Retardation effect in cyclotron resonance

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Cyclotron resonance in metals under conditions of the retardation effect is investigated theoretically for the case when the electron transit time through the skin layer considerably exceeds the period of the electromagnetic wave. The amplitude and shape of the resonance curve are found under conditions of the anomalous and normal skin effects by taking into account the real distribution of the electromagnetic field in the metal, and in particular its dependence on the character of electron reflection from the sample boundary. It is shown that in the presence of a strong retardation effect the amplitude of consecutive resonance lines decreases in power-law fashion as against the exponential decrease in the normal skin effect. The effect of anisotropy of the Fermi surface is investigated.

1. INTRODUCTION

Cyclotron resonance takes place in metals under conditions of extreme spatial inhomogeneity of the electromagnetic field, when the radius R of the electron orbit in a constant magnetic field H is much larger than the thickness δ of the skin layer (see Fig. 1). This means that when the electron is in the skin layer it interacts with the wave during a characteristic time $(8R\delta)^{1/2}/v$ (v is the electron velocity), which is only a small fraction of the Larmor period $2\pi/\Omega$. If the electromagnetic field does not manage to change noticeably during this time, then the interaction of the electron with the wave is the most effective. In other words, if the inequality

$$(8R\delta)^{1/2}/v \ll \pi/\omega \tag{1.1}$$

is satisfied, then the electron absorbs strongly energy from the electromagnetic field during the entire time of motion in the skin layer. It is seen that the inequality (1.1) imposes an upper bound on the frequency ω of the external wave, and that at a fixed frequency ω it imposes a lower bound on the magnetic field H. Therefore condition (1.1) is violated at sufficiently large ω (or in weak fields H). This means in turn that the travel time of the electron through the skin layer becomes larger than the half-period π/ω of the wave, i.e., the electron "is late" in leaving the skin layer by the instant when an appreciable temporal change takes place in the high-frequency field. The change of the electromagnetic field during the time of flight of the electron through the skin layer is called the retardation effect.

Under the conditions of strong retardation, when the inequality (1.1) is reversed, the field reverses its sign many times during the stay of the electron in the skin layer. This causes the electron to effectively absorb the wave energy only during a small fraction of the time $(8R\delta)^{1/2}/v$. As a result of the retardation, the amplitude of the cyclotron resonance decreases strongly and its line shape is altered.

Before we proceed to study the influence of the retardation effect on the resonance, let us recall briefly the main results of the theoretical investigation of cyclotron resonance when inequality (1.1) is satisfied. If the reflection of the electrons from the sample boundary is not specular, and if the resonance is sharp enough, i.e.,

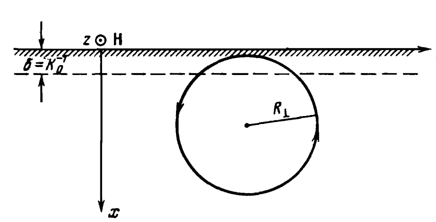
$$|\gamma|(\delta/R)^{1/2} \ll 1-\rho, \quad \nu/\Omega \ll 1-\rho, \tag{1.2}$$

then the formula for the surface impedance of the metal Z(H), in the case of isotropic and quadratic electron dispersion, takes the form

$$Z(H) \approx Z(0)[1-\exp(-2\pi\gamma)]^{1/3}. \tag{1.3}$$

Relation (1.3) was first obtained by Azbel' and one of us[1]. Here ρ is the specularity parameter $(0 \le \rho \le 1$; in the case of diffuse reflection $\rho = 0$, and if the metal boundary is specular then $\rho = 1)$; ν is the frequency of the collision of the electrons with the volume scatterers, and $\gamma = (\nu - i\omega)/\Omega$. We emphasize that in this case the conductivity of the metal, and with it also the skin layer, are formed by the resonant electrons, whereas the contribution of the nonresonant electrons is negligibly small.

Subsequently experiments have revealed frequent and noticeable deviations from theory[1]. The oscillations of Z(H), which are connected with cyclotron resonance, constitute a small fraction of the average value of the impedance. Chambers[2] has attempted to eliminate the resultant contradictions and obtained results that are in much better agreement with experiment. His phenomenological calculations are based on the assumption that the skin layer is formed mainly by electrons that do not take part in the resonance. In other words, the nonresonant electrons make a much larger contribution to the conductivity than the resonant electrons. Chambers connected his assumption with the insufficiently low value of ν/Ω (not too sharp a resonance). Chambers hypothesis found a natural microscopic justification in the work of Meĭerovich[3] and Zherebchevskiĭ and one of us[4]. In[3] there was considered the limiting case of near-specular reflection of electrons, when the second inequality of (1.2) is reversed $(|\gamma|(\delta/R)^{1/2} \ll 1 - \rho \ll \nu/\Omega)$. In[4], a cyclotron-resonance theory was constructed for specular reflection of electrons from the surface of a metal $(1 - \rho \ll |\gamma|(\delta/R)^{1/2} \ll \nu/\Omega)$. In this case the principal term of the asymptotic expansion of the current density has no resonant singularities, since it is due to the contribution of electrons that glide along the sample boundary because of multiple collisions with it[5]. The cyclotron resonance arises only in the next higher terms of the asymptotic terms of the expansion of the current density.



Thus, the status of the "ordinary" cyclotron resonance (1.1) has by now been investigated with sufficient detail. The opposite limiting case, which leads as indicated above to the retardation effect, was considered by a number of authors[6-8]. In none of these references, however, was an analysis of this case developed. The most detailed theoretical investigation of the retardation effect in cyclotron resonance was carried out by Drew[9]. He has shown that the retardation effect leads to an exponential decrease of the amplitude of the successive resonance bursts. The main shortcoming of[9] is that Drew used in his calculations an exponential decrease of the electromagnetic field in the metal. This model, generally speaking, is not equivalent to the real situation that obtains under the conditions of the anomalous skin effect.

We construct in this paper a consistent theory of cyclotron resonance in the case of a strong retardation effect. We take into account the real distribution of the high-frequency field in the metal, and in particular its dependence on the electron reflection coefficient ρ. Unlike in[9], this leads to a power-law decrease of the amplitude of the successive resonance lines. In addition, we investigate cyclotron resonance in the region of the normal skin effect, corresponding to extremely high frequencies $\omega \gg v/\delta$. We analyze the influence of the anisotropy of the Fermi surface.

We note that recent papers deal in considerable detail with the influence of the retardation effect on cyclotron resonance from the experimental point of view (see, e.g.,[10,11]). This makes this problem of even greater theoretical interest.

2. FORMULATION OF THE PROBLEM

We consider a metallic half-space placed in a constant homogeneous magnetic field **H** parallel to its surface. Assume that a plane monochromatic electromagnetic wave of frequency ω is incident on the metal-vacuum interface (the yz plane). The wave propagation is perpendicular to the surface $x = 0$ and coincides with the x axis (the x axis is directed in the interior of the metal, $z \parallel \mathbf{H}$). The dependence on the time t in the incident wave is assumed throughout to be of the form $\exp(-i\omega t)$. The electric field **E** inside the metallic half-space $x > 0$ depends only on the coordinate x, i.e., $\mathbf{E} = \mathbf{E}(x)\exp(-i\omega t)$. We introduce the Fourier transformation in accordance with the formula

$$\vec{\mathscr{E}}(k) = 2\int_0^\infty \mathbf{E}(x)\cos(kx)\,dx.$$

The problem is to find the principal values of the surface-impedance tensor of the metal

$$Z_\alpha = \frac{4\pi i\omega}{c^2}\frac{E_\alpha(0)}{E_\alpha'(0)}, \quad E_\alpha(0) = \frac{1}{\pi}\int_0^\infty dk\mathscr{E}_\alpha(k), \qquad (2.1)$$

where c is the speed of light and the prime denotes differentiation with respect to x.

