\mathbf{r}_1)d\mathbf{r}_1\int U(\mathbf{r}_2)\varphi(\mathbf{r}_2)\psi_0(\mathbf{r}_2)d\mathbf{r}_2, \tag{8}$$

$$\Gamma=2\,\mathrm{Im}\int U(\mathbf{r})\varphi(\mathbf{r})(\hat{g}-\hat{g}_0)U(\mathbf{r}')\varphi(\mathbf{r}')d\mathbf{r}\,d\mathbf{r}'. \tag{8a}$$

Formula (8) refers to the case in which there is a single impurity. If we denote its coordinate by $\mathbf{r}_j$, then it is convenient to measure the arguments from the point $\mathbf{r}_j$ in Eq. (8), i.e., to replace $U(\mathbf{r})$, $\varphi(\mathbf{r})$ by $U(\mathbf{r}-\mathbf{r}_j)$, $\varphi(\mathbf{r}-\mathbf{r}_j)$. If there are N impurities in the barrier and they are located at sufficiently large distances from one another, so that the electron interacts with each of them independently (the criterion will be indicated below), then the right side of (8) is represented by the sum over the locations of the impurities.

2. CALCULATION OF LEVEL DAMPING

Up to now we have made no model assumptions as to the shape of the potential barrier. For calculation of Γ and ψ from Eqs. (8), (8a), we need to know the specific form of the Green's function $g(\mathbf{r}\cdot\mathbf{r}')$. It is comparatively simple to calculate $g(\mathbf{r}\cdot\mathbf{r}')$ for a rectangular barrier.

However, we shall be interested in the situation in which the electric field F which creates the current is sufficiently large that it itself affects the shape of the barrier. We shall therefore assume the potential barrier to be an arbitrary function of the coordinate and also that the conditions are quasiclassical. As is seen from Eq. (8a), to find Γ, we must know the difference $g-g_0$ at distances $|\mathbf{r}-\mathbf{r}'|$ of the order of the characteristic dimension of U or φ, i.e., much smaller than the barrier width. Then $g\approx g_0$ in the first quasiclassical approximation. The corrections that make a contribution to the damping are of the order of e^{-pd}, while the quasiclassical approximation cannot take into account terms of order $(pd)^{-1}$, where d is the barrier thickness. The latter exceed e^{-pd} in value but do not make a contribution to the damping. Consequently, account of the terms of order e^{-pd} would exceed the accuracy of the method.

In order to avoid this difficulty, we consider another method of calculation of Γ, based on the approximation solution of the Schrödinger equation. We shall seek the solution of the equation

$$\Delta\psi+2(E-V(z)-U(\mathbf{r}))\psi=0$$

(here V(z) is the barrier potential) in the form of $\varphi(\mathbf{r})$ at distances $a\lesssim|\mathbf{r}-\mathbf{r}_j|\ll d$ and in the form

$$v(\mathbf{r})\exp\left(-\int^z p\,dz\right)$$

in the region $a\ll|z-z_j|\ll d$, where $v(\mathbf{r})$ is a slowly changing function of z, and a is the characteristic radius of the impurity potential $U(\mathbf{r})$.

For $v(\mathbf{r})$ we obtain the equation

$$\begin{aligned}&-2p(z)\frac{\partial v}{\partial z}+\Delta_z v=0,\\&p^2(z)=2[V(z)-V(z_0)+E_0],\quad \hbar=m=1.\end{aligned} \tag{9}$$

The Fourier transform with respect to x, y of the solution of Eq. (9) is of the form

$$v(z,\mathbf{q})=\exp\left[-\frac{q^2}{2}\int^z\frac{dz'}{p(z')}\right], \tag{10}$$

where $E_0>0$ is the binding energy of the electron on the impurity; the impurity coordinate is $\mathbf{r}_0(0,0,z_0)$. In the region $a\ll|\mathbf{r}-\mathbf{r}_0|\ll d$, the solution of (9) should be identical with the asymptotic expression for the normalized function of the bound state

$$\varphi(\mathbf{r}-\mathbf{r}_0)\approx A|\mathbf{r}-\mathbf{r}_0|^{-1}\exp[-\sqrt{2E_0}|\mathbf{r}-\mathbf{r}_0|]. \tag{11}$$

