Photocoulomb gravitons and the gravitational emittance of the sun

D. V. Gal'tsov

Moscow State University

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The cross section of the photoproduction of a graviton by a charged particle in a plasma is obtained using the techniques of classical theory. The gravitational radiation of the sun due to the transformation of thermal electromagnetic radiation is estimated.

It is commonly assumed that the principal contribution to the gravitational emittance of hot stars comes from the gravitational bremsstrahlung arising from collisions in the plasma [1-4]. The corresponding estimate of the gravitational radiation of the sun is from 10^{14} [1] to 10^{15} [2] erg/sec. In this report we shall show that at sufficiently high plasma temperatures the predominant effect is that of photocoulomb gravitational radiation. For the sun, in particular, this effect gives an emittance that is two orders of magnitude larger than the bremsstrahlung effect.

The photoproduction of a graviton by a charged particle has already been discussed by several authors [5-7], whose results do not agree even in the classical limit $\omega \ll m$ ($\hbar = c = 1$). In the classical limit, however, the cross section of graviton photoproduction can easily be obtained from the following elementary considerations. Consider an electromagnetic field that is the superposition of the field of a plane electromagnetic wave with the electric vector

$$\mathbf{E}(\mathbf{r}, t)=\mathbf{E}_0 \cos (\omega t-\mathbf{kr}), \quad \mathbf{kE}_0=0, \tag{1}$$

and the Coulomb field of a particle with charge e. The electromagnetic forces associated with this field are the source of gravitational radiation, which obviously should be identified with the photoproduction of gravitons in the classical limit. The problem is thus reduced to the solution of the equations for a weak gravitational field [8] with the right-hand side

$$T_{\alpha\beta}=\frac{e}{4\pi}(E_\alpha \nabla_\beta+E_\beta \nabla_\alpha) r^{-1}, \quad \alpha=1,2,3, \tag{2}$$

where we have omitted the nonessential diagonal terms. For the gravitational potentials in the wave zone, omitting the longitudinal part, we obtain

$$\psi_{\alpha\beta}(r\mathbf{n}, t)=\frac{2Ge}{ir}\frac{E_{0\alpha}k_\beta+E_{0\beta}k_\alpha}{(\omega\mathbf{n}-\mathbf{k})^2}e^{-i\omega(t-r)}+\text{c.c.} \tag{3}$$

By using the Landau-Lifshitz pseudotensor [8] to determine the intensity of the gravitational radiation, after taking the average over the polarizations of wave (1) we find

$$\frac{dI}{d\Omega}=\frac{Ge^2}{4\pi}\times E_0^2\frac{\omega^2 k^2(1+\cos^2\theta)}{[(\omega\mathbf{n}-\mathbf{k})^2]^2}\sin^2\theta, \tag{4}$$

where θ is the angle between the momenta of the graviton and the photon. In the vacuum case, the corresponding differential cross section is of the form

$$\frac{d\sigma}{d\Omega}=Ge^2\left(\cos^4\frac{\theta}{2}+\sin^4\frac{\theta}{2}\right)\operatorname{ctg}^2\frac{\theta}{2}, \tag{5}$$

which is in agreement with the results obtained in [7] but differs from those of [5,6]. For a plasma with dielectric constant $\epsilon = 1-\omega_L^2/\omega^2$ (we neglect spatial dispersion) we have

$$\frac{d\sigma}{d\Omega}=\frac{Ge^2\varepsilon^2(1+\cos^2\theta)\sin^2\theta}{2(x-\cos\theta)^2}, \quad x=\frac{1+\varepsilon}{2\sqrt{\varepsilon}}. \tag{6}$$

ξ	$\varphi(\xi)$	ξ	$\varphi(\xi)$	ξ	$\varphi(\xi)$
10^{-4}	145	0.05	59	1.0	14
10^{-3}	113	0.1	49	2.0	4.5
10^{-2}	81	0.2	39	3.0	1.5
0.02	71	0.3	33	4.0	0.49
0.03	66	0.4	28	5.0	0.17
0.04	62	0.5	25	10.0	0.0008

When Eq. (6) is integrated over angles, we find the total cross section

$$\sigma=8\pi Ge^2\varepsilon^2 F(x), \quad F(x)=x^2\left(\frac{x}{2}\ln\frac{x+1}{x-1}-1\right)-\frac{1}{3}. \tag{7}$$

By making use of Eq. (7) it is not hard to write a formula for the spectral distribution of the power of the gravitational radiation from a unit volume of heated plasma, due to the transformation of transverse thermal radiation:

$$\frac{d\mathscr{E}}{d\omega}=\frac{8Ge^2}{\pi\omega^3}\frac{(\omega^2-\omega_L^2)^3}{e^{\omega/T}-1}F(x)\left(n_e+\sum_i n_i Z_i^2\right), \tag{8}$$

where n_e is the electron density, n_i and Z_i are the density and charge of the ions of type i (i = 1, 2, ...), and T is the temperature in energy units. The total radiation depends on the parameter $\xi = \omega_L/T$:

$$\mathscr{E}\approx 0.30\cdot 10^{-42}T^4\left(n_e+\sum_i n_i Z_i^2\right)\varphi(\xi), \tag{9}$$

where $\mathscr{E}$ is calculated in erg/cm$^3\cdot$sec, and T in keV. Some values of the function $\varphi(\xi)$, obtained by numerical integration, are shown in the table.

For the simple uniform hydrogen-helium model of the sun we can write the total gravitational emittance related to the photocoulomb mechanism in the following form:

$$L_\odot\approx 0.3\cdot 10^{-42}\frac{M_\odot}{m_p}\frac{X+1}{2}\frac{4+3\mu(1-X)}{4-\mu(3X+1)}T^4\varphi(\xi), \tag{10}$$

where $L_\odot$ is in units of erg/sec. Here μ is the average molecular weight, X is the hydrogen content, and m_p is the mass of a proton. Substituting the parameter values $\mu = 0.589$, X = 0.76, and T = 1.3 keV in Eq. (10) and computing the value of the function corresponding to $\xi \approx 0.025$, we obtain $L_\odot \approx 1.3\times 10^{17}$ erg/sec, which exceeds by two orders of magnitude the highest estimate of the gravitational radiation from the sun due to the bremsstrahlung mechanism [2].

Note that the photocoulomb gravitational radiation rises more rapidly with increasing temperature than that due to the bremsstrahlung process, and therefore should comprise the principal contribution to the gravitational radiation of hot stars.

The inclusion of the spatial dispersion of the plasma could affect the value of the parameter ξ. However, at small ξ the cross section depends logarithmically on ξ and does not show much variation. At large values of ξ the given mechanism is ineffective.

At the position of the earth the corresponding value of the flux is $F \approx 0.5 \times 10^{-10}$ erg/cm^2 sec, i.e., it is of the same order of magnitude as the flux of gravitational radiation from the nearest double stars[9].

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