We shall assume throughout that the following conditions are satisfied:

$$\delta \ll R,\ v/\omega \ll (R\delta)^{1/2},\ \omega \approx n\Omega\,(n=1, 2, 3, \ldots),\ \nu \ll \Omega. \qquad (2.2)$$

The first of these conditions is simply the condition that the skin effect be anomalous with respect to the magnetic field. The second inequality of (2.2) (the inverse of (1.1)) points to a strong retardation effect and means that the time of flight of the electron through the skin layer is much longer than the half-period of the wave. The third condition ensures resonant interaction between the electron and the high-frequency field (proximity to cyclotron resonance). The difference between two successive resonant peaks is Ω, the width of the resonance curve is of the order of ν. Therefore the fourth condition in (2.2) is equivalent to the requirement that each of the resonance lines be well resolved ("sharp" resonance).

The connection between the Fourier transform of the α-component of the current density $j_\alpha(k)$ and the field $\vec{\mathscr{E}}(k)$ should be obtained by solving the kinetic equation for the electron distribution function. Calculations of this type were performed earlier (see, e.g.,[5]). We do not present here the explicit expression for $j_\alpha(k)$, since it is contained in[5,4]. Owing to the conditions (2.2) it is possible to separate in the exact formula for $j_\alpha(k)$ a resonant term $j_\alpha^{res}(k)$ and a term $j^0(k)$ that contains no resonant singularities:

$$j_\alpha(k) = j_\alpha^0(k) + j_\alpha^{res}(k). \qquad (2.3)$$

By virtue of the first two inequalities of (2.2), it is possible to disregard completely the dependence of the current $J^0(k)$ on the magnetic field and assume $H = 0$. Indeed, in the case of a strong retardation effect, the interaction of the electron with the high-frequency field can be regarded in the linear approximation as a sum of successive acts of absorption and emission of a wave on short sections of the trajectory in the skin layer. As a result of interference, an important role will be played only by a small [compared with $(8R\delta)^{1/2}$] segment of the trajectory covered by the electron with approximately half the period π/ω. It is therefore obvious that the magnetic field does not manage to bend the electron trajectory in such a small segment.

In the case of isotropic and quadratic conduction-electron dispersion, we have[5,4]

$$j_y^{res}(k) = \frac{3\omega_0^2}{4\pi^2}\frac{1}{\nu - i(\omega - n\Omega)}\int_0^{\pi/2} d\vartheta \sin^3\vartheta \int_0^\infty dk'\,\mathscr{E}_y(k')J_n'(kR_\perp)J_n'(k'R_\perp)$$
$$\times\left\{\pi\delta(k-k') - \frac{\sin[(k-k')R_\perp]}{k-k'} + (-1)^n\frac{\sin[(k+k')R_\perp]}{k+k'}\right\}$$
$$j_z^{res}(k) = \frac{3\omega_0^2}{4\pi^2}\frac{1}{\nu - i(\omega - n\Omega)}\int_0^{\pi/2} d\vartheta \sin\vartheta\cos^2\vartheta \int_0^\infty dk'\mathscr{E}_z(k')J_n(kR_\perp)$$
$$\times J_n(k'R_\perp)\left\{\pi\delta(k-k') - \frac{\sin[(k-k')R_\perp]}{k-k'} - (-1)^n\frac{\sin[(k+k')R_\perp]}{k+k'}\right\}. \qquad (2.4)$$

Here

$$\omega_0 = (4\pi N_0 e^2/m)^{1/2},\ R_\perp = R\sin\vartheta,\ R = v/\Omega; \qquad (2.5)$$

ω_0 is the plasma frequency of the metal, N_0 is the concentration, m is the effective mass, v is the Fermi velocity, e is the absolute value of the electron charge, $\Omega = eH/mc$ is the cyclotron frequency, $R_\perp$ is the radius of the electron orbit in the magnetic field **H**, ϑ is the polar angle with the polar axis $z(v_z = v\cos\vartheta)$, $J_n(x)$ is a Bessel function, and $J_n'(x)$ is its derivative with respect to its argument.

In the metallic half-space, there are two different groups of electrons. One consists of the so-called "volume" electrons, which do not interact with the surface of the metal. The other group is made up of the "surface" electrons, which collide in each revolution with the sample boundary. The resonant term $j_\alpha^{res}(k)$ is due to the contribution of the volume electrons, for they are the only ones that take part in the cyclotron resonance. At the same time $j_\alpha^0(k)$ receives contributions from both the nonresonant volume electrons and the surface electrons. Therefore the dependence on the reflection coefficient ρ in formula (2.3) is contained only in the term $j_\alpha^0(k)$.

In the considered limiting case of strong retardation effect (2.2), just as in the case considered by Chambers[2], the resonant term $j_\alpha^{res}(k)$ turns out to be much smaller than the nonresonant current

$$|j_\alpha^{res}(k)/j_\alpha^0(k)| \ll 1. \qquad (2.6)$$

The validity of (2.6) can be easily verified by recognizing that in formula (2.4) for $j_\alpha^{res}(k)$ there are subtracted from the δ function $\delta(k-k')$ the oscillating terms that comprise less-than-limiting expressions for the δ functions. This greatly decreases the value of $j_\alpha^{res}(k)$ in comparison with $j_\alpha^0(k)$, in spite of the presence of the resonant factor in the current $j_\alpha^{res}(k)$.

According to (2.1), the surface impedance of the metal is expressed in terms of the Fourier component of the field $\mathscr{E}_\alpha(k)$ which should be obtained from Maxwell's equations

$$k^2\mathscr{E}_\alpha(k)+2E_\alpha'(0)=4\pi i\omega c^{-2}j_\alpha(k). \qquad (2.7)$$

The inequality (2.6) enables us to solve the Maxwell equations by perturbation in terms of the parameter that determines the existence of the retardation effect. In this case we obtain for the surface impedance $Z_\alpha(H)$

$$Z_\alpha(H)=Z_\alpha(0)+\frac{8\pi\omega^2}{c^4E_\alpha'^2(0)}\int_0^\infty dk\,\mathscr{E}_\alpha(k)j_\alpha^{res}(k). \qquad (2.8)$$

Here $Z_\alpha(0)$ is the impedance and $\mathscr{E}_\alpha(k)$ is the Fourier component of the field in the absence of a magnetic field **H**. In other words, $\mathscr{E}_\alpha(k)$ is the solution of Maxwell's equation (2.7) with current density $j_\alpha^0(k)$. It is seen from (2.8) that the principal term of the asymptotic expansion in the impedance does not contain any resonant singularities, and cyclotron resonance arises in the next higher approximation of perturbation theory. The oscillations of $Z_\alpha(H)$ due to cyclotron resonance constitutes only a small fraction of the mean value of the impedance $Z_\alpha(0)$.