It is easy to understand from (11) which region of space turns out to be important for tunneling; this is the interior of a cylindrical tube with dimension along the z axis of the order of d and a diameter of the order of $(d/\sqrt{E_0})^{1/2}$. Expanding $|\mathbf{r}-\mathbf{r}_0|$ in (11) inside this region and calculating the Fourier component in x, y, we can establish its identity with the solution (10) if we set $p(z)=\sqrt{2E_0}$ in it and choose z_0 as the lower limit of the integral. This makes it possible to match the two solutions. In contrast to the one-dimensional problems, the matching here takes place in a three-dimensional region (in some part of the tunneling tube). Outside this tube, i.e., at $x^2+y^2\gg d/\sqrt{E_0}$, the constructed solution becomes inapplicable, since the second derivatives with respect to x, y contained in it will be comparable with the omitted second derivative with respect to z. However, as is

clear from what has been said, it suffices in the calculation of Γ to know the ψ function only inside the tunneling tube.

After the solution has been found in the region $|z - z_0| \gg a$, the problem reduces to a one-dimensional one (for each Fourier component of the function), since at these distances only the barrier field V(z) is significant. The further calculations do not present any difficulty and are completely analogous to what was said in Sec. 50 of [7]. Finding the current densities j_+, j_- outside the barrier for particles which travel in the positive and negative directions of the z axis, respectively, we calculate Γ by means of the continuity equation from the relation

$$\Gamma = \int j_+ ds_+ + \int j_- ds_-,$$

where the integrals are taken over planes that are normal to the z axis, in the region of large absolute values of $(z = z_0)$ outside the barrier. We finally obtain

$$\Gamma = \frac{\sqrt{2}\pi A^2}{\sqrt{E_0}} \left[\left(\int_0^{z_0} \frac{dz}{p(z)} \right)^{-1} \exp\left(-2 \int_0^{z_0} p\, dz \right) + \left(\int_{z_0}^{d} \frac{dz}{p(z)} \right)^{-1} \exp\left(-2 \int_{z_0}^{d} p\, dz \right) \right] \tag{12}$$

where 0, d are the turning points.

Considering Γ as a function of z_0, it is seen from (12) that if the barrier is sufficiently wide ($\sqrt{E_0}d \gg 1$), then $\Gamma(z_0)$ has a sharp minimum at some $z_0 = z_m$ inside the barrier. For a symmetric barrier, it is clear that $z_m = d/2$. Thus the narrowest resonance and the one with the highest amplitude appears in the wave function and, as will be shown below, in the tunnel current, due to the impurities which lie near the plane $z = z_m$. The integral

$$\int dz/p(z),$$

which enters into (12), is equal in order of magnitude to the cross section of the tunneling tube $d/\sqrt{E_0}$.

We shall now make clear the criteria for independence of the interaction of the individual impurity atoms with the tunneling electron. First of all, it is necessary to require that there be no more than a single impurity atom inside the tunneling tube. This leads to the inequality $nd^2E_0^{-1/2} \ll 1$, where n is the impurity density. Next, we estimate the role of the effects of concentration broadening of the impurity level. Fluctuating approaches of two, three and more centers lead to a smearing of the level E_0 in the impurity band. In the low-density limit ($n \ll E_0^{3/2}$), it suffices to take paired approaches of impurity atoms into account. It is essential that the overwhelming majority of configurations correspond to nonresonant interaction of the impurities because of the presence of a strong electric field in the barrier. The detuning is equal to $\mathbf{F} \cdot \mathbf{R}$, where $\mathbf{F}$ is the electric field and $\mathbf{R}$ the vector connecting the two impurity atoms. For the case of nonresonant interaction of two short-range centers, the level shift δE is equal in order of magnitude to