We substitute in (2.8) the value of $j_\alpha^{res}(k)$ from (2.4) and represent the surface impedance of the metal in the form

$$Z_\alpha(H)-Z(0)=\frac{24\omega^2\omega_0^2}{\pi c^4k_0^4v}\,\frac{\Omega}{\nu-i(\omega-n\Omega)}\int_0^{\pi/2}d\vartheta\, n_\alpha^2(\vartheta)I_\alpha(k_0R_\perp),$$
$$\alpha=y,z;\quad n_y(\vartheta)=\sin\vartheta,\quad n_z(\vartheta)=\cos\vartheta. \qquad (2.9)$$

Here

$$I_y(x)=x\int_0^\infty d\xi\int_0^\infty d\xi' F(\xi)F(\xi')J_n'(x\xi)J_n'(x\xi')\cdot$$
$$\times\left\{\pi\delta(\xi-\xi')-\frac{\sin[x(\xi-\xi')]}{\xi-\xi'}+(-1)^n\frac{\sin[x(\xi+\xi')]}{\xi+\xi'}\right\}. \qquad (2.10)$$

The expression for $I_z(x)$ differs from $I_y(x)$ in that J_n' is replaced by J_n, and the sign of the last term in the curly brackets should be reversed (cf. (2.4)). In (2.10), $\xi = k/k_0$ and $\xi' = k'/k_0$ are dimensionless wave numbers. The quantity k_0^{-1} corresponds to the depth of penetration of the electromagnetic field into the metal ($k_0^{-1}=\delta$), and the field $\mathscr{E}_\alpha(k)$ is connected with the function $F(\xi)$ by the relation

$$\mathscr{E}_\alpha(k)=-2E_\alpha'(0)k_0^{-2}F(k/k_0). \qquad (2.11)$$

The surface impedance $Z(0)$ and the skin-layer thickness k_0^{-1} in (2.9) are independent of the index α in the absence of the magnetic field **H**. This is a consequence of the isotropic and quadratic dispersion of the electrons.

3. ANOMALOUS SKIN EFFECT

Formulas (2.9) and (2.10) admit of further simplifications if it is assumed, in addition to the requirements (2.2), that the conditions of the anomalous skin effect

$$\delta\ll v/\omega,\quad \delta^{-1}=k_0=(3\pi\omega_0^2\omega/4vc^2)^{1/3} \qquad (3.1)$$

are satisfied. Inasmuch as the inequality (3.1) is equivalent to $n \ll k_0R$ near the resonance $\omega\approx n\Omega$, the Bessel functions in (2.10) can be replaced by their asymptotic expressions in accordance with the formula

$$J_n(z)\cong\sqrt{\frac{2}{\pi z}}\cos\left(z+\frac{n^2}{2z}-\frac{\pi n}{2}-\frac{\pi}{4}\right)\ \text{at}\ 1, n\ll z \qquad (3.2)$$

It is necessary to retain the term $n^2/2z$ in the argument of the cosine, since its value can greatly exceed unity under conditions of a strong retardation effect. We substitute this asymptotic form in (2.10) and obtain after simple transformations

$$I_\alpha(x)\cong\frac{1}{2\pi}\int_0^\infty\frac{d\xi}{\sqrt{\xi}}F(\xi)\int_0^\infty\frac{d\xi'}{\sqrt{\xi'}}F(\xi')\left\{\pi\delta(\xi-\xi')-\frac{\sin\left[\frac{n^2}{2x}\left(\frac{1}{\xi'}-\frac{1}{\xi}\right)\right]}{\xi-\xi'}-\frac{\cos\left[\frac{n^2}{2x}\left(\frac{1}{\xi}+\frac{1}{\xi'}\right)\right]}{\xi+\xi'}\right\}. \qquad (3.3)$$

Obviously, $I_\alpha(x)$ does not depend on the index α in the considered limiting case of the anomalous skin effect. The curly brackets in (3.3) are conveniently expressed in the form of an integral, using the equality

$$2\int_1^\infty dt\cos(xt+\varphi)\cos(x't+\varphi)=\pi\delta(x-x')-\frac{\sin(x-x')}{x-x'}-\frac{\sin(x+x'+2\varphi)}{x+x'}$$
$$(x,x'>0). \qquad (3.4)$$

As a result we obtain for $I_\alpha(x)$ the following asymptotic formula

$$I(x)\cong\frac{n^2}{2\pi x^2}\int_1^\infty dt\left[\int_0^\infty\frac{d\xi}{\sqrt{\xi}}F\left(\frac{1}{\xi}\right)\cos\left(\frac{n^2}{2x}t\xi+\frac{\pi}{4}\right)\right]^2. \qquad (3.5)$$

To calculate the integrals in (3.5), it is necessary to know the function $F(\xi)$. The explicit form of the field $\mathscr{E}_\alpha(k)$, and hence of $F(\xi)$, is determined by the character of the reflection of the electrons from the surface of the metal. $F(\xi)$ has the simplest form in the case of specular reflection. We therefore consider first precisely this case.

1. Specular reflection ($\rho = 1$). Here

$$F(\xi)=\xi/(\xi^3-i). \qquad (3.6)$$

The validity of (3.6) can be easily verified by solving Maxwell's equation (2.7) with the current density

$$j_\alpha^0(k)=3\omega_0^2\mathscr{E}_\alpha(k)/16kv.$$

We substitute (3.6) in the integral with respect to ξ in the square brackets of (3.5). After transformations, which will not be presented here, we obtain

$$\int_0^\infty\frac{\xi^{1/2}d\xi}{1-i\xi^3}\cos\left(\frac{n^2}{2x}t\xi+\frac{\pi}{4}\right)=-\frac{\pi}{6}\exp\left(-\frac{n^2}{2x}t\right)$$
$$+\frac{\pi}{3}\exp\left(-\frac{\pi i}{6}-\frac{1+i\sqrt{3}}{2}\frac{n^2}{2x}t\right)+i\int_0^\infty\frac{u^{1/2}du}{u^6-1}\exp\left(-\frac{n^2}{2x}tu\right). \qquad (3.7)$$

The inequality $v/\omega \ll (R\delta)^{1/2}$ from (2.2), which gives rise to the retardation effect, is equivalent near the resonance $\omega\approx n\Omega$ to the requirement $k_0R \ll n^2$. Consequently, the parameter $n^2t/2x$ is large in comparison with unity. This makes it possible to neglect in the right-hand side of (3.7) the first term in comparison with the second, and the integral in the sense of the principal value can be replaced by its asymptotic form. As a result we get

$$\int_0^\infty \frac{\xi^{1/2}d\xi}{1-i\xi^3}\cos\left(\frac{n^2}{2x}t\xi+\frac{\pi}{4}\right)\cong\frac{\pi}{3}\exp\left(-\frac{\pi i}{6}-\frac{1+i\sqrt{3}}{2}\frac{n^2}{2x}t\right) -i\Gamma\left(\frac{11}{2}\right)\left(\frac{n^2}{2x}t\right)^{-11/2}, \tag{3.8}$$

where $\Gamma(x)$ is the Euler gamma function. Squaring (3.8) and integrating as a result with respect to t, we obtain in accordance with (3.5) the value of I(x). We then substitute I(x) in (2.9) and obtain the final expression for the surface impedance in the case of specular reflection. The impedance of the metal, for a wave polarized transversely to the vector H, is

$$\frac{Z_y(H)-Z(0)}{|Z(0)|}\cong\frac{2\sqrt{3}}{\pi^2}\frac{\Omega}{\nu-i(\omega-n\Omega)}\left\{i\eta\Lambda^{-6}\exp\left(\frac{\pi i}{3}-\frac{1+i\sqrt{3}}{2}\Lambda\right) -2\varkappa\Lambda^{-1/2}\exp\left[\frac{\pi i}{6}-(1+i\sqrt{3})\Lambda\right]-11\mu\Lambda^{-13}\right\}, \tag{3.9}$$

where $\Lambda = n^2/2k_0R$, and the values of the constants η, κ, and μ are given in Appendix 1. In the case of longitudinal polarization ($\mathbf{E} \parallel \mathbf{H}$), the resonant increment to the impedance turns out to be smaller than (3.9) in absolute magnitude, and is given by the formula

$$\frac{Z_z(H)-Z(0)}{|Z(0)|}\cong\frac{2\sqrt{3}}{\pi^2}\frac{\Omega}{\nu-i(\omega-n\Omega)}\left\{i\eta\Lambda^{-7}\exp\left(-\frac{1+i\sqrt{3}}{2}\Lambda\right) -\varkappa\Lambda^{-3/2}\exp\left[-\frac{\pi i}{6}-(1+i\sqrt{3})\Lambda\right]-\mu\Lambda^{-10}\right\}. \tag{3.10}$$

Here

$$Z(0)=\frac{16\pi\omega}{3\sqrt{3}\,c^2k_0}\exp\left(-\frac{\pi i}{3}\right) \tag{3.11}$$

is the impedance of the metal in the absence of a magnetic field in the case of specular reflection of the electrons from the sample surface. At a fixed external-signal frequency ω, the parameter Λ varies linearly with the number of the resonance n (in inverse proportion to the magnetic field H), $\Lambda = n\omega/2k_0v$. We recall that formulas (3.9) and (3.10) are valid for higher harmonics of the resonance if the following inequalities are satisfied:

$$1\ll n\ll k_0R\ll n^2. \tag{3.12}$$

It follows from (3.9) and (3.10) that the cyclotron-resonance amplitude in the surface impedance consists of three terms. Each decreases in a different manner relative to the parameter $\Lambda = n^2/2k_0R$. Whereas in the first two terms this law is determined by the product of a power function by an exponential, the third term decreases with increasing Λ in power-law fashion. A comparison of the different terms with one another shows that in the regions $\Lambda < 4$ and $24 < \Lambda < 28$ for $Z_y(H)$ and $\Lambda < 5$ and $20 < \Lambda < 25$ for $Z_z(H)$ the principal role is played by the first term, while in the interval $4 < \Lambda < 24$ (and respectively $5 < \Lambda < 20$) it is played by the second term. Finally, at $\Lambda > 28$ ($\Lambda < 25$ for $Z_z(H)$) the third term becomes much larger than the remaining ones. We note that in the asymptotic expansions (3.9) and (3.10) the principal role is played by the third term. This leads to a power-law character of the decrease of the amplitude of the successive bursts of cyclotron resonance, and the exponent is large (Λ^{-10}).

We emphasize that the resonance-line shape depends essentially on the numerical value of the parameter Λ, which enters in the phases of the exponentials that are contained in (3.9) and (3.10). Consequently, the shape of the resonance curves for the real and imaginary parts of the impedance is described by a linear combination of the quantities $\mathrm{Re}[\nu - i(\omega - n\Omega)]^{-1}$ and $\mathrm{Im}[\nu - i(\omega - n\Omega)]^{-1}$, the coefficients of which oscillate with variation of Λ. As a result, the resonance line shape can assume a great variety of forms. The neighboring resonances are practically of the same shape because when the number n of the resonance changes by unity the parameter Λ changes by an amount $\Delta\Lambda = \omega/2k_0v \ll 1$. Finally, in the region of extremely large Λ, when the principal role is played by the last term of formula (3.9) or (3.10), the resonance curves have a pure Lorentz shape.

2. Arbitrary reflection ($0 \le \rho \le 1$). We consider now the case of an arbitrary coefficient of reflection of the electrons from the metal boundary. The current density $j_\alpha^0(k)$ becomes under the conditions of the anomalous skin effect (3.1)

$$j_\alpha^0(k)=\frac{3\omega_0^2}{16kv}\mathscr{E}_\alpha(k)-\frac{3\omega_0^2}{8\pi^2v}(1-\rho)\int_0 dk'\mathscr{E}_\alpha(k')\frac{\ln(k/k')}{k^2-k'^2}. \tag{3.13}$$

Solving Maxwell's equations (2.7) with the current density (3.13), we obtain the Fourier component of the field $\mathscr{E}_\alpha(k)$, and in accordance with (2.11) we obtain the function $F(\xi)$. In order not to clutter up the main text with the auxiliary formulas, we relegate the necessary results of the solutions of Maxwell's equations (2.7) with the current density (3.13) to Appendix 2.

Just as in the preceding subsection, we calculate first the integral with respect to ξ in the square brackets of expression (3.5) for I(x). We use the Mellin integral representation (A.2.2) for the function $F(\xi)$ and obtain

$$\int_0^\infty \frac{d\xi}{\sqrt{\xi}}F\left(\frac{1}{\xi}\right)\cos\left(\frac{n^2}{2x}t\xi+\frac{\pi}{4}\right) =\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dz\left(\frac{n^2t}{2x}\right)^{z-1/2}M(z)\Gamma\left(\frac{1}{2}-z\right)\sin\left(\frac{\pi z}{2}\right), \quad -4<c=\mathrm{Re}\,z<1/2; \tag{3.14}$$

where M(z) is the Mellin transform of the function $F(\xi)$. By virtue of the condition $n^2t/2x \gg 1$, the contour integral in (3.14) can be estimated asymptotically. The asymptotic expansion of this integral with respect to the parameter $n^2t/2x$ is determined by the contribution of the singular points of the integrand, which lie to the left of the integration contour. The principal terms of the asymptotic form are, in accordance with (A.2.3), the residues of the integrand taken at the simple poles $z = -4$ and $z = -5$. Confining ourselves to the contribution of these points, we obtain

$$\int_0^\infty \frac{d\xi}{\sqrt{\xi}}F\left(\frac{1}{\xi}\right)\cos\left(\frac{n^2}{2x}t\xi+\frac{\pi}{4}\right) \cong-\frac{2i}{\pi}\Gamma\left(\frac{9}{2}\right)M(-1)\sin^2\left(\frac{\pi z_0}{2}\right)\left(\frac{n^2}{2x}t\right)^{-9/2} -i\Gamma\left(\frac{11}{2}\right)\cos^2\left(\frac{\pi z_0}{2}\right)\left(\frac{n^2}{2x}t\right)^{-11/2}. \tag{3.15}$$

The value of M(−1) is given by formula (A.2.5) in Appendix 2. Substituting the asymptotic expression (3.15) in (3.5) and performing the operations indicated in (3.5), we obtain I(x). We substitute the value of I(x) obtained in this manner in expression (2.9). After averaging over the polar angle, we write down the following formula for the surface impedance of the metal at arbitrary reflection of the electrons from the sample boundary:

$$\frac{Z_\alpha(H)-Z(0)}{|Z(0)|}\cong -A_\alpha \exp\left(\frac{\pi i}{3}\right)\frac{\Omega}{\nu-i(\omega-n\Omega)}\sin^2\left(\frac{\pi z_0}{2}\right)\sin^2\left(\frac{\pi z_0}{3}\right)\Lambda^{-8}$$
$$\times\left[1+a_\alpha\exp\left(-\frac{\pi i}{6}\right)\frac{\cos^2(\pi z_0/2)}{\sin^2(\pi z_0/3)}\Lambda^{-1}+b_\alpha\exp\left(-\frac{\pi i}{3}\right)\frac{\cos^4(\pi z_0/2)}{\sin^4(\pi z_0/3)}\Lambda^{-2}\right]. \quad (3.16)$$

The values of the constants A_α, a_α, and b_α are given in Appendix 1. The specularity parameter is $\rho = \cos(\pi z_0)$. We note that the surface impedance in the absence of a magnetic field Z(0), at arbitrary reflection of the electrons from the metal boundary, takes the form[12]

$$Z(0)=-\frac{8i\omega}{c^2k_0}M(-1)=\frac{4\pi\sqrt{3}\,\omega}{c^2k_0}\frac{\sin^2(\pi z_0/3)}{\sin^2(\pi z_0/2)}\exp\left(-\frac{\pi i}{3}\right). \quad (3.17)$$

Formula (3.16) is valid for both specular ($\rho = 1$, $z_0 = 0$) and diffuse ($\rho = 0$, $z_0 = 1/2$) reflection of the electrons from the sample surface. If the reflection is close to specular, i.e.,

$$1-\rho\ll\Lambda^{-1}, \quad (3.18)$$

then the last term in the square brackets of (3.16) turns out to be much larger than the first two. In this case the result coincides, accurate to the exponential terms, with formulas (3.9) and (3.10) derived by us under conditions of strictly specular reflection.