$$\delta E \sim \frac{E_0}{FR} \frac{\exp(-2\sqrt{2E_0}R)}{R^3}$$

If this shift exceeds the tunnel width Γ, then resonance tunneling comes about through collectivized states in the impurity band, and not through the level E_0 of the isolated atom. We can neglect the contribution of such transitions to the total tunnel current if the fraction of atoms leading to shifts $\delta E \gtrsim \Gamma$ is small. Using (12), we obtain the estimate

$$\Gamma_{min} \sim \sqrt{E_0}\, d^{-1} \exp(-\sqrt{2E_0}d),$$

i.e., the distances $R \lesssim d/2$ are dangerous. Consequently, upon satisfaction of the inequality $nd^3 \ll 1$ (which is stricter than $nd^2E_0^{-1/2} \ll 1$) we can neglect the effects of impurity interactions.

3. TUNNEL CURRENT AND VOLT-AMPERE CHARACTERISTIC

For calculation of the current, we choose ψ_0 in (8) in the form

$$\psi_0 = 2\sqrt{p_-/p_k(z)}\, e^{i\mathbf{k}\boldsymbol{\rho}} \cos\left[\int_z^z p_k(z')\, dz' \mp \pi/4 \right] \quad \text{at} \quad z < 0,$$

$$p_k^2 = 2[E - V(z)] - k^2, \qquad p_- = p_k(z \to -\infty),$$

which corresponds to a single particle in a unit volume in a plane wave incident on the barrier with tangential momentum k. The Green's function, one argument of which corresponds to the region near the impurity and the other to the region $z > d$, is equal to

$$g(\mathbf{r}\mathbf{r}_1) = (2\pi)^{-2} \int g(\mathbf{q}; z, z_1)\, e^{i\mathbf{q}\boldsymbol{\rho}}\, d^2q,$$

$$g(\mathbf{q}; z, z_1) = \frac{1}{2\sqrt{p_q(z)p_q(z_1)}} \exp\left[-\int_{z_1}^{d} |p_q|\, dz' + i\int_{d}^{z} p_q\, dz' - \frac{i\pi}{4} \right], \tag{13}$$

$$\rho^2 = (\mathbf{r} - \mathbf{r}_1)^2 - (z - z_1)^2.$$

The total current consists of two parts, corresponding to the two terms in Eq. (8): the direct tunneling current j_0, which is associated with ψ_0, and the resonance current, which is proportional to $[(E - E_0)^2 + \Gamma^2/4]^{-1}$. The interference term need not be taken into account, since it vanishes after averaging over the positions of the impurities.

The partial resonance current is equal to

$$j_{res}(\mathbf{k}) = (2\pi)^{-2} \int \frac{np_-}{p^2(z_0)} \frac{|S_q S_k|^2}{(E - E_0)^2 + \Gamma^2/4} \exp\left(-2\int_0^d p_0\, dz \right) \times \exp\left(-k^2 \int_0^{z_0} \frac{dz}{p_0(z)} - q^2 \int_{z_0}^{d} \frac{dz}{p_0(z)} \right) d^2\mathbf{q}\, dz_0. \tag{14}$$

Here

$$p_0 = \sqrt{2(E - V(z))}, \qquad S_q \equiv \int U(\mathbf{r})\varphi(\mathbf{r}) e^{-p_q(z_0)(z - z_0)} e^{i\mathbf{q}\boldsymbol{\rho}}\, dr$$