In the case of nonspecular reflection, when inequality (3.18) is reversed, the last two terms in the square brackets of (3.16) can be neglected in comparison with the other two, Under these conditions, the amplitude of the cyclotron resonance in the impedance $Z_\alpha(H)$ decreases with increasing parameter Λ like Λ^{-8}, i.e., more slowly than in the specular case (3.18). We note that when the inequality (3.18) is satisfied, the cyclotron-resonance amplitude is proportional to Λ^{-10}. This difference in the rate of change of the resonance amplitude makes it possible to assess the character of the reflection of the conduction electrons from the surface of the metal.

4. NORMAL SKIN EFFECT

We investigate cyclotron resonance at sufficiently high frequencies ω, when the anomalous skin effect condition $n \ll k_0R$ is not satisfied. In other words, we consider the region of high-frequency normal skin effects, characterized by the inequality

$$v/\omega\ll\delta, \quad \delta^{-1}=k_0=\omega_0/c. \quad (4.1)$$

We emphasize that the condition of strong retardation $v/\omega \ll (R\delta)^{1/2}$ is satisfied automatically for resonant electrons with $\Omega = \omega/n$ in the case of normal skin effect (4.1).

Thus, we supplement the requirements (2.2) by the normal skin effect condition (4.1) and calculate the resonant increment to the surface impedance. To this end we transform formulas (2.10) for $I_\alpha(x)$ with the aid of (3.4) into the expressions

$$I_y(x)=2x^2\int_1^\infty dt\left[\int_0^\infty d\xi F(\xi)J_n'(x\xi)\sin\left(xt\xi+\frac{\pi n}{2}\right)\right]^2,$$
$$I_z(x)=2x^2\int_1^\infty dt\left[\int_0^\infty d\xi F(\xi)J_n(x\xi)\cos\left(xt\xi+\frac{\pi n}{2}\right)\right]^2. \quad (4.2)$$

The current density $j_\alpha^0(k)$ is independent in this case of the coefficient ρ of electron reflection from the metal boundary, and is given by

$$j_\alpha^0(k)=i\omega_0^2\mathscr{E}_\alpha(k)/4\pi\omega. \quad (4.3)$$

Consequently, the electric field in the metal has an exponential distribution, and the function $F(\xi)$ takes the form

$$F(\xi)=1/(\xi^2+1). \quad (4.4)$$

Substituting (4.4) in (4.2), we can readily calculate the integrals with respect to ξ in the square brackets. For example,

$$\int_0^\infty\frac{d\xi J_n(x\xi)}{\xi^2+1}\cos\left(xt\xi+\frac{\pi n}{2}\right)=\frac{\pi}{2}J_n(ix)\exp\left(-xt+\frac{i\pi n}{2}\right). \quad (4.5)$$

After taking the integral with respect to t, we obtain

$$I_y(x)=\frac{\pi^2}{4}(-1)^{n+1}xJ_n'^2(ix)\exp(-2x),\quad I_z(x)=\frac{\pi^2}{4}(-1)^nxJ_n^2(ix)\exp(-2x). \quad (4.6)$$

Formulas (4.6) in the model in which the electric field has exponential dependence on x are general and valid for both the normal and anomalous skin effects. Substituting them in (2.9) we obtain the surface impedance for any degree of anomality of the skin effect in the exponential-dependence model. In particular, if the Bessel function in (4.6) is replaced by the asymptotic form (3.2), which is valid in the case of the anomalous skin effect (3.1), then we obtain the result corresponding to the Drew model[9].

Since condition (4.1) near the resonance $\omega \approx n\Omega$ means that $k_0R \ll n$, it follows that the Bessel functions (4.6) can be replaced by the asymptotic form at large values of the index in comparison with the argument. Therefore

$$I_y(x)\cong\frac{\pi^2}{8}\left(\frac{x}{2}\right)^{2n-1}\frac{\exp(-2x)}{\Gamma^2(n)},\quad I_z(x)\cong\frac{\pi^2}{2}\left(\frac{x}{2}\right)^{2n+1}\frac{\exp(-2x)}{\Gamma^2(n+1)}. \quad (4.7)$$

Using the asymptotic expression (4.7), we obtain in accordance with (2.9) the following formulas for the surface impedance:

$$\frac{Z_y(H)-Z(0)}{|Z(0)|}\cong\frac{3}{32\sqrt{\pi}}\frac{\omega}{\nu-i(\omega-n\Omega)}\left(\frac{2n}{k_0R}\right)^{-2n+2}\frac{\exp[2(n-k_0R)]}{n^{5/2}} \quad (4.8)$$

Just as in the case of the anomalous skin effect, the resonant increment in the impedance turns out to be much less for the z polarization than for the y polarization

$$\frac{Z_z(H)-Z(0)}{|Z(0)|}\cong\frac{3}{16\sqrt{\pi}}\frac{\omega}{\nu-i(\omega-n\Omega)}\left(\frac{2n}{k_0R}\right)^{-2n}\frac{\exp[2(n-k_0R)]}{n^{5/2}}. \quad (4.9)$$

Here Z(0) is the surface impedance of the metal in the absence of a magnetic field in the region of the high-frequency normal skin effect,

$$Z(0)=-4\pi i\omega/c^2k_0. \quad (4.10)$$

At a fixed frequency ω of the external electromagnetic field, the parameter $2n/k_0R$ does not depend on the number of the resonance n (in the magnetic field H), and is simply equal to $2\omega/k_0v$. Therefore the dependence on the number n in the resonant increments (4.8) and (4.9) is practically exponential. It is seen from (4.8) and (4.9) that in the case of the normal skin effect the cyclotron-resonance amplitude decreases with increasing n much more rapidly than under the conditions of the anomalous skin effect.

We note that formulas (4.8) and (4.9) are valid for large resonance numbers n, when the following inequalities are satisfied:

$$k_0R\ll n,\quad 1\ll n. \quad (4.11)$$

In this case the parameter k_0R can be arbitrary in comparison with unity. The inequalities (4.11) are the sufficient conditions for the resonant current $j_\alpha^0(k)$ to be independent of the magnetic field H. For z-polarization, the necessary and sufficient condition for $j_z^0(k)$ to be independent of H is only $k_0 R \ll n$. It can be satisfied even for the fundamental resonance $n = 1$. In this case the resonant increment to the impedance can be written in the more general form

$$\frac{Z_z(H)-Z(0)}{|Z(0)|} \cong \frac{3}{2}\frac{\omega}{\nu-i(\omega-n\Omega)}\frac{(k_0R/2)^{2n}}{(n!)^2}\cdot \tag{4.12}$$
$$\times\int_0^{\pi/2} d\vartheta \cos^2\vartheta \sin^{2n+1}\vartheta \exp(-2k_0R\sin\vartheta) \quad \text{if} \quad k_0R\ll n.$$

We call attention to the fact that cyclotron resonance under the conditions of the normal skin effect ($k_0 R \ll n$, $k_0 R \ll 1$) was considered earlier by Meĭerovich[8]. He analyzed the limiting case when the resonant part of the current is much larger than the nonresonant part. Formulas (4.8), (4.9), and (4.12) describe resonance in the opposite limiting case (2.6). We indicate also that a formula similar to (4.8), cited by Strom, Kamgar, and Koch[13] contains an extra factor n^{2n}.