The last exponential factor in (14) appears on expansion of p_k, p_q in powers of k^2, q^2. As is seen from (14), the characteristic k^2, q^2 are of the order of p_0/d, i.e., later terms of the order of $(p_0 d)^{-1} \ll 1$ in the expansion of the exponent can be neglected. The integral over z_0 in (14) corresponds to summation over the impurities. By comparing the partial currents, we can easily establish that the resonance current exceeds the direct tunneling current at $|E - E_0| \sim \Gamma$ if

$$\frac{nd}{p_0^2} > \exp\left(-2\int_0^d p_0\, dz \right).$$

However, it makes sense to compare not the partial but the total currents, with account of the energy distribution of the electrons. The total current I is equal to

$$I = \sum_k j(k)\, [f(E_k) - f(E_k - eV_e)], \tag{15}$$

V_e is the applied potential difference. The results here depend significantly on the parameters of the tunnel structure and the impurity atoms. The level separation between E_0 and the edge ϵ of the conduction band of the

dielectric is a constant that depends weakly on the field in the barrier. Therefore, E_0 depends on the location of the impurity:

$$E_0(z_0)=\varepsilon+eV(z_0).$$

The potential V(z) is determined by the difference in the potentials between the electrons and the charge on the impurities located inside the barrier:

$$\Delta V=-4\pi qn+4\pi e\langle\psi^2(\mathbf{r})\rangle.$$

Here q is the charge of the impurity in the initial state (q = e in the case of a donor, q = 0 for an acceptor), the brackets $\langle\ldots\rangle$ denote averaging over the location of the impurities and summation over the energies.

Thus the determination of V(z) reduces to solution of a self-consistent problem, inasmuch as $\psi(\mathbf{r})$ depends weakly on V(z). In this paper we shall assume the inequality $end^2 \ll V_e$ to be satisfied, which guarantees homogeneity of the field in the barrier (for d = 30 Å, V_e = 3V, we should have $n \ll 2\times 10^{20}$ cm^{-3}).

We now investigate the form of the volt-ampere characteristics in several typical cases. (It is impossible to go any further with Eq. (14) in its general form, since the quantities S_k, S_q are determined by the specific form of the impurity potential. However, it is seen that in finding the total current, integration of (14) over E leads to the appearance of the quantity $\Gamma(z_0)$ in the denominator. This factor is exponentially small in the parameter $p_0 d$ and determines the region of predominance of the resonance current over the direct tunneling current.)

We now consider a metal—dielectric—metal system in which the impurity level is everywhere higher than the Fermi level in the absence of an applied field (see Fig. 1). If a difference in potentials V_e at which electrons make the transition from metal I to metal II is applied, then for $V_e < V_1 = \Delta - \epsilon - V_0$ and T = 0, the resonance component of the tunnel current is lacking. For $V_e > V_1$, the impurity level falls below the Fermi surface of metal I. A resonance current arises due to impurities located near z = d. The value of the resonance current, summed over the energies of the electrons, is proportional to $Dnd/p_0^2\Gamma(z_1)$, where

$$D=\exp\left(-2\int_0^d p\,dz\right),\qquad z_1=d\frac{\Delta-\varepsilon}{V_e+V_0}.$$

If $z_m < z_1 < d$, then

$$\Gamma(z_1)\sim\left(\int_{z_1}^{d} dz/p(z)\right)^{-1}\exp\left(-2\int_{z_1}^{d} p\,dz\right),$$

and the current is determined chiefly by the quantity

$$\exp\left(-2\int_0^{z_1} p\,dz\right)=\exp\left[-\frac{4\sqrt{2}\,d(\Delta^{3/2}-\varepsilon^{3/2})}{3(V_e+V_0)}\right].$$

With increasing V_e, the point z_1 is shifted to the left and the current increases exponentially until z_1 reaches z_m, where Γ has a minimum. For still larger V_e ($0 < z_1$