5. INFLUENCE OF FERMI-SURFACE ANISOTROPY

So far we have considered the influence of the retardation effect on the cyclotron resonance in metals with an isotropic and quadratic dispersion of the conduction electrons. This dispersion law holds in alkali metals, in which the Fermi surfaces are spherical. The results obtained above can be generalized without difficulty to the case of an anisotropic quadratic dispersion law, which is realized, for example, in semimetals such as bismuth. A feature of the quadratic dependence of the energy on the momentum near the Fermi surface is the absence of a dependence of the cyclotron frequency Ω on the momentum projection p_z. In many metals, the electron dispersion law differs from quadratic and therefore Ω depends on p_z. In this case not all the electrons take part in the resonance, but only those whose cyclotron frequency Ω is extremal. This leads to a decrease of the amplitude and to a change in the shape of the resonance curves. In this section we analyze the influence of the anisotropy of the Fermi surface on the cyclotron resonance under conditions of the retardation effect.

1. Cylinderical Fermi surface. We consider first an idealized model of an anisotropic metal, the Fermi surface of which is a combination of a sphere and a circular cylinder, with p_z axis parallel to the magnetic field H. The concentration of the Fermi-sphere electrons is N_0, and the density of the electrons inside that part of the cylinder which is bounded by the main cell of the reciprocal lattice will be designated by N_c. We consider resonance with the electrons of the Fermi cylinder. For these electrons $v_z \equiv 0$ and therefore $j_z^{res} = 0$, i.e., the resonance is contained only in the y-component of the impedance.

The resonant part of the Fourier component of the current density can be represented in the form

$$j_y^{res}(k) = \frac{2N_ce^2}{\pi m_c}\frac{1}{\nu-i(\omega-n\Omega)}\int_0^\infty dk'\mathscr{E}_y(k')J_n'(kR)J_n'(k'R)$$
$$\times\left\{\pi\delta(k-k') - \frac{\sin[(k-k')R]}{k-k'} + (-1)^n\frac{\sin[(k+k')R]}{k+k'}\right\}. \tag{5.1}$$

Here m_c, Ω, and R are the characteristics of the electrons on the Fermi cylinder. The nonresonant current is determined by the contribution of the electrons from both the sphere and the cylinder, and differs therefore from expressions (3.13) and (4.3) by the factors $1 + (8N_cp/3\pi N_0p_c)$ and $1 + (2N_cm/N_0m_c)$ respectively (p and p_c are the Fermi momenta on the sphere and cylinder). Renormalization of the plasma frequency in $j_y^0(k)$ leads to an obvious change in the thickness k_0^{-1} of the skin layer. The final expressions for $Z_y(H)$ are obtained by the same method as in the preceding sections. The difference between the results are due to the absence of averaging over the angle ϑ (over p_z) and in that the numerical coefficient in (5.1) is different.

Under the anomalous skin effect conditions (3.1), the resonant increment to the impedance for specular reflection is

$$\frac{Z_y(H)-Z_y(0)}{|Z_y(0)|} \cong -\frac{3\sqrt{3}}{2\pi^2}\left(1+\frac{3\pi N_0p_c}{8N_cp}\right)^{-1}\frac{\Omega}{\nu-i(\omega-n\Omega)}\left\{\frac{2}{3}\Gamma(^{11}/_2)\Lambda^{-11/2}\right.$$
$$\left.\times\exp\left(-\frac{1+i\sqrt{3}}{2}\Lambda\right)+\frac{\pi}{18}\exp\left[\frac{\pi i}{3}-(1+i\sqrt{3})\Lambda\right]+\frac{\Gamma^2(^{11}/_2)}{10\pi}\Lambda^{-10}\right\}. \tag{5.2}$$

The values of $Z_y(0)$ and Λ are determined by the same formulas as before, with allowance for the renormalization of k_0.

At arbitrary reflection of the electrons from the metal-vacuum interface we have

$$\frac{Z_y(H)-Z_y(0)}{|Z_y(0)|} \cong -\frac{\Gamma^2(^{11}/_2)}{(3\pi)^3\sqrt{3}}\exp\left(\frac{\pi i}{3}\right)\left(1+\frac{3\pi N_0p_c}{8N_cp}\right)^{-1}\frac{\Omega}{\nu-i(\omega-n\Omega)}$$
$$\times\sin^2\left(\frac{\pi z_0}{2}\right)\sin^2\left(\frac{\pi z_0}{3}\right)\Lambda^{-8}\left[1+\frac{8}{\sqrt{3}}\exp\left(-\frac{\pi i}{6}\right)\frac{\cos^2(\pi z_0/2)}{\sin^2(\pi z_0/3)}\Lambda^{-1}\right.$$
$$\left.+\frac{27}{5}\exp\left(-\frac{\pi i}{3}\right)\frac{\cos^4(\pi z_0/2)}{\sin^4(\pi z_0/3)}\Lambda^{-2}\right]. \tag{5.3}$$

Finally, in the region of the high-frequency normal skin effect (4.1), the resonant part of the surface impedance is

$$\frac{Z_y(H)-Z_y(0)}{|Z_y(0)|}$$
$$\cong\frac{1}{16\pi}\left(1+\frac{N_0m_c}{2N_cm}\right)^{-1}\frac{\omega}{\nu-i(\omega-n\Omega)}\left(\frac{2n}{k_0R}\right)^{-2n+2}\frac{\exp[2(n-k_0R)]}{n}. \tag{5.4}$$

2. Arbitrary Fermi surface. Anomalous skin effect. In the case of isotropic and quadratic dispersion, the nonresonant current $j_\alpha^0(k)$ is given by (3.13). In the case of arbitrary dispersion, formula (3.13) retains the same form, but the quantity $3\omega_0^2/16v$ is replaced by the principal value of the tensor[14]

$$B_{\alpha\beta} = \frac{2\pi e^2}{(2\pi\hbar)^3}\oint_{\substack{v_x=0\\ \varepsilon=\varepsilon_F}}\frac{n_\alpha(\lambda)n_\beta(\lambda)}{K(\lambda)}d\lambda. \tag{5.5}$$

Here $n_\alpha = v_\alpha/v$ is the unit vector of the velocity on the Fermi surface, K is the Gaussian curvature, λ and ψ are the azimuthal and polar angles in velocity space with polar axis v_x ($v_x = v\cos\psi$, $v_y = v\sin\psi\sin\lambda$, $v_z = v\sin\psi\cos\lambda$, $v = v(\psi, \lambda)$), and the integration is carried out along the line $\psi = \pi/2$, on which $v_x = 0$. In the presence of several lines $v_x = 0$ it is necessary to take in (5.5) the sum of the analogous integrals over all these lines. We note that the principal directions of the real tensor $B_{\alpha\beta}$ do not coincide, generally speaking, with the axes y and z. The expression for the resonant current $j_\alpha^{res}(k)$ can be easily generalized for an arbitrary spectrum of electrons, in analogy with the procedure used in[1]

$$j_\alpha^{res}(k) = \frac{e^2}{\pi(2\pi\hbar)^3}\oint_{\substack{v_x=0\\ \varepsilon=\varepsilon_F}}\frac{n_\alpha^2(\lambda)\Omega(\lambda)d\lambda}{K(\lambda)[\nu-i(\omega-n\Omega(\lambda))]}\int_0^\infty\frac{dk'}{(kk')^{1/2}}\mathscr{E}_\alpha(k')$$
$$\times\left\{\pi\delta(k-k') - \frac{\sin\left[\frac{n^2}{D(\lambda)}\left(\frac{1}{k'}-\frac{1}{k}\right)\right]}{k-k'} - \frac{\cos\left[\frac{n^2}{D(\lambda)}\left(\frac{1}{k}+\frac{1}{k'}\right)\right]}{k+k'}\right\} \tag{5.6}$$

where $D(\lambda) = c\,|\,p_y^{max} - p_y^{min}\,|/eH$ is the diameter of the electron orbit in the direction of the y axis with given value $p_z = p_z(\lambda)$. The off-diagonal components of the conductivity tensor in $j_\alpha^{res}(k)$ can be neglected, inasmuch as we confine ourselves to the first order of perturbation theory in $j_\alpha^{res}(k)$ when calculating the resonant correction to the impedance.