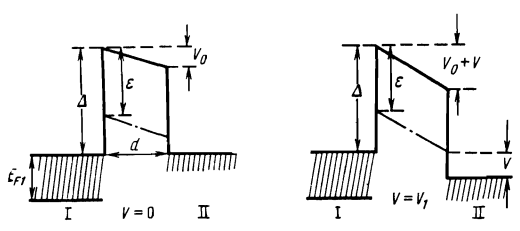

FIG. 1

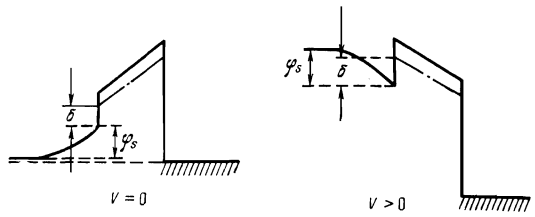

FIG. 2

$< z_m$) the resonance current is determined by impurities which lie close to the plane $z = z_m$, and the energy of the resonating electrons is shifted to the bottom of the conduction band of metal I. A region of comparatively slow change of the resonant component of the current corresponds to this voltage range on the volt-ampere characteristic. Finally, for $V_e + V_0 > 2(\Delta - \epsilon + E_{F_1})$ all the electrons of metal I depart from resonance with impurities lying near z_m. The resonance current then falls off exponentially. The total direct tunneling current will be smaller than the resonance current if the inequality

$$\frac{n}{p_0E_{F_1}}>\exp\left(-\int_0^d p\,dz\right)$$

is satisfied. Thus the total current $I_0 + I_{res}$ will have a maximum as a function of V_e. For the parameters of the tunnel structure used above, the field intensity in the dielectric in the region of the maximum will be of the order of 10^6—10^7 V/cm.

The results that have been obtained are valid in the range of temperatures $kT \ll (V_e + V_0)/p_0 d$. Here, the characteristic size of the function $f[E_0(z_0)]$ in terms of z_0 is much less than the characteristic size of $\Gamma(z_0)$. In the opposite limiting case, the distribution function can be taken out from under the integral over z_0 at the point z_m. Then, the following estimate is valid for the current:

$$I\sim f\left(\Delta-\varepsilon+\frac{V_e+V_0}{d}z_m\right)\sqrt{D}.$$

In this case, the principal part of the dependence of the current on the voltage is contained in the distribution function.

Finally, we consider the metal—dielectric—n-semiconductor structure. Its difference from the preceding case lies in the fact that the field penetrates into the semiconductor, creating a strong bending of its bands. Let the picture shown in Fig. 2 obtain in the case without an external field. The Schottky barrier in the semiconductor has a height ~1 eV. In the vicinity of the Schottky barrier, the field is screened by charged impurities. The potential curve is described by the expression

$$\varphi=\frac{2\pi e}{\varkappa}N_i(z+L)^2,\qquad -L<z<0,$$
$$\varphi=0,\qquad z<-L,$$

where κ is the dielectric constant of the semiconductor and N_i the concentration of charged impurities; L is determined by the potential φ_s at the surface of the semiconductor: $L = (\kappa\varphi_s/2\pi eN_i)^{1/2}$. For the characteristic values $N_i \lesssim 10^{17}$ cm^{-3}, we have $L \gtrsim 4\times 10^{-5}$ cm. For electrons with energies of the order of kT, such a barrier is practically impenetrable. This means that the current will be carried chiefly by electrons with energies close to the surface potential. The latter conclusion remains correct even upon the application of a potential difference, up to the point of enriching bending of the bands. Let the initial position of the impurity level in the

dielectric lie above the edge of the conduction band of the semiconductor and higher than the Fermi surface in the metal. Then the resonance component of the current will be small so long as the bending of the band corresponds to repulsion of electrons from the surface. The beginning of the resonance current takes place at the enriching bending of the band.

For the sake of argument, we shall assume that the electronic conductivity of the semiconductor is supported by donors that possess a shallow level: $E_d \sim kT_n$, where T_n is a temperature of the order of room temperature. Then the bending of the band at which the resonance current begins is much greater than kT_n, since $\delta \gg kT_n$ (see Fig. 2). Therefore, in the near-surface region, the electrons are degenerate and the surface potential φ_s is connected with the electric field intensity in the dielectric, F_i, by the following relation, which is obtained from a solution of the one-dimensional Thomas-Fermi equation:

$$e\varphi_s = \left(\frac{15\pi}{32\sqrt{2}}\right)^{2/5} \frac{\varkappa_i^{4/5}}{\varkappa^{2/5}} F_i^{4/5} \tag{16}$$

(the numerical coefficient ≈ 1.017).

Thus the applied potential difference V_e is expressed in terms of F_i by the formula

$$V_e = V_0 + F_i d + \varphi_s = V_0 + F_i\left(d + \frac{\text{const}}{eF_i^{2/5}}\right), \tag{17}$$

i.e., the difference from the situation with the metal consists in the renormalized thickness of the barrier ($d \to d_{eff}$), which is weakly dependent on φ_s:

$$d_{eff} - d \sim F_i^{-1/5} \sim \varphi_s^{-1/4}. \tag{18}$$

The volt-ampere characteristic in this case has a maximum with a width of the order of φ_s.

4. TUNNELING THROUGH AN AMORPHOUS DIELECTRIC

We shall start out from the model described in [4], according to which the energy spectrum of an amorphous system is characterized by a certain threshold energy E_c, below which all the levels correspond to localized states. Above E_c, the level density is relatively high and the states are localized, so that E_c is in a sense analogous to the edge of an allowed band. We shall consider the resonance tunneling through levels with energy $\epsilon < E_c$. Inasmuch as the density of states $\nu(\epsilon)$ is small in the range of energies of interest to us, then the fluctuation lowerings of the random potential $U(\mathbf{r})$ (the "traps") in which the required energy level ϵ is realized, can be regarded as widely-spaced impurity centers. We shall neglect the effect of the potential $U(\mathbf{r})$ outside the radius of the given fluctuation. We can then use the results obtained above for calculation of the current.

The resonance partial current of electrons with energy E through a trap located at the point z_1 is essentially proportional to

$$j_E \sim \frac{D}{[E - E_0(z_1)]^2 + \Gamma^2(E, z_1)/4},$$

where $E_0(z) = \epsilon - Fz$. Multiplying j_E by $\nu(\epsilon)$, i.e., by the number of traps in a unit energy range near ϵ in a unit volume, and integrating over ϵ and z_1, we obtain the total density of the resonance current for a given energy E:

$$I_E = \int j_E \nu(\varepsilon)\, d\varepsilon\, dz \sim 2\pi D \int_0^d \frac{\nu(E+Fz)\,dz}{\Gamma(E,z)}. \tag{19}$$

This result is easily verified by averaging the current j_E with the functional distribution $U(\mathbf{r})$, for example, in the Zittartz-Langer model[8] and that of Lifshitz.[9] We consider the weak-field limit $F \to 0$. In this limit, I_E takes the form

$$I_E \approx \frac{2\pi D}{p_0} \frac{\nu(E)}{\Gamma(E, z_m)}.$$

Inasmuch as the states fill the "forbidden band" continuously, the resonance current (even at T = 0) is present at all energies, in contrast to the case of a crystalline dielectric. The competition between the resonance current and the direct tunneling current is determined by the ratio $\nu(E)/\Gamma(E, z_m)$. Both quantities ν and Γ fall off exponentially as E moves into the forbidden band, but in principle the situation in which the resonance current prevails is also possible here.

Finally, if we do not neglect the electric field in the barrier, we obtain the following estimates: the characteristic size of Γ as a function of z has the order of $1/p_0$, and the characteristic size of $\nu(E)$ in terms of energy will be denoted by w. Then if $p_0 \gg F/w$, we have

$$I_E \approx \frac{2\pi D}{p_0} \frac{\nu(E+Fz_m)}{\Gamma(E, z_m)}.$$

In the case of transitions from the metal, the current must be integrated over the region of transitions that are energy allowed. Assuming that Γ depends weakly on E in comparison with $\nu(E)$ ($p_0/d \gg w \gg F/p_0$), we obtain the result that in the range of voltages $V \ll E_c - E_F \sim p_0^2$, the current is

$$I \sim \frac{2\pi D}{p_0} \frac{1}{\Gamma(E_F, z_m)} \int_{E_F}^{E_F+V} \nu(E+Fz_m)\,dE. \tag{20}$$

Thus, by measuring the volt-ampere characteristic, we can determine the density of states in the amorphous dielectric.

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