If we substitute (5.6) in (2.8) and use the representation (3.4), then we obtain for the correction to the impedance a formula analogous to (2.9)

$$Z_\alpha(H)-Z_\alpha(0)=-\frac{64\pi e^2\omega^2}{(2\pi\hbar)^3 k_\alpha^4 c^4}\oint\limits_{\substack{v_y=0\\ \varepsilon=\varepsilon_F}}\frac{d\lambda}{K(\lambda)}\frac{n_\alpha^2(\lambda)\Omega(\lambda)}{\nu-i(\omega-n\Omega(\lambda))}I\left(\frac{k_\alpha D(\lambda)}{2}\right). \quad (5.7)$$

Here I(x) is determined as before by formula (3.5), and

$$k_\alpha=(4\pi\omega B_\alpha c^{-2})^{1/3}. \quad (5.8)$$

The impedance $Z_\alpha(0)$ differs from (3.17) in that k_0 is replaced by k_α.

Unlike in (2.9), the integrand in (5.7) contains the product of two relatively varying functions λ: the functions $I(k_\alpha D(\lambda)/2)$ and the resonant denominator. It is easy to see, however, that the denominator is a "sharper" function than $I(k_\alpha D/2)$. Indeed, if the extrema of $\Omega(\lambda)$ and $D(\lambda)$ coincide, then the denominator is changed over an interval $\Delta\lambda \sim (\nu/\omega)^{1/2}$ much smaller than the characteristic interval $\Delta\lambda(k_\alpha D)^{1/2}/n$ over which the function $I(k_\alpha D/2)$ varies. When calculating the integral with respect to λ, we can therefore regard the function $I(k_\alpha D/2)$ as a smooth one and take it outside the integral sign at a point where the cyclotron frequency is extremal. We note that there is no resonance at the limiting point (it is exponentially small with respect to the parameter ω/ν), because $I(0) = 0$[1)].

Recognizing that in the general case of a centrally-symmetrical Fermi surface the extremum of $\Omega(\lambda)$ is reached at four values of λ, we can represent the resonant part of the impedance in the form

$$Z_\alpha(H)-Z_\alpha(0)\cong\frac{64\pi e^2\omega^2}{(2\pi\hbar)^3(k_\alpha c)^4}I\left(\frac{k_\alpha D_0}{2}\right)2\left[\frac{n_\alpha^2(\lambda_0)}{K(\lambda_0)}+\frac{n_\alpha^2(\lambda_1)}{K(\lambda_1)}\right]\Phi_r. \quad (5.9)$$

Here λ_0 and λ_1 are two nonequivalent points on the line $v_x = 0$ ($p_z(\lambda_0) = p_z(\lambda_1)$), at which the frequency $\Omega(\lambda)$ is extremal. The function $I(k_\alpha D_0/2)$ takes into account the influence of the character of the reflection of the electrons from the metal boundary. In the case of specular reflection we have

$$I(k_\alpha D_0/2)\cong-\frac{2}{3}\Gamma(^{11}/_3)\Lambda_\alpha^{-11/3}\exp\left(-\frac{1+i\sqrt{3}}{2}\Lambda_\alpha\right)$$
$$-\frac{\pi}{18}\exp\left[\frac{\pi i}{3}-(1+i\sqrt{3})\Lambda_\alpha\right]-\frac{\Gamma^2(^{11}/_3)}{10\pi}\Lambda_\alpha^{-10},\quad \Lambda_\alpha=n^2/k_\alpha D_0. \quad (5.10)$$

In the case of arbitrary reflection we have

$$I(k_\alpha D_0/2)\cong-\frac{\Gamma^2(^{11}/_3)}{54\pi}\exp\left(\frac{\pi i}{3}\right)\sin^4\left(\frac{\pi z_0}{3}\right)\Lambda_\alpha^{-8}$$
$$\times\left[1+\frac{8}{\sqrt{3}}\exp\left(-\frac{\pi i}{3}\right)\frac{\cos^2(\pi z_0/2)}{\sin^2(\pi z_0/3)}\Lambda_\alpha^{-1}\right. \quad (5.11)$$
$$\left.+\frac{27}{5}\exp\left(-\frac{\pi i}{3}\right)\frac{\cos^4(\pi z_0/2)}{\sin^4(\pi z_0/3)}\Lambda_\alpha^{-2}\right].$$

The shape of the resonance curve is described by the function

$$\Phi_r=\Omega_0\int\limits_{-\infty}^{\infty}d\lambda[\nu-i(\omega-n\Omega_0)+in\Omega_0^{(2r)}\lambda^{2r}/(2r)!]^{-1}. \quad (5.12)$$

The zero subscripts denote here the values of the functions at the point of the extremum of the frequency $\Omega(\lambda)$; the integer r = 1, 2, 3, ... characterizes the order of the extremum of $\Omega(\lambda)$. The dependence of the integral (5.12) on ν and $\Delta\omega = \omega - n\Omega_0$ can be easily obtained for arbitrary r.[10] At r = 1 we have

$$\Phi_1=\frac{\pi\Omega_0\exp(-is\varphi)}{(n|\Omega_0''|)^{1/2}(\nu^2+\Delta\omega^2)^{1/4}},\quad s=\operatorname{sgn}\Omega_0'',$$
$$\varphi=\operatorname{arctg}\left[\frac{(\nu^2+\Delta\omega^2)^{1/2}-s\Delta\omega}{(\nu^2+\Delta\omega^2)^{1/2}+s\Delta\omega}\right] \quad (5.13)$$

The presented formulas provide an analytic description of the influence of anisotropy of the Fermi surface on cyclotron resonance, with allowance for the retardation effect. They show that the line shape depends significantly on the character of the Fermi-surface anisotropy.

3. Arbitrary Fermi surface. Normal skin effect. In the limiting case of high-frequency normal skin effect, when $kv \ll \omega$ (i.e., $kR \ll n$), the spatial dispersion of the conductivity can generally be neglected. The reason is that for an arbitrary dispersion, generally speaking, all the Fourier components of the electron velocity vector, with respect to the variable τ, differ from zero (with the exception of the zeroth harmonics $v_x^{(0)}$ and $v_y^{(0)}$). In other words, resonance at the multiple frequencies $\omega = n\Omega$ exists even in a spatially homogeneous electromagnetic field[2)]. This constitutes the qualitative difference between the arbitrary dispersion law and the isotropic and quadratic law, at which the interaction of the electrons with the wave in the n-th resonance is proportional to $(kR)^{2n}$ (see Sec. 4). Inasmuch as the spatial inhomogeneity of the wave field in the metal does not play any role when the conductivity tensor is calculated, the result does not depend on the character of the reflection of the electrons from the boundary of the sample. This in turn denotes that one can use the expression for the conductivity of an unbounded sample in a homogeneous high-frequency field. It is well known that at an arbitrary dispersion law this conductivity can be represented in the form

$$\sigma_{\alpha\beta}=\frac{4\pi e^2}{(2\pi\hbar)^3}\int\limits_{\varepsilon=\varepsilon_F}m\,dp_z\sum_{n=-\infty}^{\infty}\frac{v_\alpha^{(n)*}v_\beta^{(n)}}{\nu-i(\omega-n\Omega(p_z))}, \quad (5.14)$$

where the asterisk denotes complex conjugation, and the Fourier harmonic of the α-component of the velocity is

$$2\pi v_\alpha^{(n)}=\oint d\tau\,v_\alpha(\tau)\exp(-in\Omega\tau).$$

By virtue of the condition $\omega \gg \Omega \gg \nu$ it is necessary to separate from (5.14) one resonant term $\sigma_{\alpha\beta}^{res}$, and the sum of all the remaining terms should be replaced by the integral with respect to n. This integral obviously yields the conductivity tensor $\sigma_{\alpha\beta}^0$ at H = 0. Thus

$$\sigma_{\alpha\beta}=\sigma_{\alpha\beta}^0+\sigma_{\alpha\beta}^{res},\quad \sigma_{\alpha\beta}^0=\frac{2e^2}{(2\pi\hbar)^3(\nu-i\omega)}\int\limits_{\varepsilon=\varepsilon_F}dS\frac{v_\alpha v_\beta}{v}. \quad (5.15)$$

The surface-impedance tensor is equal to

$$Z_{\alpha\beta}=(4\pi\omega c^{-2})^{1/2}\exp(-\pi i/4)\,(\sigma^0+\sigma^{res})_{\alpha\beta}^{-1/2}. \quad (5.16)$$

The actual form of the resonance and the magnitude of the resonant part of the impedance depend significantly on the relative values of the tensors σ^0 and σ^{res}. The resonant increment to the conductivity $\sigma_{\alpha\beta}^{res}$ can be analyzed exactly, as was done in the preceding subsection. Resonance takes place at the extremal frequencies $\Omega(p_z) = \Omega_0$, and near resonance we have

$$\sigma_{\alpha\beta}^{res}=\frac{8\pi e^2 m_0}{(2\pi\hbar)^3}(v_\alpha^{(n)*}v_\alpha^{(n)})_0$$

$$\times\int_{}^{\infty} dp_z \left[\nu - i(\omega - n\Omega_0) + i\frac{n\Omega_0^{(2r)}}{(2r)!}(p_z - |p_z^0|)^{2r} \right]^{-1}, \quad (5.17)$$

where $\Omega_0^{(2r)} = d^{2r}\Omega(p_z^0)/dp_z^{2r}$, and the factor 2 is the result of the two sections $p_z = |p_z^0|$ at which the frequency $\Omega(p_z)$ is extremal. Since $\sigma_{\alpha\beta}^{res}$ is proportional to the bivector $v_\alpha^{(n)*} v_\beta^{(n)}$, the resonance is maximal at the chosen polarization of the wave. For lack of space, and in view of its relative simplicity, we shall not investigate the shapes of the resonance in the numerous cases described by the general formulas (5.16) and (5.17).

APPENDIX 1

In this Appendix we present exact and also approximate values of the constants contained in the main text. The constants η, κ, and μ of formulas (3.9) and (3.10) are given by the expressions

$$\eta = \Gamma(^{11}/_2)\sqrt{2/\pi} \approx 41.76, \quad \kappa = \sqrt{\pi}/24 \approx 0.07385,$$
$$\mu = 2^6\Gamma^4(^{11}/_2)/5\pi^2\Gamma(11) \approx 2.683. \quad (A.1.1)$$

Further, we present the definitions and approximate values of the constants A_α, a_α, and b_α, introduced in formula (3.16). The constant A_α is given by

$$A_\alpha = \frac{4\Gamma^2(11/2)\sqrt{3}}{(3\pi)^4}\int_0^{\pi/2} d\vartheta\, n_\alpha^2(\vartheta)\sin^8\vartheta,$$
$$A_y = 9A_z = 2^{11}\Gamma^4(^{11}/_2)\sqrt{3}/(3\pi)^4\Gamma(11) \approx 0.93. \quad (A.1.2)$$

The constant a_α is given by the expressions

$$a_\alpha = 8\int_0^{\pi/2} d\vartheta\, n_\alpha^2(\vartheta)\sin^9\vartheta/\sqrt{3}\int_0^{\pi/2} d\vartheta\, n_\alpha^2(\vartheta)\sin^8\vartheta$$
$$a_y = 10a_z/9 = 2^4\Gamma^2(6)/11\Gamma^2(^{11}/_2)\sqrt{3} \approx 4.414. \quad (A.1.3)$$

Finally, the constant b_α is defined by

$$b_\alpha = 27\int_0^{\pi/2} d\vartheta\, n_\alpha^2(\vartheta)\sin^{10}\vartheta/5\int_0^{\pi/2} d\vartheta\, n_\alpha^2(\vartheta)\sin^8\vartheta,$$
$$b_y = 11b_z/9 = ^{99}/_{20}. \quad (A.1.4)$$

APPENDIX 2

According to (2.7), (2.11), (3.1), and (3.13), the sought function $F(\xi)$ is determined from the following integral equation:

$$\left(\xi^2 - \frac{i}{\xi}\right)F(\xi) + \frac{2i}{\pi^2}(1-\rho)\int_0^\infty d\xi' F(\xi')\frac{\ln(\xi/\xi')}{\xi^2 - \xi'^2} = 1. \quad (A.2.1)$$

This equation was solved by Hartmann and Luttinger[12]. They have shown that

$$F(\xi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dz\, M(z)\xi^z, \quad -2 < c = \mathrm{Re}\, z < 1 - z_0, \quad (A.2.2)$$

where $z_0 = \pi^{-1}\arccos\rho$, $0 \le z_0 \le 1/2$. The function $M(z)$ is regular in the band $-4 < \mathrm{Re}\, z < 1 - z_0$, with the exception of one singular point $z = -2$. At this point $z = -2$ the function $M(z)$ has a simple pole with a residue equal to unity. The function $M(z)$ satisfies the difference equation

$$\frac{M(z-3)}{M(z)} = i\frac{\cos[\pi(z+z_0)/2]\cos[\pi(z-z_0)/2]}{\cos^2(\pi z/2)}. \quad (A.2.3)$$

This equation, together with the requirements stipulated above, defines uniquely and in single fashion the function $M(z)$, which is described in the regularity band $(-4, 1 - z_0)$ by the formula

$$M(z) = \frac{\pi}{3}\frac{\exp[\pi i(z+2)/6)}{\sin[\pi(z+2)/3]}\exp\left(\frac{\pi}{36}\int^z dz'\{12z'\,\mathrm{ctg}(\pi z')\right.$$
$$-6(z'+z_0)\,\mathrm{ctg}[\pi(z'+z_0)] - 6(z'-z_0)\,\mathrm{ctg}[\pi(z'-z_0)]$$
$$-18\,\mathrm{tg}(\pi z'/2) + 9\,\mathrm{tg}[\pi(z'+z_0)/2] + 9\,\mathrm{tg}[\pi(z'-z_0)/2] \quad (A.2.4)$$
$$+8\,\mathrm{ctg}[\pi(z'-1)/3] - 4\,\mathrm{ctg}[\pi(z'-1+z_0)/3] - 4\,\mathrm{ctg}[\pi(z'-1-z_0)/3]$$
$$\left.-8\,\mathrm{ctg}[\pi(z'+1)/3] + 4\,\mathrm{ctg}[\pi(z'+1+z_0)/3] + 4\,\mathrm{ctg}[\pi(z'+1-z_0)/3]\}\right).$$

Having obtained the explicit form of $M(z)$, we can easily obtain its value at the point $z = -1$:

$$M(-1) = \frac{\pi\sqrt{3}}{2}\frac{\sin^2(\pi z_0/3)}{\sin^2(\pi z_0/2)}\exp\left(\frac{\pi i}{6}\right). \quad (A.2.5)$$

1) If the extrema of $\Omega(\lambda)$ and $D(\lambda)$ do not coincide, then the value of the integral in (5.7) and the character of the resonance depend significantly on the relative rate of change of the denominator and of the function $I(k_\alpha D/2)$. We shall not, however, analyze the ensuing different limiting cases.

2) If certain Fourier harmonics $v_\alpha^{(n)}$ are equal to zero (for example, from symmetry considerations), then the cyclotron resonance at this harmonic appears when account is taken of spatial dispersion, i.e., when account is taken of the next terms in the expansion of the integral

$$\oint d\tau v_\alpha(\tau)\exp(-in\Omega\tau + ikcp_y/eH)$$

in powers of k.

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